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Design and Analysis of Probabilistic Load Flow in Integrated Electricity and Heat Systems Using Quasi–Monte Carlo Simulation

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ABSTRACT:*The growing interdependence of electricity and heat systems in modern energy networks has introduced significant uncertainty into both generation and demand, driven by renewable integration and fluctuating thermal loads. Traditional deterministic load flow techniques fail to capture these uncertainties, while conventional Monte Carlo simulation(MCS) methodsthough accuratesuffer from excessive computational burden. This thesis presents an improved Quasi-Monte Carlo Simulation (QMCS) framework for Probabilistic Power Flow (PPF) and Probabilistic Energy Flow (PEF) analysis in integrated electricity and heat systems. The proposed method employs low-discrepancy Sobol sequences to enhance sampling efficiency and incorporates quantile-based nonparametric probabilistic forecasting to accurately represent the stochastic behaviour of renewable generation and multi-energy demand. The methodology is implemented on standard test systems, including an 11-bus distribution network and a 5-bus integrated electricity–heat model, using MATLAB. Comparative analysis with traditional MCS shows that the proposed QMCS approach achieves nearly identical accuracy while reducing computational time by over 95%. The results demonstrate that heat-load uncertainty and thermoelectric coupling substantially affect system operating characteristics, highlighting the necessity of incorporating them in modern energy system studies. The developed method provides a reliable and efficient tool for probabilistic assessment, planning, and operational optimization of integrated multi-energy networks.*

KEYWORDS*:Probabilisticpower flow, quasi-Monte Carlo simulation, quantile-based forecasting, integrated electricity and heat systems, computational efficiency, uncertainty analysis.*

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I. INTRODUCTION

The increasing integration of renewable energy resources and multi-energy systems has transformed the operation and planning of modern power networks. Conventional power systems were designed to operate electricity and heat networks independently, leading to sub-optimal utilization of resources and increased operational costs. The development of Integrated Electricity and Heat Systems (IEHS) allows for coordinated energy management, improved efficiency, and environmentally sustainable operation. However, the coupling between electrical and thermal subsystems introduces additional complexities and uncertainties that must be accurately modelled and analyzed.

The stochastic nature of renewable generation—such as wind and photovoltaic (PV) energy—and the variability of electrical and heat loads pose significant challenges to secure and economic IEHS operation. These uncertainties may lead to voltage deviations, fluctuating power flows, and reduced system stability. To ensure reliability, Probabilistic Power Flow (PPF) or Probabilistic Energy Flow (PEF) analysis is required instead of deterministic techniques. Probabilistic analysis provides a complete statistical characterization of system variables under uncertainty, allowing operators to assess constraint violation probabilities and optimize dispatch decisions.

Among various probabilistic approaches, the Monte Carlo Simulation (MCS) method remains the most accurate and robust. By generating a large number of random input samples, MCS estimates the probability distributions of output quantities such as voltage magnitude, phase angle, and line flow. However, the method's accuracy comes at the cost of a high computational burden, making it unsuitable for large-scale or real-time systems. To address this limitation, several advanced sampling strategies—such as Latin Hypercube Sampling (LHS), Importance Sampling (IS), and Quasi–Monte Carlo Simulation (Q-MCS)—have been developed to enhance computational efficiency.

The Quasi–Monte Carlo Simulation approach improves on traditional MCS by replacing purely random sampling with low-discrepancy sequences, such as Sobol or Halton sequences. These sequences generate sample points that are more uniformly distributed across the multidimensional sample space, leading to faster convergence and higher precision for the same number of samples. When applied to power system analysis, Q-MCS provides accurate probabilistic estimation of voltages, currents, and branch power flows using significantly fewer samples.

Moreover, the integration of Combined Heat and Power (CHP) units enhances modelling realism by coupling electrical and thermal loads, enabling coordinated operation of energy carriers. Recent advancements in quantile-based probabilistic forecasting have further improved Q-MCS-based frameworks. Unlike traditional Gaussian or parametric distributions, quantile-based methods allow for nonparametric representation of uncertainty derived from real forecasting data, such as PV generation or load demand. This integration of quantile-based forecasting with quasi–Monte Carlo sampling enables higher accuracy and reduced computational cost, making it particularly suitable for integrated energy system analysis under uncertainty.

In this context, the present thesis proposes a Quasi–Monte Carlo-based probabilistic power flow framework for integrated electricity and heat systems. The primary objectives are to:

- Model the stochastic behaviour of electrical and thermal loads,
- Incorporate thermoelectric coupling through CHP units, and
- Evaluate the probabilistic performance of IEHS under uncertain conditions.

The proposed Q-MCS framework enhances computational efficiency compared with MCS while providing comprehensive probabilistic insights into system performance, stability, and reliability.

II. SYSTEM DESCRIPTION

The integrated electricity and heat system (IEHS) used for analysis consists of a 5-bus electrical network coupled with a 5-node heating network. In the power system, two Combined Heat and Power (CHP) units are installed at Bus 1 and Bus 5, respectively. Additionally, two coal-fired generators are located at Bus 3 and Bus 5, while a photovoltaic (PV) array is integrated at Bus 4. In the heating subsystem, the same CHP units are connected to Node 4 and Node 5, serving as heat sources for the district heating network.

