Von Neumann Entropy in Quantum Computation and Sine qua non Relativistic Parameters- a Gesellschaft-Gemeinschaft Model

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ABSTRACT: *Von Neumann Entropy and computational complexity theory is a branch of the theory of computation in theoretical computer science and mathematics that focuses on classifying computational problems according to their inherent difficulty, and relating those classes to each other. In this context, a computational problem is understood to be a task that is in principle amenable to being solved by a computer (which basically means that the* ***problem can be stated by a set of mathematical instructions). Informally, a computational problem consists of problem instances and solutions to these problem*** *instances. For example, primality testing is the problem of determining whether a given number is prime or not. The instances of this problem are natural numbers, and the solution to an instance is yes or no based on whether the number is prime or not.A problem is regarded as inherently difficult if its solution requires significant resources, whatever the algorithm used. The theory formalizes this intuition, by introducing mathematical models of computation to study these problems and quantifying the amount of resources needed to solve them, such as time and storage. Other complexity measures are also used, such as the amount of communication (used in communication complexity), the number of gates in a circuit (used in circuit complexity) and the number of processors (used in parallel computing). One of the roles of* ***computational complexity theory is to determine the practical limits on what computers can and cannot do. Closely related fields in theoretical computer science are analysis of algorithms*** *and computability theory. A key distinction between analysis of algorithms and computational complexity theory is that the former is devoted to analyzing the amount of resources needed by a particular algorithm to solve a problem, whereas the latter asks a more general question about all possible algorithms that could be used to solve the same problem. More precisely, it tries to classify problems that can or cannot be solved with appropriately restricted resources. In turn, imposing restrictions on the available resources is what distinguishes computational complexity from computability theory: the latter theory asks what kind of problems can, in principle, be solved algorithmically. Low-energy excitations of one-dimensional spin-orbital models which consist of spin waves, orbital* ***waves, and joint spin****-orbital excitations. Among the latter we identify strongly entangled spin-orbital bound states which appear as peaks in the von Neumann entropy (vNE) spectral function introduced in this work. The strong entanglement of bound states is manifested by a universal logarithmic scaling of the vNE with system size, while the vNE of other spin-orbital excitations saturates. We suggest that spin-orbital entanglement can be experimentally explored by the measurement of the dynamical spin-orbital correlations using resonant inelastic x-ray scattering, where strong spin-orbit* ***coupling associated with the core hole plays a role. Distinguish ability of States and von Neumann Entropy have been studied by Richard Jozsa, Juergen Schlienz.Consider*** *an ensemble of pure quantum states |\psi_j>, j=1,...,n taken with prior probabilities p_j respectively. It has been shown that it is possible to increase all of the pair wise overlaps |<\psi_j|\psi_j>| i.e. make each constituent pair of the states more parallel (while keeping the prior probabilities the same), in such a way that the von Neumann entropy S is increased, and dually, make all pairs more orthogonal while decreasing S. This phenomenon* ***cannot occur for ensembles*** *in two dimensions but that it is a feature of almost all ensembles of three states in three dimensions. It is known that the von Neumann entropy characterizes the classical and quantum information capacities of the ensemble and we argue that information capacity in turn, is a manifestation of the distinguish ability of the signal states. Hence our result shows that the notion of distinguish ability within an ensemble is a global property that cannot be reduced to considering distinguish ability of each constituent pair of states.*

Key words: *Von Neumann entropy, Quantum computation, Governing equations*

Introduction

Von Neumann entropy

In quantum statistical mechanics, von Neumann entropy, named after John von Neumann, is the extension of classical entropy concepts to the field of quantum mechanics. John von Neumann rigorously established the mathematical framework for quantum mechanics in his work Mathematische Grundlagen der Quantenmechanik In it, he provided a theory of measurement, where the usual notion of wave-function collapse is described as an irreversible process (the so-called von Neumann or projective measurement).

The density matrix was introduced, with different motivations, by von Neumann and by Lev Landau. The motivation that inspired Landau was the impossibility of describing a subsystem of a composite quantum system by a state vector. On the other hand, von Neumann introduced the density matrix in order to develop both quantum statistical mechanics and a theory of quantum measurements. The density matrix formalism was developed to extend the tools of classical statistical mechanics to the quantum domain. In the classical framework, we compute the partition function of the system in order to evaluate all possible thermodynamic quantities. Von Neumann introduced the density matrix in the context of states and operators in a Hilbert space. The knowledge of the statistical density matrix operator would allow us to compute all average quantities in a conceptually similar, but mathematically different way. Let us suppose we have a set of wave functions |Ψ ⟩ which depend parametrically on a set of quantum numbers $n_1, n_2, \ldots, n_N$. The natural variable which we have is the amplitude with which a particular wavefunction of the basic set participates in the actual wavefunction of the system. Let us denote the square of this amplitude by $p(n_1, n_2, \ldots, n_N)$. The goal is to turn this quantity p into the classical density function in phase space. We have to verify that p goes over into the density function in the classical limit, and that it hasergodic properties. After checking that $p(n_1, n_2, \ldots, n_N)$ is a constant of motion, an ergodic assumption for the probabilities $p(n_1, n_2, \ldots, n_N)$ makes p a function of the energy only .

After this procedure, one finally arrives at the density matrix formalism when seeking a form where $p(n_1, n_2, \ldots, n_N)$ is invariant with respect to the representation used. In the form it is written, it will only yield the correct expectation values for quantities which are diagonal with respect to the quantum numbers $n_1, n_2, \ldots, n_N$.
Expectation values of operators which are not diagonal involve the phases of the quantum amplitudes. Suppose we encode the quantum numbers $n_1, n_2, \ldots, n_N$ into the single index i or j. Then our wave function has the form

$$|\Psi\rangle = \sum_i a_i\,|\psi_i\rangle.$$

The expectation value of an operator B which is not diagonal in these wave functions, so

$$\langle B\rangle = \sum_{i,j} a_i^* a_j\,\langle i|B|j\rangle.$$

The role, which was originally reserved for the quantities, $|a_i|^2$ is thus taken over by the density matrix of the system S.

$$\langle j|\,\rho\,|i\rangle = a_j\,a_i^*.$$

Therefore $\langle B\rangle$ reads as $\langle B\rangle = Tr(\rho\,B)$.

The invariance of the above term is described by matrix theory. A mathematical framework was described where the expectation value of quantum operators, as described by matrices, is obtained by taking the trace of the product of the density operator $\hat{\rho}$ and an operator $\hat{B}$ (Hilbert scalar product between operators). The matrix formalism here is in the statistical mechanics framework, although it applies as well for finite quantum systems, which is usually the case, where the state of the system cannot be described by a pure state, but as a statistical operator $\hat{\rho}$ of the above form. Mathematically, $\hat{\rho}$ is a positive, semi definite Hermitian matrix with unit trace

Given the density matrix ρ, von Neumann defined the entropy as $S(\rho) = -\mathrm{Tr}(\rho\ln\rho)$,
Which is a proper extension of the Gibbs entropy (up to a factor k_B) and the Shannon entropy to the quantum case. To compute S(ρ) it is convenient (see logarithm of a matrix) to compute the Eigen decomposition of $\rho = \sum_j \eta_j |j\rangle\langle j|$
The von Neumann entropy is then given by

$$S(\rho) = -\sum_j \eta_j \ln \eta_j.$$

Since, for a pure state, the density matrix is idempotent, ρ=ρ2, the entropy S(ρ) for it vanishes. Thus, if the system is finite (finite dimensional matrix representation), the entropy (ρ) quantifies the departure of the system from a pure state. In other words, it codifies the degree of mixing of the state describing a given finite system. Measurement decohere a quantum system into something noninterfering and ostensibly classical; so, e.g., the vanishing entropy of a pure state |Ψ⟩ = (|0⟩+|1⟩)/√2, corresponding to a density matrix

$\rho = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ increases to S=ln 2 =0.69 for the measurement outcome mixture

$\rho = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ As the quantum interference information is erased.

Properties
Some properties of the von Neumann entropy:
S(ρ) is only zero for pure states.
S (ρ) is maximal and equal to $\ln N$ for a maximally mixed state, N being the dimension of the Hilbert space.
S (ρ) is invariant under changes in the basis of ρ, that is, $S(\rho) = S(U\rho U^\dagger)$, with U a unitary transformation.
S (ρ) is concave, that is, given a collection of positive numbers λ_i which sum to unity ($\Sigma_i \lambda_i = 1$) and density operators ρ_i, we have

$$S\left(\sum_{i=1}^{k} \lambda_i \rho_i\right) \geq \sum_{i=1}^{k} \lambda_i S(\rho_i).$$

S (ρ) is additive for independent systems. Given two density matrices ρ_A, ρ_B describing independent systems A and B, we have $S(\rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B)$.

S(ρ) strongly sub additive for any three systems A, B, and C:

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC}).$$

This automatically means that S(ρ) is sub additive:

$$S(\rho_{AC}) + \leq S(\rho_A) + S(\rho_C).$$

Below, the concept of subadditivity is discussed, followed by its generalization to strong Subadditivity.

Subadditivity

If ρ_A, ρ_B are the reduced density matrices of the general state ρ_{AB}, then

$$|S(\rho_A) - S(\rho_B)| \leq S(\rho_{AB}) \leq S(\rho_A) + S(\rho_B).$$

This right hand inequality is known as subadditivity. The two inequalities together are sometimes known as the triangle inequality. They were proved in 1970 by Huzihiro Araki andElliott H. Lieb While in Shannon's theory the entropy of a composite system can never be lower than the entropy of any of its parts, in quantum theory this is not the case, i.e., it is possible that $S(\rho_{AB}) = 0$ while $S(\rho_A) > 0$ and $S(\rho_B) > 0$.

Intuitively, this can be understood as follows: In quantum mechanics, the entropy of the joint system can be less than the sum of the entropy of its components because the components may be entangled. For instance, the Bell state of two spin-1/2's, $|\psi\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, is a pure state with zero entropy, but each spin has maximum entropy when considered individually. The entropy in one spin can be "cancelled" by being correlated with the entropy of the other. The left-hand inequality can be roughly interpreted as saying that entropy can only be canceled by an equal amount of entropy.

If system A and system B have different amounts of entropy, the lesser can only partially cancel the greater, and some entropy must be left over. Likewise, the right-hand inequality can be interpreted as saying that the entropy of a composite system is maximized when its components are uncorrelated, in which case the total entropy is just a sum of the sub-entropies. This may be more intuitive in the phase space, instead of Hilbert space, representation, where the Von Neumann entropy amounts to minus the expected value of the ∗-logarithm of the Wigner function up to an offset shift.

Strong Subadditivity

The von Neumann entropy is also strongly sub additive. Given three Hilbert spaces, A, B, C,

$$S(\rho_{ABC}) + S(\rho_B) \leq S(\rho_{AB}) + S(\rho_{BC}).$$

This is a more difficult theorem and was proved in 1973 by Elliott H. Lieb and Mary Beth Ruskai using a matrix inequality of Elliott H. Lieb proved in 1973. By using the proof technique that establishes the left side of the triangle inequality above, one can show that the strong subadditivity inequality is equivalent to the following inequality.

$$S(\rho_A) + S(\rho_C) \leq S(\rho_{AB}) + S(\rho_{BC})$$

When ρ_{AB}, etc. are the reduced density matrices of a density matrix ρ_{ABC}. If we apply ordinary subadditivity to the left side of this inequality, and consider all permutations of A, B, C, we obtain the triangle inequality for ρ_{ABC} : Each of the three numbers $S(\rho_{AB}),\ S(\rho_{BC}),\ S(\rho_{AC})$ is less than or equal to the sum of the other two.

Uses

The von Neumann entropy is being extensively used in different forms (conditional entropies, relative entropies, etc.) in the framework of quantum information theory. Entanglement measures are based upon some quantity directly related to the von Neumann entropy. However, there have appeared in the literature several papers dealing with the possible inadequacy of the Shannon information measure, and consequently of the von Neumann entropy as an appropriate quantum generalization of Shannon entropy. The main argument is that in classical measurement the Shannon information measure is a natural measure of our ignorance about the properties of a system, whose existence is independent of measurement.

Conversely, quantum measurement cannot be claimed to reveal the properties of a system that existed before the measurement was made. This controversy has encouraged some authors to introduce the non-additivity property of Tsallis entropy (a generalization of the standard Boltzmann–Gibbs entropy) as the main reason for recovering a true quantal information measure in the quantum context, claiming that non-local correlations ought to be described because of the particularity of Tsallis entropy.

THE SYSTEM IN QUESTION IS:

1. Von Neumann Entropy And Quantum Entanglement
2. Velocity Field Of The Particle And Wave Function
3. Matter Presence In Abundance And Break Down Of Parity Conservation
4. Dissipation In Quantum Computation And Efficiency Of Quantum Algorithms
5. Decoherence And Computational Complexity
6. Coherent Superposition Of Outputs And Different Possible Inputs In The Form Of Qubits

VON NEUMANN ENTROPY AND QUANTUM ENTANGLEMENT: MODULE ONE

NOTATION :

G_{13} : Category One Of Quantum Entanglement
G_{14} : Category Two Of Quantum Entanglement
G_{15} : Category Three Of Quantum Entanglement
T_{13} : Category One Of Von Neumann Entropy
T_{14} : Category Two Of Von Neumann Entropy
T_{15} : Category Three Of Von Neumann Entropy

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WAVE FUNCTIONS AND VELOCITY FIELD OF THE PARTICLES: MODULE TWO

G_{16} : Category One Of Velocity Field Of The Particles
G_{17} : Category Two Of The Velocity Field Of The Particles
G_{18} : Category Three Of The Velocity Field Of The Particles
T_{16} : Category One Of Wave Functions Concomitant To The Velocity Fields
T_{17} :Category Two Of Wave Functions Corresponding To Category Two Of Velocity Field
T_{18} : Category Three Of Wave Functions-

BREAK DOWN OF PARITY CONSERVATION AND ABUNDANCE OF MATTER PRESCENCE: MODULE THREE:

G_{20} : Category One Of Systems Where There Is Break Down Of Parity Conservation
G_{21} : Category Two Of Systems Where There Is Break Down In Parity Conservation
G_{22} : Category Three Of Systems Where There Is Break Down Of Parity Conservation
T_{20} : Category Three Of Systems Where There Is Break Down Of Parity Conservation
T_{21} : Category One Of Systems Where There Is Abundance Of Matter
T_{22} : Category Two Of Systems Where There Is Abundance Of Matter
G_{24} : Category Three Of Systems Where There Is Abundance Of Matter

EFFICIENCY OF QUANTUM ALGORITHMS AND DISSIPATION IN QUANTUM COMPUTATION MODULE NUMBERED FOUR:

G_{25} **:** Category Two Of Efficiency Of Quantum Algorithms
G_{26} : Category Three Of efficiency Of Quantum Algorithms
G_{24} : Category One Of Efficiency Of Quantum Algorithms
T_{24} : Category Three Of Dissipation In Quantum Computation
T_{25} : Category One Of Systems With Efficiency In Quantum Algorithm
T_{26} : Category Two Of Systems With Quantum Algorithm Of Efficiency (Different From Category One)

COMPUTATIONAL COMPLEXITY AND DECOHERENCE MODULE NUMBERED FIVE

G_{28} **:** Category One Of Computational Complexity
G_{29} :Category Two Of Computational Complexity
G_{30} : Category Three Of Computational Complexity
T_{28} : Category One Of Decoherence
T_{29} : Category Two Of Decoherence
T_{30} : Category Three Of Decoherence

