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Turing Machine Operation-A Checks and Balances Model

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ABSTRACT: *A state register that stores the state of the Turing machine, one of finitely many. There is one special start state with which the state register is initialized. These states, writes Turing, **replace the** "state of mind" a person performing computations would ordinarily be in.It is like bank ledger, which has Debits and Credits. Note that in double entry computation, both debits and credits are entered in to the systems, namely the Bank Ledger and balance is posted. Individual Debits are equivalent to individual credits. On a generalizational and globalist scale, a "General Ledger" is written which records in all its wide ranging manifestation the Debits and Credits. This is also conservative. In other words Assets is equivalent to Liabilities. True, Profit is distributed among overheads and charges, and there shall be another account in the General ledger that is the account of Profit. This account is credited with the amount earned as commission, exchange, or discount of bills. Now when we write the "General Ledger" of Turing machine, the Primma Donna and terra firma of the model, we are writing the General Theory of all the variables that are incorporated in the model. So for every variable, we have an anti variable. This is the dissipation factor. Conservation laws do hold well in computers. They do not break the conservation laws. Thus energy is not dissipated in to the atmosphere when computation is being performed. To repeat we are suggesting a General Theory Of working of a simple Computer and in further papers, we want to extend this theory to both nanotechnology and Quantum Computation. Turing's work is the proponent, precursor, primogeniture, progenitor and promethaleon for the development of Quantum Computers. Computers follow conservation laws. This work is one which formed the primordial concept of diurnal dynamics and hypostasized dynamism of Quantum computers which is the locus of essence, sense and expression of the present day to day musings and mundane drooling. Verily Turing and Churchill stand out like connoisseurs, rancouteurs, and cognescenti of eminent persons, who strode like colossus the screen of collective consciousness of people. We dedicate this paper on the eve of one hundred years of Turing innovation. Model is based on **Hill and Peterson diagram.***

INTRODUCTION

Turing machine –A beckoning begorra (Extensive excerpts from Wikipedia AND PAGES OF Turing,Churchill,and other noted personalities-Emphasis is mine)

A Turing machine is a device that **manipulates s**ymbols on a strip of tape according to a table of rules. Despite its simplicity, a Turing machine can be adapted to **simulate the logic** of any computer algorithm, and is particularly **useful in** explaining the functions of a CPU inside a computer.

The "Turing" machine was **described by** Alan Turing in 1936 who called it an "a (automatic)-machine". The Turing machine is not intended as a practical computing technology, but rather as a hypothetical device representing a computing machine. Turing machines **help c**omputer scientists understand the limits of mechanical computation.

Turing gave a succinct and candid definition of the experiment in his 1948 essay, "Intelligent Machinery". Referring to his 1936 publication, Turing wrote that the Turing machine, here called a Logical Computing Machine, consisted of:

...an infinite memory capacity **obtained in the** form of an infinite tape marked out into squares, on each of which a symbol could be printed. At any moment there is one symbol **in the** machine; it is called the scanned symbol. The machine **can alter** the scanned symbol and its behavior is in part **determined b**y that symbol, but the symbols on the tape elsewhere **do not affect** the behaviour of the machine. However, the tape can be moved back and forth through the machine, this being one of the elementary operations of the machine. Any symbol on the tape may therefore eventually have an innings. (Turing 1948, p. 61)

A Turing machine that is able **to simulate any** other Turing machine is called a universal Turing machine (UTM, or simply a universal machine). A more mathematically oriented definition with a similar "universal" nature was introduced by Alonzo Church, whose work on calculus **intertwined with** Turing's in a formal theory of computation known as the Church–Turing thesis. The thesis states that Turing machines **indeed capture** the informal notion of effective method in logic and mathematics, and provide a precise definition of an algorithm or 'mechanical procedure'.

In computability theory, the Church–Turing thesis (also known as the Church–Turing conjecture, Church's thesis, Church's conjecture, and Turing's thesis) is a **combined hypothesis** ("thesis") about the nature of functions whose values are effectively calculable; or, in more modern terms, functions whose values are algorithmically computable. In simple terms, the Church–Turing thesis states that "everything algorithmically computable is computable by a Turing machine."

American mathematician Alonzo Church created a method for defining functions called the λ-calculus,

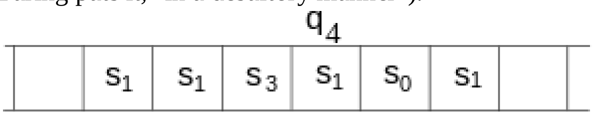
Church, along with mathematician Stephen Kleene and logician J.B. Rosser created a formal definition of a class of functions whose values **could be calculated** by recursion.

All three computational processes **(recursion, the λ-calculus, and the Turing machine**) were shown to be **equivalent—**all three approaches define the same class of functions this has led mathematicians and computer scientists to believe that the concept of computability is **accurately characterized** by these three equivalent processes. Informally the Church–Turing thesis states that if some method (algorithm**) exists** to carry out a calculation, then the same calculation can also be carried out by a Turing machine (as well as by a recursively definable function, and by a λ-function).

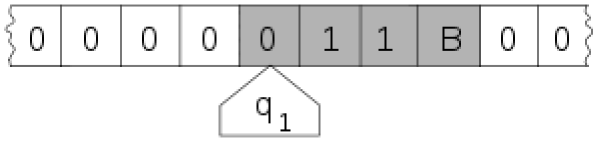
The Church–Turing thesis is a statement that characterizes the nature of computation and cannot be formally proven. Even though the three processes mentioned above **proved to be equivalent**, the fundamental premise behind the thesis—the notion of what it means for a function to be "effectively calculable" (computable)—is "a somewhat vague intuitive one" Thus, the "thesis" remains a hypothesis.

Desultory Bureaucratic burdock or a Driving dromedary?

The Turing machine mathematically models a machine that **mechanically operates on** a tape. On this tape are symbols which the machine can read and write, one at a time, using a tape head. Operation is **fully determined** by a finite set of elementary instructions such as "in state 42, if the symbol seen is 0, write a 1; if the symbol seen is 1, change into state 17; in state 17, if the symbol seen is 0, write a 1 and change to state 6;" etc. In the original article, Turing imagines not a mechanism, but a person whom he calls the "computer", who **executes these** deterministic mechanical rules slavishly (or as Turing puts it, "in a desultory manner").



The head is always over a particular square of the tape; only a finite stretch of squares is shown. The instruction to be performed (q4) is shown over the scanned square. (Drawing after Kleene (1952) p.375.)



Here, the internal state (q1) is shown inside the head, and the illustration describes the tape as being infinite and pre-filled with "0", the symbol serving as blank. The system's full state (its configuration**) consists** of the internal state, the contents of the shaded squares including the blank scanned by the head ("11B"), and the position of the head. (Drawing after Minsky (1967) p. 121).

Sequestration dispensation:

A tape which **is divided** into cells, one next to the other. Each cell **contains a** symbol from some finite alphabet. The alphabet **contains a** special blank symbol (here written as 'B') and one or more other symbols. The tape is assumed to be arbitrarily **extendable** to the left and to the right, i.e., the Turing machine is always **supplied with** as much tape as it needs for its computation. Cells that have not been written to before are assumed to be filled with the blank symbol. In some models the tape has a left end marked with a special symbol; the tape extends or is indefinitely extensible to the right.

A head that can read and write symbols on the tape and move the tape left and right one (and only one) cell at a time. In some models the head moves and the tape is stationary.

A state register **that stores** the state of the Turing machine, one of finitely many. There is one special start state with which the state register is initialized. These states, writes Turing, **replace the** "state of mind" a person performing computations would ordinarily be in.It is like bank ledger, which has Debits and Credits. Note that in double entry computation, both debits and credits are entered in to the systems, namely the Bank Ledger and balance is posted. Individual Debits are equivalent to individual Credits. On a generalizational and globalist scale, a “General Ledger” is written which records in all its wide ranging manifestation the Debits and Credits. This is also conservative. In other words Assets is equivalent to Liabilities. True, Profit is distributed among overheads and charges, and there shall be another account in the General ledger that is the account of Profit. This account is credited with the amount earned as commission, exchange, or discount of bills. Now when we write the “General Ledger” of Turing machine, the Primma Donna and terra firma of the model, we are writing the General Theory of all the variables that are incorporated in the model. So for every variable, we have an anti variable. This is the dissipation factor. Conservation laws do hold well in computers. They do not break the conservation laws. Thus energy is not dissipated in to the atmosphere when computation is being performed. To repeat we are suggesting a General Theory Of working of a simple Computer and in further papers, we want to extend this theory to both nanotechnology and Quantum Computation.

A finite table (occasionally called an action table or transition function) of instructions (usually quintuples [5-tuples]: qiaj→qi1aj1dk, but sometimes 4-tuples) that, given the state (qi) the machine is currently in and the symbol (aj) it is reading on the tape (symbol currently under the head) tells the machine to do the following in sequence (for the 5-tuple models):

Either **erase or write** a symbol (replacing aj with aj1), and then

Move the head (which is described by dk and can have values: 'L' for one step left or 'R' for one step right or 'N' for staying in the same place), and then

Assume the same or a new state as prescribed (go to state qi1).

In the 4-tuple models, erasing or writing a symbol (aj1) and moving the head left or right (dk) are specified as separate instructions. Specifically, the table tells the machine to (ia) erase or write a symbol or (ib) move the head **left or right**, and then (ii) assume the **same or a new state** as prescribed, **but not both actions** (ia) and (ib) in the same instruction. In some models, if there is no entry in the table for the current combination of symbol and state then the machine will halt; other models require all entries to be filled.

Note that every part of the machine—its state and symbol-collections—and its actions—printing, erasing and tape motion—is finite, discrete and distinguishable; **Only a virus can act as a predator to it.** It is the potentially unlimited amount of tape **that gives it an** unbounded amount of storage space.

Quantum mechanical Hamiltonian models of Turing machines are constructed here on a finite lattice of spin-½ systems. The **models do not** dissipate any energy and they operate at the quantum limit in that the system (energy uncertainty) / (computation speed) is close to the limit given by the time-energy uncertainty principle.

Regarding finite state machines as Markov chains **facilitates the** application of probabilistic methods to very large logic synthesis and formal verification problems.Variational concepts and exegetic evanescence of the subject matter is done by Hachtel, G.D. Macii, E. ; Pardo, A. ; Somenzi, F. with symbolic algorithms to compute the steady-state probabilities for very large finite state machines (up to 1027 states). These algorithms**, based on** Algebraic Decision Diagrams (ADD's) **-an extension of** BDD's **that allows** arbitrary values **to be associated** with the terminal nodes of the diagrams-**determine the** steady-state probabilities by regarding finite state machines as homogeneous, discrete-parameter Markov chains with finite state spaces, and by solving the corresponding Chapman-Kolmogorov equations. Finite state machines with state graphs **composed of a** single terminal strongly connected component systems authors **have used** two solution techniques: One is based on the Gauss-Jacobi iteration, the other one is based on simple matrix multiplication. Extension of the treatment is done to the most general case of systems which can be modeled as finite state machines with arbitrary **transition** structures; until a certain temporal point, having no relevant options **and effects for** the decision maker beyond that point. Structural morphology and easy decomposition is resorted to towards the consummation of results. Conservations Laws powerhouse performance and no breakage is done with heterogeneous synthesis of conditionalities. Accumulation. Formulation and experimentation are by word and watch word.

Logistics of misnomerliness and anathema:

In any scientific discipline there are many reasons **to use** terms that have precise definitions. Understanding the terminology of a discipline is **essential to** learning a subject and precise **terminology enables us to** communicate ideas clearly with other people. In computer science the problem is even more acute: we **need to construct** software and hardware components that must smoothly i**nteroperat**e across interfaces with clients and other components in distributed systems. The definitions of these interfaces need to be precisely specified for interoperability and good systems performance.

Using the term "computation" without qualification often **generates a** lot of confusion. Part of the problem is that the nature of systems **exhibiting** computational behavior is varied and the term computation means different things to different people depending on the kinds of computational systems they are studying and the kinds of problems they are investigating. Since computation refers to a process that **is defined** in terms of some underlying model of computation, we would achieve clearer communication if we made clear what the underlying model is.

Rather than talking about a vague notion of "computation," suggestion is to use the term in conjunction with a well-defined model of computation whose semantics is clear and which matches the problem being investigated. Computer science already has a number of useful clearly defined models of computation whose behaviors and capabilities are well understood. We should use such models as part of any definition of the term computation. However, for new domains of investigation where there are no appropriate models it may be necessary to invent new formalisms to represent the systems under study.

Courage of conviction and will for vindication:

We consider computational thinking to be the thought processes involved in formulating problems so their solutions can be represented as computational steps and algorithms. An important part of this process is finding appropriate models of computation with which to formulate the problem and derive its solutions. A familiar example would be the use of finite automata to solve string pattern matching problems. A less familiar example might be the quantum circuits and order finding formulation that Peter Schor used to devise an integer-factoring algorithm that runs in polynomial time on a quantum computer. Associated with the basic models of computation in computer science is a wealth of well-known algorithm-design and problem-solving techniques that can be used to solve common problems arising in computing.

However, as the computer systems we wish to build become more complex and as we apply computer science abstractions to new problem domains, we discover that we do not always have the appropriate models to devise solutions. In these cases, computational thinking becomes a research activity that includes inventing appropriate new models of computation.

Corrado Priami and his colleagues at the Centre for Computational and Systems Biology in Trento, Italy have been using process calculi as a model of computation to create programming languages to simulate biological processes. Priami states "the basic feature of computational thinking is abstraction of reality in such a way that the neglected details in the model make it executable by a machine." [Priami, 2007]

As we shall see, finding or devising appropriate models of computation to formulate problems is a central and often nontrivial part of computational thinking.

Hero or Zero?

In the last half century, what we think of as a computational system has expanded dramatically. In the earliest days of computing, a computer was an isolated machine with limited memory to which programs were submitted one at a time to be compiled and run. Today, in the Internet era, we have networks consisting of millions **of interconnected** computers and as we move into cloud computing, many foresee a global computing environment with billions of clients having universal on-demand access to computing services and **data hosted** in gigantic data centers located around the planet. Anything from a PC or a phone or a TV or a sensor can be a client and a data center may consist of hundreds of thousands of servers.

Needless to say, the models for studying such a universally accessible, complex, highly concurrent distributed system are very different from the ones for a single isolated computer. In fact, our aim is to build the model for infinite number of interconnected ness of computers.

Another force at play is that **because of** heat dissipation considerations the architecture of computers **is changing**. An ordinary PC today has many different computing elements such as multicore chips and graphics processing units, and an exascale supercomputer by the end of this decade is expected to be a giant parallel machine with up to a million nodes each with possibly a thousand processors. Our understanding of how to write efficient programs for these machines **is limited**. Good models of parallel computation and parallel algorithm design techniques are a vital open research area for effective parallel computing.

In addition, there is increasing interest in applying computation to studying virtually all areas of human endeavor. One fascinating example **is simulating the** highly parallel biological processes found in human cells and organs for the purposes of understanding disease and drug design. Good computational models for biological processes are still in their infancy. And it is not clear we will ever be able to find a computational model for the human brain that would account for emergent phenomena such **as consciousness or intelligence**.

Queen or show piece:

The theory of computation has been and still is one of the core areas of computer science. It explores the fundamental capabilities and limitations of models of computation. A model of computation is a mathematical **abstraction of a** computing system. The most important model of sequential computation studied in computer science is the Turing machine, first proposed by Alan Turing in 1936.

We can think of a Turing machine as a **finite-state control** attached to a tape head that can read and write symbols on the squares of a semi-infinite tape. Initially, a finite string of length n representing the input is in the leftmost n squares of the tape. An infinite sequence of blanks follows the input string. The tape head is reading the symbol in the leftmost square and the finite control is in a predefined initial state.

The Turing machine then makes a sequence of moves. In a move it reads the symbol on the tape under the tape head and **consults a** transition table in the finite-state control which specifies a symbol to be overprinted on the square under the tape head, a direction the tape head is to move (one square to the left or right), and a state **to enter** next. If the Turing machine **enters an** accepting halting state (one with no next move), the string of nonblank symbols remaining on the input tape at that point in time is its output.

Mathematically, a Turing machine **consists of seven components**: a finite set of states; a finite input alphabet (not containing the blank); a finite tape alphabet (which includes the input alphabet and the blank); a transition function that maps a state and a tape symbol into a state, tape symbol, and direction (left or right); a start state; **an accept state** from which there are no further moves; and a **reject state** from which there are no further moves.

We can characterize the **configuration o**f a Turing machine at a given moment in time by three quantities:

1. the state of the finite-state control,
2. the string of nonblank symbols on the tape, and
3. the location of the input head on the tape.

A computation of a Turing machine **on an input w** is a sequence of configurations the machine can go through starting from the initial configuration with w on the tape and terminating (if the computation terminates) in a halting configuration. We say a function f from strings to strings is computable if there is some Turing machine M that given any input string w **always halts in** the accepting state with just f (w) on its tape. We say that M computes f.

The Turing machine provides a precise **definition for** the term algorithm: an algorithm for a function f is just a Turing machine that computes f.

There are scores of models of computation that are equivalent to Turing machines in the sense that these models compute exactly the same set of functions that Turing machines can compute. Among these Turing-complete models of computation are **multitape Turing machines, lambda-calculus, random access machines, production systems, cellular automata, and all general-purpose programming languages**.

The reason there are so many different models of computation equivalent to Turing machines is that we rarely want to implement an algorithm as a Turing machine program; we would like to use a computational notation such as a programming language that is easy to write and easy to understand. But no matter what notation we choose, the famous Church-Turing thesis hypothesizes that any function that can be computed can be computed by a Turing machine.

Note that if there is one algorithm to compute a function f, then there is an infinite number. Much of computer science is devoted to finding efficient algorithms to compute a given function.

For clarity, we should point out that we have defined a computation as a sequence of configurations a Turing machine can go through on a given input. This sequence could be infinite if the machine does not halt or one of a number of possible sequences in case the machine is nondeterministic.