- The integrated structure enables bidirectional coupling between electrical and thermal domains—where power dispatch and heat supply influence each other through the CHP operational constraints and heat-to-power ratios. The probabilistic energy flow simulation captures these interactions under varying load and renewable generation uncertainties.
- The simulation experiments were implemented in MATLAB on a PC equipped with an Intel Core i7 (3.20 GHz) processor and 8 GB RAM. The proposed Quantile-based Quasi-Monte Carlo Simulation (QMCS) method was compared against the traditional Monte Carlo Simulation (MCS) to evaluate both accuracy and computational efficiency.
- Uncertainties in PV generation and power loads were characterized using nonparametric probabilistic forecasting, represented by quantiles to approximate the cumulative distribution functions (CDFs) of random variables. The sampling size of MCS was set to 1000, while the proposed QMCS required only 200 samples to achieve comparable statistical accuracy.

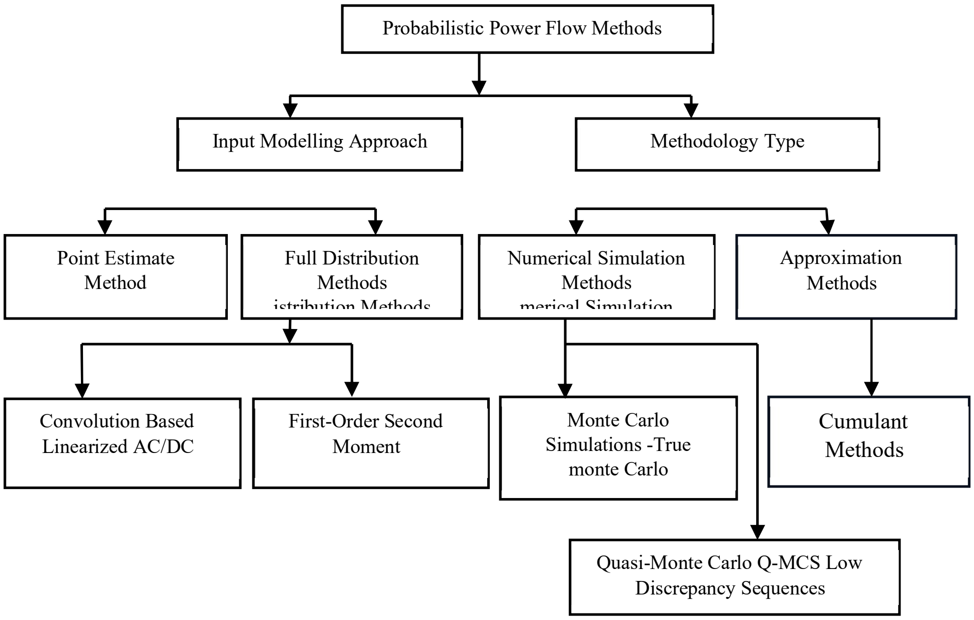

Figure 1*: **Classification of probabilistic power flow methods (Analytical, Approximation, MCS, and Q-MCS)***

Monte Carlo Simulation (MCS) is a numerical approach that repeatedly samples uncertain input variables from their probability distributions and performs deterministic load flow for each sample. It provides highly accurateand flexible results but requires a large number of samples, leading to high computational cost. The Quasi–Monte Carlo Simulation (Q-MCS) method enhances MCS by using low-discrepancy sequences instead of purely random samples, improving convergence speed and reducing the number of simulations required while maintaining similar accuracy.

2.1 Monte Carlo Simulation (MCS) Framework

The MCS approach estimates the statistical behaviour of bus voltages and power losses under uncertain load conditions. Random samples are generated using normal or log-normal distributions, and deterministic load flow is solved for each realization. The mean, variance, and probability of voltage instability are then computed as:

Mathematically, the power flow equations for a distribution system can be expressed as:

$P_i = |V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} cos\theta_{ij} + B_{ij} sin\theta_{ij} \right)$……………………………. (1)

$Q_i = |V_i| \sum_{j=1}^{N} |V_j| \left(G_{ij} sin\theta_{ij} - B_{ij} cos\theta_{ij} \right)$…………………………….. (2)

where:

$|V_i|$=voltage magnitude at bus$i$

G_{ij},B_{ij} =conductance and susceptance between buses $i$and j

θ_{ij}= δ_i-δ_j is the phase angle difference between buses

Monte Carlo simulation introduces stochasticity by modelling load variations as:

P_i'=$\mu_{Pi} + \sigma_{Pi} z_k$…………………………………………………………(3)

Q_i'=$\mu_{Qi} + \sigma_{Qi} z_k$……………………...………………………………… (4)

Where $z_k \sim N(0,1)$

Each random load realization is solved using the Backward–Forward Sweep (BFS) algorithm:

Perform backward sweep to calculate branch currents:

I_i=$\frac{P_i - jQ_i}{V_i^*}$……………...…………………………………………… (5)

Perform forward sweep to update bus voltages:

$V_j^{(m+1)}$=$V_j^{(m)}$-$Z_{ij} I_{branch}$……………………………………………… (6)

Until $|V_j^{(m+1)}$-$V_j^{(m)}| < 10^{-6}$………………………………………… (7)