DIFFERENT POSSSIBLE INPUTS (QUBITS) AND QUANTUM SUPERPOSITION OF OUTPUTS MODULE NUMBERED SIX

G_{32} : Category One Of Different Possible Qubits Inputs
G_{33} : Category Two Of Different Possible Qubits Inputs
G_{34} : Category Three Of Different Possible Qubits Inputs
T_{32} : Category One Of Coherent Superposition Of Outputs
T_{33} : Category Two Of Coherent Superposition Of Outputs
T_{34} **:** Category Three Of Coherent Superposition Of Outputs

ACCENTUATION COEFFCIENTS OF THE HOLISTIC SYSTEM

Von Neumann Entropy And Quantum Entanglement
Velocity Field Of The Particle And Wave Function
Matter Presence In Abundance And Break Down Of Parity Conservation
Dissipation In Quantum Computation And Efficiency Of Quantum Algorithms
Decoherence And Computational Complexity
Coherent Superposition Of Outputs And Different Possible Inputs In The Form Of Qubits

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$
$(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$: $(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}$,
$(a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}, (a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$

DISSIPATION COEFFCIENTS:
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
, $(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$
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GOVERNING EQUATIONS:OF THE SYSTEM VONNEUMANN ENTROPY AND QUANTUM ENTANGLEMENT

The differential system of this model is now -

$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13}$ -
$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14}$ -
$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15}$ -
$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13}$ -
$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14}$ -
$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ -
$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor -
$-(b''_{13})^{(1)}(G,t) =$ First detritions factor -

GOVERNING EQUATIONS OF THE SYSTEM VELOCITY FIELD OF THE PARTICLE AND WAVE FUNCTION:

The differential system of this model is now -

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ -
$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ -
$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ -
$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ -
$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ -
$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ -
$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor -
$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor -

GOVERNING EQUATIONS:OF BREAK DOWN OF PARITY CONSERVATION AND MATTER ABUNDANCE:

The differential system of this model is now -

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ -
$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ -
$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ -
$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ -
$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ -
$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ -
$+(a''_{20})^{(3)}(T_{21},t) =$ *First augmentation factor* -
$-(b''_{20})^{(3)}(G_{23},t) =$ *First detritions factor* -

GOVERNING EQUATIONS:OF DISSIPATION IN QUANTUM COMPUTATION AND EFFICIENCY OF QUANTUM ALGORITHMS:

The differential system of this model is now -

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ -

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25},t)\right]G_{25}$ -

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25},t)\right]G_{26}$ -

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}((G_{27}),t)\right]T_{24}$ -

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}((G_{27}),t)\right]T_{25}$ -

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}((G_{27}),t)\right]T_{26}$ -

$+(a_{24}^{''})^{(4)}(T_{25},t) =$ ***First augmentation factor-***

$-(b_{24}^{''})^{(4)}((G_{27}),t) =$ ***First detritions factor -***

GOVERNING EQUATIONS:OF THE SYSTEM DECOHERENCE AND COMPUTATIONAL COMPLEXITY:

The differential system of this model is now -

$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29},t)\right]G_{28}$ -

$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29},t)\right]G_{29}$ -

$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29},t)\right]G_{30}$ -

$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}((G_{31}),t)\right]T_{28}$ -

$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}((G_{31}),t)\right]T_{29}$ -

$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}((G_{31}),t)\right]T_{30}$ -

$+(a_{28}^{''})^{(5)}(T_{29},t) =$ ***First augmentation factor -***

$-(b_{28}^{''})^{(5)}((G_{31}),t) =$ ***First detritions factor -***

GOVERNING EQUATIONS:COHERENT SUPERPOSITION OF OUTPUTS AND DIFFERENT POSSIBILITIES OF QUBIT INPUTS

The differential system of this model is now -

$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33},t)\right]G_{32}$ -

$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33},t)\right]G_{33}$ -

$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33},t)\right]G_{34}$ -

$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}((G_{35}),t)\right]T_{32}$ -

$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)}((G_{35}),t)\right]T_{33}$ -

$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}((G_{35}),t)\right]T_{34}$ -

$+(a_{32}^{''})^{(6)}(T_{33},t) =$ **First augmentation factor -**

$-(b_{32}^{''})^{(6)}((G_{35}),t) =$ **First detritions factor -**

CONCATENATED GOVERNING SYSTEMS OF THE HOLISTIC GLOBAL SYSTEM:

(1) **Von Neumann Entropy And Quantum Entanglement**
(2) **Velocity Field Of The Particle And Wave Function**
(3) **Matter Presence In Abundance And Break Down Of Parity Conservation**
(4) **Dissipation In Quantum Computation And Efficiency Of Quantum Algorithms**
(5) **Decoherence And Computational Complexity**

Coherent Superposition Of Outputs And Different Possible Inputs In The Form Of Qubits-

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{array}{c}(a_{13}^{'})^{(1)}\boxed{+(a_{13}^{''})^{(1)}(T_{14},t)}\boxed{+(a_{16}^{''})^{(2,2,)}(T_{17},t)}\boxed{+(a_{20}^{''})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a_{24}^{''})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a_{28}^{''})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a_{32}^{''})^{(6,6,6,6,)}(T_{33},t)}\end{array}\right]G_{13} -$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{array}{c}(a_{14}^{'})^{(1)}\boxed{+(a_{14}^{''})^{(1)}(T_{14},t)}\boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)}\boxed{+(a_{21}^{''})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a_{25}^{''})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a_{29}^{''})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a_{33}^{''})^{(6,6,6,6,)}(T_{33},t)}\end{array}\right]G_{14} -$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{array}{c}(a_{15}^{'})^{(1)}\boxed{+(a_{15}^{''})^{(1)}(T_{14},t)}\boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)}\boxed{+(a_{22}^{''})^{(3,3,)}(T_{21},t)}\\ \boxed{+(a_{26}^{''})^{(4,4,4,4,)}(T_{25},t)}\boxed{+(a_{30}^{''})^{(5,5,5,5,)}(T_{29},t)}\boxed{+(a_{34}^{''})^{(6,6,6,6,)}(T_{33},t)}\end{array}\right]G_{15} -$$

Where $\boxed{(a_{13}^{''})^{(1)}(T_{14},t)}$, $\boxed{(a_{14}^{''})^{(1)}(T_{14},t)}$, $\boxed{(a_{15}^{''})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{16}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{17}^{''})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a_{18}^{''})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3-

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b_{13}')^{(1)}\boxed{-(b_{13}'')^{(1)}(G,t)} \boxed{-(b_{16}'')^{(2,2,)}(G_{19},t)} \boxed{-(b_{20}'')^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)} \end{matrix}\right] T_{13} \text{ -}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{matrix} (b_{14}')^{(1)}\boxed{-(b_{14}'')^{(1)}(G,t)} \boxed{-(b_{17}'')^{(2,2,)}(G_{19},t)} \boxed{-(b_{21}'')^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)} \end{matrix}\right] T_{14} \text{ -}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b_{15}')^{(1)}\boxed{-(b_{15}'')^{(1)}(G,t)} \boxed{-(b_{18}'')^{(2,2,)}(G_{19},t)} \boxed{-(b_{22}'')^{(3,3,)}(G_{23},t)} \\ \boxed{-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)} \end{matrix}\right] T_{15} \text{ -}$$

Where $\boxed{-(b_{13}'')^{(1)}(G,t)}$, $\boxed{-(b_{14}'')^{(1)}(G,t)}$, $\boxed{-(b_{15}'')^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{16}'')^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{17}'')^{(2,2,)}(G_{19},t)}$, $\boxed{-(b_{18}'')^{(2,2,)}(G_{19},t)}$ are second detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{20}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{21}'')^{(3,3,)}(G_{23},t)}$, $\boxed{-(b_{22}'')^{(3,3,)}(G_{23},t)}$ are third detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{24}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{25}'')^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b_{26}'')^{(4,4,4,4,)}(G_{27},t)}$ are fourth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{28}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{29}'')^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b_{30}'')^{(5,5,5,5,)}(G_{31},t)}$ are fifth detrition coefficients for category 1, 2 and 3

$\boxed{-(b_{32}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{33}'')^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b_{34}'')^{(6,6,6,6,)}(G_{35},t)}$ are sixth detrition coefficients for category 1, 2 and 3

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[\begin{matrix} (a_{16}')^{(2)}\boxed{+(a_{16}'')^{(2)}(T_{17},t)} \boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)} \boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)} \end{matrix}\right] G_{16} \text{ -}$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[\begin{matrix} (a_{17}')^{(2)}\boxed{+(a_{17}'')^{(2)}(T_{17},t)} \boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)} \boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)} \end{matrix}\right] G_{17} \text{ -}$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[\begin{matrix} (a_{18}')^{(2)}\boxed{+(a_{18}'')^{(2)}(T_{17},t)} \boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)} \boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)} \end{matrix}\right] G_{18} \text{ -}$$

Where $\boxed{+(a_{16}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{17}'')^{(2)}(T_{17},t)}$, $\boxed{+(a_{18}'')^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a_{13}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{14}'')^{(1,1,)}(T_{14},t)}$, $\boxed{+(a_{15}'')^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{20}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{21}'')^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a_{22}'')^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{24}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{25}'')^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a_{26}'')^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{28}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{29}'')^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a_{30}'')^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a_{32}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{33}'')^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a_{34}'')^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3 -

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{matrix} (b_{16}')^{(2)}\boxed{-(b_{16}'')^{(2)}(G_{19},t)} \boxed{-(b_{13}'')^{(1,1,)}(G,t)} \boxed{-(b_{20}'')^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b_{24}'')^{(4,4,4,4,4)}(G_{27},t)} \boxed{-(b_{28}'')^{(5,5,5,5,5)}(G_{31},t)} \boxed{-(b_{32}'')^{(6,6,6,6,6)}(G_{35},t)} \end{matrix}\right] T_{16} \text{ -}$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{matrix} (b_{17}')^{(2)}\boxed{-(b_{17}'')^{(2)}(G_{19},t)} \boxed{-(b_{14}'')^{(1,1,)}(G,t)} \boxed{-(b_{21}'')^{(3,3,3,)}(G_{23},t)} \\ \boxed{-(b_{25}'')^{(4,4,4,4,4)}(G_{27},t)} \boxed{-(b_{29}'')^{(5,5,5,5,5)}(G_{31},t)} \boxed{-(b_{33}'')^{(6,6,6,6,6)}(G_{35},t)} \end{matrix}\right] T_{17} \text{ -}$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)} T_{17} - \left[\begin{matrix} (b'_{18})^{(2)} -(b''_{18})^{(2)}(G_{19},t) -(b''_{15})^{(1,1,)}(G,t) -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \end{matrix} \right] T_{18} \text{ -}$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detrition coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detrition coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detrition coefficients for category 1,2 and 3 -

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)} G_{21} - \left[\begin{matrix} (a'_{20})^{(3)} +(a''_{20})^{(3)}(T_{21},t) +(a''_{16})^{(2,2,2)}(T_{17},t) +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \end{matrix} \right] G_{20} \text{ -}$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)} G_{20} - \left[\begin{matrix} (a'_{21})^{(3)} +(a''_{21})^{(3)}(T_{21},t) +(a''_{17})^{(2,2,2)}(T_{17},t) +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) \end{matrix} \right] G_{21} \text{ -}$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)} G_{21} - \left[\begin{matrix} (a'_{22})^{(3)} +(a''_{22})^{(3)}(T_{21},t) +(a''_{18})^{(2,2,2)}(T_{17},t) +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) \end{matrix} \right] G_{22} \text{ -}$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3 -

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)} T_{21} - \left[\begin{matrix} (b'_{20})^{(3)} -(b''_{20})^{(3)}(G_{23},t) -(b''_{16})^{(2,2,2)}(G_{19},t) -(b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) -(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) -(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \end{matrix} \right] T_{20} \text{ -}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)} T_{20} - \left[\begin{matrix} (b'_{21})^{(3)} -(b''_{21})^{(3)}(G_{23},t) -(b''_{17})^{(2,2,2)}(G_{19},t) -(b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) -(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) -(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \end{matrix} \right] T_{21} \text{ -}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)} T_{21} - \left[\begin{matrix} (b'_{22})^{(3)} -(b''_{22})^{(3)}(G_{23},t) -(b''_{18})^{(2,2,2)}(G_{19},t) -(b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) -(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) -(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \end{matrix} \right] T_{22} \text{ -}$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detrition coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detrition coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detrition coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detrition coefficients for category 1, 2 and 3*

$\boxed{-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)}$ *are sixth detrition coefficients for category 1, 2 and 3* -

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$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \begin{bmatrix} (a'_{24})^{(4)} \boxed{+(a''_{24})^{(4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{24} \text{ -}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \begin{bmatrix} (a'_{25})^{(4)} \boxed{+(a''_{25})^{(4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{25} \text{ -}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \begin{bmatrix} (a'_{26})^{(4)} \boxed{+(a''_{26})^{(4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{26} \text{ -}$$

$Where$ $\boxed{(a''_{24})^{(4)}(T_{25},t)}$, $\boxed{(a''_{25})^{(4)}(T_{25},t)}$, $\boxed{(a''_{26})^{(4)}(T_{25},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,)}(T_{29},t)}$ $are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ $are\ third\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3 -

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)} \boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \end{bmatrix} T_{24} \text{ -}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)} \boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \end{bmatrix} T_{25} \text{ -}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)} \boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \end{bmatrix} T_{26} \text{ -}$$

$Where$ $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$

$are\ fourth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$

$are\ fifth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$

$are\ sixth\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$ -

$$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)} \boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{28} \text{ -}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)} \boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{29} \text{ -}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)} \boxed{+(a''_{30})^{(5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \end{bmatrix} G_{30} \text{ -}$$

$Where$ $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ *are second augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3 -

$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{matrix} (b'_{28})^{(5)} \boxed{-(b''_{28})^{(5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{28} -$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{matrix} (b'_{29})^{(5)} \boxed{-(b''_{29})^{(5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{29} -$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{matrix} (b'_{30})^{(5)} \boxed{-(b''_{30})^{(5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)} \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{30} -$$

where $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ *are second detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3-

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{matrix} (a'_{32})^{(6)} \boxed{+(a''_{32})^{(6)}(T_{33},t)} \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{32} -$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{matrix} (a'_{33})^{(6)} \boxed{+(a''_{33})^{(6)}(T_{33},t)} \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{33} -$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{matrix} (a'_{34})^{(6)} \boxed{+(a''_{34})^{(6)}(T_{33},t)} \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \end{matrix}\right] G_{34} -$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ *are first augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ *are second augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients -

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \end{matrix}\right] T_{32} -$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[\begin{array}{c} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \end{array} \right] T_{33} \text{ -}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[\begin{array}{c} (b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35},t)} \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \end{array} \right] T_{34} \text{ -}$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are third detrition coefficients for category* 1,2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3-

Where we suppose-

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$
$i,j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G,t) \leq \ (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$-

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14},t) = (p_i)^{(1)}$

$\lim_{G \to \infty} (b''_i)^{(1)} (G,t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants and $\boxed{i = 13,14,15}$-

They satisfy Lipschitz condition:

$|(a''_i)^{(1)}(T'_{14},t) - (a''_i)^{(1)}(T_{14},t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)} t}$

$|(b''_i)^{(1)}(G',t) - (b''_i)^{(1)}(G,t)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)} t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14},t)$ and $(a''_i)^{(1)}(T_{14},t)$. (T'_{14},t) and (T_{14},t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14},t)$, the first augmentation coefficient would be absolutely continuous. -

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$-

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ and $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a'_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a'_i)^{(1)} + \ (\hat{A}_{13})^{(1)} + \ (\hat{P}_{13})^{(1)} \ (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [\ (b_i)^{(1)} + (b'_i)^{(1)} + \ (\hat{B}_{13})^{(1)} + \ (\hat{Q}_{13})^{(1)} \ (\hat{k}_{13})^{(1)}] < 1 \text{ -}$$