The reason we went through this explanation is to point out how much detail is involved in precisely defining the term computation for the Turing machine, one of the simplest models of computation. It is not surprising, then, as we move to more complex models, the amount of effort needed to precisely formulate computation in terms of those models grows substantially.

Sublime synthesis not dismal anchorage:

Many real-world computational systems compute more than just a single function—the world has moved to **interactive computing** [Goldin, Smolka, Wegner, 2006]. The term reactive system **is used to** describe a system that maintains an ongoing interaction with its environment. Examples of reactive systems include operating systems and embedded systems.

A distributed system is one that consists of autonomous computing systems that communicate with one another through some kind **of network using** message passing. Examples of distributed systems **include** telecommunications systems, the Internet, air-traffic control systems, and parallel computers. Many distributed systems are also reactive systems.

Perhaps the most intriguing examples of ***reactive*** distributed computing systems are biological systems such as cells and organisms. We could even consider the human brain to be ***a biological computing*** system. Formulation of appropriate models of computation for understanding biological processes is a formidable scientific challenge in the intersection of biology and computer science.

Distributed systems can ***exhibit*** behaviors such as deadlock, live lock, race conditions, and the like that cannot be usefully studied **using a** sequential model of computation. Moreover, solving problems such as determining the throughput, latency, and performance of a distributed system cannot be productively formulated with a single-thread model of computation. For these reasons, computer scientists **have developed a number** of models of concurrent computation which can **be used** to study these phenomena and to architect tools and components for building distributed systems. Many authors have studied these aspects in wider detail (See for example Alfred V. Aho),

There are many theoretical models for concurrent computation. One is the message-passing Actor model, consisting of computational entities called actors [Hewitt, Bishop, Steiger, 1973].

An actor can send and receive messages, make local decisions, create more actors, and fix the behavior to be used for the next message it receives. These actions may be executed in parallel and in no fixed order. The Actor model was devised to study the behavioral properties of parallel computing machines consisting of large numbers of independent processors **communicating by** passing messages through a network. Other well-studied models of concurrent computation include Petri nets and the process calculi such as pi-calculus and mu-calculus.

Many variants of computational models for distributed systems are being devised to study and understand the behaviors of biological systems. For example, Dematte, Priami, and Romanel [2008] describe a language called BlenX that is based on a process calculus called Beta-binders for modeling and simulating biological systems.

We do not have the space to describe these concurrent models in any detail. However, it is still an open research area to find practically useful concurrent models of computation that combine control and data for many areas of distributed computing.

Comprehensive envelope of expression not an identarian instance of semantic jugglery:

In addition to aiding education and understanding, there are many practical benefits to having appropriate models of computation for the systems we are trying to build. In cloud computing, for example, there are still a host of poorly understood concerns for systems of this scale. We need to better understand the architectural tradeoffs needed to achieve the desired levels of reliability, performance, scalability and adaptivity in the services these systems are expected to provide. We do not have appropriate abstractions to describe these properties in such a way that they can be automatically mapped from a model of computation into an implementation (or the other way around).

In cloud computing, there are a host of research challenges for system developers and tool builders. As examples, we need programming languages, compilers, verification tools, defect detection tools, and service management tools that can scale to the huge number of clients and servers involved in the networks and data centers of the future. Cloud computing is one important area that can benefit from innovative computational thinking.

The Finale:

Mathematical abstractions called models of computation are at the heart of computation and computational thinking. Computation is a process that is defined in terms of an underlying model of computation and computational thinking is the thought processes involved in formulating problems so their solutions can be represented as computational steps and algorithms. Useful models of computation for solving problems arising in sequential computation can range from

simple finite-state machines to Turing-complete models such as random access machines. Useful models of concurrent computation for solving problems arising in the design and analysis of complex distributed systems are still a subject of current research.

The P versus NP problem is to determine whether every language accepted by some nondeterministic algorithm in polynomial time is also accepted by some (deterministic) algorithm in polynomial time. To define the problem precisely it is necessary to give a formal model of a computer. The standard computer model in computability theory is the Turing machine, introduced by Alan Turing in 1936 [Tur36]. Although the model was introduced before physical computers were built, it nevertheless continues to be accepted as the proper computer model for the purpose of defining the notion of computable function.

Examples of Turing machines

3-state busy beaver

Formal definition

Hopcroft and Ullman (1979, p. 148) formally define a (one-tape) Turing machine as a 7-**tuple** $M = \langle Q, \Gamma, b, \Sigma, \delta, q_0, F \rangle$ where

Q Is a finite, non-empty set of states

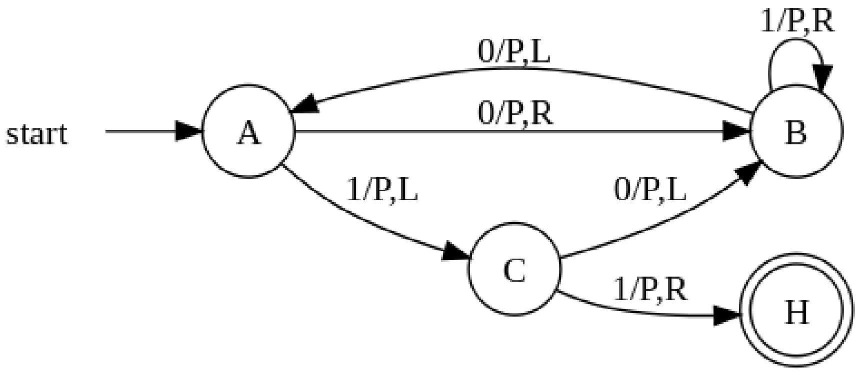
Γ Is a finite, non-empty set of the tape alphabet/symbols
$b \in \Gamma$ is the blank symbol (the only symbol allowed to occur on the tape infinitely often at any step during the computation)
$\Sigma \subseteq \Gamma \setminus \{b\}$ is the set of input symbols
$q_0 \in Q$ is the initial state
$F \subseteq Q$ is the set of final or accepting states.
$\delta : Q \setminus F \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$ is a partial function called the transition function, where L is left shift, R is right shift. (A relatively uncommon variant allows "no shift", say N, as a third element of the latter set.)
Anything that operates according to these specifications is a Turing machine.
The 7-tuple for the 3-state busy beaver looks like this (see more about this busy beaver at Turing machine examples):
$Q = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{HALT}\}$
$\Gamma = \{0, 1\}$
$b = 0$ ("Blank")
$\Sigma = \{1\}$
$q_0 = \mathrm{A}$ (the initial state)
$F = \{\mathrm{HALT}\}$
$\delta =$ see state-table below
Initially all tape cells are marked with 0.
State table for 3 state, 2 symbol busy beaver
Tape symbol-Current state A-Current state B-Current state C
-Write symbol-Move tape-Next state-Write symbol-Move tape-Next state-Write symbol-Move tape-Next state
0-1-R-B-1-L-A-1-L-B
1-1-L-C-1-R-B-1-R-HALT
In the words of van Emde Boas (1990), p. 6: "The set-theoretical object his formal seven-tuple description similar to the above] provides only partial information on how the machine will behave and what its computations will look like."
For instance,
There will need to be some decision on what the symbols actually look like, and a failproof way of reading and writing symbols indefinitely.
The shift left and shift right operations may shift the tape head across the tape, but when ***actually building a*** Turing machine it is more practical to make the tape slide back and forth under the head instead.
The tape can be finite, and ***automatically extended*** with blanks as needed (which is closest to the mathematical definition), but it is more common to think of it as stretching infinitely at both ends and being pre-filled with blanks except on the explicitly given finite fragment the tape head is on. (This is, of course, not implementable in practice.) The tape cannot be fixed in length, since that would not correspond to the given definition and would seriously limit the range of computations the machine can perform to those of alinear bounded automaton.

Contradictions and complementarities:
Definitions in literature sometimes differ slightly, to make arguments or proofs easier or clearer, but this is always done in such a way that the resulting machine has the same computational power. For example, changing the set $\{L, R\}$ to $\{L, R, N\}$, where N ("None" or "No-operation") would allow the machine to stay on the same tape cell instead of moving left or right, does not increase the machine's computational power.
The most common convention represents each "Turing instruction" in a "Turing table" by one of nine 5-tuples, per the convention of Turing/Davis (Turing (1936) in Undecidable, p. 126-127 and Davis (2000) p. 152):
(**Definition 1**): (qi, Sj, Sk/E/N, L/R/N, qm)
(Current state qi , symbol scanned Sj , print symbol Sk/erase E/none N , move_tape_one_square left L/right R/none N , new state qm)
Other authors (Minsky (1967) p. 119, Hopcroft and Ullman (1979) p. 158, Stone (1972) p. 9) adopt a different convention, with new state qm listed immediately after the scanned symbol Sj:
(**Definition 2):** (qi, Sj, qm, Sk/E/N, L/R/N)
(Current state qi , symbol scanned Sj , new state qm , print symbol Sk/erase E/none N , move_tape_one_square left L/right R/none N)
For the remainder of this article "definition 1" (the Turing/Davis convention) will be used.
Example: state table for the 3-state 2-symbol busy beaver reduced to 5-tuples
Current state-Scanned symbol--Print symbol-Move tape-Final (i.e. next) state-5-tuples
A-0--1-R-B-(A, 0, 1, R, B)
A-1--1-L-C-(A, 1, 1, L, C)
B-0--1-L-A-(B, 0, 1, L, A)
B-1--1-R-B-(B, 1, 1, R, B)

C-0--1-L-B-(C, 0, 1, L, B)
C-1--1-N-H-(C, 1, 1, N, H)
In the following table, Turing's original model allowed only the first three lines that he called N1, N2, N3 (cf Turing in Undecidable, p. 126). He allowed for erasure of the "scanned square" by naming a 0th symbol S0 = "erase" or "blank", etc. However, he did not allow for non-printing, so every instruction-line includes "print symbol Sk" or "erase" (cf footnote 12 in Post (1947), Undecidable p. 300). The abbreviations are Turing's (Undecidable p. 119). Subsequent to Turing's original paper in 1936–1937, machine-models have allowed all nine possible types of five-tuples:
-Current m-configuration (Turing state)-Tape symbol-Print-operation-Tape-motion-Final m-configuration (Turing state)-5-tuple-5-tuple comments-4-tuple
N1-qi-Sj-Print(Sk)-Left L-qm-(qi, Sj, Sk, L, qm)-"blank" = S0, 1=S1, etc.-
N2-qi-Sj-Print(Sk)-Right R-qm-(qi, Sj, Sk, R, qm)-"blank" = S0, 1=S1, etc.-
N3-qi-Sj-Print(Sk)-None N-qm-(qi, Sj, Sk, N, qm)-"blank" = S0, 1=S1, etc.-(qi, Sj, Sk, qm)
4-qi-Sj-None N-Left L-qm-(qi, Sj, N, L, qm)--(qi, Sj, L, qm)
5-qi-Sj-None N-Right R-qm-(qi, Sj, N, R, qm)--(qi, Sj, R, qm)
6-qi-Sj-None N-None N-qm-(qi, Sj, N, N, qm)-Direct "jump"-(qi, Sj, N, qm)
7-qi-Sj-Erase-Left L-qm-(qi, Sj, E, L, qm)--
8-qi-Sj-Erase-Right R-qm-(qi, Sj, E, R, qm)--
9-qi-Sj-Erase-None N-qm-(qi, Sj, E, N, qm)--(qi, Sj, E, qm)
Any Turing table (list of instructions) can be constructed from the above nine 5-tuples. For technical reasons, the three non-printing or "N" instructions (4, 5, 6) can usually be dispensed with. For examples see Turing machine examples.
Less frequently the use of 4-tuples is encountered: these represent a further atomization of the Turing instructions (cf Post (1947), Boolos & Jeffrey (1974, 1999), Davis-Sigal-Weyuker (1994)); also see more at Post–Turing machine.

The "state"

The word "state" used in context of Turing machines can be a source of confusion, as it can mean two things. Most commentators after Turing have used "state" to mean the name/designator of the current instruction to be performed—i.e. the contents of the state register. But Turing (1936) made a strong distinction between a record of what he called the machine's "m-configuration", (its internal state) and the machine's (or person's) "state of progress" through the computation - the current state of the total system. What Turing called "the state formula" includes both the current instruction and all the symbols on the tape:
Thus the state of progress of the computation at any stage is completely determined by the note of instructions and the symbols on the tape. That is, the state of the system may be described by a single expression (sequence of symbols) consisting of the symbols on the tape followed by Δ (which we suppose not to appear elsewhere) and then by the note of instructions. This expression is called the 'state formula'.
—Undecidable, p.139–140, emphasis added
Earlier in his paper Turing carried this even further: he gives an example where he places a symbol of the current "m-configuration"—the instruction's label—beneath the scanned square, together with all the symbols on the tape (Undecidable, p. 121); this he calls "the complete configuration" (Undecidable, p. 118). To print the "complete configuration" on one line he places the state-label/m-configuration to the left of the scanned symbol.
A variant of this is seen in Kleene (1952) where Kleene shows how to write the Gödel number of a machine's "situation": he places the "m-configuration" symbol q4 over the scanned square in roughly the center of the 6 non-blank squares on the tape (see the Turing-tape figure in this article) and puts it to the right of the scanned square. But Kleene refers to "q4" itself as "the machine state" (Kleene, p. 374-375). Hopcroft and Ullman call this composite the "instantaneous description" and follow the Turing convention of putting the "current state" (instruction-label, m-configuration) to the left of the scanned symbol (p. 149).
Example: total state of 3-state 2-symbol busy beaver after 3 "moves" (taken from example "run" in the figure below):
1A1
This means: after three moves the tape has ... 000110000 ... on it, the head is scanning the right-most 1, and the state is A. Blanks (in this case represented by "0"s) can be part of the total state as shown here: B01 ; the tape has a single 1 on it, but the head is scanning the 0 ("blank") to its left and the state is B.
"State" in the context of Turing machines should be clarified as to which is being described: (i) the current instruction, or (ii) the list of symbols on the tape together with the current instruction, or (iii) the list of symbols on the tape together with the current instruction placed to the left of the scanned symbol or to the right of the scanned symbol.
Turing's biographer Andrew Hodges (1983: 107) has noted and discussed this confusion.
Turing machine "state" diagrams
The table for the 3-state busy beaver ("P" = print/write a "1")
Tape symbol-Current state A-Current state B-Current state C
-Write symbol-Move tape-Next state-Write symbol-Move tape-Next state-Write symbol-Move tape-Next state
0-P-R-B-P-L-A-P-L-B
1-P-L-C-P-R-B-P-R-HALT



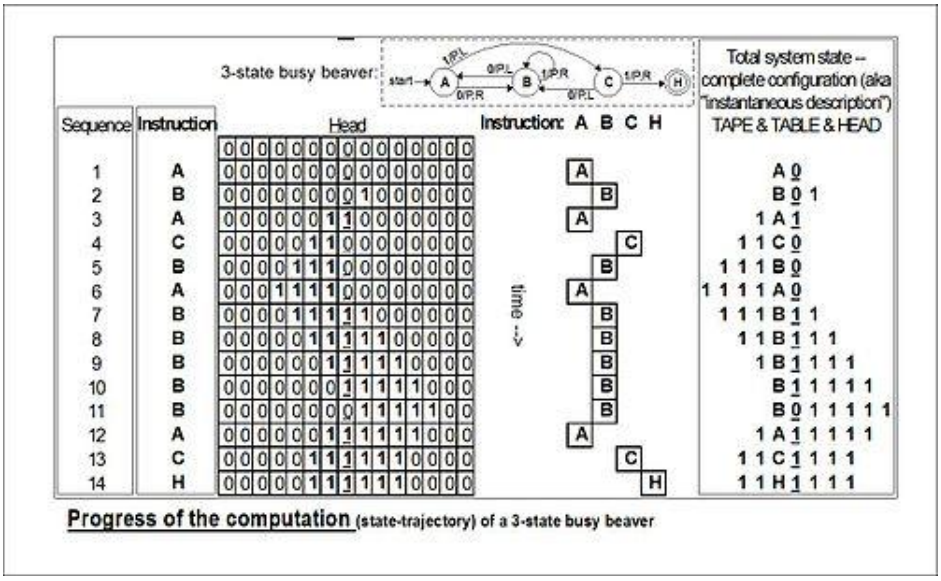
The "3-state busy beaver" Turing machine in a finite state representation. Each circle represents a "state" of the TABLE—an "m-configuration" or "instruction". "Direction" of a state transition is shown by an arrow. The label (e.g. 0/P, R) near the outgoing state (at the "tail" of the arrow) specifies the scanned symbol that causes a particular transition (e.g. 0) followed by a slash /, followed by the subsequent "behaviors" of the machine, e.g. "P Print" then move tape "R Right". No general accepted format exists. The convention shown is after McClusky (1965), Booth (1967), Hill and Peterson (1974).

To the right: the above TABLE as expressed as a "state transition" diagram.

Usually large TABLES are better left as tables (Booth, p. 74). They are more readily simulated by computer in tabular form (Booth, p. 74). However, certain concepts—e.g. machines with "reset" states and machines with repeating patterns (cf Hill and Peterson p. 244ff)—can be more readily seen when viewed as a drawing.

Whether a drawing represents an improvement on its TABLE must be decided by the reader for the particular context.

See Finite state machine for more.



The evolution of the busy-beaver's computation starts at the top and proceeds to the bottom.

The reader should again be cautioned that such diagrams represent a snapshot of their TABLE frozen in time, not the course ("trajectory") of a computation through time and/or space. While every time the busy beaver machine "runs" it will always follow the same state-trajectory, this is not true for the "copy" machine that can be provided with variable input "parameters".

The diagram "Progress of the computation" shows the 3-state busy beaver's "state" (instruction) progress through its computation from start to finish. On the far right is the Turing "complete configuration" (Kleene "situation", Hopcroft–Ullman "instantaneous description") at each step. If the machine were to be stopped and cleared to blank both the "state register" and entire tape, these "configurations" could be used to rekindle a computation anywhere in its progress (cf Turing (1936) Undecidable pp. 139–140).