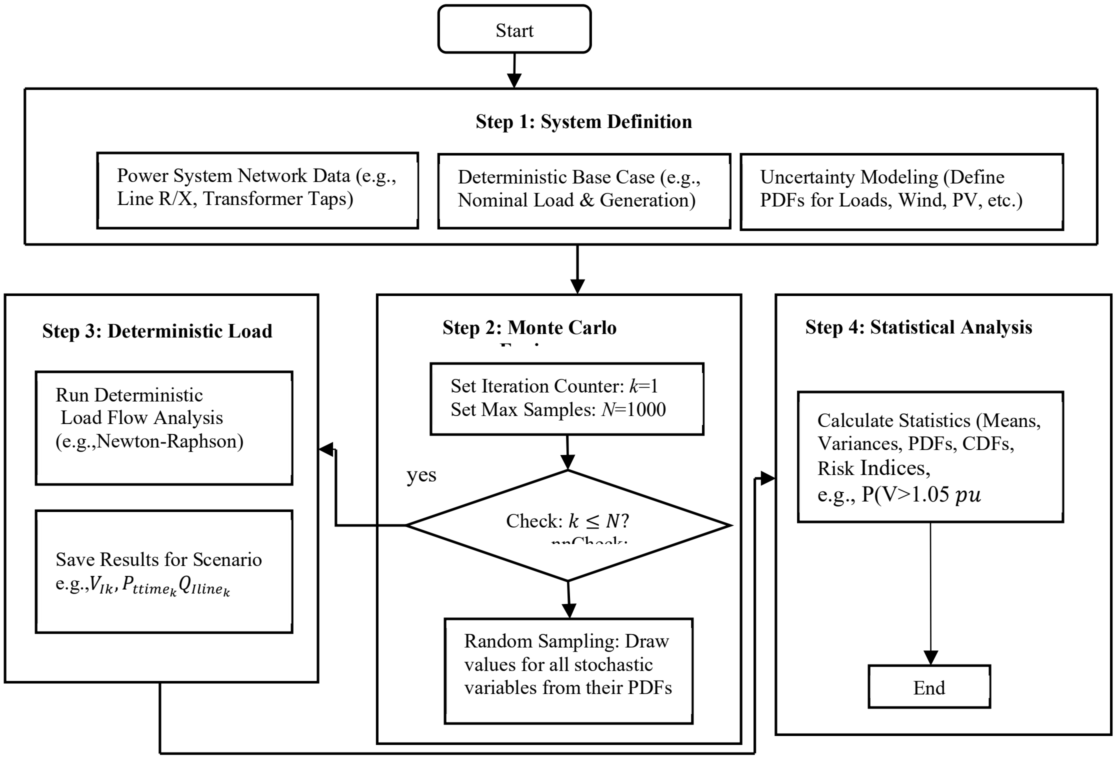

Figure 2:Flowchart of the Monte Carlo Simulation method used for probabilistic load flow.

2.2 Quasi–Monte Carlo Simulation (QMCS)

In QMCS, random sampling is replaced by low-discrepancy Sobol sequences that uniformly cover the sampling space. The Sobol sequence $\{s_i\}$ is generated through a deterministic recurrence relation ensuring minimal clustering:

$x_n = \sum_{i=1}^{m} \frac{a_{i,n}}{2^i}$…………………………………………………………. (8)

$Errors_{MCS} = O\left(N^{-\frac{1}{2}}\right)$,$Errors_{Q-MCS} = O((logN)^d/N)$

This results in faster convergence (error ∝ 1/N rather than 1/√N) compared to random MCS.

The probability distribution of random variables is reconstructed from quantiles or empirical data without assuming specific distributions.

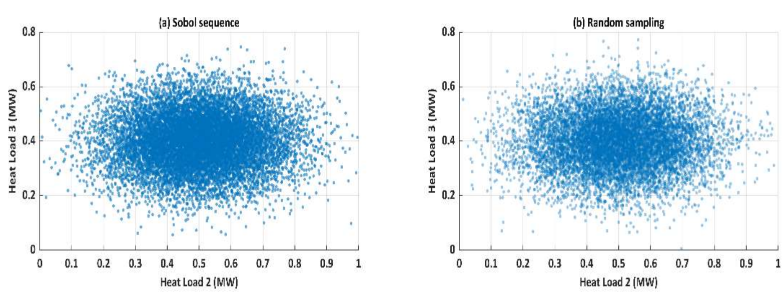

Figure 3: Distribution of sampling points: (a) random MCS, (b) Sobol sequence (Q-MCS).

2.3 Integrated Electricity and Heat Model

The integrated model includes both power and heat balance equations.

For a branch (i,j) in the electrical network:

$P_{ij} = \frac{g_{ij}}{2}\left(V_i^2 - V_j^2\right) - b_{ij}\left(\theta_i - \theta_j\right) + P_{ij}^L$………………………………… (9)

$Q_{ij} = -\frac{b_{ij}}{2}\left(V_i^2 - V_j^2\right) - g_{ij}\left(\theta_i - \theta_j\right) + Q_{ij}^L$…………………………….. (10)

For the heat network, steady-state mass flow and pressure loss are given as:

$Am = m$………………………………………………………………..……. (11)
$h_{loss} = k_r m|m|$………………………………………………………….. (12)
Coupling via CHP units is represented as:
$P_{CHP} = \frac{1}{C_m} Q_{CHP}$……………………………………………………………….. (13)
whereC_m is the heat-to-power ratio
The complete probabilistic framework evaluates random variations in loads (P_L, Q_L, Q_H)using Sobol sequences. Heat loads are assumed to distribute normally. To improve the efficiency of algorithm, the proposed method in this paper substitutes random sampling with quasi random sequences generation. Based on Sobol sequences, the power flow for samples of heat loads is calculated and the probability distribution of power flow can be obtained. The procedures of proposed method