Where we suppose-

$(a_i)^{(2)}, (a'_i)^{(2)}, (a''_i)^{(2)}, (b_i)^{(2)}, (b'_i)^{(2)}, (b''_i)^{(2)} > 0, \quad i,j = 16,17,18$-

The functions $(a''_i)^{(2)}, (b''_i)^{(2)}$ are positive continuous increasing and bounded.-

Definition of $(p_i)^{(2)}, \ (r_i)^{(2)}$:-

$\left(a_i''\right)^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq \left(\hat{A}_{16}\right)^{(2)}$ -

$\left(b_i''\right)^{(2)}(G_{19},t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)}$ -

$\lim_{T_2\to\infty}\left(a_i''\right)^{(2)}(T_{17},t) = (p_i)^{(2)}$-

$\lim_{G\to\infty}\left(b_i''\right)^{(2)}\left((G_{19}),t\right) = (r_i)^{(2)}$ -

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$:

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$-

They satisfy Lipschitz condition:-

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ -

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}((G_{19}),t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and $(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) and (T_{17},t) are points belonging to the interval $\left[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}\right]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous. -

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$:-

(F) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$-

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$:

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)} and\ (\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$ satisfy the inequalities -

$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ -

$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$ -

Where we suppose-

(G) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}$, $(\mathrm{r_i})^{(3)}$:

$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$

$(b_i'')^{(3)}(G_{23},t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$-

$lim_{T_2\to\infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$

$\lim_{G\to\infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$:

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$-

They satisfy Lipschitz condition:

$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$

$|(b_i'')^{(3)}(G_{23}{}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}{}'||e^{-(\hat{M}_{20})^{(3)}t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and $(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $\left[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}\right]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the THIRD first augmentation coefficient would be absolutely continuous. -

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$:

(H) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$-

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)} and\ (\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$

$\frac{1}{(\hat{M}_{20})^{(3)}}[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$ -

Where we suppose-

(I) $(a_i)^{(4)}, (a_i^{'})^{(4)}, (a_i^{''})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (b_i^{''})^{(4)} > 0, \quad i,j = 24,25,26$

(J) The functions $(a_i^{''})^{(4)}, (b_i^{''})^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, \ (r_i)^{(4)}$:

$(a_i^{''})^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$

$(b_i^{''})^{(4)}((G_{27}), t) \leq \ (r_i)^{(4)} \leq (b_i^{'})^{(4)} \leq (\hat{B}_{24})^{(4)}$-

(K) $lim_{T_2 \to \infty} (a_i^{''})^{(4)} (T_{25}, t) = (p_i)^{(4)}$

$\lim_{G \to \infty} (b_i^{''})^{(4)} ((G_{27}), t) = \ (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$-

They satisfy Lipschitz condition:

$|(a_i^{''})^{(4)}(T_{25}^{'}, t) - (a_i^{''})^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)}|T_{25} - T_{25}^{'}|e^{-(\hat{M}_{24})^{(4)}t}$

$|(b_i^{''})^{(4)}((G_{27})^{'}, t) - (b_i^{''})^{(4)}((G_{27}), t)| < (\hat{k}_{24})^{(4)}||(G_{27}) - (G_{27})^{'}||e^{-(\hat{M}_{24})^{(4)}t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(4)}(T_{25}^{'}, t)$ and $(a_i^{''})^{(4)}(T_{25}, t)$. $(T_{25}^{'}, t)$ And (T_{25}, t) are points belonging to the interval $[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}]$. It is to be noted that $(a_i^{''})^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i^{''})^{(4)}(T_{25}, t)$, the FOURTH **augmentation coefficient** would be absolutely continuous. -

Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$:

(L) $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}} \ , \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ -

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$:

(M) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and \ (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i^{'})^{(4)}, (b_i)^{(4)}, (b_i^{'})^{(4)}, (p_i)^{(4)}, \ (r_i)^{(4)}, i = 24,25,26,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{24})^{(4)}}[(a_i)^{(4)} + (a_i^{'})^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$

$\frac{1}{(\hat{M}_{24})^{(4)}}[(b_i)^{(4)} + (b_i^{'})^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}] < 1$ -

Where we suppose-

(N) $(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, \quad i,j = 28,29,30$

(O) The functions $(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$(a_i^{''})^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$

$(b_i^{''})^{(5)}((G_{31}), t) \leq \ (r_i)^{(5)} \leq (b_i^{'})^{(5)} \leq (\hat{B}_{28})^{(5)}$-

(P) $lim_{T_2 \to \infty} (a_i^{''})^{(5)} (T_{29}, t) = (p_i)^{(5)}$

$\lim_{G \to \infty} (b_i^{''})^{(5)} (G_{31}, t) = \ (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$-

They satisfy Lipschitz condition:

$|(a_i^{''})^{(5)}(T_{29}^{'}, t) - (a_i^{''})^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}^{'}|e^{-(\hat{M}_{28})^{(5)}t}$

$|(b_i^{''})^{(5)}((G_{31})^{'}, t) - (b_i^{''})^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\hat{M}_{28})^{(5)}t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'}, t)$ and $(a_i^{''})^{(5)}(T_{29}, t)$. $(T_{29}^{'}, t)$ and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i^{''})^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i^{''})^{(5)}(T_{29}, t)$, the FIFTH **augmentation coefficient** would be absolutely continuous. -

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$:

(Q) $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} \ , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$-

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$:

(R) There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and \ (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i^{'})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$\frac{1}{(\hat{M}_{28})^{(5)}}[(a_i)^{(5)}+(a_i^{'})^{(5)}+(\hat{A}_{28})^{(5)}+(\hat{P}_{28})^{(5)}(\hat{k}_{28})^{(5)}]<1$

$\frac{1}{(\hat{M}_{28})^{(5)}}[(b_i)^{(5)}+(b_i^{'})^{(5)}+(\hat{B}_{28})^{(5)}+(\hat{Q}_{28})^{(5)}(\hat{k}_{28})^{(5)}]<1$ -

Where we suppose-

$(a_i)^{(6)},(a_i^{'})^{(6)},(a_i^{''})^{(6)},(b_i)^{(6)},(b_i^{'})^{(6)},(b_i^{''})^{(6)}>0, \quad i,j=32,33,34$

(S) The functions $(a_i^{''})^{(6)},(b_i^{''})^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, (r_i)^{(6)}$:

$(a_i^{''})^{(6)}(T_{33},t)\le(p_i)^{(6)}\le(\hat{A}_{32})^{(6)}$

$(b_i^{''})^{(6)}((G_{35}),t)\le (r_i)^{(6)}\le(b_i^{'})^{(6)}\le(\hat{B}_{32})^{(6)}$-

(T) $lim_{T_2\to\infty}(a_i^{''})^{(6)}(T_{33},t)=(p_i)^{(6)}$

$\lim_{G\to\infty}(b_i^{''})^{(6)}((G_{35}),t)=(r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)}$:

Where $(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)},(p_i)^{(6)}, (r_i)^{(6)}$ are positive constants and $i=32,33,34$-

They satisfy Lipschitz condition:

$|(a_i^{''})^{(6)}(T_{33}^{'},t)-(a_i^{''})^{(6)}(T_{33},t)|\le(\hat{k}_{32})^{(6)}|T_{33}-T_{33}^{'}|e^{-(\hat{M}_{32})^{(6)}t}$

$|(b_i^{''})^{(6)}((G_{35})^{'},t)-(b_i^{''})^{(6)}((G_{35}),t)|<(\hat{k}_{32})^{(6)}||(G_{35})-(G_{35})^{'}||e^{-(\hat{M}_{32})^{(6)}t}$ -

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'},t)$ and $(a_i^{''})^{(6)}(T_{33},t)$. $(T_{33}^{'},t)$ and (T_{33},t) are points belonging to the interval $[(\hat{k}_{32})^{(6)},(\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)}=6$ then the function $(a_i^{''})^{(6)}(T_{33},t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. -

Definition of $(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$:

$(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$, are positive constants

$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$-

Definition of $(\hat{P}_{32})^{(6)},(\hat{Q}_{32})^{(6)}$:

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)},(\hat{A}_{32})^{(6)} and (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)},(a_i^{'})^{(6)},(b_i)^{(6)},(b_i^{'})^{(6)},(p_i)^{(6)}, (r_i)^{(6)},i=32,33,34,$

satisfy the inequalities

$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)}+(a_i^{'})^{(6)}+(\hat{A}_{32})^{(6)}+(\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}]<1$

$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)}+(b_i^{'})^{(6)}+(\hat{B}_{32})^{(6)}+(\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}]<1$ -

Theorem 1: if the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0),T_i(0)$:

$G_i(t)\le(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$, $G_i(0)=G_i^0>0$

$T_i(t)\le(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$, $T_i(0)=T_i^0>0$-

if the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0),T_i(0)$

$G_i(t)\le(\hat{P}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}$, $G_i(0)=G_i^0>0$

$T_i(t)\le(\hat{Q}_{16})^{(2)}e^{(\hat{M}_{16})^{(2)}t}$, $T_i(0)=T_i^0>0$-

if the conditions above are fulfilled, there exists a solution satisfying the conditions

$G_i(t)\le(\hat{P}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}$, $G_i(0)=G_i^0>0$

$T_i(t)\le(\hat{Q}_{20})^{(3)}e^{(\hat{M}_{20})^{(3)}t}$, $T_i(0)=T_i^0>0$-

if the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0),T_i(0)$:

$G_i(t)\le(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}t}$, $G_i(0)=G_i^0>0$

$T_i(t)\le(\hat{Q}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}t}$, $T_i(0)=T_i^0>0$ -

if the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0),T_i(0)$:

$G_i(t)\le(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$, $G_i(0)=G_i^0>0$

$T_i(t)\le(\hat{Q}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$, $T_i(0)=T_i^0>0$-

if the conditions above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0),T_i(0)$:

$G_i(t)\le(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t}$, $G_i(0)=G_i^0>0$

$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$-

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$,$T_i^0 \leq (\hat{Q}_{13})^{(1)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ -

By

$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$ -

$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$ -

$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$ -

$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$ -

$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$ -

$\bar{\mathrm{T}}_{15}(\mathrm{t}) = \mathrm{T}_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$-

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$,$T_i^0 \leq (\hat{Q}_{16})^{(2)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ -

By

$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}(s_{(16)}) - \left((a_{16}')^{(2)} + a_{16}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{16}(s_{(16)})\right] ds_{(16)}$ -

$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}(s_{(16)}) - \left((a_{17}')^{(2)} + (a_{17}'')^{(2)}(T_{17}(s_{(16)}), s_{(17)})\right) G_{17}(s_{(16)})\right] ds_{(16)}$ -

$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}(s_{(16)}) - \left((a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}(s_{(16)}), s_{(16)})\right) G_{18}(s_{(16)})\right] ds_{(16)}$ -

$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}(s_{(16)}) - \left((b_{16}')^{(2)} - (b_{16}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{16}(s_{(16)})\right] ds_{(16)}$ -

$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}(s_{(16)}) - \left((b_{17}')^{(2)} - (b_{17}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{17}(s_{(16)})\right] ds_{(16)}$ -

$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}(s_{(16)}) - \left((b_{18}')^{(2)} - (b_{18}'')^{(2)}(G(s_{(16)}), s_{(16)})\right) T_{18}(s_{(16)})\right] ds_{(16)}$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$-

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$,$T_i^0 \leq (\hat{Q}_{20})^{(3)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ -

By

$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a_{20}')^{(3)} + a_{20}'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{20}(s_{(20)})\right] ds_{(20)}$ -

$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}')^{(3)} + (a_{21}'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{21}(s_{(20)})\right] ds_{(20)}$ -

$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}(s_{(20)}), s_{(20)})\right) G_{22}(s_{(20)})\right] ds_{(20)}$ -

$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}')^{(3)} - (b_{20}'')^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{20}(s_{(20)})\right] ds_{(20)}$ -

$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}')^{(3)} - (b_{21}'')^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{21}(s_{(20)})\right] ds_{(20)}$ -

$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}')^{(3)} - (b_{22}'')^{(3)}(G(s_{(20)}), s_{(20)})\right) T_{22}(s_{(20)})\right] ds_{(20)}$

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$-

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$,$T_i^0 \leq (\hat{Q}_{24})^{(4)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ -

By

$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}')^{(4)} + a_{24}'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{24}(s_{(24)})\right] ds_{(24)}$ -

$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}')^{(4)} + (a_{25}'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{25}(s_{(24)})\right] ds_{(24)}$ -

$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}(s_{(24)}), s_{(24)})\right) G_{26}(s_{(24)})\right] ds_{(24)}$ -

$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}')^{(4)} - (b_{24}'')^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$ -

$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}')^{(4)} - (b_{25}'')^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$ -

$\bar{T}_{26}(t) = T_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}')^{(4)} - (b_{26}'')^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$-

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ -

By

$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a_{28}')^{(5)} + a_{28}''^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$ -

$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a_{29}')^{(5)} + (a_{29}'')^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{29}(s_{(28)}) \right] ds_{(28)}$ -

$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a_{30}')^{(5)} + (a_{30}'')^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{30}(s_{(28)}) \right] ds_{(28)}$ -

$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b_{28}')^{(5)} - (b_{28}'')^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{28}(s_{(28)}) \right] ds_{(28)}$ -

$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b_{29}')^{(5)} - (b_{29}'')^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{29}(s_{(28)}) \right] ds_{(28)}$ -

$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b_{30}')^{(5)} - (b_{30}'')^{(5)} (G(s_{(28)}), s_{(28)}) \right) T_{30}(s_{(28)}) \right] ds_{(28)}$

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$-

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy -

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$, -

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ -

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ -

By

$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a_{32}')^{(6)} + a_{32}''^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$ -

$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a_{33}')^{(6)} + (a_{33}'')^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$ -

$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a_{34}')^{(6)} + (a_{34}'')^{(6)} (T_{33}(s_{(32)}), s_{(32)}) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$ -

$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b_{32}')^{(6)} - (b_{32}'')^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$ -

$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b_{33}')^{(6)} - (b_{33}'')^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$ -

$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b_{34}')^{(6)} - (b_{34}'')^{(6)} (G(s_{(32)}), s_{(32)}) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$-

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{13}(t) \leq G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} \left(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \right) \right] ds_{(13)} =$$

$$\left(1 + (a_{13})^{(1)} t\right) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left(e^{(\hat{M}_{13})^{(1)} t} - 1 \right) -$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \leq \frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\left((\hat{P}_{13})^{(1)} + G_{14}^0 \right) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0} \right)} + (\hat{P}_{13})^{(1)} \right]$$

(G_i^0) is as defined in the statement of theorem 1-

Analogous inequalities hold also for G_{14} , $G_{15}, T_{13}, T_{14}, T_{15}$-

The operator $\mathcal{A}^{(2)}$ maps the space of functions satisfying GLOBAL EQATIONS into itself .Indeed it is obvious that-

$$G_{16}(t) \leq G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} \left(G_{17}^0 + (\hat{P}_{16})^{(6)} e^{(\hat{M}_{16})^{(2)} s_{(16)}} \right) \right] ds_{(16)} = \left(1 + (a_{16})^{(2)} t\right) G_{17}^0 + \frac{(a_{16})^{(2)} (\hat{P}_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left(e^{(\hat{M}_{16})^{(2)} t} - 1 \right)$$

-

From which it follows that

$$(G_{16}(t) - G_{16}^0) e^{-(\hat{M}_{16})^{(2)} t} \leq \frac{(a_{16})^{(2)}}{(\hat{M}_{16})^{(2)}} \left[\left((\hat{P}_{16})^{(2)} + G_{17}^0 \right) e^{\left(-\frac{(\hat{P}_{16})^{(2)} + G_{17}^0}{G_{17}^0} \right)} + (\hat{P}_{16})^{(2)} \right] -$$

