International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol.2, Issue.4, July-Aug. 2012 pp-1929-1933 ISSN: 2249-6645

Edge Coloring of a Complement Fuzzy Graph

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ABSTRACT: Graph coloring is one of the most important problems of combinatorial optimization. Many problems of practical interest can be modeled as coloring problems. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph. Edge coloring is a function which assigns colors to the edges so that incident edges receive different colors. Let G=(V,μ, σ) be a simple connected undirected graph where V is a set of vertices and each vertices has a membership value μ and each edge has a membership value σ. Minimum number of color needed to color the graph is known as chromatic number of that graph. Graph coloring is a NP complete problem. In our paper, we introduce an algorithm to find the complement of any fuzzy graph with O (n^2) time and also coloring this complement fuzzy graph using α cut.

Keywords: *Complement fuzzy graph, edge color, α cut of fuzzy graph*

I. INTRODUCTION

We know that graphs are simple model of relation. A graph is a convenient way of representing information involving relationship between objects. The object is represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both we need to design fuzzy graph model. One of the most important properties of fuzzy graph model is fuzzy graph coloring which is used to solve problems of combinatorial optimization like traffic light control, exam-scheduling, register allocation etc. Two types of coloring namely vertex coloring and edge coloring are usually associated with any graph. The paper is organized in five sections. 1st section includes introduction of fuzzy graph model. Section two defines fuzzy graph and its properties. Section three, we find the complement of a fuzzy graph and define a coloring function which is based on α cut to color the complement fuzzy graph and finding the chromatic number for this fuzzy graph.

II. FUZZY GRAPH

A fuzzy graph is a pair of functions G: (σ, μ) where σ is a fuzzy subset X and μ is a symmetric relation on σ i.e. σ: X → [0, 1] and μ: X × X → [0, 1] such that μ(x, y) ≤ σ(x) ∧ σ(y) for all x, y in X. we denote the crisp graph of G: (σ, μ) by G*: (σ*, μ*) where σ* is referred to as the nonempty set X of nodes and μ* = E∊ X × X. Now crisp graph (X, E) is a special case of a fuzzy graph with each vertex and edge of (X, E) having degree of membership value 1where loops are not consider and μ is reflexive.

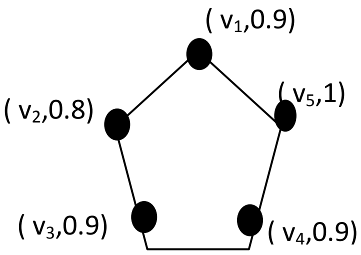


Fig 1 Fuzzy graph (G)

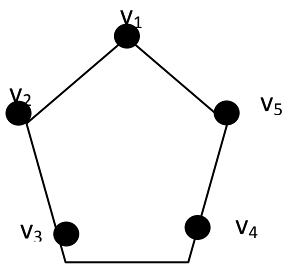
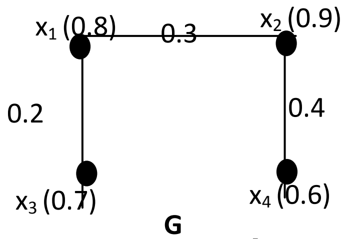


Fig 2 Crisp graph(G^*)

2.1 COMPLEMENT OF FUZZY GRAPH

Complement of a fuzzy graph has been defined by Moderson [1]. Complement of a fuzzy graph G: (σ, μ) as a fuzzy graph G^c: (σ^c, μ^c) where $\sigma^c = \sigma$ and μ^c (x, y) = 0 if μ(x, y)> 0 and μ^c (x, y) = σ(x) ∧ σ(y) otherwise. From the definition G^c is a fuzzy graph even if G is not and $(G^c)^c$ = G if and only if G is a strong fuzzy graph. Also, automorphism group of G and G^c are not identical. But there is some drawbacks in the definition of complement of a fuzzy graph mentioned above. In fig 5 $(G^c)^c \neq$ G and note that they are identical provided G is a strong fuzzy graph