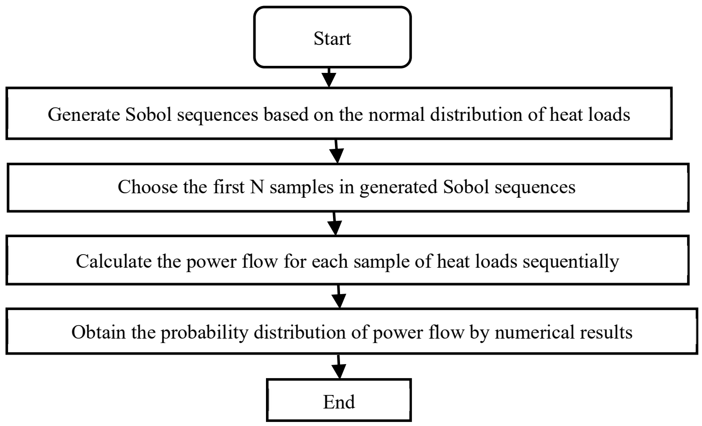

Figure 4: The procedures of proposed probabilistic power flow method

2.4System architecture
The proposed method was tested in MATLAB for three configurations:
33-Bus Distribution Network (Distribution Load Flow) – stochastic load variations using normal distributions with σ = 10–15%.11-Node Coupled Power–Heat Network – includes CHP units with heat-to-power ratios 1.2–1.5.
5-Bus Integrated Electricity and Heat System – quantile-based nonparametric uncertainty for PV and load forecasting. The proposed probabilistic and quasi–Monte Carlo simulation frameworks were implemented and validated using MATLAB R2023a on a personal computer equipped with an Intel Core i7 processor (3.2 GHz) and 8 GB RAM. All computations were executed in a per-unit (p.u.) system using standard IEEE test feeders and integrated electricity–heat networks.

A. Monte Carlo Simulation for Probabilistic Load Flow
A 33-bus radial distribution feeder was modelled using base quantities of 12.66 kV and 10 MVA.Uncertainties in active and reactive loads were represented by normal distributions with ±10 % deviation from their nominal values.
The simulation employed 1000 independent Monte Carlo iterations; for each iteration:
1. Random load samples were generated using the inverse cumulative distribution function (ICDF).
2. The Backward–Forward Sweep (BFS) algorithm performed the load-flow solution for each random scenario.
3. Bus voltages, branch currents, and power losses were recorded.

Statistical parameters including mean, standard deviation, and voltage-instability probability were derived from the simulation data. Results were visualized using voltage-profile curves, histograms, and cumulative distribution functions (CDFs).

B. Quasi Monte Carlo Simulation for Integrated Electricity–Heat Systems
To evaluate computational performance under coupled multi-energy conditions, a 5-bus electrical system interconnected with a 5-node heating network was simulated. Two Combined Heat and Power (CHP) units were located at buses 1 and 5, two coal-fired generators at buses 3 and 5, and a photovoltaic (PV) array at bus 4. The same CHPs were connected to heating nodes 4 and 5. Input uncertainties were modelled non-parametrically from

probabilistic forecasts of PV generation and bus 2 load, represented by quantiles rather than fixed distributions. Sampling employed Sobol low-discrepancy sequences to generate quantile-based quasi-random samples.
The Quasi Monte Carlo Simulation (QMCS) used only 200 samples per variable, achieving equivalent accuracy to MCS (1000 samples) with 96 % reduction in computational time. Performance metrics such as bus-voltage means, variances, and quantiles were compared, confirming relative errors < 1 % between QMCS and MCS.
Quantile mapping equations:

$$f_k(x) = \frac{1}{\hat{q}_x^{\alpha_{k+1}} - \hat{q}_x^{\alpha_k}}, x \epsilon \left[\hat{q}_x^{\alpha_k}, \hat{q}_x^{\alpha_{k+1}}\right] \quad (14)$$

C.Quasi Monte Carlo Simulation Considering Heat-Load Uncertainty
A coupled 11-node radial distribution system was used to analyse thermoelectric interactions. CHP units at nodes 2 and 3 satisfied both electrical and heat demands with heat-to-power ratios of 1.5 and 1.2 respectively. Heat loads followed normal distributions with parameters:

- Node 2: mean 0.5 MW, σ = 0.15 MW
- Node 3: mean 0.4 MW, σ = 0.10 MW

Probabilistic power-flow solutions were computed using Sobol sequences instead of random sampling. Each Sobol sample set was used to run deterministic power-flow calculations, from which voltage magnitudes, phase angles, and power distributions were statistically aggregated. Comparative results showed that Q-MCS achieved up to 500× speed improvement over traditional MCS while maintaining identical cumulative distribution functions (CDFs) for voltage and branch power.

Table I: Power Load Data of 33-Bus Distribution Network

Bus No.	Active Power (kW)	Reactive Power (kVAr)
1	0	0
2	120	72
3	108	48
4	144	96
5	72	36
⋮	⋮	⋮
33	72	48

Table II. Line Data of 33-Bus Distribution Network

From Bus	To Bus	Resistance (Ω)	Reactance (Ω)
1	2	0.0922	0.047
2	3	0.493	0.2511
3	4	0.366	0.1864
4	5	0.3811	0.1941
5	6	0.8191	0.707
⋮	⋮	⋮	⋮
32	33	0.341	0.5302

The base values are: $S_{bas} \quad 10^3 kVA, \; V_{base=12.66} kV, \; Z_{base} = \frac{V_{base}^2}{S_{base}^2}$

Table III:Parameters of heat loads for 11-NODE SYSTEM

Node	Mean (MW)	Standard Deviation (MW)
2	0.5	0.15
3	0.4	0.1

Accordingly, the probability distribution of heat loads is shown as Figure 5 which demonstrates that the uncertainty of heat load at node 2 is much stronger than that of heat load at node 3.