Analogous inequalities hold also for G_{17} , $G_{18}, T_{16}, T_{17}, T_{18}$-

(a) The operator $\mathcal{A}^{(3)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{20}(t) \leq G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} \left(G_{21}^0 + (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} s_{(20)}} \right) \right] ds_{(20)} =$$

$\left(1+(a_{20})^{(3)}t\right)G_{21}^{0}+\frac{(a_{20})^{(3)}(\hat{P}_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left(e^{(\hat{M}_{20})^{(3)}t}-1\right)$ -

From which it follows that

$$(G_{20}(t)-G_{20}^{0})e^{-(\hat{M}_{20})^{(3)}t}\leq\frac{(a_{20})^{(3)}}{(\hat{M}_{20})^{(3)}}\left[\left((\hat{P}_{20})^{(3)}+G_{21}^{0}\right)e^{\left(-\frac{(\hat{P}_{20})^{(3)}+G_{21}^{0}}{G_{21}^{0}}\right)}+(\hat{P}_{20})^{(3)}\right]$$ -

Analogous inequalities hold also for $G_{21}, G_{22}, T_{20}, T_{21}, T_{22}$ -

(b) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{24}(t)\leq G_{24}^{0}+\int_{0}^{t}\left[(a_{24})^{(4)}\left(G_{25}^{0}+(\hat{P}_{24})^{(4)}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\right)\right]ds_{(24)}=$$

$$\left(1+(a_{24})^{(4)}t\right)G_{25}^{0}+\frac{(a_{24})^{(4)}(\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left(e^{(\hat{M}_{24})^{(4)}t}-1\right)$$ -

From which it follows that

$$(G_{24}(t)-G_{24}^{0})e^{-(\hat{M}_{24})^{(4)}t}\leq\frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{P}_{24})^{(4)}+G_{25}^{0}\right)e^{\left(-\frac{(\hat{P}_{24})^{(4)}+G_{25}^{0}}{G_{25}^{0}}\right)}+(\hat{P}_{24})^{(4)}\right]$$

(G_i^0) is as defined in the statement of theorem -

(c) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{28}(t)\leq G_{28}^{0}+\int_{0}^{t}\left[(a_{28})^{(5)}\left(G_{29}^{0}+(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}s_{(28)}}\right)\right]ds_{(28)}=$$

$$\left(1+(a_{28})^{(5)}t\right)G_{29}^{0}+\frac{(a_{28})^{(5)}(\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left(e^{(\hat{M}_{28})^{(5)}t}-1\right)$$ -

From which it follows that

$$(G_{28}(t)-G_{28}^{0})e^{-(\hat{M}_{28})^{(5)}t}\leq\frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}}\left[\left((\hat{P}_{28})^{(5)}+G_{29}^{0}\right)e^{\left(-\frac{(\hat{P}_{28})^{(5)}+G_{29}^{0}}{G_{29}^{0}}\right)}+(\hat{P}_{28})^{(5)}\right]$$

(G_i^0) is as defined in the statement of theorem -

(d) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that

$$G_{32}(t)\leq G_{32}^{0}+\int_{0}^{t}\left[(a_{32})^{(6)}\left(G_{33}^{0}+(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\right)\right]ds_{(32)}=$$

$$\left(1+(a_{32})^{(6)}t\right)G_{33}^{0}+\frac{(a_{32})^{(6)}(\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left(e^{(\hat{M}_{32})^{(6)}t}-1\right)$$ -

From which it follows that

$$(G_{32}(t)-G_{32}^{0})e^{-(\hat{M}_{32})^{(6)}t}\leq\frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\hat{P}_{32})^{(6)}+G_{33}^{0}\right)e^{\left(-\frac{(\hat{P}_{32})^{(6)}+G_{33}^{0}}{G_{33}^{0}}\right)}+(\hat{P}_{32})^{(6)}\right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{25}, G_{26}, T_{24}, T_{25}, T_{26}$ -

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}<1$ and to choose

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have-

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right]\leq(\hat{P}_{13})^{(1)}$$ -

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right]\leq(\hat{Q}_{13})^{(1)}$$ -

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself-

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$ -

Indeed if we denote

<u>Definition of</u> $\tilde{G},\tilde{T}$: $(\tilde{G},\tilde{T})=\mathcal{A}^{(1)}(G,T)$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}ds_{(13)}+$$

$$\int_0^t\{(a_{13}')^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$$(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}+$$

$G_{13}^{(2)}|(a_{13}^{''})^{(1)}(T_{14}^{(1)},s_{(13)}) - (a_{13}^{''})^{(1)}(T_{14}^{(2)},s_{(13)})| \ e^{-(\widehat{M}_{13})^{(1)}s_{(13)}}e^{(\widehat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows-

$$\left|G^{(1)} - G^{(2)}\right|e^{-(\widehat{M}_{13})^{(1)}t} \leq \frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a_{13}^{'})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\ G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows-

Remark 1: The fact that we supposed $(a_{13}^{''})^{(1)}$ and $(b_{13}^{''})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)}e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(1)}$ and $(b_i^{''})^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.-

Remark 2: There does not exist any t where $G_i\,(t) = 0\ and\ T_i\,(t) = 0$

From 19 to 24 it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i^{'})^{(1)}-(a_i^{''})^{(1)}(T_{14}(s_{(13)}),s_{(13)})\}ds_{(13)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{(-(b_i^{'})^{(1)}t)} > 0$ for $\mathrm{t} > 0$-

Definition of $\left((\widehat{M}_{13})^{(1)}\right)_1, \left((\widehat{M}_{13})^{(1)}\right)_2\ and\ \left((\widehat{M}_{13})^{(1)}\right)_3$:

Remark 3: if G_{13} is bounded, the same property have also $G_{14}\ and\ G_{15}$. indeed if

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a_{14}^{'})^{(1)}G_{14}$ and by integrating

$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1/(a_{14}^{'})^{(1)}$

In the same way , one can obtain

$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2/(a_{15}^{'})^{(1)}$

If $G_{14}\ or\ G_{15}$ is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.-

Remark 4: If G_{13} *is* bounded, from below, the same property holds for $G_{14}\ and\ G_{15}$. The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below.-

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(1)}\,(G(t),t)) = (b_{14}^{'})^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$(b_{14})^{(1)} - (b_i^{''})^{(1)}(G(t),t) < \varepsilon_1, T_{13}\,(t) > (m)^{(1)}$ -

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}^{''})^{(1)}\,(G(t),t) = (b_{15}^{'})^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions -

It is now sufficient to take $\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}, \frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}} < 1$ and to choose

$(\widehat{P}_{16})^{(2)}\ and\ (\widehat{Q}_{16})^{(2)}$ large to have-

$$\frac{(a_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[(\widehat{P}_{16})^{(2)} + ((\widehat{P}_{16})^{(2)} + G_j^0)e^{-\left(\frac{(\widehat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{16})^{(2)}\ -$$

$$\frac{(b_i)^{(2)}}{(\widehat{M}_{16})^{(2)}}\left[((\widehat{Q}_{16})^{(2)} + T_j^0)e^{-\left(\frac{(\widehat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{16})^{(2)}\right] \leq (\widehat{Q}_{16})^{(2)}\ -$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions $G_i\,, T_i$ satisfying GLOBAL EQUATIONS into itself-

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric

$$d\left(\left((G_{19})^{(1)},(T_{19})^{(1)}\right),\left((G_{19})^{(2)},(T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{16})^{(2)}t}\}\ -$$

Indeed if we denote

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$-

It results

$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)}\left|G_{17}^{(1)} - G_{17}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\,ds_{(16)} +$

$\int_0^t\{(a_{16}^{'})^{(2)}\left|G_{16}^{(1)} - G_{16}^{(2)}\right|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{-(\widehat{M}_{16})^{(2)}s_{(16)}} +$

$(a''_{16})^{(2)}\big(T_{17}^{(1)},s_{(16)}\big)\big|G_{16}^{(1)}-G_{16}^{(2)}\big|e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}+$
$G_{16}^{(2)}|(a''_{16})^{(2)}\big(T_{17}^{(1)},s_{(16)}\big)-(a''_{16})^{(2)}\big(T_{17}^{(2)},s_{(16)}\big)|\ e^{-(\widehat{M}_{16})^{(2)}s_{(16)}}e^{(\widehat{M}_{16})^{(2)}s_{(16)}}\}ds_{(16)}$ -
Where $s_{(16)}$ represents integrand that is integrated over the interval $[0,t]$
From the hypotheses it follows-
$\big|(G_{19})^{(1)}-(G_{19})^{(2)}\big|e^{-(\widehat{M}_{16})^{(2)}t}\leq$
$\frac{1}{(\widehat{M}_{16})^{(2)}}\big((a_{16})^{(2)}+(a'_{16})^{(2)}+(\widehat{A}_{16})^{(2)}+(\widehat{P}_{16})^{(2)}(\widehat{k}_{16})^{(2)}\big)d\Big(\big((G_{19})^{(1)},(T_{19})^{(1)};\ (G_{19})^{(2)},(T_{19})^{(2)}\big)\Big)$ -
And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows-
Remark 1: The fact that we supposed $(a''_{16})^{(2)}$ and $(b''_{16})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ and $(\widehat{Q}_{16})^{(2)}e^{(\widehat{M}_{16})^{(2)}t}$ respectively of $\mathbb{R}_+$.
If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(2)}$ and $(b''_i)^{(2)}, i=16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.-
Remark 2: There does not exist any t where $G_i(t)=0$ and $T_i(t)=0$
From CONCATENATED SYTEM OF GLOBAL EQUATIONS it results
$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(2)}-(a''_i)^{(2)}(T_{17}(s_{(16)}),s_{(16)})\}ds_{(16)}\right]}\geq 0$
$T_i(t)\geq T_i^0 e^{(-(b'_i)^{(2)}t)}>0$ for $t>0$-
Definition of $\big((\widehat{M}_{16})^{(2)}\big)_1,\big((\widehat{M}_{16})^{(2)}\big)_2$ and $\big((\widehat{M}_{16})^{(2)}\big)_3$:
Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if
$G_{16}<(\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt}\leq\big((\widehat{M}_{16})^{(2)}\big)_1-(a'_{17})^{(2)}G_{17}$ and by integrating
$G_{17}\leq\big((\widehat{M}_{16})^{(2)}\big)_2=G_{17}^0+2(a_{17})^{(2)}\big((\widehat{M}_{16})^{(2)}\big)_1/(a'_{17})^{(2)}$
In the same way , one can obtain
$G_{18}\leq\big((\widehat{M}_{16})^{(2)}\big)_3=G_{18}^0+2(a_{18})^{(2)}\big((\widehat{M}_{16})^{(2)}\big)_2/(a'_{18})^{(2)}$
If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.-
Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below.-
Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(2)}((G_{19})(t),t))=(b'_{17})^{(2)}$ then $T_{17}\to\infty$.
Definition of $(m)^{(2)}$ and ε_2 :
Indeed let t_2 be so that for $t>t_2$
$(b_{17})^{(2)}-(b''_i)^{(2)}((G_{19})(t),t)<\varepsilon_2, T_{16}(t)>(m)^{(2)}$ -
Then $\frac{dT_{17}}{dt}\geq(a_{17})^{(2)}(m)^{(2)}-\varepsilon_2 T_{17}$ which leads to
$T_{17}\geq\left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1-e^{-\varepsilon_2 t})+T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t}=\frac{1}{2}$ it results -
$T_{17}\geq\left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right),\quad t=\log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b''_{18})^{(2)}\big((G_{19})(t),t\big)=(b'_{18})^{(2)}$
We now state a more precise theorem about the behaviors at infinity of the solutions -
It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}<1$ and to choose
$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have-
$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)}+\big((\widehat{P}_{20})^{(3)}+G_j^0\big)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{20})^{(3)}\ -$$
$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\big((\widehat{Q}_{20})^{(3)}+T_j^0\big)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{20})^{(3)}\right]\leq(\widehat{Q}_{20})^{(3)}\ -$$
In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself-
The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric
$d\Big(\big((G_{23})^{(1)},(T_{23})^{(1)}\big),\big((G_{23})^{(2)},(T_{23})^{(2)}\big)\Big)=$
$\sup_i\{\max_{t\in\mathbb{R}_+}\big|G_i^{(1)}(t)-G_i^{(2)}(t)\big|e^{-(\widehat{M}_{20})^{(3)}t},\max_{t\in\mathbb{R}_+}\big|T_i^{(1)}(t)-T_i^{(2)}(t)\big|e^{-(\widehat{M}_{20})^{(3)}t}\}$ -
Indeed if we denote
Definition of $\widetilde{G_{23}},\widetilde{T_{23}}$: $\big(\widetilde{(G_{23})},\widetilde{(T_{23})}\big)=\mathcal{A}^{(3)}\big((G_{23}),(T_{23})\big)$-
It results

$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_{i}^{(2)}\right| \leq \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$
$\int_0^t \{(a_{20}^{'})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$
$(a_{20}^{''})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$
$G_{20}^{(2)} \left|(a_{20}^{''})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a_{20}^{''})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right|\ e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\} ds_{(20)}$
Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$
From the hypotheses it follows-
$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \leq$
$\frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a_{20}^{'})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)}\right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)}\right)\right)$
And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows-

Remark 1: The fact that we supposed $(a_{20}^{''})^{(3)}$ and $(b_{20}^{''})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$.
If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(3)}$ and $(b_i^{''})^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on* t) and hypothesis can replaced by a usual Lipschitz condition.-

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$
From 19 to 24 it results
$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(3)} - (a_i^{''})^{(3)}(T_{21}(s_{(20)}), s_{(20)})\} ds_{(20)}\right]} \geq 0$
$T_i(t) \geq T_i^0 e^{(-(b_i^{'})^{(3)} t)} > 0$ for t > 0-

Definition of $\left((\widehat{M}_{20})^{(3)}\right)_1, \left((\widehat{M}_{20})^{(3)}\right)_2$ *and* $\left((\widehat{M}_{20})^{(3)}\right)_3$:

Remark 3: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if
$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \leq \left((\widehat{M}_{20})^{(3)}\right)_1 - (a_{21}^{'})^{(3)} G_{21}$ and by integrating
$G_{21} \leq \left((\widehat{M}_{20})^{(3)}\right)_2 = G_{21}^0 + 2(a_{21})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_1 / (a_{21}^{'})^{(3)}$
In the same way , one can obtain
$G_{22} \leq \left((\widehat{M}_{20})^{(3)}\right)_3 = G_{22}^0 + 2(a_{22})^{(3)}\left((\widehat{M}_{20})^{(3)}\right)_2 / (a_{22}^{'})^{(3)}$
If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.-

Remark 4: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below.-

Remark 5: If T_{20} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(3)}\left((G_{23})(t), t\right)) = (b_{21}^{'})^{(3)}$ then $T_{21} \to \infty$.