Register machine,

Machines that might be thought to have more computational capability than a simple universal Turing machine can be shown to have no more power (Hopcroft and Ullman p. 159, cf Minsky (1967)). They might compute faster, perhaps, or use less memory, or their instruction set might be smaller, but they cannot compute more powerfully (i.e. more mathematical functions). (Recall that the Church–Turing thesis hypothesizes this to be true for any kind of machine: that anything that can be "computed" can be computed by some Turing machine.)

A Turing machine is equivalent to a pushdown automaton that has been made more flexible and concise by relaxing the last-in-first-out requirement of its stack.
At the other extreme, some very simple models turn out to be Turing-equivalent, i.e. to have the same computational power as the Turing machine model.
Common equivalent models are the multi-tape Turing machine, multi-track Turing machine, machines with input and output, and the non-deterministic Turing machine(NDTM) as opposed to the deterministic Turing machine (DTM) for which the action table has at most one entry for each combination of symbol and state.
Read-only, right-moving Turing machines are equivalent to NDFAs (as well as DFAs by conversion using the NDFA to DFA conversion algorithm).
For practical and didactical intentions the equivalent register machine can be used as a usual assembly programming language.

Choice c-machines, Oracle o-machines

Early in his paper (1936) Turing makes a distinction between an "automatic machine"—its "motion ... completely determined by the configuration" and a "choice machine":
...whose motion is only partially determined by the configuration ... When such a machine reaches one of these ambiguous configurations; it cannot go on until some arbitrary choice has been made by an external operator. This would be the case if we were using machines to deal with axiomatic systems.
—Undecidable, p. 118
Turing (1936) does not elaborate further except in a footnote in which he describes how to use an a-machine to "find all the provable formulae of the [Hilbert] calculus" rather than use a choice machine. He "supposes[s] that the choices are always between two possibilities 0 and 1. Each proof will then be determined by a sequence of choices i1, i2, ..., in (i1 = 0 or 1, i2 = 0 or 1, ..., in = 0 or 1), and hence the number 2n + i12n-1 + i22n-2 + ... +in completely determines the proof. The automatic machine carries out successively proof 1, proof 2, proof 3, ..." (Footnote ‡, Undecidable, p. 138)
This is indeed the technique by which a deterministic (i.e. a-) Turing machine can be used to mimic the action of a nondeterministic Turing machine; Turing solved the matter in a footnote and appears to dismiss it from further consideration.
An oracle machine or o-machine is a Turing a-machine that pauses its computation at state "o" while, to complete its calculation, it "awaits the decision" of "the oracle"—an unspecified entity "apart from saying that it cannot be a machine" (Turing (1939), Undecidable p. 166–168). The concept is now actively used by mathematicians.

Universal Turing machines

As Turing wrote in Undecidable, p. 128 (italics added):
It is possible to invent a single machine which can be used to compute any computable sequence. If this machine U is supplied with the tape on the beginning of which is written the string of quintuples separated by semicolons of some computing machine M, then U will compute the same sequence as M.
This finding is now taken for granted, but at the time (1936) it was considered astonishing. The model of computation that Turing called his "universal machine"—"U" for short—is considered by some (cf Davis (2000)) to have been the fundamental theoretical breakthrough that led to the notion of the Stored-program computer.
Turing's paper ... contains, in essence, the invention of the modern computer and some of the programming techniques that accompanied it.
—Minsky (1967), p. 104
In terms of computational complexity, a multi-tape universal Turing machine need only be slower by logarithmic factor compared to the machines it simulates. This result was obtained in 1966 by F. C. Hennie and R. E. Stearns. (Arora and Barak, 2009, theorem 1.9)

Comparison with real machines

It is often said that Turing machines, unlike simpler automata, are as powerful as real machines, and are able to execute any operation that a real program can. What is missed in this statement is that, because a real machine can only be in finitely many configurations, in fact this "real machine" is nothing but a linear bounded automaton. On the other hand, Turing machines **are equivalent to** machines that have an unlimited amount of storage space for their computations. In fact, Turing machines are not intended to model computers, but rather they are intended to model computation itself; historically, computers, which compute only on their (fixed) internal storage, were developed only later.
There are a number of ways to explain why Turing machines are useful models of real computers:
Anything a real computer can compute, a Turing machine can also compute. For example: "A Turing machine can simulate any type of subroutine found in programming languages, including recursive procedures and any of the known parameter-passing mechanisms" (Hopcroft and Ullman p. 157). A large enough FSA can also model any real computer, disregarding IO. Thus, a statement about the limitations of Turing machines will also apply to real computers.
The difference lies only with the ability of a Turing machine to manipulate an unbounded amount of data. However, given a finite amount of time, a Turing machine (like a real machine) can only manipulate a finite amount of data.
Like a Turing machine, a real machine can have its storage space enlarged as needed, by acquiring more disks or other storage media. If the supply of these runs short, the Turing machine may become less useful as a model. But the fact is that

neither Turing machines nor real machines need astronomical amounts of storage space in order to perform useful computation. The processing time required is usually much more of a problem.
Descriptions of real machine programs using simpler abstract models are often much more complex than descriptions using Turing machines. For example, a Turing machine describing an algorithm may have a few hundred states, while the equivalent deterministic finite automaton on a given real machine has quadrillions. This makes the DFA representation infeasible to analyze.
Turing machines describe algorithms independent of how much memory they use. There is a limit to the memory possessed by any current machine, but this limit can rise arbitrarily in time. Turing machines allow us to make statements about algorithms which will (theoretically) hold forever, regardless of advances in conventional computing machine architecture.
Turing machines simplify the statement of algorithms. Algorithms running on Turing-equivalent abstract machines are usually more general than their counterparts running on real machines, because they have arbitrary-precision data types available and never have to deal with unexpected conditions (including, but not limited to, running out of memory).
One way in which Turing machines are a poor model for programs is that many real programs, such as operating systems and word processors, are written to receive unbounded input over time, and therefore do not halt. Turing machines do not model such ongoing computation well (but can still model portions of it, such as individual procedures).

Computational complexity theory
A limitation of Turing machines is that they do not model the strengths of a particular arrangement well. For instance, modern stored-program computers are actually instances of a more specific form of abstract machine known as the random access stored program machine or RASP machine model. Like the Universal Turing machine the RASP stores its "program" in "memory" external to its finite-state machine's "instructions". Unlike the universal Turing machine, the RASP has an infinite number of distinguishable, numbered but unbounded "registers"—memory "cells" that can contain any integer (cf. Elgot and Robinson (1964), Hartmanis (1971), and in particular Cook-Rechow (1973); references at random access machine). The RASP's finite-state machine is equipped with the capability for indirect addressing (e.g. the contents of one register can be used as an address to specify another register); thus the RASP's "program" can address any register in the register-sequence. The upshot of this distinction is that there are computational optimizations that can be performed based on the memory indices, which are not possible in a general Turing machine; thus when Turing machines are used as the basis for bounding running times, a 'false lower bound' can be proven on certain algorithms' running times (due to the false simplifying assumption of a Turing machine). An example of this is binary search, an algorithm that can be shown to perform more quickly when using the RASP model of computation rather than the Turing machine model.

Concurrency
Another limitation of Turing machines is that they do not model concurrency well. For example, there is a bound on the size of integer that can be computed by an always-halting nondeterministic Turing machine starting on a blank tape. (See article on unbounded nondeterminism.) By contrast, there are always-halting concurrent systems with no inputs that can compute an integer of unbounded size. (A process can be created with local storage that is initialized with a count of 0 that concurrently sends itself both a stop and a go message. When it receives a go message, it increments its count by 1 and sends itself a go message. When it receives a stop message, it stops with an unbounded number in its local storage.)

"A" AND "B"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.

MODULE NUMBERED ONE
NOTATION :
G_{13} : CATEGORY ONE OF"A"
G_{14} : CATEGORY TWO OF"A"
G_{15} : CATEGORY THREE OF 'A'
T_{13} : CATEGORY ONE OF 'B'
T_{14} : CATEGORY TWO OF 'B'
T_{15} :CATEGORY THREE OF 'B'

"B" AND "A"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED TWO
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO.
==
G_{16} : CATEGORY ONE OF 'B' (NOTE THAT THEY REPRESENT CONFIGURATIONS,INSTRUCTIONS OR STATES)
G_{17} : CATEGORY TWO OF 'B'

G_{18} : CATEGORY THREE OF 'B'
T_{16} :CATEGORY ONE OF 'A'
T_{17} : CATEGORY TWO OF 'A'
T_{18} : CATEGORY THREE OF'A'

==

A" AND "C"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED THREE
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G_{20} : CATEGORY ONE OF'A'
G_{21} :CATEGORY TWO OF'A'
G_{22} : CATEGORY THREE OF'A'
T_{20} : CATEGORY ONE OF 'C'
T_{21} :CATEGORY TWO OF 'C'
T_{22} : CATEGORY THREE OF 'C'

"C" AND "B"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED FOUR
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO
:
==

G_{24} : CATEGORY ONE OF "C"(EVALUATIVE PARAMETRICIZATION OF SITUATIONAL ORIENTATIONS AND ESSENTIAL COGNITIVE ORIENTATION AND CHOICE VARIABLES OF THE SYSTEM TO WHICH CONFIGURATION IS APPLICABLE)
G_{25} : CATEGORY TWO OF "C"
G_{26} : CATEGORY THREE OF "C"
T_{24} :CATEGORY ONE OF "B"
T_{25} :CATEGORY TWO OF "B"(SYSTEMIC INSTRUMENTAL CHARACTERISATIONS AND ACTION ORIENTATIONS AND FUNCTIONAL IMPERATIVES OF CHANGE MANIFESTED THEREIN)
T_{26} : CATEGORY THREE OF"B"

"C" AND "H"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED FIVE
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO:
===

G_{28} : CATEGORY ONE OF "C"
G_{29} : CATEGORY TWO OF"C"
G_{30} :CATEGORY THREE OF "C"
T_{28} :CATEGORY ONE OF "H"
T_{29} :CATEGORY TWO OF "H"
T_{30} :CATEGORY THREE OF "H"
===

"B" AND "B"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
THE SYSTEM HERE IS ONE OF SELF TRANSFORMATIONAL,SYSTEM CHANGING,STRUCTURALLY MUTATIONAL,SYLLOGISTICALLY CHANGEABLE AND CONFIGURATIONALLY ALTERABLE(VERY VERY IMPORTANT SYSTEM IN ALMOST ALL SUBJECTS BE IT IN QUANTUM SYSTEMS OR DISSIPATIVE STRUCTURES
MODULE NUMBERED SIX
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED

EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO:

==

G_{32} : CATEGORY ONE OF "B"
G_{33} : CATEGORY TWO OF "B"
G_{34} : CATEGORY THREE OF"B"
INTERACTS WITH:*ITSELF*:
T_{32} : CATEGORY ONE OF "B"
T_{33} : CATEGORY TWO OF"B"
T_{34} : CATEGORY THREE OF "B"

"INPUT" AND "A"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED SEVEN
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO:

==

G_{36} : CATEGORY ONE OF "INPUT"
G_{37} : CATEGORY TWO OF "INPUT"
G_{38} : CATEGORY THREE OF"INPUT" (INPUT FEEDING AND CONCOMITANT GENERATION OF ENERGY DIFFERENTIAL-TIME LAG OR INSTANTANEOUSNESSMIGHT EXISTS WHEREBY ACCENTUATION AND ATTRITIONS MODEL MAY ASSUME ZERO POSITIONS)
T_{36} : CATEGORY ONE OF "A"
T_{37} : CATEGORY TWO OF "A"
T_{38} : CATEGORY THREE OF"A"

==

$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}, (b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$ $(a_{16})^{(2)}, (a_{17})^{(2)}, (a_{18})^{(2)}$ $(b_{16})^{(2)}, (b_{17})^{(2)}, (b_{18})^{(2)}$:
$(a_{20})^{(3)}, (a_{21})^{(3)}, (a_{22})^{(3)}$, $(b_{20})^{(3)}, (b_{21})^{(3)}, (b_{22})^{(3)}$
$(a_{24})^{(4)}, (a_{25})^{(4)}, (a_{26})^{(4)}, (b_{24})^{(4)}, (b_{25})^{(4)}, (b_{26})^{(4)}, (b_{28})^{(5)}, (b_{29})^{(5)}, (b_{30})^{(5)}, (a_{28})^{(5)}, (a_{29})^{(5)}, (a_{30})^{(5)}$,
$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}, (b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$
are Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}, (b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}, (a'_{16})^{(2)}, (a'_{17})^{(2)}, (a'_{18})^{(2)}$, $(b'_{16})^{(2)}, (b'_{17})^{(2)}, (b'_{18})^{(2)}$
, $(a'_{20})^{(3)}, (a'_{21})^{(3)}, (a'_{22})^{(3)}, (b'_{20})^{(3)}, (b'_{21})^{(3)}, (b'_{22})^{(3)}$
$(a'_{24})^{(4)}, (a'_{25})^{(4)}, (a'_{26})^{(4)}, (b'_{24})^{(4)}, (b'_{25})^{(4)}, (b'_{26})^{(4)}, (b'_{28})^{(5)}, (b'_{29})^{(5)}, (b'_{30})^{(5)}$ $(a'_{28})^{(5)}, (a'_{29})^{(5)}, (a'_{30})^{(5)}$,
$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}, (b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$

are Dissipation coefficients-

"A" AND "B"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.

MODULE NUMBERED ONE

The differential system of this model is now (Module Numbered one)-1
$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t)\right]G_{13}$ -2
$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t)\right]G_{14}$ -3
$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t)\right]G_{15}$ -4
$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)} - (b''_{13})^{(1)}(G,t)\right]T_{13}$ -5
$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)} - (b''_{14})^{(1)}(G,t)\right]T_{14}$ -6
$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)} - (b''_{15})^{(1)}(G,t)\right]T_{15}$ -7
$+(a''_{13})^{(1)}(T_{14},t) =$ First augmentation factor -8
$-(b''_{13})^{(1)}(G,t) =$ First detritions factor-

"B" AND "A"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.

MODULE NUMBERED TWO

NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO.

The differential system of this model is now (Module numbered two)-9

$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17},t)\right]G_{16}$ -10

$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17},t)\right]G_{17}$ -11

$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17},t)\right]G_{18}$ -12

$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)} - (b''_{16})^{(2)}\left((G_{19}),t\right)\right]T_{16}$ -13

$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)} - (b''_{17})^{(2)}\left((G_{19}),t\right)\right]T_{17}$ -14

$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)} - (b''_{18})^{(2)}\left((G_{19}),t\right)\right]T_{18}$ -15

$+(a''_{16})^{(2)}(T_{17},t) =$ First augmentation factor -16

$-(b''_{16})^{(2)}\left((G_{19}),t\right) =$ First detritions factor -17

:

==**A" AND "C"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.**

MODULE NUMBERED THREE

The differential system of this model is now (Module numbered three)-18

$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right]G_{20}$ -19

$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21},t)\right]G_{21}$ -20

$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21},t)\right]G_{22}$ -21

$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23},t)\right]T_{20}$ -22

$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23},t)\right]T_{21}$ -23

$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23},t)\right]T_{22}$ -24

$+(a''_{20})^{(3)}(T_{21},t) =$ First augmentation factor-

$-(b''_{20})^{(3)}(G_{23},t) =$ First detritions factor -25

"C" AND "B"(SEE FIGURE REPRESENTS AN "M CONFIGURATION" OR "INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.