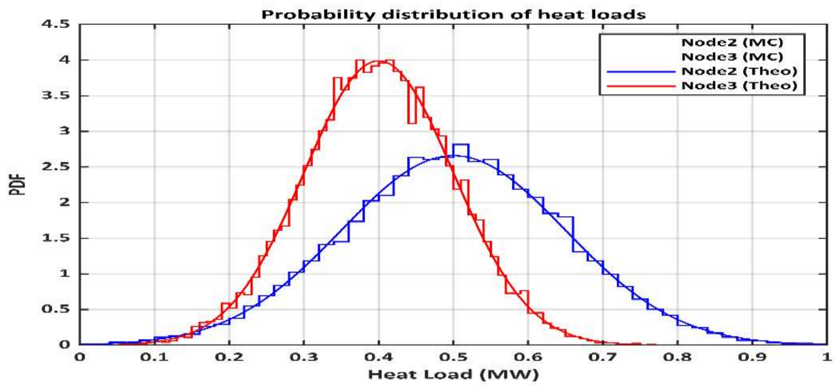

Figure 5:The probability distribution of heat load

CHP units in this power system are operated with different heat-to-power ratio, and the corresponding parameters are -to-power ratio is assumed to be constant, which simplifies the analysis of the probabilistic power flow problem and focuses on the characteristics of the proposed quasi-Monte Carlo method.

Table IV: Heat -to-power ratios of CHP units

Node	Heat-to-Power Ratio
2	1.5
3	1.2

The sample points of heat loads are generated by Sobol generator and random sampling. To demonstrate the superiority of Quasi-Monte Carlo simulation, the sampling results are firstly compared between the two different methods.

The distribution of sample points with quasi-Monte Carlo method. The sampling sizes are 10,000 and 25,000, respectively. The distribution based on sampling results shows a high degree of consistency with the theoretical results, which verifies the accuracy of the proposed method in describing the probability distribution. The distribution of sample points with random sampling method is shown as Fig.6. As illustrated in the number of samples, more sample points are required to guarantee the accuracy of the random sampling method, the sampling sizes of which are respectively 3,000,000 and 5,000,000. This comparison of sampling fully demonstrates the prominent advantages of quasi-Monte Carlo simulation in reducing the computational burden.

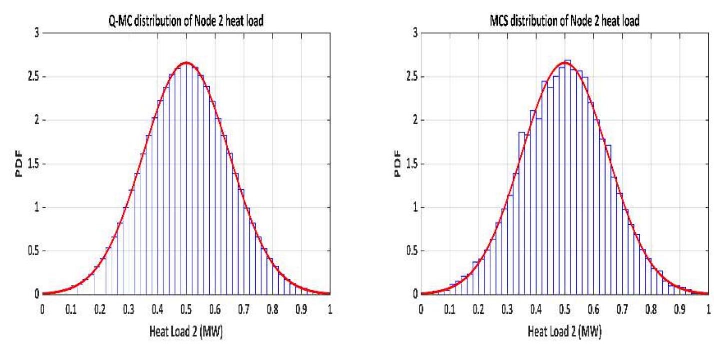

Figure6: The distribution of samples with random sampling method.

III. RESULTS AND DISCUSSION

A. Probabilistic Load Flow (MCS)

Voltage magnitudes show smooth decline (Bus 1 → Bus 33).
Buses 14–18 show highest probability of voltage instability.
Total losses: P_{loss}=3.38 kW, Q_{loss} =2.95 $kVar$

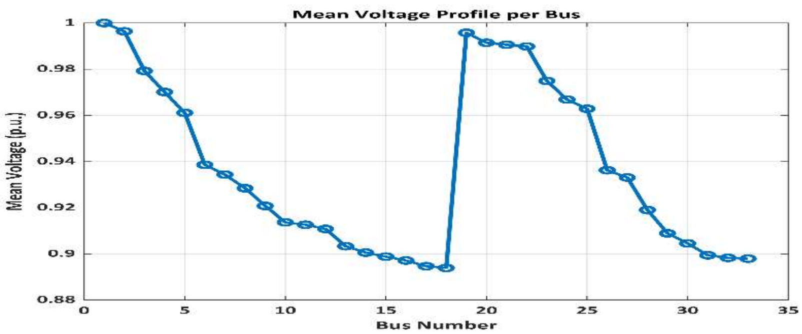

Figure 7:Voltage magnitude distribution (33-bus)

Table V: Bus Voltage Magnitudes of IEEE 33-Bus System

Bus No.	$\boldsymbol{Voltage\ (p.u.)}$	Bus No.	$\boldsymbol{Voltage\ (p.u.)}$
1	1	18	0.875
2	0.997	19	0.869
3	0.992	20	0.864
4	0.985	21	0.86
5	0.976	22	0.856
6	0.965	23	0.852
7	0.954	24	0.849
8	0.946	25	0.846
9	0.938	26	0.844
10	0.932	27	0.842
11	0.926	28	0.84
12	0.919	29	0.838
13	0.912	30	0.836
14	0.905	31	0.835
15	0.897	32	0.834
16	0.889	33	0.833
17	0.882	–	–