Definition of $(m)^{(3)}$ and ε_3 :
Indeed let t_3 be so that for $t > t_3$
$(b_{21})^{(3)} - (b_i^{''})^{(3)}\left((G_{23})(t), t\right) < \varepsilon_3, T_{20}(t) > (m)^{(3)}$ -
Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to
$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results
$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}^{''})^{(3)}\left((G_{23})(t), t\right) = (b_{22}^{'})^{(3)}$
We now state a more precise theorem about the behaviors at infinity of the solutions-
It is now sufficient to take $\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}} < 1$ and to choose
$(\widehat{\mathrm{P}}_{24})^{(4)}$ and $(\widehat{\mathrm{Q}}_{24})^{(4)}$ large to have-

$$\frac{(a_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[(\widehat{P}_{24})^{(4)} + \left((\widehat{P}_{24})^{(4)} + G_j^0\right) e^{-\left(\frac{(\widehat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{24})^{(4)} \text{ -}$$

$$\frac{(b_i)^{(4)}}{(\widehat{M}_{24})^{(4)}}\left[\left((\widehat{Q}_{24})^{(4)} + T_j^0\right) e^{-\left(\frac{(\widehat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{24})^{(4)}\right] \leq (\widehat{Q}_{24})^{(4)} \text{ -}$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself-
The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric
$d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right), \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) =$
$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\widehat{M}_{24})^{(4)} t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\widehat{M}_{24})^{(4)} t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{27})}, \widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})} \right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)} \left|G_{25}^{(1)} - G_{25}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} \, ds_{(24)} +$
$\int_0^t \{(a_{24}^{'})^{(4)} \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} +$
$(a_{24}^{''})^{(4)}(T_{25}^{(1)}, s_{(24)}) \left|G_{24}^{(1)} - G_{24}^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}} +$
$G_{24}^{(2)} |(a_{24}^{''})^{(4)}(T_{25}^{(1)}, s_{(24)}) - (a_{24}^{''})^{(4)}(T_{25}^{(2)}, s_{(24)})| \; e^{-(\widehat{M}_{24})^{(4)} s_{(24)}} e^{(\widehat{M}_{24})^{(4)} s_{(24)}}\} ds_{(24)}$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0, t]$

From the hypotheses it follows-

$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)} t} \leq$
$\frac{1}{(\widehat{M}_{24})^{(4)}} \left((a_{24})^{(4)} + (a_{24}^{'})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)} (\widehat{k}_{24})^{(4)}\right) d \left(\left((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)}\right) \right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows-

Remark 1: The fact that we supposed $(a_{24}^{''})^{(4)}$ and $(b_{24}^{''})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ *and* $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)} t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.-

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$

From THE CONCATENATED SYTEM OF GLOBAL EQUATIONS it results

$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(4)} - (a_i^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i(t) \geq T_i^0 e^{(-(b_i^{'})^{(4)} t)} > 0$ for t > 0-

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ *and* $\left((\widehat{M}_{24})^{(4)}\right)_3$:

Remark 3: if G_{24} is bounded, the same property have also G_{25} *and* G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}^{'})^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}^{'})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)} \left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}^{'})^{(4)}$

If G_{25} *or* G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.-

Remark 4: If G_{24} *is* bounded, from below, the same property holds for G_{25} *and* G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below.-

Remark 5: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(4)}((G_{27})(t), t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$.

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}(t) > (m)^{(4)}$:

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $\lim_{t\to\infty}(b_{26}^{''})^{(4)}\left((G_{27})(t), t\right) = (b_{26}^{'})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for $G_{29}, G_{30}, T_{28}, T_{29}, T_{30}$:

It is now sufficient to take $\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}, \frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}} < 1$ and to choose

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have-

$$\frac{(a_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)} + ((\widehat{P}_{28})^{(5)} + G_j^0) e^{-\left(\frac{(\widehat{P}_{28})^{(5)} + G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{28})^{(5)} \text{ -}$$

$$\frac{(b_i)^{(5)}}{(\widehat{M}_{28})^{(5)}}\left[((\widehat{Q}_{28})^{(5)} + T_j^0) e^{-\left(\frac{(\widehat{Q}_{28})^{(5)} + T_j^0}{T_j^0}\right)} + (\widehat{Q}_{28})^{(5)}\right] \leq (\widehat{Q}_{28})^{(5)} \text{ -}$$

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS into itself-

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric

$d\left(\left((G_{31})^{(1)}, (T_{31})^{(1)}\right), \left((G_{31})^{(2)}, (T_{31})^{(2)}\right)\right) =$

$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$

Indeed if we denote

Definition of $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$

$\int_0^t\{(a_{28}')^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$

$(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$

$G_{28}^{(2)}\left|(a_{28}'')^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a_{28}'')^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)\right|\ e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows-

$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq$

$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a_{28}')^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(\left((G_{31})^{(1)},(T_{31})^{(1)};\ (G_{31})^{(2)},(T_{31})^{(2)}\right)\right)$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (35,35,36) the result follows-

Remark 1: The fact that we supposed $(a_{28}'')^{(5)}$ and $(b_{28}'')^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ *and* $(\widehat{Q}_{28})^{(5)}e^{(\widehat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(5)}$ and $(b_i'')^{(5)}, i=28,29,30$ depend only on T_{29} and respectively on $(G_{31})(\textit{and not on } t)$ and hypothesis can replaced by a usual Lipschitz condition.-

Remark 2: There does not exist any t where $G_i\,(t)=0$ *and* $T_i\,(t)=0$

From 19 to 28 it results

$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(5)}-(a_i'')^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]}\geq 0$

$T_i\,(t)\geq T_i^0e^{(-(b_i')^{(5)}t)}>0$ for $\mathrm{t}>0$-

Definition of $\left((\widehat{M}_{28})^{(5)}\right)_1,\left((\widehat{M}_{28})^{(5)}\right)_2$ *and* $\left((\widehat{M}_{28})^{(5)}\right)_3$:

Remark 3: if G_{28} is bounded, the same property have also G_{29} *and* G_{30} . indeed if

$G_{28}<(\widehat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt}\leq\left((\widehat{M}_{28})^{(5)}\right)_1-(a_{29}')^{(5)}G_{29}$ and by integrating

$G_{29}\leq\left((\widehat{M}_{28})^{(5)}\right)_2=G_{29}^0+2(a_{29})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_1/(a_{29}')^{(5)}$

In the same way , one can obtain

$G_{30}\leq\left((\widehat{M}_{28})^{(5)}\right)_3=G_{30}^0+2(a_{30})^{(5)}\left((\widehat{M}_{28})^{(5)}\right)_2/(a_{30}')^{(5)}$

If G_{29} *or* G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.-

Remark 4: If G_{28} *is* bounded, from below, the same property holds for G_{29} *and* G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below.-

Remark 5: If T_{28} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(5)}\,((G_{31})(t),t))=(b_{29}')^{(5)}$ then $T_{29}\to\infty$.

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t>t_5$

$(b_{29})^{(5)}-(b_i'')^{(5)}((G_{31})(t),t)<\varepsilon_5, T_{28}\,(t)>(m)^{(5)}$-

Then $\frac{dT_{29}}{dt}\geq(a_{29})^{(5)}(m)^{(5)}-\varepsilon_5T_{29}$ which leads to

$T_{29}\geq\left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1-e^{-\varepsilon_5t})+T_{29}^0e^{-\varepsilon_5t}$ If we take t such that $e^{-\varepsilon_5t}=\frac{1}{2}$ it results

$T_{29}\geq\left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right),\quad t=log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t\to\infty}(b_{30}'')^{(5)}\left((G_{31})(t),t\right)=(b_{30}')^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$-

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}<1$ and to choose

$(\widehat{\mathrm{P}}_{32})^{(6)}$ and $(\widehat{\mathrm{Q}}_{32})^{(6)}$ large to have-

$$\frac{(a_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)}+\left((\widehat{P}_{32})^{(6)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{32})^{(6)}\text{ -}$$

$$\frac{(b_i)^{(6)}}{(\widehat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{32})^{(6)}\right]\leq(\widehat{Q}_{32})^{(6)}\text{ -}$$

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions $G_i\ ,T_i$ satisfying GLOBAL EQUATIONS into itself-

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d\left(\left((G_{35})^{(1)},(T_{35})^{(1)}\right),\left((G_{35})^{(2)},(T_{35})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})},\widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})},\widetilde{(T_{35})}\right)=\mathcal{A}^{(6)}\left((G_{35}),(T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{32})^{(6)}\left|G_{33}^{(1)}-G_{33}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\,ds_{(32)}+$$
$$\int_0^t\{(a_{32}')^{(6)}\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}+$$
$$(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)\left|G_{32}^{(1)}-G_{32}^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}+$$
$$G_{32}^{(2)}\left|(a_{32}'')^{(6)}\left(T_{33}^{(1)},s_{(32)}\right)-(a_{32}'')^{(6)}\left(T_{33}^{(2)},s_{(32)}\right)\right|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}}e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows-

$$\left|(G_{35})^{(1)}-(G_{35})^{(2)}\right|e^{-(\widehat{M}_{32})^{(6)}t}\leq$$
$$\frac{1}{(\widehat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+\ (a_{32}')^{(6)}+(\widehat{A}_{32})^{(6)}+(\widehat{P}_{32})^{(6)}(\widehat{k}_{32})^{(6)}\right)d\left(\left((G_{35})^{(1)},(T_{35})^{(1)};\ (G_{35})^{(2)},(T_{35})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\ and\ T_i$. Taking into account the hypothesis the result follows-

Remark 1: The fact that we supposed $(a_{32}'')^{(6)}$ and $(b_{32}'')^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}\ and\ (\widehat{Q}_{32})^{(6)}e^{(\widehat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(6)}$ and $(b_i'')^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on $(G_{35})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.-

Remark 2: There does not exist any t where $G_i\ (t)=0\ and\ T_i\ (t)=0$

From 69 to 32 it results

$$G_i\ (t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(6)}-(a_i'')^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\geq 0$$

$T_i\ (t)\geq T_i^0e^{(-(b_i')^{(6)}t)}>0$ for t > 0-

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1,\left((\widehat{M}_{32})^{(6)}\right)_2\ and\ \left((\widehat{M}_{32})^{(6)}\right)_3$:

Remark 3: if G_{32} is bounded, the same property have also $G_{33}\ and\ G_{34}$. indeed if

$G_{32}<(\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt}\leq\left((\widehat{M}_{32})^{(6)}\right)_1-(a_{33}')^{(6)}G_{33}$ and by integrating

$$G_{33}\leq\left((\widehat{M}_{32})^{(6)}\right)_2=G_{33}^0+2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a_{33}')^{(6)}$$

In the same way , one can obtain

$$G_{34}\leq\left((\widehat{M}_{32})^{(6)}\right)_3=G_{34}^0+2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a_{34}')^{(6)}$$

If $G_{33}\ or\ G_{34}$ is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.-

Remark 4: If $G_{32}\ is$ bounded, from below, the same property holds for $G_{33}\ and\ G_{34}$. The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below.-

Remark 5: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t))=(b_{33}')^{(6)}$ then $T_{33}\ \to\infty$.

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t\ >t_6$

$(b_{33})^{(6)}-(b_i'')^{(6)}\left((G_{35})(t),t\right)<\varepsilon_6,T_{32}\ (t)>(m)^{(6)}$-

Then $\frac{dT_{33}}{dt}\geq(a_{33})^{(6)}(m)^{(6)}-\varepsilon_6T_{33}$ which leads to

$T_{33}\ \geq\left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1-e^{-\varepsilon_6t})+T_{33}^0e^{-\varepsilon_6t}$ If we take t such that $e^{-\varepsilon_6t}=\ \frac{1}{2}$ it results

$T_{33}\ \geq\left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right),\quad t=log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $\lim_{t\to\infty}(b_{34}'')^{(6)}\left((G_{35})(t),t(t),t\right)=(b_{34}')^{(6)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of the system-

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$:

(a) $\sigma_1)^{(1)},(\sigma_2)^{(1)},(\tau_1)^{(1)},(\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)}\leq-(a_{13}')^{(1)}+(a_{14}')^{(1)}-(a_{13}'')^{(1)}(T_{14},t)+(a_{14}'')^{(1)}(T_{14},t)\leq-(\sigma_1)^{(1)}$$
$$-(\tau_2)^{(1)}\leq-(b_{13}')^{(1)}+(b_{14}')^{(1)}-(b_{13}'')^{(1)}(G,t)-(b_{14}'')^{(1)}(G,t)\leq-(\tau_1)^{(1)}\ -$$

Definition of $(\nu_1)^{(1)},(\nu_2)^{(1)},(u_1)^{(1)},(u_2)^{(1)},\nu^{(1)},u^{(1)}$:

By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ -

Definition of $(\bar{\nu}_1)^{(1)},, (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$:

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the roots of the equations $(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0$ and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ -

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$:-

(b) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by

$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)},\ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}, if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $\boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$ -

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}, if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $\boxed{(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}}$

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ where $(u_1)^{(1)}, (\bar{u}_1)^{(1)}$ are defined respectively-

Then the solution of CONCATENATED GLOBAL EQUATIONS satisfies the inequalities

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{13}(t) \leq G_{13}^0 e^{(S_1)^{(1)}t}$

where $(p_i)^{(1)}$ is defined by equation 25

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \leq G_{14}(t) \leq \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)}t}$ -

$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)}t}\right] + G_{15}^0 e^{-(S_2)^{(1)}t} \leq G_{15}(t) \leq \frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)}t} - e^{-(a'_{15})^{(1)}t}\right] + G_{15}^0 e^{-(a'_{15})^{(1)}t}\right)$ -

$\boxed{T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}}$ -

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)}t} \leq T_{13}(t) \leq \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ -

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)}t} - e^{-(b'_{15})^{(1)}t}\right] + T_{15}^0 e^{-(b'_{15})^{(1)}t} \leq T_{15}(t) \leq$

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)}t}\right] + T_{15}^0 e^{-(R_2)^{(1)}t}$ -

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:-

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$ -

Behavior of the solutions

If we denote and define-

Definition of $(\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$:

$\sigma_1)^{(2)}, (\sigma_2)^{(2)}, (\tau_1)^{(2)}, (\tau_2)^{(2)}$ four constants satisfying-

$-(\sigma_2)^{(2)} \leq -(a'_{16})^{(2)} + (a'_{17})^{(2)} - (a''_{16})^{(2)}(T_{17}, t) + (a''_{17})^{(2)}(T_{17}, t) \leq -(\sigma_1)^{(2)}$ -

$-(\tau_2)^{(2)} \leq -(b'_{16})^{(2)} + (b'_{17})^{(2)} - (b''_{16})^{(2)}\left((G_{19}), t\right) - (b''_{17})^{(2)}\left((G_{19}), t\right) \leq -(\tau_1)^{(2)}$ -

Definition of $(\nu_1)^{(2)}, (\nu_2)^{(2)}, (u_1)^{(2)}, (u_2)^{(2)}$:-

By $(\nu_1)^{(2)} > 0$, $(\nu_2)^{(2)} < 0$ and respectively $(u_1)^{(2)} > 0$, $(u_2)^{(2)} < 0$ the roots-
of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$ -
and $(b_{14})^{(2)}\left(u^{(2)}\right)^2 + (\tau_1)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ and-

Definition of $(\bar{\nu}_1)^{(2)},, (\bar{\nu}_2)^{(2)}, (\bar{u}_1)^{(2)}, (\bar{u}_2)^{(2)}$:-

By $(\bar{\nu}_1)^{(2)} > 0$, $(\bar{\nu}_2)^{(2)} < 0$ and respectively $(\bar{u}_1)^{(2)} > 0$, $(\bar{u}_2)^{(2)} < 0$ the-
roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)} = 0$-
and $(b_{17})^{(2)}\left(u^{(2)}\right)^2 + (\tau_2)^{(2)}u^{(2)} - (b_{16})^{(2)} = 0$ -

Definition of $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$:--

If we define $(m_1)^{(2)}, (m_2)^{(2)}, (\mu_1)^{(2)}, (\mu_2)^{(2)}$ by-

$(m_2)^{(2)} = (\nu_0)^{(2)}, (m_1)^{(2)} = (\nu_1)^{(2)},\ \boldsymbol{if}\ (\nu_0)^{(2)} < (\nu_1)^{(2)}$ -

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\bar{\nu}_1)^{(2)}, \boldsymbol{if}\ (\nu_1)^{(2)} < (\nu_0)^{(2)} < (\bar{\nu}_1)^{(2)}$, and $\boxed{(\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}}$-