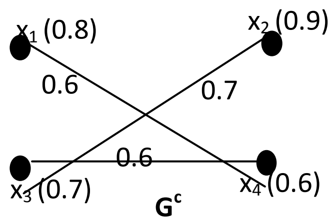


Fig 3 Fuzzy graph Fig 4 Complement Fuzzy graph

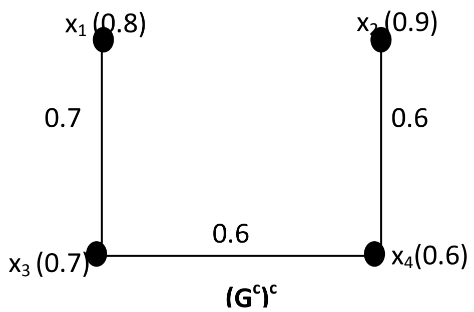


Fig .5 Complement of complement Fuzzy graph

Now the complement of a fuzzy graph G: (σ, μ) is the fuzzy graph $\bar{G}$: ($\bar{\sigma}$, $\bar{\mu}$) where $\bar{\sigma} \equiv \sigma$ and $\bar{\mu}$ (x, y) = σ(x) ∧ σ(y) - μ(x, y) ∀ x, y ∊ X (1)

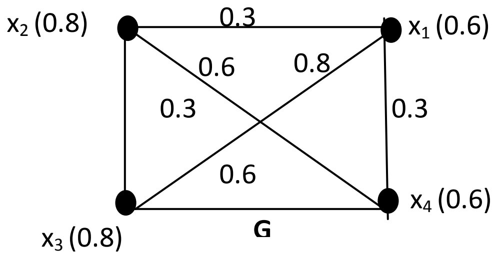


Fig .6 Fuzzy graph

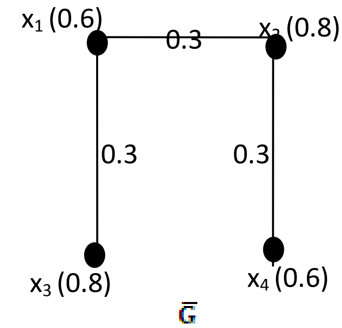


Fig. 7 Complement Fuzzy graph

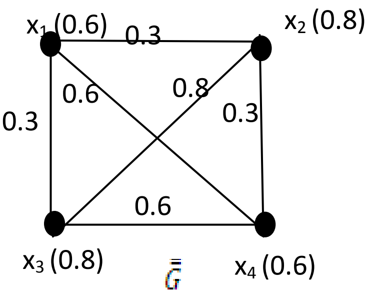


Fig .8 Complement of complement Fuzzy graph

Now, $\bar{\bar{\sigma}} = \bar{\bar{\sigma}} = \sigma$ and $\bar{\bar{\mu}}$ (x, y) = $\bar{\sigma}$(x) ∧ $\bar{\sigma}$(y) – $\bar{\mu}$ (x, y) = σ (x) ∧ σ(y) – ((σ (x) ∧ σ(y) – μ(x, y)) = μ(x, y) ∀ x, y.

Hence $\bar{\bar{G}}$ = G

Fig. 8 shows the Complement of complement fuzzy graph is a fuzzy graph

2.2 α CUT OF A FUZZY GRAPH

α cut set of fuzzy set A is denoted as A_α is made up of members whose membership is not less than α .A_α ={x∊X | $\mu_A(x) \geq \alpha$.}. α cut set of fuzzy set is crisp set. In this paper, α cut set depend on vertex and edge membership value. The α cut of fuzzy graph defined as $G_\alpha =(V_\alpha, E_\alpha)$ where V_α ={v∈V|$\sigma \geq \alpha$} and E_α ={e∈E| $\mu \geq \alpha$}

III. COLORING OF COMPLEMENT FUZZY GRAPH

In our previous paper [8] we have done a C program to find the complement a fuzzy graph using the condition (1). In this paper we find the complement of the fuzzy graph using the C program. We find all the different membership value of vertices and edges in the complement of a fuzzy graph. This membership value will work as a α cut of this complement fuzzy graph. Depend upon the values of α cut we find different types of fuzzy graphs for the same complement fuzzy graph. Then we color all the edges of the complement fuzzy graph so that no incident edges will not get the same color and find the minimum number of color will need to color the complement fuzzy graph is known as chromatic number.