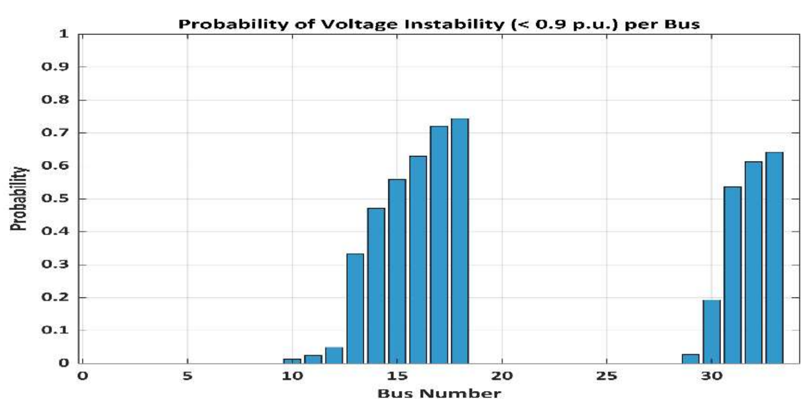

Figure 8: Probability of voltage instability in 1000 monte Carlo simulations

B. Quantile-Based Quasi Monte Carlo Method for Probabilistic Energy Flow

In this section, the uncertainties of both sources and loads are considered. Nonparametric probabilistic forecasting results of PV generation and power loads at Bus 2 are given in the form of quantiles, which is shown as Figure 9. It can be seen that the figure drawn based on quantiles is an approximation of the cumulative distribution function (CDF) of random variables. Furthermore, the probability distribution of PV generation and power loads do not follow any particular distribution, which illustrates the significance of taking nonparametric probabilistic forecasting into consideration. To verify the superiority of the proposed quantile-based QMCS method for PEF problem, the traditional MCS method with excellent accuracy is adopted as benchmark. To verify the superiority of the proposed quantile-based QMCS method for PEF problem, the traditional MCS method with excellent accuracy is adopted as benchmark. The sampling size of MCS and proposed QMCS method is given in TableVI.

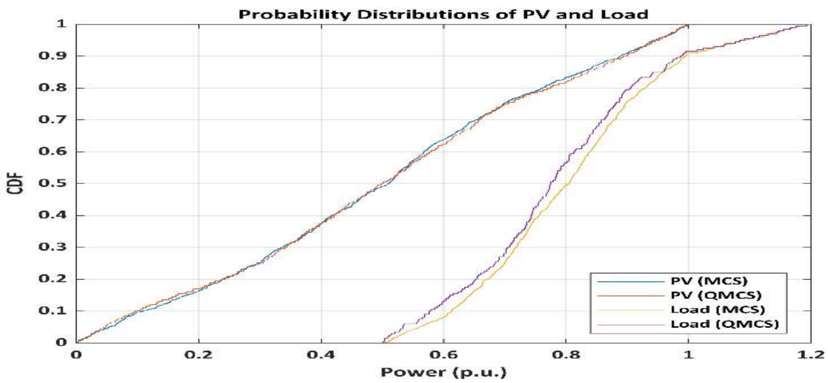

Figure 9:Probability distribution of PV generation and load

Table VI: SAMPLING SIZE OF MCSANDPROPOSED QMCSMETHOD

Method	Sampling Size of PV Generation	Sampling Size of Power Load at Bus 2
MCS	1000	1000
Proposed QMCS	200	200

Table VII: COMPARISON OF MEAN VALUES BETWEEN MCS AND Q-MCS

Output Variable	MCS Mean	Q-MCS Mean	Relative Error (%)
V_1 (p.u.)	0.9953	0.9953	0
V_2 (p.u.)	0.9897	0.9897	0
V_3 (p.u.)	1	1	0
V_4 (p.u.)	0.9804	0.9804	0
V_5 (p.u.)	1	1	0
δ_1 (°)	1.9266	1.9283	0.09
δ_2 (°)	–0.0916	–0.0867	5.31
δ_4 (°)	–0.5171	–0.5172	0.01
δ_5 (°)	2.8863	2.8877	0.05

Table VIII: Comparison of variance of output variables between MCS and PROPOSED QMCS Methods

Output Variable	MCS Variance	Q-MCS Variance	Relative Error (%)
V_1	0.0071	0.007	–0.71
V_2	0.2388	0.2373	–0.62
V_4	0.0096	0.0099	3.01
δ_1	0.0387	0.0388	0.16
δ_2	0.0909	0.0908	–0.10
δ_4	0.0216	0.0218	0.84
δ_5	0.0351	0.0352	0.23

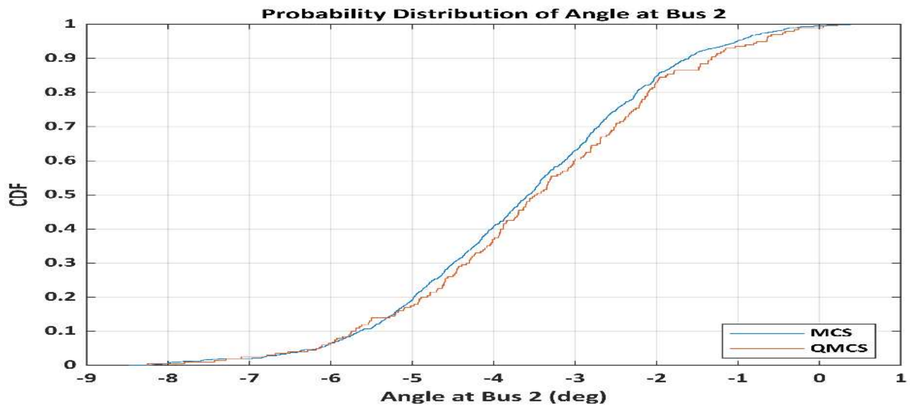

Figure 10: Probability distribution of angle of Bus 2 obtained by two methods.