$(m_2)^{(2)} = (\nu_1)^{(2)}, (m_1)^{(2)} = (\nu_0)^{(2)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(2)} < (\nu_0)^{(2)}$ -

and analogously

$(\mu_2)^{(2)} = (u_0)^{(2)}, (\mu_1)^{(2)} = (u_1)^{(2)}, \ \boldsymbol{if}\ (u_0)^{(2)} < (u_1)^{(2)}$

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (\bar{u}_1)^{(2)}\ , \boldsymbol{if}\ (u_1)^{(2)} < (u_0)^{(2)} < (\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)} = \frac{T_{16}^0}{T_{17}^0}}$-

$(\mu_2)^{(2)} = (u_1)^{(2)}, (\mu_1)^{(2)} = (u_0)^{(2)}, \ \boldsymbol{if}\ (\bar{u}_1)^{(2)} < (u_0)^{(2)}$ -

Then the solution of CONCATENATED GLOBAL EQUATIONS satisfies the inequalities

$G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{16}(t) \leq G_{16}^0 e^{(S_1)^{(2)}t}$-

$(p_i)^{(2)}$ is defined -

$\frac{1}{(m_1)^{(2)}} G_{16}^0 e^{((S_1)^{(2)} - (p_{16})^{(2)})t} \leq G_{17}(t) \leq \frac{1}{(m_2)^{(2)}} G_{16}^0 e^{(S_1)^{(2)}t}$ -

$\left(\frac{(a_{18})^{(2)} G_{16}^0}{(m_1)^{(2)}((S_1)^{(2)} - (p_{16})^{(2)} - (S_2)^{(2)})}\left[e^{((S_1)^{(2)} - (p_{16})^{(2)})t} - e^{-(S_2)^{(2)}t}\right] + G_{18}^0 e^{-(S_2)^{(2)}t} \leq G_{18}(t) \leq \frac{(a_{18})^{(2)} G_{16}^0}{(m_2)^{(2)}((S_1)^{(2)} - (a_{18}')^{(2)})}\left[e^{(S_1)^{(2)}t} - e^{-(a_{18}')^{(2)}t}\right] + G_{18}^0 e^{-(a_{18}')^{(2)}t}\right)$ -

$\boxed{T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}}$ -

$\frac{1}{(\mu_1)^{(2)}} T_{16}^0 e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}} T_{16}^0 e^{((R_1)^{(2)} + (r_{16})^{(2)})t}$ -

$\frac{(b_{18})^{(2)} T_{16}^0}{(\mu_1)^{(2)}((R_1)^{(2)} - (b_{18}')^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}')^{(2)}t}\right] + T_{18}^0 e^{-(b_{18}')^{(2)}t} \leq T_{18}(t) \leq$

$\frac{(a_{18})^{(2)} T_{16}^0}{(\mu_2)^{(2)}((R_1)^{(2)} + (r_{16})^{(2)} + (R_2)^{(2)})}\left[e^{((R_1)^{(2)} + (r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^0 e^{-(R_2)^{(2)}t}$ -

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:--

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}')^{(2)}$

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$ -

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}')^{(2)}$

$(R_2)^{(2)} = (b_{18}')^{(2)} - (r_{18})^{(2)}$-

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \leq -(a_{20}')^{(3)} + (a_{21}')^{(3)} - (a_{20}'')^{(3)}(T_{21}, t) + (a_{21}'')^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \leq -(b_{20}')^{(3)} + (b_{21}')^{(3)} - (b_{20}'')^{(3)}(G, t) - (b_{21}'')^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$ -

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$:

(b) By $(\nu_1)^{(3)} > 0, (\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0, (\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}\left(u^{(3)}\right)^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ -

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:-

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}, \ \boldsymbol{if}\ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}\ , \boldsymbol{if}\ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)},$

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)}, \ \boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$ -

and analogously

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)}, \ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}\ , \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)}, \ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)} - (p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined-

$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$ -

$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq \frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a_{22}')^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t}] + G_{22}^0 e^{-(a_{22}')^{(3)}t}\right)$ -

$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}}$ -

$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}$ -

$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b_{22}')^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t}\right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq$

$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$ -

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:-

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{20}')^{(3)}$

$(R_2)^{(3)} = (b_{22}')^{(3)} - (r_{22})^{(3)}$-

Behavior of the solutions

If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a_{24}')^{(4)} + (a_{25}')^{(4)} - (a_{24}'')^{(4)}(T_{25}, t) + (a_{25}'')^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \leq -(b_{24}')^{(4)} + (b_{25}')^{(4)} - (b_{24}'')^{(4)}((G_{27}), t) - (b_{25}'')^{(4)}((G_{27}), t) \leq -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$:

(e) By $(\nu_1)^{(4)} > 0, (\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(\nu^{(4)})^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$:

By $(\bar{\nu}_1)^{(4)} > 0, (\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0, (\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}(\nu^{(4)})^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$

and $(b_{25})^{(4)}(u^{(4)})^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$

Definition of $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}, (m_2)^{(4)}, (\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \; \textbf{\textit{if}} \; (\nu_0)^{(4)} < (\nu_1)^{(4)}$

$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)}, \textbf{\textit{if}} \; (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)}$, and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}, \; \textbf{\textit{if}} \; (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$

and analogously

$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \; \textbf{\textit{if}} \; (u_0)^{(4)} < (u_1)^{(4)}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)}, \textbf{\textit{if}} \; (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)}$, and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \textbf{\textit{if}} \; (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$are defined

Then the solution of CONCATENATED GLOBAL EQUATIONS satisfies the inequalities

$G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$

where $(p_i)^{(4)}$ is defined

$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{((S_1)^{(4)}-(p_{24})^{(4)})t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$

$\left(\frac{(a_{26})^{(4)}G_{24}^0}{(m_1)^{(4)}((S_1)^{(4)}-(p_{24})^{(4)}-(S_2)^{(4)})}\left[e^{((S_1)^{(4)}-(p_{24})^{(4)})t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq\right.$

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}}$

$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{((R_1)^{(4)}+(r_{24})^{(4)})t}$

$\frac{(b_{26})^{(4)}T_{24}^0}{(\mu_1)^{(4)}((R_1)^{(4)}-(b_{26}')^{(4)})}\left[e^{(R_1)^{(4)}t} - e^{-(b_{26}')^{(4)}t}\right] + T_{26}^0 e^{-(b_{26}')^{(4)}t} \leq T_{26}(t) \leq$

$\frac{(a_{26})^{(4)}T_{24}^0}{(\mu_2)^{(4)}((R_1)^{(4)}+(r_{24})^{(4)}+(R_2)^{(4)})}\left[e^{((R_1)^{(4)}+(r_{24})^{(4)})t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:-

$(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a_{24}^{'})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b_{24}^{'})^{(4)}$ *and* $(R_2)^{(4)} = (b_{26}^{'})^{(4)} - (r_{26})^{(4)}$

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \le -(a_{28}^{'})^{(5)} + (a_{29}^{'})^{(5)} - (a_{28}^{''})^{(5)}(T_{29}, t) + (a_{29}^{''})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$

$-(\tau_2)^{(5)} \le -(b_{28}^{'})^{(5)} + (b_{29}^{'})^{(5)} - (b_{28}^{''})^{(5)}((G_{31}), t) - (b_{29}^{''})^{(5)}((G_{31}), t) \le -(\tau_1)^{(5)}$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$:

(h) By $(\nu_1)^{(5)} > 0, (\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$:

By $(\bar{\nu}_1)^{(5)} > 0, (\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$

and $(b_{29})^{(5)}\left(u^{(5)}\right)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)},\ \boldsymbol{if}\ (\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)},\ \boldsymbol{if}\ (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$ and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)},\ \boldsymbol{if}\ (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)},\ \boldsymbol{if}\ (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$ and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)},\ \boldsymbol{if}\ (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined

Then the solution of CONCATENATED SYSTEM OF GLOBAL EQUATIONS satisfies the inequalities

$G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{28}(t) \le G_{28}^0 e^{(S_1)^{(5)}t}$

where $(p_i)^{(5)}$ is defined

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)} - (p_{28})^{(5)})t} \le G_{29}(t) \le \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$

$\left(\frac{(a_{30})^{(5)}G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)} - (p_{28})^{(5)} - (S_2)^{(5)})}\left[e^{((S_1)^{(5)} - (p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \le G_{30}(t) \le\right.$

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}}$

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \le T_{28}(t) \le \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)} + (r_{28})^{(5)})t}$

$\frac{(b_{30})^{(5)}T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)} - (b_{30}^{'})^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b_{30}^{'})^{(5)}t}\right] + T_{30}^0 e^{-(b_{30}^{'})^{(5)}t} \le T_{30}(t) \le$

$\frac{(a_{30})^{(5)}T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)} + (r_{28})^{(5)} + (R_2)^{(5)})}\left[e^{((R_1)^{(5)} + (r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:-

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a_{28}^{'})^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b_{28}^{'})^{(5)}$

$(R_2)^{(5)} = (b_{30}^{'})^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)} \le -(a_{32}^{'})^{(6)} + (a_{33}^{'})^{(6)} - (a_{32}^{''})^{(6)}(T_{33}, t) + (a_{33}^{''})^{(6)}(T_{33}, t) \le -(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)} \le -(b_{32}^{'})^{(6)} + (b_{33}^{'})^{(6)} - (b_{32}^{''})^{(6)}((G_{35}), t) - (b_{33}^{''})^{(6)}((G_{35}), t) \le -(\tau_1)^{(6)}$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$:

(k) By $(\nu_1)^{(6)} > 0, (\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations

$(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$:

By $(\bar{\nu}_1)^{(6)} > 0$, $(\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0$, $(\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$

and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)},\ \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$ and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)},\ \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$ and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution of CONCATENATED SYSTEM OF GLOBAL EQUATIONS satisfies the inequalities

$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$

where $(p_i)^{(6)}$ is defined

$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$

$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le\right.$

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}}$

$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}$

$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b_{34}')^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b_{34}')^{(6)}t}\right] + T_{34}^0 e^{-(b_{34}')^{(6)}t} \le T_{34}(t) \le$

$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:-

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a_{32}')^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b_{32}')^{(6)}$

$(R_2)^{(6)} = (b_{34}')^{(6)} - (r_{34})^{(6)}$

Proof : From GLOBAL EQUATIONS we obtain

$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a_{13}')^{(1)} - (a_{14}')^{(1)} + (a_{13}'')^{(1)}(T_{14}, t)\right) - (a_{14}'')^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$

Definition of $\nu^{(1)}$:- $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows

$-\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right) \le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:-

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)} - (\nu_0)^{(1)}\right)t\right]}}$, $\boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get

$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)}\right)t\right]}}$, $\boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$$(\nu_1)^{(1)} \leq \frac{(\nu_1)^{(1)}+(C)^{(1)}(\nu_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}}{1+(C)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\nu_1)^{(1)}-(\nu_2)^{(1)}\right)t\right]}} \leq \nu^{(1)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\bar{\nu}_1)^{(1)}$$

(c) If $0 < (\nu_1)^{(1)} \leq (\bar{\nu}_1)^{(1)} \leq \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(\nu_1)^{(1)} \leq \nu^{(1)}(t) \leq \frac{(\bar{\nu}_1)^{(1)}+(\bar{C})^{(1)}(\bar{\nu}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{\nu}_1)^{(1)}-(\bar{\nu}_2)^{(1)}\right)t\right]}} \leq (\nu_0)^{(1)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(1)}(t)$:-

$(m_2)^{(1)} \leq \nu^{(1)}(t) \leq (m_1)^{(1)}$, $\boxed{\nu^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}$, $\boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}}$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{13})^{(1)} = (a''_{14})^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(\nu_1)^{(1)} = (\bar{\nu}_1)^{(1)}$ if in addition $(\nu_0)^{(1)} = (\nu_1)^{(1)}$ then $\nu^{(1)}(t) = (\nu_0)^{(1)}$ and as a consequence $G_{13}(t) = (\nu_0)^{(1)}G_{14}(t)$ this also defines $(\nu_0)^{(1)}$ for the special case

Analogously if $(b''_{13})^{(1)} = (b''_{14})^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$ and then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$ This is an important consequence of the relation between $(\nu_1)^{(1)}$ and $(\bar{\nu}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

From CONCATENATED SYSTEM OF GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(2)}}{dt} = (a_{16})^{(2)} - \left((a'_{16})^{(2)} - (a'_{17})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right) - (a''_{17})^{(2)}(T_{17},t)\nu^{(2)} - (a_{17})^{(2)}\nu^{(2)}$$

Definition of $\nu^{(2)}$:- $\boxed{\nu^{(2)} = \frac{G_{16}}{G_{17}}}$

It follows

$$-\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_2)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right) \leq \frac{d\nu^{(2)}}{dt} \leq -\left((a_{17})^{(2)}\left(\nu^{(2)}\right)^2 + (\sigma_1)^{(2)}\nu^{(2)} - (a_{16})^{(2)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(2)}, (\nu_0)^{(2)}$:-

(d) For $0 < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\nu_1)^{(2)} < (\bar{\nu}_1)^{(2)}$

$$\nu^{(2)}(t) \geq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_0)^{(2)}\right)t\right]}} \quad , \quad \boxed{(C)^{(2)} = \frac{(\nu_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\nu_2)^{(2)}}}$$

it follows $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\nu_1)^{(2)}$

In the same manner , we get

$$\nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(2)} = \frac{(\bar{\nu}_1)^{(2)}-(\nu_0)^{(2)}}{(\nu_0)^{(2)}-(\bar{\nu}_2)^{(2)}}}$$

From which we deduce $(\nu_0)^{(2)} \leq \nu^{(2)}(t) \leq (\bar{\nu}_1)^{(2)}$

(e) If $0 < (\nu_1)^{(2)} < (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0} < (\bar{\nu}_1)^{(2)}$ we find like in the previous case,

$$(\nu_1)^{(2)} \leq \frac{(\nu_1)^{(2)}+(C)^{(2)}(\nu_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}}{1+(C)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\nu_1)^{(2)}-(\nu_2)^{(2)}\right)t\right]}} \leq \nu^{(2)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\bar{\nu}_1)^{(2)}$$

(f) If $0 < (\nu_1)^{(2)} \leq (\bar{\nu}_1)^{(2)} \leq (\nu_0)^{(2)} = \frac{G_{16}^0}{G_{17}^0}$, we obtain

$$(\nu_1)^{(2)} \leq \nu^{(2)}(t) \leq \frac{(\bar{\nu}_1)^{(2)}+(\bar{C})^{(2)}(\bar{\nu}_2)^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}}{1+(\bar{C})^{(2)}e^{\left[-(a_{17})^{(2)}\left((\bar{\nu}_1)^{(2)}-(\bar{\nu}_2)^{(2)}\right)t\right]}} \leq (\nu_0)^{(2)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(2)}(t)$:-

$(m_2)^{(2)} \leq \nu^{(2)}(t) \leq (m_1)^{(2)}$, $\boxed{\nu^{(2)}(t) = \frac{G_{16}(t)}{G_{17}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(2)}(t)$:-

$(\mu_2)^{(2)} \leq u^{(2)}(t) \leq (\mu_1)^{(2)}$, $\boxed{u^{(2)}(t) = \frac{T_{16}(t)}{T_{17}(t)}}$

Now, using this result and replacing it in CONCATENATED SYSTEM OF GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

: From CONCATENATED GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21}, t)\right) - (a''_{21})^{(3)}(T_{21}, t)\nu^{(3)} - (a_{21})^{(3)}\nu^{(3)}$$

Definition of $\nu^{(3)}$:- $\boxed{\nu^{(3)} = \frac{G_{20}}{G_{21}}}$

It follows

$$-\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right) \leq \frac{d\nu^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(\nu^{(3)}\right)^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)}\right)$$