MODULE NUMBERED FOUR

NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO

The differential system of this model is now (Module numbered Four)-26

$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right]G_{24}$ -27

$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\left(a'_{25}\right)^{(4)} + \left(a''_{25}\right)^{(4)}(T_{25},t)\right] G_{25}$ -28

$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25},t)\right]G_{26}$ -29

$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \left[(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27}),t\right)\right]T_{24}$ -30

$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \left[\left(b'_{25}\right)^{(4)} - \left(b''_{25}\right)^{(4)}\left((G_{27}),t\right)\right] T_{25}$ -31

$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \left[(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27}),t\right)\right]T_{26}$ -32

$+(a''_{24})^{(4)}(T_{25},t) =$ **First augmentation factor**-33

$-(b''_{24})^{(4)}\left((G_{27}),t\right) =$ **First detritions factor** -34

“C” AND “H”(SEE FIGURE REPRESENTS AN “M CONFIGURATION” OR “INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED FIVE
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO

The differential system of this model is now (Module number five)-35

$\frac{dG_{28}}{dt} = (a_{28})^{(5)} G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29}, t) \right] G_{28}$ -36

$\frac{dG_{29}}{dt} = (a_{29})^{(5)} G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29}, t) \right] G_{29}$ -37

$\frac{dG_{30}}{dt} = (a_{30})^{(5)} G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29}, t) \right] G_{30}$ -38

$\frac{dT_{28}}{dt} = (b_{28})^{(5)} T_{29} - \left[(b'_{28})^{(5)} - (b''_{28})^{(5)}\left((G_{31}), t\right) \right] T_{28}$ -39

$\frac{dT_{29}}{dt} = (b_{29})^{(5)} T_{28} - \left[(b'_{29})^{(5)} - (b''_{29})^{(5)}\left((G_{31}), t\right) \right] T_{29}$ -40

$\frac{dT_{30}}{dt} = (b_{30})^{(5)} T_{29} - \left[(b'_{30})^{(5)} - (b''_{30})^{(5)}\left((G_{31}), t\right) \right] T_{30}$ -41

$+(a''_{28})^{(5)}(T_{29}, t) =$ **First augmentation factor** -42

$-(b''_{28})^{(5)}\left((G_{31}), t\right) =$ **First detritions factor** -43

B” AND “B”(SEE FIGURE REPRESENTS AN “M CONFIGURATION” OR “INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
THE SYSTEM HERE IS ONE OF SELF TRANSFORMATIONAL,SYSTEM CHANGING,STRUCTURALLY MUTATIONAL,SYLLOGISTICALLY CHANGEABLE AND CONFIGURATIONALLY ALTERABLE(VERY VERY IMPORTANT SYSTEM IN ALMOST ALL SUBJECTS BE IT IN QUANTUM SYSTEMS OR DISSIPATIVE STRUCTURES
MODULE NUMBERED SIX
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO
:

The differential system of this model is now (Module numbered Six)-44

45

$\frac{dG_{32}}{dt} = (a_{32})^{(6)} G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33}, t) \right] G_{32}$ -46

$\frac{dG_{33}}{dt} = (a_{33})^{(6)} G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33}, t) \right] G_{33}$ -47

$\frac{dG_{34}}{dt} = (a_{34})^{(6)} G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33}, t) \right] G_{34}$ -48

$\frac{dT_{32}}{dt} = (b_{32})^{(6)} T_{33} - \left[(b'_{32})^{(6)} - (b''_{32})^{(6)}\left((G_{35}), t\right) \right] T_{32}$ -49

$\frac{dT_{33}}{dt} = (b_{33})^{(6)} T_{32} - \left[(b'_{33})^{(6)} - (b''_{33})^{(6)}\left((G_{35}), t\right) \right] T_{33}$ -50

$\frac{dT_{34}}{dt} = (b_{34})^{(6)} T_{33} - \left[(b'_{34})^{(6)} - (b''_{34})^{(6)}\left((G_{35}), t\right) \right] T_{34}$ -51

$+(a''_{32})^{(6)}(T_{33}, t) =$ **First augmentation factor**-52

“INPUT” AND “A”(SEE FIGURE REPRESENTS AN “M CONFIGURATION” OR “INSTRUCTIONS) OR STATE: THE CONVENTION SHOWN IS AFTER MCCLUSKY,BOOTH,HILL AND PETERSON.
MODULE NUMBERED SEVEN
NOTE: THE ACCENTUATION COEFFICIENT AND DISSIPATION COEFFICIENT NEED NOT BE THE SAME ,IT MAY BE ZERO, OR MIGHT BE SAME,I ALL THE THREE CASES THE MODEL COULD BE CHANGED EAILY BY REPLACING THE COEFFICIENTS BY EQUALITY SIGN OR GIVING IT THE POSITION OF ZERO
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:

The differential system of this model is now (SEVENTH MODULE)
-53

$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) \right] G_{36}$ -54

$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t) \right] G_{37}$ -55

$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37},t)\right]G_{38}$ -56

$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - \left[(b'_{36})^{(7)} - (b''_{36})^{(7)}\big((G_{39}),t\big)\right]T_{36}$ -57

$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - \left[(b'_{37})^{(7)} - (b''_{37})^{(7)}\big((G_{39}),t\big)\right]T_{37}$ -58

59

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b'_{38})^{(7)} - (b''_{38})^{(7)}\big((G_{39}),t\big)\right]T_{38}$ -60

$+(a''_{36})^{(7)}(T_{37},t) =$ **First augmentation factor** -61

$-(b''_{36})^{(7)}\big((G_{39}),t\big) =$ **First detritions factor**

FIRST MODULE CONCATENATION:

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[\begin{matrix} (a'_{13})^{(1)} \boxed{+(a''_{13})^{(1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7)}(T_{37},t)} \end{matrix}\right] G_{13}$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[\begin{matrix} (a'_{14})^{(1)} \boxed{+(a''_{14})^{(1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7)}(T_{37},t)} \end{matrix}\right] G_{14}$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\begin{matrix} (a'_{15})^{(1)} \boxed{+(a''_{15})^{(1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7)}(T_{37},t)} \end{matrix}\right] G_{15}$$

Where $\boxed{(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{(a''_{15})^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{16})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,)}(T_{17},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{36})^{(7)}(T_{37},t)} \boxed{+(a''_{37})^{(7)}(T_{37},t)} \boxed{+(a''_{38})^{(7)}(T_{37},t)}$ ARESEVENTHAUGMENTATION COEFFICIENTS

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[\begin{matrix} (b'_{13})^{(1)} \boxed{-(b''_{16})^{(1)}(G,t)} \boxed{-(b''_{36})^{(7,)}(G_{39},t)} \boxed{-(b''_{20})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{36})^{(7,)}(G_{39},t)} \end{matrix}\right] T_{13}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[\begin{matrix} (b'_{14})^{(1)} \boxed{-(b''_{14})^{(1)}(G,t)} \boxed{-(b''_{17})^{(2,2,)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{37})^{(7,)}(G_{39},t)} \end{matrix}\right] T_{14}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\begin{matrix} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \\ \boxed{-(b''_{38})^{(7,)}(G_{39},t)} \end{matrix}\right] T_{15}$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS

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$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \begin{bmatrix} (b'_{15})^{(1)} \boxed{-(b''_{15})^{(1)}(G,t)} \boxed{-(b''_{18})^{(2,2,)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,)}(G_{23},t)} \\ \boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)} \end{bmatrix} T_{15} \quad -63$$

Where $\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-(b''_{15})^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3

$\boxed{-(b''_{16})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,)}(G_{19},t)}$ are second detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{20})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,)}(G_{23},t)}$ are third detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{24})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,4,)}(G_{27},t)}$ are fourth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{28})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5,5,)}(G_{31},t)}$ are fifth detritions coefficients for category 1, 2 and 3

$\boxed{-(b''_{32})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6,6,)}(G_{35},t)}$ are sixth detritions coefficients for category 1, 2 and 3 *-64*

<u>SECOND MODULE CONCATENATION</u>:-

$$65\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \begin{bmatrix} (a'_{16})^{(2)} \boxed{+(a''_{16})^{(2)}(T_{17},t)} \boxed{+(a''_{13})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{36})^{(7,7,)}(T_{37},t)} \end{bmatrix} G_{16} \quad -66$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \begin{bmatrix} (a'_{17})^{(2)} \boxed{+(a''_{17})^{(2)}(T_{17},t)} \boxed{+(a''_{14})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{37})^{(7,7,)}(T_{37},t)} \end{bmatrix} G_{17} \quad -67$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \begin{bmatrix} (a'_{18})^{(2)} \boxed{+(a''_{18})^{(2)}(T_{17},t)} \boxed{+(a''_{15})^{(1,1,)}(T_{14},t)} \boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)} \boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)} \boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{38})^{(7,7,)}(T_{37},t)} \end{bmatrix} G_{18} \quad -68$$

Where $\boxed{+(a''_{16})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2)}(T_{17},t)}$ are first augmentation coefficients for category 1, 2 and 3

$\boxed{+(a''_{13})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,)}(T_{14},t)}$ are second augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{20})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3)}(T_{21},t)}$ are third augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{24})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,4,4)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,4,4)}(T_{25},t)}$ are fourth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{28})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5,5,5)}(T_{29},t)}$ are fifth augmentation coefficient for category 1, 2 and 3

$\boxed{+(a''_{32})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6,6,6)}(T_{33},t)}$ are sixth augmentation coefficient for category 1, 2 and 3 *-69*

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$+(a''_{36})^{(7,7,)}(T_{37},t)$ $+(a''_{37})^{(7,7,)}(T_{37},t)$ $+(a''_{38})^{(7,7,)}(T_{37},t)$ ARE SEVENTH DETRITION COEFFICIENTS-*71*

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[\begin{matrix} (b'_{16})^{(2)} -(b''_{16})^{(2)}(G_{19},t) & -(b''_{13})^{(1,1,)}(G,t) & -(b''_{20})^{(3,3,3,)}(G_{23},t) \\ -(b''_{24})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{28})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{32})^{(6,6,6,6,6)}(G_{35},t) \\ & -(b''_{36})^{(7,7)}(G_{39},t) & \end{matrix} \right] T_{16} \quad -72$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[\begin{matrix} (b'_{17})^{(2)} -(b''_{17})^{(2)}(G_{19},t) & -(b''_{14})^{(1,1,)}(G,t) & -(b''_{21})^{(3,3,3,)}(G_{23},t) \\ -(b''_{25})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{29})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{33})^{(6,6,6,6,6)}(G_{35},t) \\ & -(b''_{37})^{(7,7)}(G_{39},t) & \end{matrix} \right] T_{17} \quad -73$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[\begin{matrix} (b'_{18})^{(2)} -(b''_{18})^{(2)}(G_{19},t) & -(b''_{15})^{(1,1,)}(G,t) & -(b''_{22})^{(3,3,3,)}(G_{23},t) \\ -(b''_{26})^{(4,4,4,4,4)}(G_{27},t) & -(b''_{30})^{(5,5,5,5,5)}(G_{31},t) & -(b''_{34})^{(6,6,6,6,6)}(G_{35},t) \\ & -(b''_{38})^{(7,7)}(G_{39},t) & \end{matrix} \right] T_{18} \quad -74$$

where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$ are first detrition coefficients for category 1, 2 and 3

$-(b''_{13})^{(1,1,)}(G,t)$, $-(b''_{14})^{(1,1,)}(G,t)$, $-(b''_{15})^{(1,1,)}(G,t)$ are second detrition coefficients for category 1,2 and 3

$-(b''_{20})^{(3,3,3,)}(G_{23},t)$, $-(b''_{21})^{(3,3,3,)}(G_{23},t)$, $-(b''_{22})^{(3,3,3,)}(G_{23},t)$ are third detrition coefficients for category 1,2 and 3

$-(b''_{24})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4)}(G_{27},t)$ are fourth detritions coefficients for category 1,2 and 3

$-(b''_{28})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5)}(G_{31},t)$ are fifth detritions coefficients for category 1,2 and 3

$-(b''_{32})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6)}(G_{35},t)$ are sixth detritions coefficients for category 1,2 and 3

$-(b''_{36})^{(7,7)}(G_{39},t)$ $-(b''_{36})^{(7,7)}(G_{39},t)$ $-(b''_{36})^{(7,7)}(G_{39},t)$ *are seventh detrition coefficients*

THIRD MODULE CONCATENATION:-*75*

$$\frac{dG_{20}}{dt} = (a_{20})^{(3)}G_{21} - \left[\begin{matrix} (a'_{20})^{(3)} +(a''_{20})^{(3)}(T_{21},t) & +(a''_{16})^{(2,2,2)}(T_{17},t) & +(a''_{13})^{(1,1,1,)}(T_{14},t) \\ +(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{36})^{(7.7.7.)}(T_{37},t) & \end{matrix} \right] G_{20} \quad -76$$

$$\frac{dG_{21}}{dt} = (a_{21})^{(3)}G_{20} - \left[\begin{matrix} (a'_{21})^{(3)} +(a''_{21})^{(3)}(T_{21},t) & +(a''_{17})^{(2,2,2)}(T_{17},t) & +(a''_{14})^{(1,1,1,)}(T_{14},t) \\ +(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{37})^{(7.7.7.)}(T_{37},t) & \end{matrix} \right] G_{21} \quad -77$$

$$\frac{dG_{22}}{dt} = (a_{22})^{(3)}G_{21} - \left[\begin{matrix} (a'_{22})^{(3)} +(a''_{22})^{(3)}(T_{21},t) & +(a''_{18})^{(2,2,2)}(T_{17},t) & +(a''_{15})^{(1,1,1,)}(T_{14},t) \\ +(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t) & +(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t) & +(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t) \\ & +(a''_{38})^{(7.7.7.)}(T_{37},t) & \end{matrix} \right] G_{22} \quad -78$$

$+(a''_{20})^{(3)}(T_{21},t)$, $+(a''_{21})^{(3)}(T_{21},t)$, $+(a''_{22})^{(3)}(T_{21},t)$ are first augmentation coefficients for category 1, 2 and 3

$+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3

$+(a''_{13})^{(1,1,1,)}(T_{14},t)$, $+(a''_{14})^{(1,1,1,)}(T_{14},t)$, $+(a''_{15})^{(1,1,1,)}(T_{14},t)$ are third augmentation coefficients for category 1, 2 and 3

$+(a''_{24})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{25})^{(4,4,4,4,4,4)}(T_{25},t)$, $+(a''_{26})^{(4,4,4,4,4,4)}(T_{25},t)$ are fourth augmentation coefficients for category 1, 2 and 3

$+(a''_{28})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{29})^{(5,5,5,5,5,5)}(T_{29},t)$, $+(a''_{30})^{(5,5,5,5,5,5)}(T_{29},t)$ are fifth augmentation coefficients for category 1, 2 and 3

$+(a''_{32})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{33})^{(6,6,6,6,6,6)}(T_{33},t)$, $+(a''_{34})^{(6,6,6,6,6,6)}(T_{33},t)$ are sixth augmentation coefficients for category 1, 2 and 3 *-79*

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$+(a''_{36})^{(7.7.7.)}(T_{37},t)$ $+(a''_{37})^{(7.7.7.)}(T_{37},t)$ $+(a''_{38})^{(7.7.7.)}(T_{37},t)$ are seventh augmentation coefficient*-81*

$$\frac{dT_{20}}{dt} = (b_{20})^{(3)}T_{21} - \left[\begin{matrix}(b'_{20})^{(3)} -(b''_{20})^{(3)}(G_{23},t) - (b''_{36})^{(7,7,7)}(G_{19},t) - (b''_{13})^{(1,1,1,)}(G,t) \\ -(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t) - (b''_{28})^{(5,5,5,5,5,5)}(G_{31},t) - (b''_{32})^{(6,6,6,6,6,6)}(G_{35},t) \\ - (b''_{36})^{(7,7,7)}(G_{39},t)\end{matrix}\right] T_{20} \quad \text{-82}$$

$$\frac{dT_{21}}{dt} = (b_{21})^{(3)}T_{20} - \left[\begin{matrix}(b'_{21})^{(3)} -(b''_{21})^{(3)}(G_{23},t) - (b''_{17})^{(2,2,2)}(G_{19},t) - (b''_{14})^{(1,1,1,)}(G,t) \\ -(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t) - (b''_{29})^{(5,5,5,5,5,5)}(G_{31},t) - (b''_{33})^{(6,6,6,6,6,6)}(G_{35},t) \\ - (b''_{37})^{(7,7,7)}(G_{39},t)\end{matrix}\right] T_{21} \quad \text{-83}$$

$$\frac{dT_{22}}{dt} = (b_{22})^{(3)}T_{21} - \left[\begin{matrix}(b'_{22})^{(3)} -(b''_{22})^{(3)}(G_{23},t) - (b''_{18})^{(2,2,2)}(G_{19},t) - (b''_{15})^{(1,1,1,)}(G,t) \\ -(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t) - (b''_{30})^{(5,5,5,5,5,5)}(G_{31},t) - (b''_{34})^{(6,6,6,6,6,6)}(G_{35},t) \\ - (b''_{38})^{(7,7,7)}(G_{39},t)\end{matrix}\right] T_{22} \quad \text{-84}$$

$-(b''_{20})^{(3)}(G_{23},t)$, $-(b''_{21})^{(3)}(G_{23},t)$, $-(b''_{22})^{(3)}(G_{23},t)$ *are first detritions coefficients for category 1, 2 and 3*

$-(b''_{16})^{(2,2,2)}(G_{19},t)$, $-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$ *are second detritions coefficients for category 1, 2 and 3*

$-(b''_{13})^{(1,1,1,)}(G,t)$, $-(b''_{14})^{(1,1,1,)}(G,t)$, $-(b''_{15})^{(1,1,1,)}(G,t)$ *are third detrition coefficients for category 1,2 and 3*

$-(b''_{24})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{25})^{(4,4,4,4,4,4)}(G_{27},t)$, $-(b''_{26})^{(4,4,4,4,4,4)}(G_{27},t)$ *are fourth detritions coefficients for category 1, 2 and 3*

$-(b''_{28})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{29})^{(5,5,5,5,5,5)}(G_{31},t)$, $-(b''_{30})^{(5,5,5,5,5,5)}(G_{31},t)$ *are fifth detritions coefficients for category 1, 2 and 3*

$-(b''_{32})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{33})^{(6,6,6,6,6,6)}(G_{35},t)$, $-(b''_{34})^{(6,6,6,6,6,6)}(G_{35},t)$ *are sixth detritions coefficients for category 1, 2 and 3 -85*

$-(b''_{36})^{(7,7,7)}(G_{39},t)$ $-(b''_{37})^{(7,7,7)}(G_{39},t)$ $-(b''_{38})^{(7,7,7)}(G_{39},t)$ are seventh detritions coefficients

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FOURTH MODULE CONCATENATION:*-86*

$$\frac{dG_{24}}{dt} = (a_{24})^{(4)}G_{25} - \left[\begin{matrix}(a'_{24})^{(4)} +(a''_{24})^{(4)}(T_{25},t) +(a''_{28})^{(5,5,)}(T_{29},t) +(a''_{32})^{(6,6,)}(T_{33},t) \\ +(a''_{13})^{(1,1,1,1)}(T_{14},t) +(a''_{16})^{(2,2,2,2)}(T_{17},t) +(a''_{20})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{36})^{(7,7,7,7,)}(T_{37},t)\end{matrix}\right] G_{24} \quad \text{-87}$$

$$\frac{dG_{25}}{dt} = (a_{25})^{(4)}G_{24} - \left[\begin{matrix}(a'_{25})^{(4)} +(a''_{25})^{(4)}(T_{25},t) +(a''_{29})^{(5,5,)}(T_{29},t) +(a''_{33})^{(6,6)}(T_{33},t) \\ +(a''_{14})^{(1,1,1,1)}(T_{14},t) +(a''_{17})^{(2,2,2,2)}(T_{17},t) +(a''_{21})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{37})^{(7,7,7,7,)}(T_{37},t)\end{matrix}\right] G_{25} \quad \text{-88}$$

$$\frac{dG_{26}}{dt} = (a_{26})^{(4)}G_{25} - \left[\begin{matrix}(a'_{26})^{(4)} +(a''_{26})^{(4)}(T_{25},t) +(a''_{30})^{(5,5,)}(T_{29},t) +(a''_{34})^{(6,6,)}(T_{33},t) \\ +(a''_{15})^{(1,1,1,1)}(T_{14},t) +(a''_{18})^{(2,2,2,2)}(T_{17},t) +(a''_{22})^{(3,3,3,3)}(T_{21},t) \\ +(a''_{38})^{(7,7,7,7,)}(T_{37},t)\end{matrix}\right] G_{26} \quad \text{-89}$$