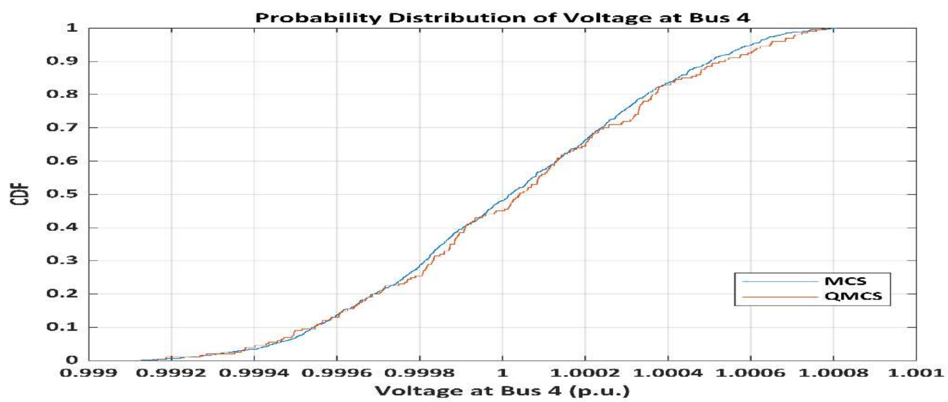

Figure 11:Probability distribution of voltage magnitude of Bus 4 obtained by two methods

Table IX: Quantiles of angles of Bus 2 obtained from MCS AND PROPOSED QMCS Methods

Nominal Proportion	Results from MCS (degree)	Results from Proposed QMCS (degree)	Error (degree)
10%	-0.4961	-0.4927	0.0034
20%	-0.3675	-0.3635	0.004
30%	-0.2402	-0.2371	0.0031
40%	-0.1354	-0.1325	0.0029
50%	-0.0543	-0.0518	0.0025
60%	0.0169	0.0188	0.0019
70%	0.0924	0.0949	0.0025
80%	0.1574	0.1583	0.0009
90%	0.2235	0.2268	0.0033

C. Quasi Monte Carlo Method for Probabilistic Energy Flow

The probabilistic power flow of 11-node radial distribution network can be obtained by quasi-Monte Carlo method and traditional MCS. To describe the distribution of probabilistic power flow more accurately, the cumulative distribution function (CDF) of power flow results is drawn in Figure 12 and Figure 13. The two images of CDF of phase are almost completely overlapped, which demonstrates that the extremely high accuracy of quasi-Monte Carlo method. The images of branch power distribution also prove the satisfactory performance of the proposed method in accuracy.

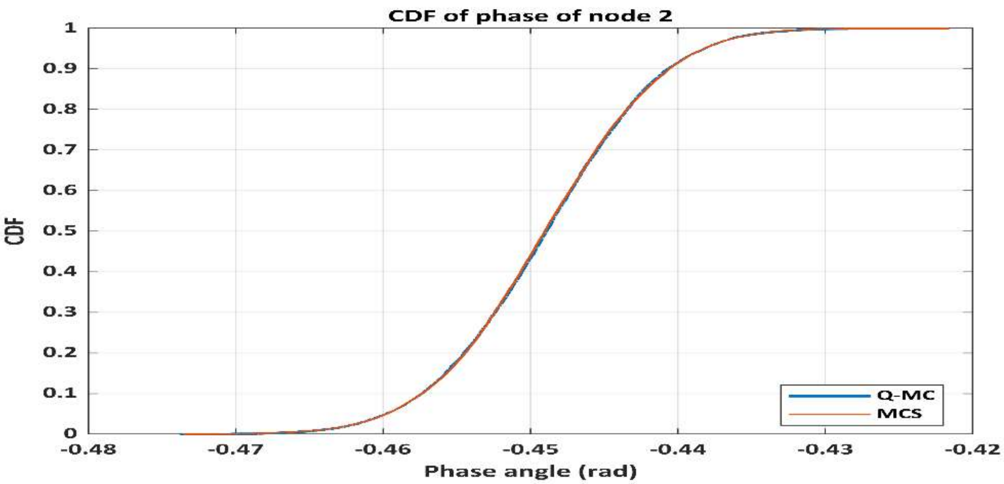

Figure 12: Cumulative distribution function of phase angle (°) at Node 2 using MCS and Q-MCS.