From which one obtains

(a) For $0 < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\nu_1)^{(3)} < (\bar{\nu}_1)^{(3)}$

$$\nu^{(3)}(t) \geq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_0)^{(3)}\right)t\right]}} \quad , \quad \boxed{(C)^{(3)} = \frac{(\nu_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\nu_2)^{(3)}}}$$

it follows $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\nu_1)^{(3)}$

In the same manner , we get

$$\nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(3)} = \frac{(\bar{\nu}_1)^{(3)} - (\nu_0)^{(3)}}{(\nu_0)^{(3)} - (\bar{\nu}_2)^{(3)}}}$$

Definition of $(\bar{\nu}_1)^{(3)}$:-

From which we deduce $(\nu_0)^{(3)} \leq \nu^{(3)}(t) \leq (\bar{\nu}_1)^{(3)}$

(b) If $0 < (\nu_1)^{(3)} < (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{\nu}_1)^{(3)}$ we find like in the previous case,

$$(\nu_1)^{(3)} \leq \frac{(\nu_1)^{(3)} + (C)^{(3)}(\nu_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}}{1 + (C)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\nu_1)^{(3)} - (\nu_2)^{(3)}\right)t\right]}} \leq \nu^{(3)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\bar{\nu}_1)^{(3)}$$

(c) If $0 < (\nu_1)^{(3)} \leq (\bar{\nu}_1)^{(3)} \leq (\nu_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain

$$(\nu_1)^{(3)} \leq \nu^{(3)}(t) \leq \frac{(\bar{\nu}_1)^{(3)} + (\bar{C})^{(3)}(\bar{\nu}_2)^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}}{1 + (\bar{C})^{(3)} e^{\left[-(a_{21})^{(3)}\left((\bar{\nu}_1)^{(3)} - (\bar{\nu}_2)^{(3)}\right)t\right]}} \leq (\nu_0)^{(3)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(3)}(t)$:-

$(m_2)^{(3)} \leq \nu^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{\nu^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(\nu_1)^{(3)} = (\bar{\nu}_1)^{(3)}$ if in addition $(\nu_0)^{(3)} = (\nu_1)^{(3)}$ then $\nu^{(3)}(t) = (\nu_0)^{(3)}$ and as a consequence $G_{20}(t) = (\nu_0)^{(3)} G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then

$(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)} T_{21}(t)$ This is an important consequence of the relation between $(\nu_1)^{(3)}$ and $(\bar{\nu}_1)^{(3)}$

: From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25}, t)\right) - (a''_{25})^{(4)}(T_{25}, t)\nu^{(4)} - (a_{25})^{(4)}\nu^{(4)}$$

Definition of $\nu^{(4)}$:- $\boxed{\nu^{(4)} = \frac{G_{24}}{G_{25}}}$

It follows

$$-\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right) \leq \frac{d\nu^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_4)^{(4)}\nu^{(4)} - (a_{24})^{(4)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}}{4 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{4 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)} - (\nu_0)^{(4)}}{(\nu_0)^{(4)} - (\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case,

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)} + (C)^{(4)}(\nu_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}}{1 + (C)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)} - (\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)} + (\bar{C})^{(4)}(\bar{\nu}_2)^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}}{1 + (\bar{C})^{(4)} e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)} - (\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}$, $\boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}$, $\boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)} G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** .

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)} T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$.

From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a'_{28})^{(5)} - (a'_{29})^{(5)} + (a''_{28})^{(5)}(T_{29}, t)\right) - (a''_{29})^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case,

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)}+(C)^{(5)}(\nu_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}}{1+(C)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)}-(\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$
$$\frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)}+(\bar{C})^{(5)}(\bar{\nu}_2)^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}}{1+(\bar{C})^{(5)}e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)}-(\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}, \quad \boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}, \quad \boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$$

Now, using this result and replacing it in CONCATENATED GOVERNING EQUATIONS OF THE GLOBAL SYSTEM we get easily the result stated in the theorem.

Particular case :

If $(a_{28}'')^{(5)} = (a_{29}'')^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)}G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b_{28}'')^{(5)} = (b_{29}'')^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)}T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

From GLOBAL EQUATIONS we obtain

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a_{32}')^{(6)} - (a_{33}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right) - (a_{33}'')^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

Definition of $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get

$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case,

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$
$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)} G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then

$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)} T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

We can prove the following

Theorem 3: If $(a''_i)^{(1)} and\ (b''_i)^{(1)}$ are independent on t , and the conditions (with the notations 25,26,27,28)

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$

$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$,

$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b'_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$

$with\ (p_{13})^{(1)}, (r_{14})^{(1)}$ as defined are satisfied , then the system

If $(a''_i)^{(2)} and\ (b''_i)^{(2)}$ are independent on t , and the conditions

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} < 0$

$(a'_{16})^{(2)}(a'_{17})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16})^{(2)}(p_{16})^{(2)} + (a'_{17})^{(2)}(p_{17})^{(2)} + (p_{16})^{(2)}(p_{17})^{(2)} > 0$

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} > 0$,

$(b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} - (b'_{16})^{(2)}(r_{17})^{(2)} - (b'_{17})^{(2)}(r_{17})^{(2)} + (r_{16})^{(2)}(r_{17})^{(2)} < 0$

$with\ (p_{16})^{(2)}, (r_{17})^{(2)}$ as defined are satisfied , then the system

If $(a''_i)^{(3)} and\ (b''_i)^{(3)}$ are independent on t , and the conditions

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} < 0$

$(a'_{20})^{(3)}(a'_{21})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20})^{(3)}(p_{20})^{(3)} + (a'_{21})^{(3)}(p_{21})^{(3)} + (p_{20})^{(3)}(p_{21})^{(3)} > 0$

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} > 0$,

$(b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} - (b'_{20})^{(3)}(r_{21})^{(3)} - (b'_{21})^{(3)}(r_{21})^{(3)} + (r_{20})^{(3)}(r_{21})^{(3)} < 0$

$with\ (p_{20})^{(3)}, (r_{21})^{(3)}$ as defined by equation 25 are satisfied , then the system

If $(a''_i)^{(4)} and\ (b''_i)^{(4)}$ are independent on t , and the conditions WE CAN UNMISTAKABLY PROVE THAT:

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} < 0$

$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a_{24})^{(4)}(p_{24})^{(4)} + (a'_{25})^{(4)}(p_{25})^{(4)} + (p_{24})^{(4)}(p_{25})^{(4)} > 0$

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} > 0$,

$(b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} - (b'_{24})^{(4)}(r_{25})^{(4)} - (b'_{25})^{(4)}(r_{25})^{(4)} + (r_{24})^{(4)}(r_{25})^{(4)} < 0$

$with\ (p_{24})^{(4)}, (r_{25})^{(4)}$ as defined are satisfied , then the system

If $(a''_i)^{(5)} and\ (b''_i)^{(5)}$ are independent on t , and the conditions

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} < 0$

$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a_{28})^{(5)}(p_{28})^{(5)} + (a'_{29})^{(5)}(p_{29})^{(5)} + (p_{28})^{(5)}(p_{29})^{(5)} > 0$

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} > 0$,

$(b'_{28})^{(5)}(b'_{29})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - (b'_{28})^{(5)}(r_{29})^{(5)} - (b'_{29})^{(5)}(r_{29})^{(5)} + (r_{28})^{(5)}(r_{29})^{(5)} < 0$

$with\ (p_{28})^{(5)}, (r_{29})^{(5)}$ as defined are satisfied , then the system

If $(a''_i)^{(6)} and\ (b''_i)^{(6)}$ are independent on t , and the conditions

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined are satisfied , then the system

$(a_{13})^{(1)} G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right] G_{13} = 0$

$(a_{14})^{(1)} G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right] G_{14} = 0$

$(a_{15})^{(1)} G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right] G_{15} = 0$

$(b_{13})^{(1)} T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ] T_{13} = 0$

$(b_{14})^{(1)} T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ] T_{14} = 0$

$(b_{15})^{(1)} T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ] T_{15} = 0$

has a unique positive solution , which is an equilibrium solution

$(a_{16})^{(2)}G_{17} - \left[(a_{16}^{'})^{(2)} + (a_{16}^{''})^{(2)}(T_{17})\right]G_{16} = 0$
$(a_{17})^{(2)}G_{16} - \left[(a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}(T_{17})\right]G_{17} = 0$
$(a_{18})^{(2)}G_{17} - \left[(a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}(T_{17})\right]G_{18} = 0$
$(b_{16})^{(2)}T_{17} - [(b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}(G_{19})\,]T_{16} = 0$
$(b_{17})^{(2)}T_{16} - [(b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}(G_{19})\,]T_{17} = 0$
$(b_{18})^{(2)}T_{17} - [(b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}(G_{19})\,]T_{18} = 0$
has a unique positive solution , which is an equilibrium solution for THE SYSTEM
$(a_{20})^{(3)}G_{21} - \left[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21})\right]G_{20} = 0$
$(a_{21})^{(3)}G_{20} - \left[(a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)}(T_{21})\right]G_{21} = 0$
$(a_{22})^{(3)}G_{21} - \left[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21})\right]G_{22} = 0$
$(b_{20})^{(3)}T_{21} - [(b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G_{23})\,]T_{20} = 0$
$(b_{21})^{(3)}T_{20} - [(b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)}(G_{23})\,]T_{21} = 0$
$(b_{22})^{(3)}T_{21} - [(b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)}(G_{23})\,]T_{22} = 0$
has a unique positive solution , which is an equilibrium solution for THE HOLISTIC SYSTEM
$(a_{24})^{(4)}G_{25} - \left[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25})\right]G_{24} = 0$

$(a_{25})^{(4)}G_{24} - \left[(a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)}(T_{25})\right]G_{25} = 0$

$(a_{26})^{(4)}G_{25} - \left[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25})\right]G_{26} = 0$

$(b_{24})^{(4)}T_{25} - [(b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)}\big((G_{27})\big)\,]T_{24} = 0$
$(b_{25})^{(4)}T_{24} - [(b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)}\big((G_{27})\big)\,]T_{25} = 0$

$(b_{26})^{(4)}T_{25} - [(b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)}\big((G_{27})\big)\,]T_{26} = 0$
has a unique positive solution , which is an equilibrium solution for the system HOLISTIC SYSTEM

$(a_{28})^{(5)}G_{29} - \left[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29})\right]G_{28} = 0$

$(a_{29})^{(5)}G_{28} - \left[(a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29})\right]G_{29} = 0$

$(a_{30})^{(5)}G_{29} - \left[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29})\right]G_{30} = 0$

$(b_{28})^{(5)}T_{29} - [(b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}(G_{31})\,]T_{28} = 0$

$(b_{29})^{(5)}T_{28} - [(b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}(G_{31})\,]T_{29} = 0$

$(b_{30})^{(5)}T_{29} - [(b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}(G_{31})\,]T_{30} = 0$

has a unique positive solution , which is an equilibrium solution for the system (HOLISTIC SYSTEM)

$(a_{32})^{(6)}G_{33} - \left[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33})\right]G_{32} = 0$

$(a_{33})^{(6)}G_{32} - \left[(a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33})\right]G_{33} = 0$

$(a_{34})^{(6)}G_{33} - \left[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33})\right]G_{34} = 0$

$(b_{32})^{(6)}T_{33} - [(b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}(G_{35})\,]T_{32} = 0$

$(b_{33})^{(6)}T_{32} - [(b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)}(G_{35})\,]T_{33} = 0$

$(b_{34})^{(6)}T_{33} - [(b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}(G_{35})\,]T_{34} = 0$

has a unique positive solution , which is an equilibrium solution for the system (GLOBAL)

Proof:
(a) Indeed the first two equations have a nontrivial solution G_{13}, G_{14} if
$F(T) =$
$(a_{13}^{'})^{(1)}(a_{14}^{'})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13}^{'})^{(1)}(a_{14}^{''})^{(1)}(T_{14}) + (a_{14}^{'})^{(1)}(a_{13}^{''})^{(1)}(T_{14}) + (a_{13}^{''})^{(1)}(T_{14})(a_{14}^{''})^{(1)}(T_{14}) = 0$
(a) Indeed the first two equations have a nontrivial solution G_{16}, G_{17} if
F$(T_{19}) =$
$(a_{16}^{'})^{(2)}(a_{17}^{'})^{(2)} - (a_{16})^{(2)}(a_{17})^{(2)} + (a_{16}^{'})^{(2)}(a_{17}^{''})^{(2)}(T_{17}) + (a_{17}^{'})^{(2)}(a_{16}^{''})^{(2)}(T_{17}) + (a_{16}^{''})^{(2)}(T_{17})(a_{17}^{''})^{(2)}(T_{17}) = 0$
(a) Indeed the first two equations have a nontrivial solution G_{20}, G_{21} if
$F(T_{23}) =$
$(a_{20}^{'})^{(3)}(a_{21}^{'})^{(3)} - (a_{20})^{(3)}(a_{21})^{(3)} + (a_{20}^{'})^{(3)}(a_{21}^{''})^{(3)}(T_{21}) + (a_{21}^{'})^{(3)}(a_{20}^{''})^{(3)}(T_{21}) + (a_{20}^{''})^{(3)}(T_{21})(a_{21}^{''})^{(3)}(T_{21}) = 0$
(a) Indeed the first two equations have a nontrivial solution G_{24}, G_{25} if

$F(T_{27}) =$
$(a'_{24})^{(4)}(a'_{25})^{(4)} - (a_{24})^{(4)}(a_{25})^{(4)} + (a'_{24})^{(4)}(a''_{25})^{(4)}(T_{25}) + (a'_{25})^{(4)}(a''_{24})^{(4)}(T_{25}) + (a''_{24})^{(4)}(T_{25})(a''_{25})^{(4)}(T_{25}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{28}, G_{29} if

$F(T_{31}) =$
$(a'_{28})^{(5)}(a'_{29})^{(5)} - (a_{28})^{(5)}(a_{29})^{(5)} + (a'_{28})^{(5)}(a''_{29})^{(5)}(T_{29}) + (a'_{29})^{(5)}(a''_{28})^{(5)}(T_{29}) + (a''_{28})^{(5)}(T_{29})(a''_{29})^{(5)}(T_{29}) = 0$

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$F(T_{35}) =$
$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$

Definition and uniqueness of T_{14}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*) = 0$. With this value , we obtain from the three first equations

$G_{13} = \frac{(a_{13})^{(1)}G_{14}}{[(a'_{13})^{(1)}+(a''_{13})^{(1)}(T_{14}^*)]}$, $G_{15} = \frac{(a_{15})^{(1)}G_{14}}{[(a'_{15})^{(1)}+(a''_{15})^{(1)}(T_{14}^*)]}$

Definition and uniqueness of T_{17}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(2)}(T_{17})$ being increasing, it follows that there exists a unique T_{17}^* for which $f(T_{17}^*) = 0$. With this value , we obtain from the three first equations

$G_{16} = \frac{(a_{16})^{(2)}G_{17}}{[(a'_{16})^{(2)}+(a''_{16})^{(2)}(T_{17}^*)]}$, $G_{18} = \frac{(a_{18})^{(2)}G_{17}}{[(a'_{18})^{(2)}+(a''_{18})^{(2)}(T_{17}^*)]}$

Definition and uniqueness of T_{21}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

$G_{20} = \frac{(a_{20})^{(3)}G_{21}}{[(a'_{20})^{(3)}+(a''_{20})^{(3)}(T_{21}^*)]}$, $G_{22} = \frac{(a_{22})^{(3)}G_{21}}{[(a'_{22})^{(3)}+(a''_{22})^{(3)}(T_{21}^*)]}$