$Where$ $(a''_{24})^{(4)}(T_{25},t)$, $(a''_{25})^{(4)}(T_{25},t)$, $(a''_{26})^{(4)}(T_{25},t)$ $are\ first\ augmentation\ coefficients\ for\ category\ 1, 2\ and\ 3$

$+(a''_{28})^{(5,5,)}(T_{29},t)$, $+(a''_{29})^{(5,5,)}(T_{29},t)$, $+(a''_{30})^{(5,5,)}(T_{29},t)$ $are\ second\ augmentation\ coefficient\ for\ category\ 1, 2\ and\ 3$

$\boxed{+(a''_{32})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,)}(T_{33},t)}$ *are third augmentation coefficient for category* 1, 2 *and* 3

$\boxed{+(a''_{13})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{16})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1, 2,and 3

$\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}\boxed{+(a''_{36})^{(7,7,7,7,)}(T_{37},t)}$ARE SEVENTH augmentation coefficients-*90*

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-92

$$\frac{dT_{24}}{dt} = (b_{24})^{(4)}T_{25} - \begin{bmatrix} (b'_{24})^{(4)}\boxed{-(b''_{24})^{(4)}(G_{27},t)} \boxed{-(b''_{28})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{32})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,7,,,)}(G_{39},t)} \end{bmatrix} T_{24} \quad \text{-93}$$

$$\frac{dT_{25}}{dt} = (b_{25})^{(4)}T_{24} - \begin{bmatrix} (b'_{25})^{(4)}\boxed{-(b''_{25})^{(4)}(G_{27},t)} \boxed{-(b''_{29})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{33})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,77,,)}(G_{39},t)} \end{bmatrix} T_{25} \quad \text{-94}$$

$$\frac{dT_{26}}{dt} = (b_{26})^{(4)}T_{25} - \begin{bmatrix} (b'_{26})^{(4)}\boxed{-(b''_{26})^{(4)}(G_{27},t)} \boxed{-(b''_{30})^{(5,5,)}(G_{31},t)} \boxed{-(b''_{34})^{(6,6,)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,,)}(G_{39},t)} \end{bmatrix} T_{26} \quad \text{-95}$$

Where $\boxed{-(b''_{24})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4)}(G_{27},t)}$ *are first detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{28})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,)}(G_{31},t)}$ *are second detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{32})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,)}(G_{35},t)}$ *are third detrition coefficients for category* 1, 2 *and* 3

$\boxed{-(b''_{13})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1)}(G,t)}$
are fourth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{16})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2)}(G_{19},t)}$
are fifth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{20})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3)}(G_{23},t)}$
are sixth detrition coefficients for category 1, 2 *and* 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,,)}(G_{39},t)}\boxed{-(b''_{37})^{(7,7,7,7,7,,)}(G_{39},t)}\boxed{-(b''_{38})^{(7,7,7,7,7,,)}(G_{39},t)}$ *ARE* SEVENTH DETRITION COEFFICIENTS-*96*
-97

FIFTH MODULE CONCATENATION:-

98 $$\frac{dG_{28}}{dt} = (a_{28})^{(5)}G_{29} - \begin{bmatrix} (a'_{28})^{(5)}\boxed{+(a''_{28})^{(5)}(T_{29},t)} \boxed{+(a''_{24})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,,7,,7,7)}(T_{37},t)} \end{bmatrix} G_{28} \quad \text{-99}$$

$$\frac{dG_{29}}{dt} = (a_{29})^{(5)}G_{28} - \begin{bmatrix} (a'_{29})^{(5)}\boxed{+(a''_{29})^{(5)}(T_{29},t)} \boxed{+(a''_{25})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,,,7,,7,7)}(T_{37},t)} \end{bmatrix} G_{29} \quad \text{-100}$$

$$\frac{dG_{30}}{dt} = (a_{30})^{(5)}G_{29} - \begin{bmatrix} (a'_{30})^{(5)}\boxed{+(a''_{30})^{(5)}(T_{29},t)} \boxed{+(a''_{26})^{(4,4,)}(T_{25},t)} \boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)} \boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)} \boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,,7,,7,7)}(T_{37},t)} \end{bmatrix} G_{30} \quad \text{-101}$$

$Where$ $\boxed{+(a''_{28})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5)}(T_{29},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

And $\boxed{+(a''_{24})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,)}(T_{25},t)}$ $are\ second\ augmentation\ coefficient\ for\ category\ 1,2\ and$

$\boxed{+(a''_{32})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6,6,6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6,6,6)}(T_{33},t)}$ $are\ third\ augmentation\ coefficient\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{13})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1)}(T_{14},t)}$ are fourth augmentation coefficients for category 1,2, and 3

$\boxed{+(a''_{16})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2)}(T_{17},t)}$ are fifth augmentation coefficients for category 1,2,and 3

$\boxed{+(a''_{20})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3)}(T_{21},t)}$ are sixth augmentation coefficients for category 1,2, 3 *-102*

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$$\frac{dT_{28}}{dt} = (b_{28})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{28})^{(5)}\boxed{-(b''_{28})^{(5)}(G_{31},t)}\ \boxed{-(b''_{24})^{(4,4,)}(G_{23},t)}\ \boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,,7,7,7,)}(G_{38},t)} \end{array}\right] T_{28} \quad -104$$

$$\frac{dT_{29}}{dt} = (b_{29})^{(5)}T_{28} - \left[\begin{array}{c} (b'_{29})^{(5)}\boxed{-(b''_{29})^{(5)}(G_{31},t)}\ \boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{37})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{29} \quad -105$$

$$\frac{dT_{30}}{dt} = (b_{30})^{(5)}T_{29} - \left[\begin{array}{c} (b'_{30})^{(5)}\boxed{-(b''_{30})^{(5)}(G_{31},t)}\ \boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}\ \boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}\ \boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}\ \boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,7,7,)}(G_{38},t)} \end{array}\right] T_{30} \quad -106$$

$where$ $\boxed{-(b''_{28})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5)}(G_{31},t)}$ $are\ first\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{24})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,)}(G_{27},t)}$ $are\ second\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{32})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6,6,6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6,6,6)}(G_{35},t)}$ $are\ third\ detrition\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{-(b''_{13})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,)}(G,t)}$ are fourth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1,2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1,2, and 3*-107*

<u>SIXTH MODULE CONCATENATION</u>-*108*

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{32})^{(6)}\boxed{+(a''_{32})^{(6)}(T_{33},t)}\ \boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)} \end{array}\right] G_{32} \quad -109$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[\begin{array}{c} (a'_{33})^{(6)}\boxed{+(a''_{33})^{(6)}(T_{33},t)}\ \boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{37})^{(7,7,7,,7,7,7,)}(T_{37},t)} \end{array}\right] G_{33} \quad -110$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[\begin{array}{c} (a'_{34})^{(6)}\boxed{+(a''_{34})^{(6)}(T_{33},t)}\ \boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}\ \boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)} \\ \boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}\ \boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}\ \boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)} \\ \boxed{+(a''_{38})^{(7,7,7,7,7,7,)}(T_{37},t)} \end{array}\right] G_{34} \quad -111$$

$\boxed{+(a''_{32})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{33})^{(6)}(T_{33},t)}$, $\boxed{+(a''_{34})^{(6)}(T_{33},t)}$ $are\ first\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{28})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{29})^{(5,5,5)}(T_{29},t)}$, $\boxed{+(a''_{30})^{(5,5,5)}(T_{29},t)}$ $are\ second\ augmentation\ coefficients\ for\ category\ 1,2\ and\ 3$

$\boxed{+(a''_{24})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{25})^{(4,4,4,)}(T_{25},t)}$, $\boxed{+(a''_{26})^{(4,4,4,)}(T_{25},t)}$ *are third augmentation coefficients for category 1, 2 and 3*

$\boxed{+(a''_{13})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1,1,1,1,1,1)}(T_{14},t)}$, $\boxed{+(a''_{15})^{(1,1,1,1,1,1)}(T_{14},t)}$ - are fourth augmentation coefficients

$\boxed{+(a''_{16})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{17})^{(2,2,2,2,2,2)}(T_{17},t)}$, $\boxed{+(a''_{18})^{(2,2,2,2,2,2)}(T_{17},t)}$ - fifth augmentation coefficients

$\boxed{+(a''_{20})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{21})^{(3,3,3,3,3,3)}(T_{21},t)}$, $\boxed{+(a''_{22})^{(3,3,3,3,3,3)}(T_{21},t)}$ sixth augmentation coefficients

$\boxed{+(a''_{36})^{(7,7,7,7,7,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,,77,7,,7,)}(T_{37},t)}$ $\boxed{+(a''_{36})^{(7,7,7,,7,7,7,)}(T_{37},t)}$ ARE SVENTH AUGMENTATION COEFFICIENTS-*112*

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$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[\begin{matrix} (b'_{32})^{(6)} \boxed{-(b''_{32})^{(6)}(G_{35},t)} \boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{36})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{matrix}\right] T_{32} \quad \text{-114}$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[\begin{matrix} (b'_{33})^{(6)} \boxed{-(b''_{33})^{(6)}(G_{35},t)} \boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)} \\ -(b''_{37})^{(7,7,7,,7,7,7)}(G_{39},t) \end{matrix}\right] T_{33} \quad \text{-115}$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[\begin{matrix} (b'_{34})^{(6)} \boxed{-(b''_{34})^{(6)}(G_{35},t)} \boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)} \boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)} \\ \boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)} \boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)} \boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)} \\ \boxed{-(b''_{38})^{(7,7,7,,7,7,7)}(G_{39},t)} \end{matrix}\right] T_{34} \quad \text{-116}$$

$\boxed{-(b''_{32})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{33})^{(6)}(G_{35},t)}$, $\boxed{-(b''_{34})^{(6)}(G_{35},t)}$ *are first detrition coefficient s for catego*

$\boxed{-(b''_{28})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{29})^{(5,5,5)}(G_{31},t)}$, $\boxed{-(b''_{30})^{(5,5,5)}(G_{31},t)}$ *are second detrition coefficients for*

$\boxed{-(b''_{24})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{25})^{(4,4,4,)}(G_{27},t)}$, $\boxed{-(b''_{26})^{(4,4,4,)}(G_{27},t)}$ *are t h ird detrition coefficients for*

$\boxed{-(b''_{13})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{14})^{(1,1,1,1,1,1)}(G,t)}$, $\boxed{-(b''_{15})^{(1,1,1,1,1,1)}(G,t)}$ are fourth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{16})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{17})^{(2,2,2,2,2,2)}(G_{19},t)}$, $\boxed{-(b''_{18})^{(2,2,2,2,2,2)}(G_{19},t)}$ are fifth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{20})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{21})^{(3,3,3,3,3,3)}(G_{23},t)}$, $\boxed{-(b''_{22})^{(3,3,3,3,3,3)}(G_{23},t)}$ are sixth detrition coefficients for category 1, 2, and 3

$\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ $\boxed{-(b''_{36})^{(7,7,7,7,7,7)}(G_{39},t)}$ ARE SEVENTH DETRITION COEFFICIENTS-*117*

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SEVENTH MODULE CONCATENATION:*-119*

$$\frac{dG_{36}}{dt} =$$

$$(a_{36})^{(7)}G_{37} - \left[\left[(a'_{36})^{(7)} \boxed{+(a''_{36})^{(7)}(T_{37},t)} + \boxed{(a''_{16})^{(7)}(T_{17},t)} + \boxed{\boxed{(a''_{20})^{(7)}(T_{21},t)}} + \boxed{(a''_{24})^{(7)}(T_{23},t)G_{36}} + \right.\right.$$

□28''7□29,□ + □32''7□33,□ +□13''7□14,□ □36-120

121

$$\frac{dG_{37}}{dt} =$$

$$(a_{37})^{(7)}G_{36} - \left[\left[(a'_{37})^{(7)} + \boxed{(a''_{37})^{(7)}(T_{37},t)} + \boxed{(a''_{14})^{(7)}(T_{14},t)} + \boxed{(a''_{21})^{(7)}(T_{21},t)} + \boxed{(a''_{17})^{(7)}(T_{17},t)} + \right.\right.$$

□25''7□25,□ +□33''7□33,□ + □29''7□29,□ □37

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$\frac{dG_{38}}{dt} =$

$(a_{38})^{(7)} G_{37} - \left[(a'_{38})^{(7)} + \boxed{(a''_{38})^{(7)} (T_{37}, t)} + \boxed{(a''_{15})^{(7)} (T_{14}, t)} + \boxed{(a''_{22})^{(7)} (T_{21}, t) +} \boxed{+(a''_{18})^{(7)} (T_{17}, t)} + \right.$

□26″7□25,□ + □34″7□33,□ + □30″7□29,□ □38

-123
124

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$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - \left[(b'_{36})^{(7)} - \boxed{(b''_{36})^{(7)} ((G_{39}), t)} - \boxed{(b''_{16})^{(7)} ((G_{19}), t)} - \boxed{(b''_{13})^{(7)} ((G_{14}), t)} - \right.$

□20″7□231,□ − □24″7□27,□ −□28″7□31,□ −□32″7□35,□ □36

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$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - \left[(b'_{36})^{(7)} - \boxed{(b''_{37})^{(7)} ((G_{39}), t)} - \boxed{(b''_{17})^{(7)} ((G_{19}), t)} - \boxed{(b''_{19})^{(7)} ((G_{14}), t)} - \right.$

□21″7□231,□ − □25″7□27,□ −□29″7□31,□ −□33″7□35,□
□37

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Where we suppose

(A) $(a_i)^{(1)}, (a'_i)^{(1)}, (a''_i)^{(1)}, (b_i)^{(1)}, (b'_i)^{(1)}, (b''_i)^{(1)} > 0,$
$i, j = 13,14,15$

(B) The functions $(a''_i)^{(1)}, (b''_i)^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}, \ (r_i)^{(1)}$:

$(a''_i)^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b''_i)^{(1)}(G, t) \leq \ (r_i)^{(1)} \leq (b'_i)^{(1)} \leq (\hat{B}_{13})^{(1)}$

(C) $lim_{T_2 \to \infty} (a''_i)^{(1)} (T_{14}, t) = (p_i)^{(1)}$
$\lim_{G \to \infty} (b''_i)^{(1)} (G, t) = \ (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants
and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$|(a''_i)^{(1)}(T'_{14}, t) - (a''_i)^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T'_{14}| e^{-(\hat{M}_{13})^{(1)} t}$

$|(b''_i)^{(1)}(G', t) - (b''_i)^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)} t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a''_i)^{(1)}(T'_{14}, t)$ and $(a''_i)^{(1)}(T_{14}, t)$. (T'_{14}, t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a''_i)^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a''_i)^{(1)}(T_{14}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)}$ *and* $(\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, (r_i)^{(1)}, i = 13,14,15,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1$$

$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - \left[(b_{38}')^{(7)} - \boxed{(b_{38}'')^{(7)}((G_{39}),t)} - \boxed{(b_{18}'')^{(7)}((G_{19}),t)} - \boxed{(b_{20}'')^{(7)}((G_{14}),t)} - \right.$ 128

b22''7G23,t − b26''7G27,t −b30''7G31,t −b34''7G35,t 129

T38

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$+(a_{36}'')^{(7)}(T_{37},t) =$ **First augmentation factor** 134

$(1)(a_i)^{(2)}, (a_i')^{(2)}, (a_i'')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (b_i'')^{(2)} > 0, \quad i,j = 16,17,18$ 135

(F) (2) The functions $(a_i'')^{(2)}, (b_i'')^{(2)}$ are positive continuous increasing and bounded. 136

Definition of $(p_i)^{(2)}, (r_i)^{(2)}$: 137

$(a_i'')^{(2)}(T_{17},t) \leq (p_i)^{(2)} \leq (\hat{A}_{16})^{(2)}$ 138

$(b_i'')^{(2)}(G_{19},t) \leq (r_i)^{(2)} \leq (b_i')^{(2)} \leq (\hat{B}_{16})^{(2)}$ 139

(G) (3) $\lim_{T_2\to\infty}(a_i'')^{(2)}(T_{17},t) = (p_i)^{(2)}$ 140

$\lim_{G\to\infty}(b_i'')^{(2)}((G_{19}),t) = (r_i)^{(2)}$ 141

Definition of $(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}$: 142

Where $\boxed{(\hat{A}_{16})^{(2)}, (\hat{B}_{16})^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}}$ are positive constants and $\boxed{i = 16,17,18}$

They satisfy Lipschitz condition: 143

$|(a_i'')^{(2)}(T_{17}',t) - (a_i'')^{(2)}(T_{17},t)| \leq (\hat{k}_{16})^{(2)}|T_{17} - T_{17}'|e^{-(\hat{M}_{16})^{(2)}t}$ 144

$|(b_i'')^{(2)}((G_{19})',t) - (b_i'')^{(2)}((G_{19}),t)| < (\hat{k}_{16})^{(2)}||(G_{19}) - (G_{19})'||e^{-(\hat{M}_{16})^{(2)}t}$ 145

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(2)}(T_{17}',t)$ and$(a_i'')^{(2)}(T_{17},t)$. (T_{17}',t) And (T_{17},t) are points belonging to the interval $[(\hat{k}_{16})^{(2)}, (\hat{M}_{16})^{(2)}]$. It is to be noted that $(a_i'')^{(2)}(T_{17},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{16})^{(2)} = 1$ then the function $(a_i'')^{(2)}(T_{17},t)$, the SECOND augmentation coefficient would be absolutely continuous. 146

Definition of $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$: 147

(H) (4) $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}$, are positive constants 148