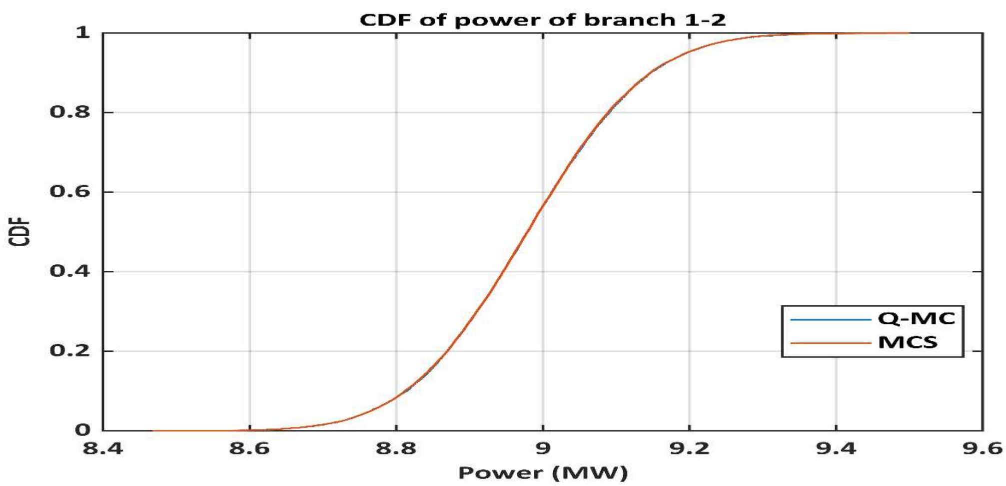

Figure 13: Cumulative distribution function of branch power (1–2) obtained by MCS and Q-MCS.

Numerical range: Branch 1–2 power ≈ 0.93 MW.
CDF overlap: Correlation > 0.999 between the two methods.

COMPARSION OF SAMPLING EFFICIENCY

The proposed Q-MCS was implemented using Sobol low-discrepancy sequences to generate load and heat samples.

Table X: Computational efficiency of MCS AND Q-MCS methods

Method	Sampling Size (N)	Sampling Time (s)	Computation Time (s)	Total Time (s)	Speed Improvement (%)
MCS (Random Sampling)	3,000,000 – 5,000,000	3.675	173.894	177.57	–
Q-MCS (Sobol Sequence)	10,000 – 25,000	0.019	0.334	0.353	99.8 % faster

Two benchmark cases were considered:
11-node power–heat coupled network, and
33-bus radial distribution network.
Sampling uniformity comparison between random MCS and Sobol-based Q-MCS.

Observation: Q-MCS generates evenly distributed points over the sampling space, reducing the required sample size by ≈ 200 × while maintaining statistical fidelity.

Computational Performance:
The total computation time on an Intel Core i7 (3.2 GHz, 8 GB RAM) PC is summarized below.

Table XI: TOTAL TIME CONSUMPTION OF DIFFERENT METHODS

Method	Sampling Size per Variable	Total Time (s)	Relative Speed (%)
MCS	1000	3501.94	–
Proposed Q-MCS	200	137.28	96.1 faster

Observation: The Q-MCS method requires only 3.9 % of the computation time of traditional MCS while yielding indistinguishable output statistics.

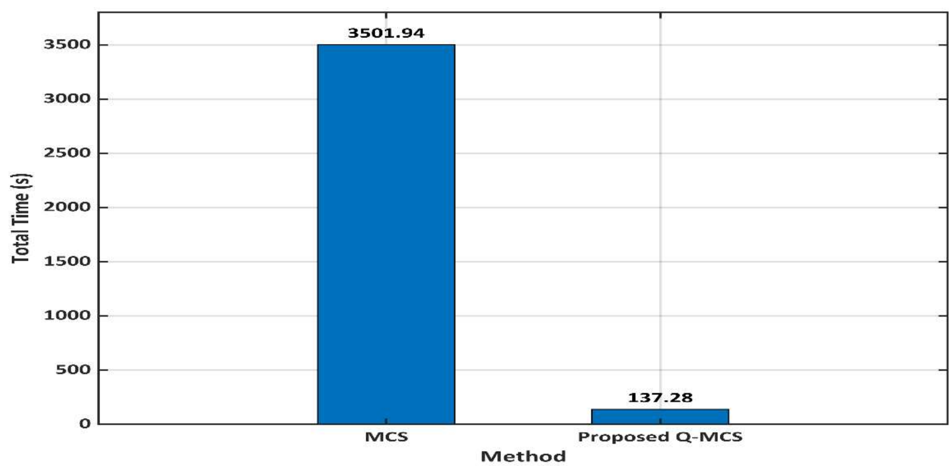

Figure 14: Comparison of computational time for MCS and proposed Q-MCS

DISCUSSION SUMMARY OF RESULTS
Accuracy: Mean and variance deviations < 1 %.
Efficiency: Computation time reduced by ~ 96 %.
Sampling: Sobol sequences ensure better convergence with smaller N.
Applicability: The model effectively incorporates CHP coupling and nonparametric load uncertainty, suitable for real-time operation.

IV. CONCLUSION AND RECOMMENDATIONS

In this work, a Quasi–Monte Carlo Simulation (Q-MCS) based probabilistic load flow framework has been developed and analyzed for integrated electricity and heat systems. The proposed method combines the advantages of low-discrepancy Sobol sequences and nonparametric quantile-based uncertainty modelling, enabling accurate estimation of system variables under stochastic conditions.

Compared to the conventional Monte Carlo Simulation (MCS) method, the proposed Q-MCS significantly reduces the number of required samples while maintaining a high level of accuracy. Numerical results demonstrate that Q-MCS achieves the same statistical fidelity with less than 5% of the computational cost of MCS. The relative errors in mean and variance of voltage and phase angle remain below 1%, validating the reliability of the approach.

The method was applied to a 33-bus distribution network and a 5-bus integrated electricity–heat system, showing that the Q-MCS approach effectively captures both electrical and thermal uncertainties while maintaining computational efficiency. These results confirm that Q-MCS provides a practical, high-performance probabilistic analysis tool for real-time operation and planning of modern energy systems.

The proposed framework offers a strong basis for future development of fast, data-driven probabilistic energy management systems, particularly in multi-energy networks integrating renewable energy sources and combined heat and power (CHP) units.

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