For solving this problem we have done the calculation into three cases. in 1st case we take a fuzzy graph (G) which have five vertices and five edges. All the vertices and edges have fuzzy membership value. In the second case we find the complement of this fuzzy graph (G_1). In third section we define the edge coloring function to color the complement fuzzy graph.

CASE 1:

We consider a fuzzy graph with have five vertices, v_1, v_2, v_3, v_4, v_5 and corresponding membership values 0.9, 0.75, 0.95, 0.95, 0.9. Graph consist of five edges e_1, e_2, e_3, e_4, e_5 with their corresponding membership value 0.75, 0.9, 0.85, 0.15, 0.6. Corresponding fuzzy graph is shown in fig. 9

$$\mu_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \\ 0.0 & 0.75 & 0.0 & 0.0 & 0.9 \\ 0.75 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.6 & 0.0 & 0.15 & 0.0 \\ 0.7 & 0.5 & 0.14 & 0.0 & 0.85 \\ 0.5 & 0.6 & 0.85 & 0.5 & 0.9 \end{array}$$

Adjacent matrix 1

$$E_1 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \\ 0 & e_1 & 0 & 0 & e_5 \\ e_1 & 0 & e_3 & 0 & 0 \\ 0 & e_2 & 0 & e_4 & 0 \\ 0 & 0 & e_3 & 0 & e_4 \\ & & & & \end{array}$$

Adjacent matrix 2

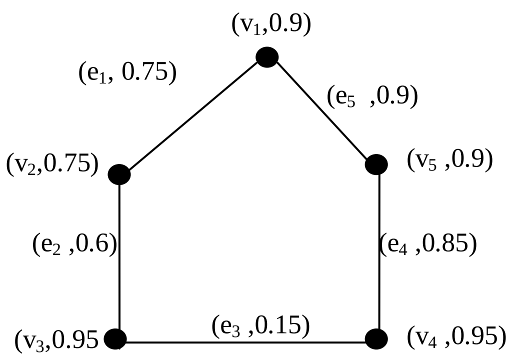


fig.9 G (fuzzy graph)

Adjacent matrix 1 represents the membership value of edges and adjacent matrix2 represents the existence of edges between the vertices

CASE 2:

We find the complement of a fuzzy graph using [8].

μ_2 =

	v_1	v_2	v_3	v_4	v_5
v_1	0.0	0.0	0.9	0.9	0.0
v_2	0.0	0.0	0.15	0.75	0.75
v_3	0.9	0.15	0.0	0.0	0.9
v_4	0.75	0.75	0.0	0.0	0.05
v_5	0.0	0.75	0.9	0.05	0.0

Adjacent matrix 3

E_2 =

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	e_1	e_2	0
v_2	0	0	e_4	e_6	e_3
v_3	e_1	e_4	0	0	e_5
v_4	e_2	e_6	0	0	e_7
v_5	0	e_3	e_5	e_7	0

Adjacent matrix 4

Adjacent matrix 3 represents the membership value of edges and adjacent matrix4 represents the existence of edges edge between the vertices

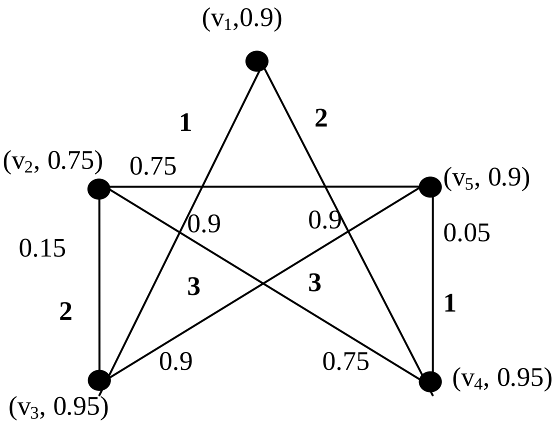


Fig.10 G_1 (complement fuzzy graph)

CASE 3:

Given a fuzzy graph $G=(V_F,E_F)$,its edge chromatic number is fuzzy number $\chi(G)=\{(x_\alpha ,\alpha)\}$ where x_α is the edge chromatic number of G_α and α values are the different membership value of vertex and edge of graph G. In this fuzzy graph, there are six α cuts. They are {0.05, 0.15, 0.75, 0.9, 0.95}.For every value of α, we find graph G_α and find its fuzzy edge chromatic number.