$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$

Definition of $(\hat{P}_{13})^{(2)}, (\hat{Q}_{13})^{(2)}$: 149

There exists two constants $(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ which together with $(\hat{M}_{16})^{(2)}, (\hat{k}_{16})^{(2)}, (\hat{A}_{16})^{(2)}$ *and* $(\hat{B}_{16})^{(2)}$ and the constants $(a_i)^{(2)}, (a_i')^{(2)}, (b_i)^{(2)}, (b_i')^{(2)}, (p_i)^{(2)}, (r_i)^{(2)}, i = 16,17,18,$

satisfy the inequalities

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(a_i)^{(2)} + (a_i')^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$$ 150

$$\frac{1}{(\hat{M}_{16})^{(2)}}[(b_i)^{(2)} + (b_i')^{(2)} + (\hat{B}_{16})^{(2)} + (\hat{Q}_{16})^{(2)}(\hat{k}_{16})^{(2)}] < 1$$ 151

Where we suppose 152

(I) (5) $(a_i)^{(3)}, (a_i')^{(3)}, (a_i'')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (b_i'')^{(3)} > 0, \quad i,j = 20,21,22$ 153

The functions $(a_i'')^{(3)}, (b_i'')^{(3)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(3)}, (r_i)^{(3)}$:

$$(a_i'')^{(3)}(T_{21},t) \leq (p_i)^{(3)} \leq (\hat{A}_{20})^{(3)}$$

$$(b_i'')^{(3)}(G_{23},t) \leq (r_i)^{(3)} \leq (b_i')^{(3)} \leq (\hat{B}_{20})^{(3)}$$

$$lim_{T_2 \to \infty}(a_i'')^{(3)}(T_{21},t) = (p_i)^{(3)}$$ 154

$$\lim_{G \to \infty}(b_i'')^{(3)}(G_{23},t) = (r_i)^{(3)}$$ 155

Definition of $(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}$: 156

Where $\boxed{(\hat{A}_{20})^{(3)}, (\hat{B}_{20})^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}}$ are positive constants and $\boxed{i = 20,21,22}$

They satisfy Lipschitz condition: 157

$$|(a_i'')^{(3)}(T_{21}',t) - (a_i'')^{(3)}(T_{21},t)| \leq (\hat{k}_{20})^{(3)}|T_{21} - T_{21}'|e^{-(\hat{M}_{20})^{(3)}t}$$ 158

$$|(b_i'')^{(3)}(G_{23}',t) - (b_i'')^{(3)}(G_{23},t)| < (\hat{k}_{20})^{(3)}||G_{23} - G_{23}'||e^{-(\hat{M}_{20})^{(3)}t}$$ 159

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(3)}(T_{21}',t)$ and$(a_i'')^{(3)}(T_{21},t)$. (T_{21}',t) And (T_{21},t) are points belonging to the interval $[(\hat{k}_{20})^{(3)}, (\hat{M}_{20})^{(3)}]$. It is to be noted that $(a_i'')^{(3)}(T_{21},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{20})^{(3)} = 1$ then the function $(a_i'')^{(3)}(T_{21},t)$, the THIRD augmentation coefficient, would be absolutely continuous. 160

Definition of $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$: 161

(J) (6) $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}$, are positive constants

$\frac{(a_i)^{(3)}}{(\hat{M}_{20})^{(3)}}$, $\frac{(b_i)^{(3)}}{(\hat{M}_{20})^{(3)}} < 1$

There exists two constants There exists two constants $(\hat{P}_{20})^{(3)}$ and $(\hat{Q}_{20})^{(3)}$ which together with $(\hat{M}_{20})^{(3)}, (\hat{k}_{20})^{(3)}, (\hat{A}_{20})^{(3)}$ *and* $(\hat{B}_{20})^{(3)}$ and the constants $(a_i)^{(3)}, (a_i')^{(3)}, (b_i)^{(3)}, (b_i')^{(3)}, (p_i)^{(3)}, (r_i)^{(3)}, i = 20,21,22,$ 162 163
satisfy the inequalities 164

$$\frac{1}{(\hat{M}_{20})^{(3)}}[(a_i)^{(3)} + (a_i')^{(3)} + (\hat{A}_{20})^{(3)} + (\hat{P}_{20})^{(3)}(\hat{k}_{20})^{(3)}] < 1$$ 165

$\frac{1}{(\hat{M}_{20})^{(3)}}\left[(b_i)^{(3)} + (b_i')^{(3)} + (\hat{B}_{20})^{(3)} + (\hat{Q}_{20})^{(3)} (\hat{k}_{20})^{(3)}\right] < 1$ 166

167

Where we suppose 168

$(a_i)^{(4)}, (a_i')^{(4)}, (a_i'')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (b_i'')^{(4)} > 0, \quad i,j = 24,25,26$ 169

(L) (7) The functions $(a_i'')^{(4)}, (b_i'')^{(4)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(4)}, (r_i)^{(4)}$:

$(a_i'')^{(4)}(T_{25}, t) \leq (p_i)^{(4)} \leq (\hat{A}_{24})^{(4)}$

$(b_i'')^{(4)}\big((G_{27}), t\big) \leq (r_i)^{(4)} \leq (b_i')^{(4)} \leq (\hat{B}_{24})^{(4)}$

170

(M) (8) $lim_{T_2\to\infty} (a_i'')^{(4)} (T_{25}, t) = (p_i)^{(4)}$
$\lim_{G\to\infty} (b_i'')^{(4)} \big((G_{27}), t\big) = (r_i)^{(4)}$

Definition of $(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}$:

Where $\boxed{(\hat{A}_{24})^{(4)}, (\hat{B}_{24})^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}}$ are positive constants and $\boxed{i = 24,25,26}$

They satisfy Lipschitz condition: 171

$|(a_i'')^{(4)}(T_{25}', t) - (a_i'')^{(4)}(T_{25}, t)| \leq (\hat{k}_{24})^{(4)} |T_{25} - T_{25}'| e^{-(\hat{M}_{24})^{(4)} t}$

$|(b_i'')^{(4)}((G_{27})', t) - (b_i'')^{(4)}\big((G_{27}), t\big)| < (\hat{k}_{24})^{(4)} ||(G_{27}) - (G_{27})'|| e^{-(\hat{M}_{24})^{(4)} t}$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(4)}(T_{25}', t)$ 172
and$(a_i'')^{(4)}(T_{25}, t)$. (T_{25}', t) And (T_{25}, t) are points belonging to the interval $\left[(\hat{k}_{24})^{(4)}, (\hat{M}_{24})^{(4)}\right]$. It is to be noted that $(a_i'')^{(4)}(T_{25}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{24})^{(4)} = 4$ then the function $(a_i'')^{(4)}(T_{25}, t)$, the FOURTH **augmentation coefficient WOULD** be absolutely continuous.

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Definition of $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}$: 174

$(\hat{M}_{24})176^{175}{}^{(4)}, (\hat{k}_{24})^{(4)}$, are positive constants

$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}, \frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$

Definition of $(\hat{P}_{24})^{(4)}, (\hat{Q}_{24})^{(4)}$: 175

(P) (9) There exists two constants $(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ which together with $(\hat{M}_{24})^{(4)}, (\hat{k}_{24})^{(4)}, (\hat{A}_{24})^{(4)} and (\hat{B}_{24})^{(4)}$ and the constants $(a_i)^{(4)}, (a_i')^{(4)}, (b_i)^{(4)}, (b_i')^{(4)}, (p_i)^{(4)}, (r_i)^{(4)}, i = 24,25,26,$
satisfy the inequalities

$\frac{1}{(\hat{M}_{24})^{(4)}}\left[(a_i)^{(4)} + (a_i')^{(4)} + (\hat{A}_{24})^{(4)} + (\hat{P}_{24})^{(4)} (\hat{k}_{24})^{(4)}\right] < 1$

$\frac{1}{(\hat{M}_{24})^{(4)}}\left[(b_i)^{(4)} + (b_i')^{(4)} + (\hat{B}_{24})^{(4)} + (\hat{Q}_{24})^{(4)} (\hat{k}_{24})^{(4)}\right] < 1$

Where we suppose 176

$(a_i)^{(5)}, (a_i^{'})^{(5)}, (a_i^{''})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (b_i^{''})^{(5)} > 0, \quad i,j = 28,29,30$ 177

(R) (10) The functions $(a_i^{''})^{(5)}, (b_i^{''})^{(5)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(5)}, \ (r_i)^{(5)}$:

$$(a_i^{''})^{(5)}(T_{29}, t) \leq (p_i)^{(5)} \leq (\hat{A}_{28})^{(5)}$$

$$(b_i^{''})^{(5)}((G_{31}), t) \leq \ (r_i)^{(5)} \leq (b_i^{'})^{(5)} \leq (\hat{B}_{28})^{(5)}$$

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(S) (11) $lim_{T_2 \to \infty} (a_i^{''})^{(5)} (T_{29}, t) = (p_i)^{(5)}$

$\lim_{G \to \infty} (b_i^{''})^{(5)} (G_{31}, t) = \ (r_i)^{(5)}$

Definition of $(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}$:

Where $\boxed{(\hat{A}_{28})^{(5)}, (\hat{B}_{28})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}}$ are positive constants and $\boxed{i = 28,29,30}$

They satisfy Lipschitz condition: 179

$$|(a_i^{''})^{(5)}(T_{29}^{'}, t) - (a_i^{''})^{(5)}(T_{29}, t)| \leq (\hat{k}_{28})^{(5)}|T_{29} - T_{29}^{'}|e^{-(\hat{M}_{28})^{(5)}t}$$

$$|(b_i^{''})^{(5)}((G_{31})^{'}, t) - (b_i^{''})^{(5)}((G_{31}), t)| < (\hat{k}_{28})^{(5)}||(G_{31}) - (G_{31})^{'}||e^{-(\hat{M}_{28})^{(5)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(5)}(T_{29}^{'}, t)$ 180
and$(a_i^{''})^{(5)}(T_{29}, t)$. $(T_{29}^{'}, t)$ and (T_{29}, t) are points belonging to the interval $[(\hat{k}_{28})^{(5)}, (\hat{M}_{28})^{(5)}]$. It is to be noted that $(a_i^{''})^{(5)}(T_{29}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{28})^{(5)} = 5$ then the function $(a_i^{''})^{(5)}(T_{29}, t)$, theFIFTH **augmentation coefficient** attributable would be absolutely continuous.

Definition of $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$: 181

$(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}$, are positive constants

$\frac{(a_i)^{(5)}}{(\hat{M}_{28})^{(5)}} , \frac{(b_i)^{(5)}}{(\hat{M}_{28})^{(5)}} < 1$

Definition of $(\hat{P}_{28})^{(5)}, (\hat{Q}_{28})^{(5)}$: 182

There exists two constants $(\hat{P}_{28})^{(5)}$ and $(\hat{Q}_{28})^{(5)}$ which together with $(\hat{M}_{28})^{(5)}, (\hat{k}_{28})^{(5)}, (\hat{A}_{28})^{(5)} and (\hat{B}_{28})^{(5)}$ and the constants $(a_i)^{(5)}, (a_i^{'})^{(5)}, (b_i)^{(5)}, (b_i^{'})^{(5)}, (p_i)^{(5)}, \ (r_i)^{(5)}, i = 28,29,30,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (a_i)^{(5)} + (a_i^{'})^{(5)} + \ (\hat{A}_{28})^{(5)} + \ (\hat{P}_{28})^{(5)} \ (\hat{k}_{28})^{(5)}] < 1$$

$$\frac{1}{(\hat{M}_{28})^{(5)}}[\ (b_i)^{(5)} + (b_i^{'})^{(5)} + \ (\hat{B}_{28})^{(5)} + \ (\hat{Q}_{28})^{(5)} \ (\hat{k}_{28})^{(5)}] < 1$$

Where we suppose 183

$(a_i)^{(6)}, (a_i^{'})^{(6)}, (a_i^{''})^{(6)}, (b_i)^{(6)}, (b_i^{'})^{(6)}, (b_i^{''})^{(6)} > 0, \quad i,j = 32,33,34$ 184

(12) The functions $(a_i^{''})^{(6)}, (b_i^{''})^{(6)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(6)}, \ (r_i)^{(6)}$:

$$(a_i^{''})^{(6)}(T_{33}, t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$

$$(b_i^{''})^{(6)}((G_{35}), t) \leq \ (r_i)^{(6)} \leq (b_i^{'})^{(6)} \leq (\hat{B}_{32})^{(6)}$$

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(13) $lim_{T_2\to\infty}(a_i^{''})^{(6)}(T_{33},t)=(p_i)^{(6)}$
$\lim_{G\to\infty}(b_i^{''})^{(6)}\big((G_{35}),t\big)=\ (r_i)^{(6)}$

Definition of $(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)},(\hat{B}_{32})^{(6)},(p_i)^{(6)},\ (r_i)^{(6)}}$ are positive constants and $\boxed{i=32,33,34}$

They satisfy Lipschitz condition: 186

$$|(a_i^{''})^{(6)}(T_{33}^{'},t)-(a_i^{''})^{(6)}(T_{33},t)|\le(\hat{k}_{32})^{(6)}|T_{33}-T_{33}^{'}|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i^{''})^{(6)}((G_{35})^{'},t)-(b_i^{''})^{(6)}\big((G_{35}),t\big)|<(\hat{k}_{32})^{(6)}||(G_{35})-(G_{35})^{'}||e^{-(\hat{M}_{32})^{(6)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i^{''})^{(6)}(T_{33}^{'},t)$ and$(a_i^{''})^{(6)}(T_{33},t)$. $(T_{33}^{'},t)$ and (T_{33},t) are points belonging to the interval $\big[(\hat{k}_{32})^{(6)},(\hat{M}_{32})^{(6)}\big]$. It is to be noted that $(a_i^{''})^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)}=6$ then the function $(a_i^{''})^{(6)}(T_{33},t)$, the SIXTH **augmentation coefficient** would be absolutely continuous. 187

Definition of $(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$: 188

$(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)}$, are positive constants
$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\ ,\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}<1$

Definition of $(\hat{P}_{32})^{(6)},(\hat{Q}_{32})^{(6)}$: 189

There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)},(\hat{k}_{32})^{(6)},(\hat{A}_{32})^{(6)} and\ (\hat{B}_{32})^{(6)}$ and the constants $(a_i)^{(6)},(a_i^{'})^{(6)},(b_i)^{(6)},(b_i^{'})^{(6)},(p_i)^{(6)},\ (r_i)^{(6)},i=32,33,34,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\,(a_i)^{(6)}+(a_i^{'})^{(6)}+\ (\hat{A}_{32})^{(6)}+\ (\hat{P}_{32})^{(6)}\ (\hat{k}_{32})^{(6)}]<1$$

$$\frac{1}{(\hat{M}_{32})^{(6)}}[\ (b_i)^{(6)}+(b_i^{'})^{(6)}+\ (\hat{B}_{32})^{(6)}+\ (\hat{Q}_{32})^{(6)}\ \ (\hat{k}_{32})^{(6)}]<1$$

Where we suppose 190

(V) $(a_i)^{(7)},\left(a_i^{'}\right)^{(7)},\left(a_i^{''}\right)^{(7)},(b_i)^{(7)},\left(b_i^{'}\right)^{(7)},\left(b_i^{''}\right)^{(7)}>0,$ 191
$i,j=36,37,38$

(W) The functions $\left(a_i^{''}\right)^{(7)},\left(b_i^{''}\right)^{(7)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(7)},\ (r_i)^{(7)}$:

$$\left(a_i^{''}\right)^{(7)}(T_{37},t)\le(p_i)^{(7)}\le(\hat{A}_{36})^{(7)}$$

$$\left(b_i^{''}\right)^{(7)}(G,t)\le\ (r_i)^{(7)}\le(b_i^{'})^{(7)}\le(\hat{B}_{36})^{(7)}$$

$\lim_{T_2\to\infty}\left(a_i^{''}\right)^{(7)}(T_{37},t)=(p_i)^{(7)}$ 192

$\lim_{G\to\infty}\left(b_i^{''}\right)^{(7)}\big((G_{39}),t\big)=\ (r_i)^{(7)}$

Definition of $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$:

Where $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$ are positive constants and $i = 36,37,38$

They satisfy Lipschitz condition: 193

$$|(a_i'')^{(7)}(T_{37}', t) - (a_i'')^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T_{37} - T_{37}'|e^{-(\hat{M}_{36})^{(7)}t}$$

$$|(b_i'')^{(7)}((G_{39})', t) - (b_i'')^{(7)}\big((G_{39}), (T_{39})\big)| < (\hat{k}_{36})^{(7)}||(G_{39}) - (G_{39})'||e^{-(\hat{M}_{36})^{(7)}t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(7)}(T_{37}', t)$ and$(a_i'')^{(7)}(T_{37}, t)$. (T_{37}', t) and (T_{37}, t) are points belonging to the interval $\big[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}\big]$. It is to be noted that $(a_i'')^{(7)}(T_{37}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{36})^{(7)} = 7$ then the function $(a_i'')^{(7)}(T_{37}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous. 194

Definition of $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$: 195

(X) $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$, are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1$$

Definition of $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$: 196

(Y) There exists two constants $(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ which together with $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)} and (\hat{B}_{36})^{(7)}$ and the constants $(a_i)^{(7)}, (a_i')^{(7)}, (b_i)^{(7)}, (b_i')^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36,37,38,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(a_i)^{(7)} + (a_i')^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}}[(b_i)^{(7)} + (b_i')^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

Definition of $G_i(0), T_i(0)$: 197

$$G_i(t) \leq \big(\hat{P}_{28} \big)^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$ 198

Definition of $G_i(0), T_i(0)$: 199

$$G_i(t) \leq \big(\hat{P}_{32} \big)^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)}t} , \quad \boxed{T_i(0) = T_i^0 > 0}$$

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Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq \big(\hat{P}_{36} \big)^{(7)} e^{(\hat{M}_{36})^{(7)}t} , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof: Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 200

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$, $T_i^0 \leq (\hat{Q}_{13})^{(1)}$, 201