Definition and uniqueness of T_{25}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$G_{24} = \frac{(a_{24})^{(4)}G_{25}}{[(a'_{24})^{(4)}+(a''_{24})^{(4)}(T_{25}^*)]}$, $G_{26} = \frac{(a_{26})^{(4)}G_{25}}{[(a'_{26})^{(4)}+(a''_{26})^{(4)}(T_{25}^*)]}$

Definition and uniqueness of T_{29}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$G_{28} = \frac{(a_{28})^{(5)}G_{29}}{[(a'_{28})^{(5)}+(a''_{28})^{(5)}(T_{29}^*)]}$, $G_{30} = \frac{(a_{30})^{(5)}G_{29}}{[(a'_{30})^{(5)}+(a''_{30})^{(5)}(T_{29}^*)]}$

Definition and uniqueness of T_{33}^* :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i'')^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^*)]}$, $G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^*)]}$

(e) By the same argument, the equations(SOLUTIONALOF THE GLOBAL EQUATIONS) admit solutions G_{13}, G_{14} if

$\varphi(G) = (b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$
$\left[(b'_{13})^{(1)}(b''_{14})^{(1)}(G) + (b'_{14})^{(1)}(b''_{13})^{(1)}(G)\right] + (b''_{13})^{(1)}(G)(b''_{14})^{(1)}(G) = 0$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(f) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if

$\varphi(G_{19}) = (b'_{16})^{(2)}(b'_{17})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$
$\left[(b'_{16})^{(2)}(b''_{17})^{(2)}(G_{19}) + (b'_{17})^{(2)}(b''_{16})^{(2)}(G_{19})\right] + (b''_{16})^{(2)}(G_{19})(b''_{17})^{(2)}(G_{19}) = 0$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi((G_{19})^*) = 0$

(g) By the same argument, the equations 92,93 admit solutions G_{20}, G_{21} if

$\varphi(G_{23}) = (b'_{20})^{(3)}(b'_{21})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$
$\left[(b'_{20})^{(3)}(b''_{21})^{(3)}(G_{23}) + (b'_{21})^{(3)}(b''_{20})^{(3)}(G_{23})\right] + (b''_{20})^{(3)}(G_{23})(b''_{21})^{(3)}(G_{23}) = 0$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^* such that $\varphi((G_{23})^*) = 0$

(h) By the same argument, the equations SOLUTIONAL SYSTEM OF THE GLOBAL EQUATIONS admit solutions G_{24}, G_{25} if

$\varphi(G_{27}) = (b'_{24})^{(4)}(b'_{25})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$
$\left[(b'_{24})^{(4)}(b''_{25})^{(4)}(G_{27}) + (b'_{25})^{(4)}(b''_{24})^{(4)}(G_{27})\right] + (b''_{24})^{(4)}(G_{27})(b''_{25})^{(4)}(G_{27}) = 0$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^* such that $\varphi((G_{27})^*) = 0$

(i) By the same argument, the equations SOLUTIONAL SYSTEM OF THE GLOBAL EQUATIONS admit solutions G_{28}, G_{29} if

$$\varphi(G_{31}) = (b_{28}')^{(5)}(b_{29}')^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} - \left[(b_{28}')^{(5)}(b_{29}'')^{(5)}(G_{31}) + (b_{29}')^{(5)}(b_{28}'')^{(5)}(G_{31})\right] + (b_{28}'')^{(5)}(G_{31})(b_{29}'')^{(5)}(G_{31}) = 0$$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^* such that $\varphi((G_{31})^*) = 0$

(j) By the same argument, the equations SOLUTIONAL SYSTEM OF THE GLOBAL EQUATIONS admit solutions G_{32}, G_{33} if

$$\varphi(G_{35}) = (b_{32}')^{(6)}(b_{33}')^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - \left[(b_{32}')^{(6)}(b_{33}'')^{(6)}(G_{35}) + (b_{33}')^{(6)}(b_{32}'')^{(6)}(G_{35})\right] + (b_{32}'')^{(6)}(G_{35})(b_{33}'')^{(6)}(G_{35}) = 0$$

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^* such that $\varphi(G^*) = 0$

Finally we obtain the unique solution of THE HOLISTIC SYSTEM

G_{14}^* given by $\varphi(G^*) = 0$, T_{14}^* given by $f(T_{14}^*) = 0$ and

$$G_{13}^* = \frac{(a_{13})^{(1)}G_{14}^*}{[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14}^*)]} \quad , \quad G_{15}^* = \frac{(a_{15})^{(1)}G_{14}^*}{[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}^*)]}$$

$$T_{13}^* = \frac{(b_{13})^{(1)}T_{14}^*}{[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G^*)]} \quad , \quad T_{15}^* = \frac{(b_{15})^{(1)}T_{14}^*}{[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G^*)]}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL SYSTEM OF GOVERNING EQUATIONS

Finally we obtain the unique solution of THE HOLISTIC SYSTEM

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(\mathrm{T}_{17}^*) = 0$ and

$$G_{16}^* = \frac{(a_{16})^{(2)}G_{17}^*}{[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)}G_{17}^*}{[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}^*)]}$$

$$\mathrm{T}_{16}^* = \frac{(\mathrm{b}_{16})^{(2)}\mathrm{T}_{17}^*}{[(\mathrm{b}_{16}')^{(2)} - (\mathrm{b}_{16}'')^{(2)}((G_{19})^*)]} \quad , \quad \mathrm{T}_{18}^* = \frac{(\mathrm{b}_{18})^{(2)}\mathrm{T}_{17}^*}{[(\mathrm{b}_{18}')^{(2)} - (\mathrm{b}_{18}'')^{(2)}((G_{19})^*)]}$$

Obviously, these values represent an equilibrium solution of THE HOLISTIC SYSTEM

Finally we obtain the unique solution of SOLUTIONAL EQUATIONS OF THE GLOBAL SYSTEM

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)}G_{21}^*}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)}G_{21}^*}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)}T_{21}^*}{[(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)}T_{21}^*}{[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution of THE GLOBAL GOVERNING EQUATIONS

Finally we obtain the unique solution of SOLUTIONS FOR THE GLOBAL GOVERNING EQUATIONS

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)}G_{25}^*}{[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)}G_{25}^*}{[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)}T_{25}^*}{[(b_{24}')^{(4)} - (b_{24}'')^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)}T_{25}^*}{[(b_{26}')^{(4)} - (b_{26}'')^{(4)}((G_{27})^*)]}$$

Obviously, these values represent an equilibrium solution of GLOBAL GOVERNING EQUATIONS

Finally we obtain the unique solution of THE HOLISTIC SYSTEM

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)}G_{29}^*}{[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)}G_{29}^*}{[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)}T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)}T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]}$$

Obviously, these values represent an equilibrium solution of THE HOLISTIC SYSTEM

Finally we obtain the unique solution of SLOUTIONAL EQUATIONS OF THE CONCATENATED EQUATIONS

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)}G_{33}^*}{[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)}G_{33}^*}{[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)}T_{33}^*}{[(b_{32}')^{(6)} - (b_{32}'')^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)}T_{33}^*}{[(b_{34}')^{(6)} - (b_{34}'')^{(6)}((G_{35})^*)]}$$

Obviously, these values represent an equilibrium solution of the GLOBAL SYSTEM

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$ *and* $(b_i'')^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i'')^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations OF SOLUTIONALEQUATIONS OF THE GLOBAL SYSTEM and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}')^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}')^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14}$$

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}')^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14}$$

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}')^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}')^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}')^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(2)}$ and $(b_i'')^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{17}'')^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i'')^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij}$$

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}')^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17}$$

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}')^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17}$$

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}')^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17}$$

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}')^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}')^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}')^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(3)}$ *and* $(b_i'')^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)}G_{20}^*\mathbb{T}_{21}$$

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)}G_{21}^*\mathbb{T}_{21}$$

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)}G_{22}^*\mathbb{T}_{21}$$

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)}T_{20}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)}T_{21}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)}T_{22}^*\mathbb{G}_j\right)$$

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \quad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)}G_{24}^*\mathbb{T}_{25}$$

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)}G_{25}^*\mathbb{T}_{25}$$

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)}G_{26}^*\mathbb{T}_{25}$$

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)}T_{24}^*\mathbb{G}_j\right)$$

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)}T_{25}^*\mathbb{G}_j\right)$$

$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}^{'})^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)}T_{26}^{*}\mathbb{G}_j\right)$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(5)}$ and $(b_i^{''})^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{29}^{''})^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i^{''})^{(5)}}{\partial G_j}\left((G_{31})^*\right) = s_{ij}$

$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}^{'})^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)}G_{28}^*\mathbb{T}_{29}$

$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}^{'})^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)}G_{29}^*\mathbb{T}_{29}$

$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}^{'})^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)}G_{30}^*\mathbb{T}_{29}$

$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}^{'})^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)}T_{28}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}^{'})^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)}T_{29}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}^{'})^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)}T_{30}^{*}\mathbb{G}_j\right)$

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(6)}$ and $(b_i^{''})^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{33}^{''})^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i^{''})^{(6)}}{\partial G_j}\left((G_{35})^*\right) = s_{ij}$

$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}^{'})^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^*\mathbb{T}_{33}$

$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}^{'})^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^*\mathbb{T}_{33}$

$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}^{'})^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^*\mathbb{T}_{33}$

$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}^{'})^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}^{'})^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j\right)$

$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}^{'})^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j\right)$

The characteristic equation of this system is

$\left((\lambda)^{(1)} + (b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)$

$\left[\left(\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$

$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$

$+\left(\left((\lambda)^{(1)} + (a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)$

$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$

$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$

$+\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$

$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$

$+$

$\left((\lambda)^{(2)} + (b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)$

$\left[\left(\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^*\right)\right]$

$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^*\right)$

$+\left(\left((\lambda)^{(2)} + (a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^*\right)$

$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^*\right)$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$

$\left(\left((\lambda)^{(2)}\right)^2 + \left((b_{16}^{'})^{(2)} + (b_{17}^{'})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right)(\lambda)^{(2)}\right)$

$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)(q_{18})^{(2)}G_{18}$

$$+\left((\lambda)^{(2)}+(a_{16}^{'})^{(2)}+(p_{16})^{(2)}\right)\left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}_{17}^{*}+(a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}_{16}^{*}\right)$$
$$\left(\left((\lambda)^{(2)}+(b_{16}^{'})^{(2)}-(r_{16})^{(2)}\right)s_{(17),(18)}\mathrm{T}_{17}^{*}+(b_{17})^{(2)}s_{(16),(18)}\mathrm{T}_{16}^{*}\right)\}=0$$
$$+$$
$$\left((\lambda)^{(3)}+(b_{22}^{'})^{(3)}-(r_{22})^{(3)}\right)\{\left((\lambda)^{(3)}+(a_{22}^{'})^{(3)}+(p_{22})^{(3)}\right)$$
$$\left[\left(\left((\lambda)^{(3)}+(a_{20}^{'})^{(3)}+(p_{20})^{(3)}\right)(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(3)}+(b_{20}^{'})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(21)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(21)}T_{21}^{*}\right)$$
$$+\left(\left((\lambda)^{(3)}+(a_{21}^{'})^{(3)}+(p_{21})^{(3)}\right)(q_{20})^{(3)}G_{20}^{*}+(a_{20})^{(3)}(q_{21})^{(1)}G_{21}^{*}\right)$$
$$\left(\left((\lambda)^{(3)}+(b_{20}^{'})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(20)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(20)}T_{20}^{*}\right)$$
$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((a_{20}^{'})^{(3)}+(a_{21}^{'})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$
$$\left(\left((\lambda)^{(3)}\right)^{2}+\left((b_{20}^{'})^{(3)}+(b_{21}^{'})^{(3)}-(r_{20})^{(3)}+(r_{21})^{(3)}\right)(\lambda)^{(3)}\right)$$
$$+\left(\left((\lambda)^{(3)}\right)^{2}+\left((a_{20}^{'})^{(3)}+(a_{21}^{'})^{(3)}+(p_{20})^{(3)}+(p_{21})^{(3)}\right)(\lambda)^{(3)}\right)(q_{22})^{(3)}G_{22}$$
$$+\left((\lambda)^{(3)}+(a_{20}^{'})^{(3)}+(p_{20})^{(3)}\right)\left((a_{22})^{(3)}(q_{21})^{(3)}G_{21}^{*}+(a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G_{20}^{*}\right)$$
$$\left(\left((\lambda)^{(3)}+(b_{20}^{'})^{(3)}-(r_{20})^{(3)}\right)s_{(21),(22)}T_{21}^{*}+(b_{21})^{(3)}s_{(20),(22)}T_{20}^{*}\right)\}=0$$
$$+$$
$$\left((\lambda)^{(4)}+(b_{26}^{'})^{(4)}-(r_{26})^{(4)}\right)\{\left((\lambda)^{(4)}+(a_{26}^{'})^{(4)}+(p_{26})^{(4)}\right)$$
$$\left[\left(\left((\lambda)^{(4)}+(a_{24}^{'})^{(4)}+(p_{24})^{(4)}\right)(q_{25})^{(4)}G_{25}^{*}+(a_{25})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(4)}+(b_{24}^{'})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(25)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(25)}T_{25}^{*}\right)$$
$$+\left(\left((\lambda)^{(4)}+(a_{25}^{'})^{(4)}+(p_{25})^{(4)}\right)(q_{24})^{(4)}G_{24}^{*}+(a_{24})^{(4)}(q_{25})^{(4)}G_{25}^{*}\right)$$
$$\left(\left((\lambda)^{(4)}+(b_{24}^{'})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(24)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(24)}T_{24}^{*}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^{2}+\left((b_{24}^{'})^{(4)}+(b_{25}^{'})^{(4)}-(r_{24})^{(4)}+(r_{25})^{(4)}\right)(\lambda)^{(4)}\right)$$
$$+\left(\left((\lambda)^{(4)}\right)^{2}+\left((a_{24}^{'})^{(4)}+(a_{25}^{'})^{(4)}+(p_{24})^{(4)}+(p_{25})^{(4)}\right)(\lambda)^{(4)}\right)(q_{26})^{(4)}G_{26}$$
$$+\left((\lambda)^{(4)}+(a_{24}^{'})^{(4)}+(p_{24})^{(4)}\right)\left((a_{26})^{(4)}(q_{25})^{(4)}G_{25}^{*}+(a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G_{24}^{*}\right)$$
$$\left(\left((\lambda)^{(4)}+(b_{24}^{'})^{(4)}-(r_{24})^{(4)}\right)s_{(25),(26)}T_{25}^{*}+(b_{25})^{(4)}s_{(24),(26)}T_{24}^{*}\right)\}=0$$
$$+$$
$$\left((\lambda)^{(5)}+(b_{30}^{'})^{(5)}-(r_{30})^{(5)}\right)\{\left((\lambda)^{(5)}+(a_{30}^{'})^{(5)}+(p_{30})^{(5)}\right)$$
$$\left[\left(\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)(q_{29})^{(5)}G_{29}^{*}+(a_{29})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(29)}T_{29}^{*}\right)$$
$$+\left(\left((\lambda)^{(5)}+(a_{29}^{'})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G_{28}^{*}+(a_{28})^{(5)}(q_{29})^{(5)}G_{29}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(28)}T_{28}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b_{28}^{'})^{(5)}+(b_{29}^{'})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$
$$+\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^{*}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(30)}T_{28}^{*}\right)\}=0$$
$$+$$
$$\left((\lambda)^{(6)}+(b_{34}^{'})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a_{34}^{'})^{(6)}+(p_{34})^{(6)}\right)$$
$$\left[\left(\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$
$$+\left(\left((\lambda)^{(6)}+(a_{33}^{'})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G_{32}^{*}+(a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b_{32}^{'})^{(6)}+(b_{33}^{'})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a_{32}')^{(6)}+(a_{33}')^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$
$$+\left((\lambda)^{(6)}+(a_{32}')^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}+(b_{32}')^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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