For α= 0.05 Fuzzy graph G = (V, σ, μ) where σ={0.9,0.75,0.95,0.95, 0.9} and

μ_3 =

	v_1	v_2	v_3	v_4	v_5
v_1	0.0	0.0	0.9	0.9	0.0
v_2	0.0	0.0	0.15	0.75	0.75
v_3	0.9	0.15	0.0	0.0	0.9
v_4	0.75	0.75	0.0	0.0	0.05
v_5	0.0	0.75	0.9	0.05	0.0

Adjacent matrix 5

E_3 =

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	e_1	e_2	0
v_2	0	0	e_4	e_6	e_3
v_3	e_1	e_4	0	0	e_5
v_4	e_2	e_6	0	0	e_7
v_5	0	e_3	e_5	e_7	0

Adjacent matrix 6

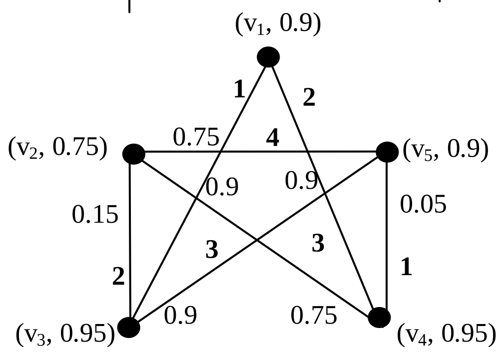


Fig.11 $\chi_{(0.05)} = 4$

For α cut value 0.05 we find the graph $G_{0.05}$ (Fig. 11). Then we proper color all the edges of this graph and find the chromatic number of this graph is 4.

For α= 0.15 Fuzzy graph G =(V, σ, μ) where σ={0.9,0.75,0.95,0.95, 0.9} and

μ_4 =

	v_1	v_2	v_3	v_4	v_5
v_1	0.0	0.0	0.9	0.9	0.0
v_2	0.0	0.0	0.15	0.75	0.75
v_3	0.9	0.15	0.0	0.0	0.9
v_4	0.75	0.75	0.0	0.0	0.0
v_5	0.0	0.75	0.9	0.0	0.0

Adjacent matrix 7

v_1 v_2 v_3 v_4 v_5

$E_4 =$

v_1	0	0	e_1	e_2	0
v_2	0	0	e_4	e_6	e_3
v_3	e_1	e_4	0	0	e_5
v_4	e_2	e_6	0	0	0
v_5	0	e_3	e_5	0	0

Adjacent matrix 8

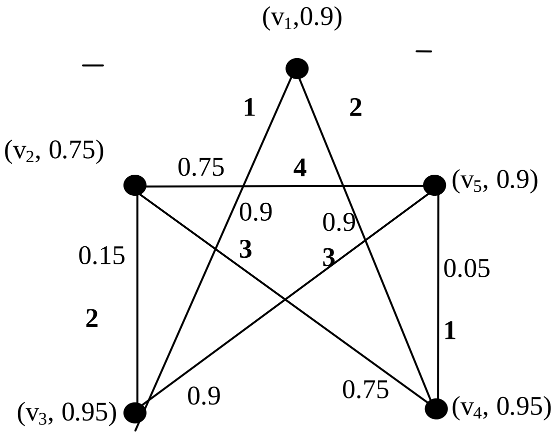


Fig. 12 $\chi_{(0.15)} = 4$

For α cut value 0.15 we find the graph $G_{0.15}$ (Fig. 12). Then we proper color all the edges of this graph and find the chromatic number of this graph is 4.

For α= 0.75 Fuzzy graph G =(V, σ, μ) where σ ={0.9,0.75,0.95,0.95, 0.9}

$\mu_5 =$

	v_1	v_2	v_3	v_4	v_5
v_1	0.0	0.0	0.9	0.9	0.0
v_2	0.0	0.0	0.0	0.75	0.75
v_3	0.9	0.0	0.0	0.0	0.9
v_4	0.75	0.75	0.0	0.0	0.0
v_5	0.0	0.75	0.9	0.0	0.0

Adjacent matrix 9

$E_5 =$

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	e_1	e_2	0
v_2	0	0	0	e_6	e_3
v_3	e_1	0	0	0	e_5
v_4	e_2	e_6	0	0	0
v_5	0	e_3	e_5	0	0