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 202

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t}$ 203

By 204

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \left[(a_{13})^{(1)} G_{14}(s_{(13)}) - \left((a_{13}')^{(1)} + a_{13}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{13}(s_{(13)})\right] ds_{(13)}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \left[(a_{14})^{(1)} G_{13}(s_{(13)}) - \left((a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{14}(s_{(13)})\right] ds_{(13)}$$ 205

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \left[(a_{15})^{(1)} G_{14}(s_{(13)}) - \left((a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\right) G_{15}(s_{(13)})\right] ds_{(13)}$$ 206

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \left[(b_{13})^{(1)} T_{14}(s_{(13)}) - \left((b_{13}')^{(1)} - (b_{13}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{13}(s_{(13)})\right] ds_{(13)}$$ 207

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \left[(b_{14})^{(1)} T_{13}(s_{(13)}) - \left((b_{14}')^{(1)} - (b_{14}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{14}(s_{(13)})\right] ds_{(13)}$$ 208

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \left[(b_{15})^{(1)} T_{14}(s_{(13)}) - \left((b_{15}')^{(1)} - (b_{15}'')^{(1)}(G(s_{(13)}), s_{(13)})\right) T_{15}(s_{(13)})\right] ds_{(13)}$$ 209

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

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if the conditions IN THE FOREGOING above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0)$, $T_i(0)$:

$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{36})^{(7)}$, $T_i^0 \leq (\hat{Q}_{36})^{(7)}$,

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}(s_{(36)}) - \left((a_{36}')^{(7)} + a_{36}'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{36}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}(s_{(36)}) - \left((a_{37}')^{(7)} + (a_{37}'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\right) G_{37}(s_{(36)})\right] ds_{(36)}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}\left(s_{(36)}\right) - \left((a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)}\left(T_{37}\left(s_{(36)}\right), s_{(36)}\right)\right) G_{38}\left(s_{(36)}\right)\right] ds_{(36)}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}\left(s_{(36)}\right) - \left((b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)}\left(G\left(s_{(36)}\right), s_{(36)}\right)\right) T_{36}\left(s_{(36)}\right)\right] ds_{(36)}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}\left(s_{(36)}\right) - \left((b_{37}^{'})^{(7)} - (b_{37}^{''})^{(7)}\left(G\left(s_{(36)}\right), s_{(36)}\right)\right) T_{37}\left(s_{(36)}\right)\right] ds_{(36)}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}\left(s_{(36)}\right) - \left((b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)}\left(G\left(s_{(36)}\right), s_{(36)}\right)\right) T_{38}\left(s_{(36)}\right)\right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(2)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 211

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{16})^{(2)}$, $T_i^0 \leq (\hat{Q}_{16})^{(2)}$, 212

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 213

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ 214

By 215

$$\bar{G}_{16}(t) = G_{16}^0 + \int_0^t \left[(a_{16})^{(2)} G_{17}\left(s_{(16)}\right) - \left((a_{16}^{'})^{(2)} + a_{16}^{''})^{(2)}\left(T_{17}\left(s_{(16)}\right), s_{(16)}\right)\right) G_{16}\left(s_{(16)}\right)\right] ds_{(16)}$$

$$\bar{G}_{17}(t) = G_{17}^0 + \int_0^t \left[(a_{17})^{(2)} G_{16}\left(s_{(16)}\right) - \left((a_{17}^{'})^{(2)} + (a_{17}^{''})^{(2)}\left(T_{17}\left(s_{(16)}\right), s_{(17)}\right)\right) G_{17}\left(s_{(16)}\right)\right] ds_{(16)} \quad 216$$

$$\bar{G}_{18}(t) = G_{18}^0 + \int_0^t \left[(a_{18})^{(2)} G_{17}\left(s_{(16)}\right) - \left((a_{18}^{'})^{(2)} + (a_{18}^{''})^{(2)}\left(T_{17}\left(s_{(16)}\right), s_{(16)}\right)\right) G_{18}\left(s_{(16)}\right)\right] ds_{(16)} \quad 217$$

$$\bar{T}_{16}(t) = T_{16}^0 + \int_0^t \left[(b_{16})^{(2)} T_{17}\left(s_{(16)}\right) - \left((b_{16}^{'})^{(2)} - (b_{16}^{''})^{(2)}\left(G\left(s_{(16)}\right), s_{(16)}\right)\right) T_{16}\left(s_{(16)}\right)\right] ds_{(16)} \quad 218$$

$$\bar{T}_{17}(t) = T_{17}^0 + \int_0^t \left[(b_{17})^{(2)} T_{16}\left(s_{(16)}\right) - \left((b_{17}^{'})^{(2)} - (b_{17}^{''})^{(2)}\left(G\left(s_{(16)}\right), s_{(16)}\right)\right) T_{17}\left(s_{(16)}\right)\right] ds_{(16)} \quad 219$$

$$\bar{T}_{18}(t) = T_{18}^0 + \int_0^t \left[(b_{18})^{(2)} T_{17}\left(s_{(16)}\right) - \left((b_{18}^{'})^{(2)} - (b_{18}^{''})^{(2)}\left(G\left(s_{(16)}\right), s_{(16)}\right)\right) T_{18}\left(s_{(16)}\right)\right] ds_{(16)} \quad 220$$

Where $s_{(16)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(3)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 221

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{20})^{(3)}$, $T_i^0 \leq (\hat{Q}_{20})^{(3)}$, 222

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 223

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{20})^{(3)} e^{(\hat{M}_{20})^{(3)} t}$ 224

By 225

$\bar{G}_{20}(t) = G_{20}^0 + \int_0^t \left[(a_{20})^{(3)} G_{21}(s_{(20)}) - \left((a_{20}^{'})^{(3)} + a_{20}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{20}(s_{(20)}) \right] ds_{(20)}$

$\bar{G}_{21}(t) = G_{21}^0 + \int_0^t \left[(a_{21})^{(3)} G_{20}(s_{(20)}) - \left((a_{21}^{'})^{(3)} + (a_{21}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{21}(s_{(20)}) \right] ds_{(20)}$ 226

$\bar{G}_{22}(t) = G_{22}^0 + \int_0^t \left[(a_{22})^{(3)} G_{21}(s_{(20)}) - \left((a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \right) G_{22}(s_{(20)}) \right] ds_{(20)}$ 227

$\bar{T}_{20}(t) = T_{20}^0 + \int_0^t \left[(b_{20})^{(3)} T_{21}(s_{(20)}) - \left((b_{20}^{'})^{(3)} - (b_{20}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{20}(s_{(20)}) \right] ds_{(20)}$ 228

$\bar{T}_{21}(t) = T_{21}^0 + \int_0^t \left[(b_{21})^{(3)} T_{20}(s_{(20)}) - \left((b_{21}^{'})^{(3)} - (b_{21}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{21}(s_{(20)}) \right] ds_{(20)}$ 229

$\bar{\mathrm{T}}_{22}(\mathrm{t}) = \mathrm{T}_{22}^0 + \int_0^t \left[(b_{22})^{(3)} T_{21}(s_{(20)}) - \left((b_{22}^{'})^{(3)} - (b_{22}^{''})^{(3)} (G(s_{(20)}), s_{(20)}) \right) T_{22}(s_{(20)}) \right] ds_{(20)}$ 230

Where $s_{(20)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(4)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 231

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{24})^{(4)}$, $T_i^0 \leq (\hat{Q}_{24})^{(4)}$, 232

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 233

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} t}$ 234

By 235

$\bar{G}_{24}(t) = G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} G_{25}(s_{(24)}) - \left((a_{24}^{'})^{(4)} + a_{24}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{24}(s_{(24)}) \right] ds_{(24)}$

$\bar{G}_{25}(t) = G_{25}^0 + \int_0^t \left[(a_{25})^{(4)} G_{24}(s_{(24)}) - \left((a_{25}^{'})^{(4)} + (a_{25}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{25}(s_{(24)}) \right] ds_{(24)}$ 236

$\bar{G}_{26}(t) = G_{26}^0 + \int_0^t \left[(a_{26})^{(4)} G_{25}(s_{(24)}) - \left((a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)} (T_{25}(s_{(24)}), s_{(24)}) \right) G_{26}(s_{(24)}) \right] ds_{(24)}$ 237

$\bar{T}_{24}(t) = T_{24}^0 + \int_0^t \left[(b_{24})^{(4)} T_{25}(s_{(24)}) - \left((b_{24}^{'})^{(4)} - (b_{24}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{24}(s_{(24)}) \right] ds_{(24)}$ 238

$\bar{T}_{25}(t) = T_{25}^0 + \int_0^t \left[(b_{25})^{(4)} T_{24}(s_{(24)}) - \left((b_{25}^{'})^{(4)} - (b_{25}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{25}(s_{(24)}) \right] ds_{(24)}$ 239

$\bar{\mathrm{T}}_{26}(\mathrm{t}) = \mathrm{T}_{26}^0 + \int_0^t \left[(b_{26})^{(4)} T_{25}(s_{(24)}) - \left((b_{26}^{'})^{(4)} - (b_{26}^{''})^{(4)} (G(s_{(24)}), s_{(24)}) \right) T_{26}(s_{(24)}) \right] ds_{(24)}$ 240

Where $s_{(24)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(5)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 241

242

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{28})^{(5)}$, $T_i^0 \leq (\hat{Q}_{28})^{(5)}$, 243

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 244

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} t}$ 245

By 246

$\bar{G}_{28}(t) = G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} G_{29}(s_{(28)}) - \left((a_{28}^{'})^{(5)} + a_{28}^{''})^{(5)} (T_{29}(s_{(28)}), s_{(28)}) \right) G_{28}(s_{(28)}) \right] ds_{(28)}$

$$\bar{G}_{29}(t) = G_{29}^0 + \int_0^t \left[(a_{29})^{(5)} G_{28}(s_{(28)}) - \left((a_{29}^{'})^{(5)} + (a_{29}^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{29}(s_{(28)})\right] ds_{(28)}$$ 247

$$\bar{G}_{30}(t) = G_{30}^0 + \int_0^t \left[(a_{30})^{(5)} G_{29}(s_{(28)}) - \left((a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29}(s_{(28)}), s_{(28)})\right) G_{30}(s_{(28)})\right] ds_{(28)}$$ 248

$$\bar{T}_{28}(t) = T_{28}^0 + \int_0^t \left[(b_{28})^{(5)} T_{29}(s_{(28)}) - \left((b_{28}^{'})^{(5)} - (b_{28}^{''})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{28}(s_{(28)})\right] ds_{(28)}$$ 249

$$\bar{T}_{29}(t) = T_{29}^0 + \int_0^t \left[(b_{29})^{(5)} T_{28}(s_{(28)}) - \left((b_{29}^{'})^{(5)} - (b_{29}^{''})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{29}(s_{(28)})\right] ds_{(28)}$$ 250

$$\bar{T}_{30}(t) = T_{30}^0 + \int_0^t \left[(b_{30})^{(5)} T_{29}(s_{(28)}) - \left((b_{30}^{'})^{(5)} - (b_{30}^{''})^{(5)}(G(s_{(28)}), s_{(28)})\right) T_{30}(s_{(28)})\right] ds_{(28)}$$ 251

Where $s_{(28)}$ is the integrand that is integrated over an interval $(0, t)$

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy 252

$G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{32})^{(6)}$, $T_i^0 \leq (\hat{Q}_{32})^{(6)}$, 253

$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 254

$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 255

By 256

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a_{32}^{'})^{(6)} + a_{32}^{''})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{32}(s_{(32)})\right] ds_{(32)}$$

$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a_{33}^{'})^{(6)} + (a_{33}^{''})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{33}(s_{(32)})\right] ds_{(32)}$$ 257

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}(s_{(32)}), s_{(32)})\right) G_{34}(s_{(32)})\right] ds_{(32)}$$ 258

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b_{32}^{'})^{(6)} - (b_{32}^{''})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{32}(s_{(32)})\right] ds_{(32)}$$ 259

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b_{33}^{'})^{(6)} - (b_{33}^{''})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{33}(s_{(32)})\right] ds_{(32)}$$ 260

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b_{34}^{'})^{(6)} - (b_{34}^{''})^{(6)}(G(s_{(32)}), s_{(32)})\right) T_{34}(s_{(32)})\right] ds_{(32)}$$ 261

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

: if the conditions IN THE FOREGOING are fulfilled, there exists a solution satisfying the conditions 262

Definition of $G_i(0), T_i(0)$:

$G_i(t) \leq \left(\hat{P}_{36}\right)^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{G_i(0) = G_i^0 > 0}$

$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} t}$, $\boxed{T_i(0) = T_i^0 > 0}$

Proof:

Consider operator $\mathcal{A}^{(7)}$ defined on the space of sextuples of continuous functions G_i , $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0 \ , \ T_i(0) = T_i^0 \ , \ G_i^0 \leq (\ \hat{P}_{36}\ )^{(7)} \ , T_i^0 \leq (\ \hat{Q}_{36}\ )^{(7)},$ 263

$0 \leq G_i(t) - G_i^0 \leq (\ \hat{P}_{36}\ )^{(7)} e^{(\ \hat{M}_{36}\ )^{(7)} t}$ 264

$0 \leq T_i(t) - T_i^0 \leq (\ \hat{Q}_{36}\ )^{(7)} e^{(\ \hat{M}_{36}\ )^{(7)} t}$ 265

By 266

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} G_{37}\left(s_{(36)}\right) - \left((a_{36}')^{(7)} + a_{36}'')^{(7)} \left(T_{37}\left(s_{(36)}\right), s_{(36)} \right) \right) G_{36}\left(s_{(36)}\right) \right] ds_{(36)}$$

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$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t \left[(a_{37})^{(7)} G_{36}\left(s_{(36)}\right) - \left((a_{37}')^{(7)} + (a_{37}'')^{(7)} \left(T_{37}\left(s_{(36)}\right), s_{(36)} \right) \right) G_{37}\left(s_{(36)}\right) \right] ds_{(36)}$$

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$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t \left[(a_{38})^{(7)} G_{37}\left(s_{(36)}\right) - \left((a_{38}')^{(7)} + (a_{38}'')^{(7)} \left(T_{37}\left(s_{(36)}\right), s_{(36)} \right) \right) G_{38}\left(s_{(36)}\right) \right] ds_{(36)}$$

269

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t \left[(b_{36})^{(7)} T_{37}\left(s_{(36)}\right) - \left((b_{36}')^{(7)} - (b_{36}'')^{(7)} \left(G\left(s_{(36)}\right), s_{(36)} \right) \right) T_{36}\left(s_{(36)}\right) \right] ds_{(36)}$$

270

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t \left[(b_{37})^{(7)} T_{36}\left(s_{(36)}\right) - \left((b_{37}')^{(7)} - (b_{37}'')^{(7)} \left(G\left(s_{(36)}\right), s_{(36)} \right) \right) T_{37}\left(s_{(36)}\right) \right] ds_{(36)}$$

271

$$\bar{\mathrm{T}}_{38}(\mathrm{t}) = \mathrm{T}_{38}^0 + \int_0^t \left[(b_{38})^{(7)} T_{37}\left(s_{(36)}\right) - \left((b_{38}')^{(7)} - (b_{38}'')^{(7)} \left(G\left(s_{(36)}\right), s_{(36)} \right) \right) T_{38}\left(s_{(36)}\right) \right] ds_{(36)}$$

Where $s_{(36)}$ is the integrand that is integrated over an interval $(0, t)$

Analogous inequalities hold also for $G_{21}\ , G_{22}, T_{20}, T_{21}, T_{22}$ 272

(a) The operator $\mathcal{A}^{(4)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 273

$$G_{24}(t) \leq G_{24}^0 + \int_0^t \left[(a_{24})^{(4)} \left(G_{25}^0 + (\hat{P}_{24})^{(4)} e^{(\hat{M}_{24})^{(4)} s_{(24)}} \right) \right] ds_{(24)} =$$

$$\left(1 + (a_{24})^{(4)} t\right) G_{25}^0 + \frac{(a_{24})^{(4)} (\hat{P}_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left(e^{(\hat{M}_{24})^{(4)} t} - 1 \right)$$

From which it follows that 274

$$(G_{24}(t) - G_{24}^0) e^{-(\hat{M}_{24})^{(4)} t} \leq \frac{(a_{24})^{(4)}}{(\hat{M}_{24})^{(4)}} \left[\left((\hat{P}_{24})^{(4)} + G_{25}^0 \right) e^{\left(-\frac{(\hat{P}_{24})^{(4)} + G_{25}^0}{G_{25}^0} \right)} + (\hat{P}_{24})^{(4)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(b) The operator $\mathcal{A}^{(5)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 275

$$G_{28}(t) \leq G_{28}^0 + \int_0^t \left[(a_{28})^{(5)} \left(G_{29}^0 + (\hat{P}_{28})^{(5)} e^{(\hat{M}_{28})^{(5)} s_{(28)}} \right) \right] ds_{(28)} =$$

$$\left(1 + (a_{28})^{(5)} t\right) G_{29}^0 + \frac{(a_{28})^{(5)} (\hat{P}_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left(e^{(\hat{M}_{28})^{(5)} t} - 1 \right)$$

From which it follows that 276

$$(G_{28}(t) - G_{28}^0) e^{-(\hat{M}_{28})^{(5)} t} \leq \frac{(a_{28})^{(5)}}{(\hat{M}_{28})^{(5)}} \left[\left((\hat{P}_{28})^{(5)} + G_{29}^0 \right) e^{\left(-\frac{(\hat{P}_{28})^{(5)} + G_{29}^0}{G_{29}^0} \right)} + (\hat{P}_{28})^{(5)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

(c) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying GLOBAL EQUATIONS into itself .Indeed it is obvious that 277

$$G_{32}(t) \leq G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\hat{P}_{32})^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} =$$

$$\left(1 + (a_{32})^{(6)} t\right) G_{33}^0 + \frac{(a_{32})^{(6)} (\hat{P}_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that 278