Adjacent matrix 10

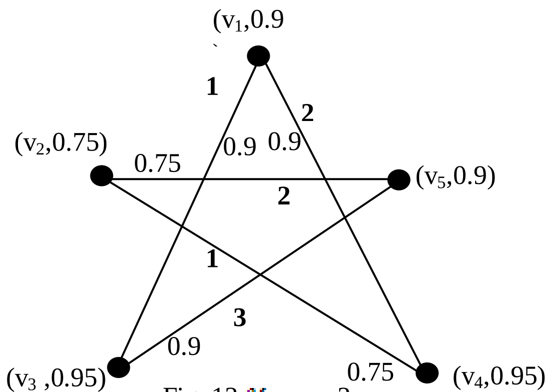


Fig. 13 $\chi_{(0.75)} = 3$

Now for α cut value 0.75 we find the graph $G_{0.75}$ (Fig. 13). Then we proper color all the edges of this graph and find the chromatic number of this graph is 3.

For α= 0.9 Fuzzy graph G = (V, σ, μ) where σ={0.9, 0.95,0.95, 0.9} and

$\mu_6 =$

	v_1	v_2	v_3	v_4	v_5
v_1	0.0	0.0	0.9	0.9	0.0
v_2	0.0	0.0	0.0	0.0	0.0
v_3	0.9	0.0	0.0	0.0	0.9
v_4	0.0	0.0	0.0	0.0	0.0
v_5	0.0	0.0	0.9	0.0	0.0

Adjacent matrix 11

$E_6 =$

	v_1	v_2	v_3	v_4	v_5
v_1	0	0	e_1	e_2	0
v_2	0	0	0	0	0
v_3	e_1	0	0	0	e_5
v_4	0	0	0	0	0
v_5	0	0	e_5	0	0

Adjacent matrix 12

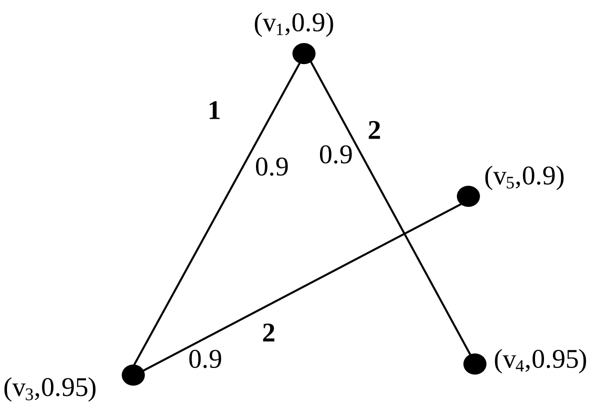


Fig. 14 $\chi_{(0.9)} = 2$

For α cut value 0.9 we find the graph $G_{0.9}$ (Fig. 14). Then we proper color all the edges of this graph and find the chromatic number of this graph is 2.

For α= 0.95 Fuzzy graph G =(V, σ, μ) where σ={0.95} and

$$\mu_7 = \begin{array}{c|cc|} & v_3 & v_4 \\ \hline v_3 & 0.0 & 0.0 \\ v_4 & 0.0 & 0.0 \end{array}$$

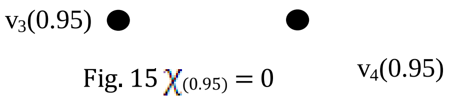


Fig. 15 $\chi_{(0.95)} = 0$

For α cut value 0.95 we find the graph $G_{0.95}$ (Fig. 15). Then we proper color all the edges of this graph and find the chromatic number of this graph is 0.

In the above example, five crisp graph $G_\alpha = (V_\alpha, E_\alpha)$ are obtained by considering different values of α. Now for the edge chromatic number χ_α for any α,it can be shown that the chromatic number of fuzzy graph G is

χ(G) = {(4, 0.05), (4, 0.15), (3, 0.75), (2, 0.9), (0, 0.95)}

IV. CONCLUSION

In this paper we find the complement fuzzy graph and color all the edges of that complement fuzzy graph. Here edge chromatic number depends on α cut value. In our next paper, we shall try to design an algorithm on edge coloring function to any fuzzy graph.

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