$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \leq \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\hat{P}_{32})^{(6)} + G_{33}^0 \right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\hat{P}_{32})^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem1

Analogous inequalities hold also for G_{25} , $G_{26}, T_{24}, T_{25}, T_{26}$

(d) The operator $\mathcal{A}^{(7)}$ maps the space of functions satisfying 37,35,36 into itself .Indeed it is obvious that 279

$$G_{36}(t) \leq G_{36}^0 + \int_0^t \left[(a_{36})^{(7)} \left(G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)} s_{(36)}} \right) \right] ds_{(36)} =$$

$$\left(1 + (a_{36})^{(7)} t\right) G_{37}^0 + \frac{(a_{36})^{(7)} (\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left(e^{(\hat{M}_{36})^{(7)} t} - 1 \right)$$

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From which it follows that

$$(G_{36}(t)-G_{36}^0)e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}}\left[\left((\hat{P}_{36})^{(7)}+G_{37}^0\right)e^{\left(-\frac{(\hat{P}_{36})^{(7)}+G_{37}^0}{G_{37}^0}\right)}+(\hat{P}_{36})^{(7)}\right]$$

(G_i^0) is as defined in the statement of theorem 7

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose 281

$(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ large to have

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$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[(\hat{P}_{13})^{(1)}+\left((\hat{P}_{13})^{(1)}+G_j^0\right)e^{-\left(\frac{(\hat{P}_{13})^{(1)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{13})^{(1)}$$

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$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}}\left[\left((\hat{Q}_{13})^{(1)}+T_j^0\right)e^{-\left(\frac{(\hat{Q}_{13})^{(1)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{13})^{(1)}\right] \leq (\hat{Q}_{13})^{(1)}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i , T_i satisfying GLOBAL EQUATIONS into itself 285

The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric 286

$$d\left(\left(G^{(1)},T^{(1)}\right),\left(G^{(2)},T^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\hat{M}_{13})^{(1)}t}\}$$

Indeed if we denote 287

Definition of $\tilde{G},\tilde{T}$:

$$\left(\tilde{G},\tilde{T}\right) = \mathcal{A}^{(1)}(G,T)$$

It results

$$\left|\tilde{G}_{13}^{(1)}-\tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{13})^{(1)}\left|G_{14}^{(1)}-G_{14}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\,ds_{(13)} +$$

$$\int_0^t\{(a_{13}')^{(1)}\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{-(\hat{M}_{13})^{(1)}s_{(13)}} +$$

$$(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)\left|G_{13}^{(1)}-G_{13}^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}} +$$

$$G_{13}^{(2)}\left|(a_{13}'')^{(1)}\left(T_{14}^{(1)},s_{(13)}\right)-(a_{13}'')^{(1)}\left(T_{14}^{(2)},s_{(13)}\right)\right|\; e^{-(\hat{M}_{13})^{(1)}s_{(13)}}e^{(\hat{M}_{13})^{(1)}s_{(13)}}\}ds_{(13)}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)}-G^{(2)}\right|e^{-(\hat{M}_{13})^{(1)}t} \leq$$ 288

$$\frac{1}{(\hat{M}_{13})^{(1)}}\left((a_{13})^{(1)}+(a_{13}')^{(1)}+(\hat{A}_{13})^{(1)}+(\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}\right)d\left(\left(G^{(1)},T^{(1)};\;G^{(2)},T^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{13}'')^{(1)}$ and $(b_{13}'')^{(1)}$ depending also on t can be considered as not 289 conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ and $(\hat{Q}_{13})^{(1)}e^{(\hat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(1)}$ and $(b_i'')^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on $G(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 290

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i')^{(1)} - (a_i'')^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$ 291

$$T_i(t) \geq T_i^0 e^{(-(b_i')^{(1)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\hat{M}_{13})^{(1)}\right)_1, and \left((\hat{M}_{13})^{(1)}\right)_3$: 292

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if

$G_{13} < (\hat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\hat{M}_{13})^{(1)}\right)_1 - (a_{14}')^{(1)}G_{14}$ and by integrating

$$G_{14} \leq \left((\hat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\hat{M}_{13})^{(1)}\right)_1/(a_{14}')^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\hat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\hat{M}_{13})^{(1)}\right)_2/(a_{15}')^{(1)}$$

If G_{14} or G_{15} is bounded, the same property follows for G_{13} , G_{15} and G_{13} , G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} . The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. 293

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(1)}(G(t),t)) = (b_{14}')^{(1)}$ then $T_{14} \to \infty$. 294

Definition of $(m)^{(1)}$ and ε_1 :

Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b_i'')^{(1)}(G(t),t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to 295

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$ If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$ By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b_{15}'')^{(1)}(G(t),t) = (b_{15}')^{(1)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}$, $\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}} < 1$ and to choose 297

$(\hat{P}_{16})^{(2)}$ and $(\hat{Q}_{16})^{(2)}$ large to have

$$\frac{(a_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[(\hat{P}_{16})^{(2)} + ((\hat{P}_{16})^{(2)} + G_j^0)e^{-\left(\frac{(\hat{P}_{16})^{(2)}+G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{16})^{(2)}$$ 298

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$$\frac{(b_i)^{(2)}}{(\hat{M}_{16})^{(2)}}\left[((\hat{Q}_{16})^{(2)} + T_j^0)e^{-\left(\frac{(\hat{Q}_{16})^{(2)}+T_j^0}{T_j^0}\right)} + (\hat{Q}_{16})^{(2)}\right] \leq (\hat{Q}_{16})^{(2)}$$

In order that the operator $\mathcal{A}^{(2)}$ transforms the space of sextuples of functions G_i, T_i satisfying 300

The operator $\mathcal{A}^{(2)}$ is a contraction with respect to the metric 301

$$d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)}\right), \left((G_{19})^{(2)}, (T_{19})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} \left|G_i^{(1)}(t) - G_i^{(2)}(t)\right| e^{-(\hat{M}_{16})^{(2)}t}, \max_{t\in\mathbb{R}_+} \left|T_i^{(1)}(t) - T_i^{(2)}(t)\right| e^{-(\hat{M}_{16})^{(2)}t}\}$$

Indeed if we denote 302

Definition of $\widetilde{G_{19}}, \widetilde{T_{19}}$: $\left(\widetilde{G_{19}}, \widetilde{T_{19}}\right) = \mathcal{A}^{(2)}(G_{19}, T_{19})$

It results 303

$$\left|\tilde{G}_{16}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{16})^{(2)} \left|G_{17}^{(1)} - G_{17}^{(2)}\right| e^{-(\hat{M}_{16})^{(2)} s_{(16)}} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\, ds_{(16)} +$$

$$\int_0^t \{(a_{16}^{'})^{(2)} \left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\hat{M}_{16})^{(2)} s_{(16)}} e^{-(\hat{M}_{16})^{(2)} s_{(16)}} +$$

$$(a_{16}^{''})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right)\left|G_{16}^{(1)} - G_{16}^{(2)}\right| e^{-(\hat{M}_{16})^{(2)} s_{(16)}} e^{(\hat{M}_{16})^{(2)} s_{(16)}} +$$

$$G_{16}^{(2)} \left|(a_{16}^{''})^{(2)}\left(T_{17}^{(1)}, s_{(16)}\right) - (a_{16}^{''})^{(2)}\left(T_{17}^{(2)}, s_{(16)}\right)\right|\ e^{-(\hat{M}_{16})^{(2)} s_{(16)}} e^{(\hat{M}_{16})^{(2)} s_{(16)}}\} ds_{(16)}$$

Where $s_{(16)}$ represents integrand that is integrated over the interval $[0, t]$ 304

From the hypotheses it follows

$$\left|(G_{19})^{(1)} - (G_{19})^{(2)}\right| e^{-(\hat{M}_{16})^{(2)} t} \leq$$ 305

$$\frac{1}{(\hat{M}_{16})^{(2)}}\left((a_{16})^{(2)} + (a_{16}^{'})^{(2)} + (\hat{A}_{16})^{(2)} + (\hat{P}_{16})^{(2)}(\hat{k}_{16})^{(2)}\right) d\left(\left((G_{19})^{(1)}, (T_{19})^{(1)};\ (G_{19})^{(2)}, (T_{19})^{(2)}\right)\right)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows 306

Remark 1: The fact that we supposed $(a_{16}^{''})^{(2)}$ and $(b_{16}^{''})^{(2)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ and $(\hat{Q}_{16})^{(2)} e^{(\hat{M}_{16})^{(2)} t}$ respectively of $\mathbb{R}_+$. 307

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(2)}$ and $(b_i^{''})^{(2)}, i = 16,17,18$ depend only on T_{17} and respectively on (G_{19})(and not on t) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 308

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(2)} - (a_i^{''})^{(2)}(T_{17}(s_{(16)}), s_{(16)})\} ds_{(16)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b_i^{'})^{(2)} t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{16})^{(2)}\right)_1, \left((\widehat{M}_{16})^{(2)}\right)_2$ and $\left((\widehat{M}_{16})^{(2)}\right)_3$: 309

Remark 3: if G_{16} is bounded, the same property have also G_{17} and G_{18} . indeed if

$G_{16} < (\widehat{M}_{16})^{(2)}$ it follows $\frac{dG_{17}}{dt} \leq \left((\widehat{M}_{16})^{(2)}\right)_1 - (a'_{17})^{(2)} G_{17}$ and by integrating

$G_{17} \leq \left((\widehat{M}_{16})^{(2)}\right)_2 = G_{17}^0 + 2(a_{17})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_1 / (a'_{17})^{(2)}$

In the same way , one can obtain

$G_{18} \leq \left((\widehat{M}_{16})^{(2)}\right)_3 = G_{18}^0 + 2(a_{18})^{(2)}\left((\widehat{M}_{16})^{(2)}\right)_2 / (a'_{18})^{(2)}$ 310

If G_{17} or G_{18} is bounded, the same property follows for G_{16} , G_{18} and G_{16} , G_{17} respectively.

Remark 4: If G_{16} is bounded, from below, the same property holds for G_{17} and G_{18} . The proof is analogous with the preceding one. An analogous property is true if G_{17} is bounded from below. 311

Remark 5: If T_{16} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(2)}((G_{19})(t),t)) = (b'_{17})^{(2)}$ then $T_{17} \to \infty$. 312

Definition of $(m)^{(2)}$ and ε_2 :

Indeed let t_2 be so that for $t > t_2$

$(b_{17})^{(2)} - (b''_i)^{(2)}((G_{19})(t),t) < \varepsilon_2, T_{16}(t) > (m)^{(2)}$

Then $\frac{dT_{17}}{dt} \geq (a_{17})^{(2)}(m)^{(2)} - \varepsilon_2 T_{17}$ which leads to 313

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{\varepsilon_2}\right)(1 - e^{-\varepsilon_2 t}) + T_{17}^0 e^{-\varepsilon_2 t}$ If we take t such that $e^{-\varepsilon_2 t} = \frac{1}{2}$ it results

$T_{17} \geq \left(\frac{(a_{17})^{(2)}(m)^{(2)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_2}$ By taking now ε_2 sufficiently small one sees that T_{17} is unbounded. The same property holds for T_{18} if $\lim_{t\to\infty}(b''_{18})^{(2)}\left((G_{19})(t),t\right) = (b'_{18})^{(2)}$ 314

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}, \frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}} < 1$ and to choose 316

$(\widehat{P}_{20})^{(3)}$ and $(\widehat{Q}_{20})^{(3)}$ large to have

$$\frac{(a_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[(\widehat{P}_{20})^{(3)} + \left((\widehat{P}_{20})^{(3)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{20})^{(3)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{20})^{(3)}$$ 317

$$\frac{(b_i)^{(3)}}{(\widehat{M}_{20})^{(3)}}\left[\left((\widehat{Q}_{20})^{(3)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{20})^{(3)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{20})^{(3)}\right] \leq (\widehat{Q}_{20})^{(3)}$$ 318

In order that the operator $\mathcal{A}^{(3)}$ transforms the space of sextuples of functions G_i, T_i into itself 319

The operator $\mathcal{A}^{(3)}$ is a contraction with respect to the metric 320

$$d\left(\left((G_{23})^{(1)},(T_{23})^{(1)}\right),\left((G_{23})^{(2)},(T_{23})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{20})^{(3)}t}\}$$

Indeed if we denote 321

Definition of $\widetilde{G_{23}}, \widetilde{T_{23}}$ $:\left(\left(\widetilde{G_{23}}\right), \left(\widetilde{T_{23}}\right) \right) = \mathcal{A}^{(3)}\left((G_{23}), (T_{23}) \right)$

It results 322

$$\left|\tilde{G}_{20}^{(1)} - \tilde{G}_i^{(2)}\right| \le \int_0^t (a_{20})^{(3)} \left|G_{21}^{(1)} - G_{21}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}}\, ds_{(20)} +$$

$$\int_0^t \{ (a'_{20})^{(3)} \left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} +$$ 323

$$(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right)\left|G_{20}^{(1)} - G_{20}^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} +$$

$$G_{20}^{(2)} \left|(a''_{20})^{(3)}\left(T_{21}^{(1)}, s_{(20)}\right) - (a''_{20})^{(3)}\left(T_{21}^{(2)}, s_{(20)}\right)\right| \; e^{-(\widehat{M}_{20})^{(3)} s_{(20)}} e^{(\widehat{M}_{20})^{(3)} s_{(20)}} \} ds_{(20)}$$

Where $s_{(20)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{20})^{(3)} t} \le \frac{1}{(\widehat{M}_{20})^{(3)}} \left((a_{20})^{(3)} + (a'_{20})^{(3)} + (\widehat{A}_{20})^{(3)} + (\widehat{P}_{20})^{(3)} (\widehat{k}_{20})^{(3)} \right) d\left(\left((G_{23})^{(1)}, (T_{23})^{(1)};\ (G_{23})^{(2)}, (T_{23})^{(2)} \right) \right)$$ 324

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a''_{20})^{(3)}$ and $(b''_{20})^{(3)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ *and* $(\widehat{Q}_{20})^{(3)} e^{(\widehat{M}_{20})^{(3)} t}$ respectively of $\mathbb{R}_+$. 325

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(3)}$ and $(b''_i)^{(3)}, i = 20,21,22$ depend only on T_{21} and respectively on (G_{23})(*and not on t*) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ *and* $T_i(t) = 0$ 326

From 19 to 24 it results

$$G_i(t) \ge G_i^0 e^{\left[-\int_0^t \{ (a'_i)^{(3)} - (a''_i)^{(3)} (T_{21}(s_{(20)}), s_{(20)}) \} ds_{(20)} \right]} \ge 0$$

$$T_i(t) \ge T_i^0 e^{\left(-(b'_i)^{(3)} t \right)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{20})^{(3)} \right)_1, \left((\widehat{M}_{20})^{(3)} \right)_2$ *and* $\left((\widehat{M}_{20})^{(3)} \right)_3$: 327

Remark 3: if G_{20} is bounded, the same property have also G_{21} *and* G_{22} . indeed if

$G_{20} < (\widehat{M}_{20})^{(3)}$ it follows $\frac{dG_{21}}{dt} \le \left((\widehat{M}_{20})^{(3)} \right)_1 - (a'_{21})^{(3)} G_{21}$ and by integrating

$$G_{21} \le \left((\widehat{M}_{20})^{(3)} \right)_2 = G_{21}^0 + 2(a_{21})^{(3)} \left((\widehat{M}_{20})^{(3)} \right)_1 / (a'_{21})^{(3)}$$

In the same way , one can obtain

$$G_{22} \le \left((\widehat{M}_{20})^{(3)} \right)_3 = G_{22}^0 + 2(a_{22})^{(3)} \left((\widehat{M}_{20})^{(3)} \right)_2 / (a'_{22})^{(3)}$$

If G_{21} *or* G_{22} is bounded, the same property follows for G_{20} , G_{22} and G_{20} , G_{21} respectively.

Remark 4: If G_{20} *is* bounded, from below, the same property holds for G_{21} *and* G_{22} . The proof is analogous with the preceding one. An analogous property is true if G_{21} is bounded from below. 328

Remark 5: If T_{20} is bounded from below and $\lim_{t\to\infty} ((b''_i)^{(3)} \left((G_{23})(t), t \right) = (b'_{21})^{(3)}$ then $T_{21} \to \infty$. 329

Definition of $(m)^{(3)}$ and ε_3 :

Indeed let t_3 be so that for $t > t_3$ 330

$(b_{21})^{(3)} - (b_i'')^{(3)}\big((G_{23})(t), t\big) < \varepsilon_3, T_{20}\,(t) > (m)^{(3)}$

Then $\frac{dT_{21}}{dt} \geq (a_{21})^{(3)}(m)^{(3)} - \varepsilon_3 T_{21}$ which leads to 331

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{\varepsilon_3}\right)(1 - e^{-\varepsilon_3 t}) + T_{21}^0 e^{-\varepsilon_3 t}$ If we take t such that $e^{-\varepsilon_3 t} = \frac{1}{2}$ it results

$T_{21} \geq \left(\frac{(a_{21})^{(3)}(m)^{(3)}}{2}\right), \quad t = log\frac{2}{\varepsilon_3}$ By taking now ε_3 sufficiently small one sees that T_{21} is unbounded. The same property holds for T_{22} if $\lim_{t\to\infty}(b_{22}'')^{(3)}\left((G_{23})(t), t\right) = (b_{22}')^{(3)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

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It is now sufficient to take $\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}$, $\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}} < 1$ and to choose 333

$(\hat{P}_{24})^{(4)}$ and $(\hat{Q}_{24})^{(4)}$ large to have

$$\frac{(a_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[(\hat{P}_{24})^{(4)} + \left((\hat{P}_{24})^{(4)} + G_j^0\right)e^{-\left(\frac{(\hat{P}_{24})^{(4)} + G_j^0}{G_j^0}\right)}\right] \leq (\hat{P}_{24})^{(4)} \quad 334$$

$$\frac{(b_i)^{(4)}}{(\hat{M}_{24})^{(4)}}\left[\left((\hat{Q}_{24})^{(4)} + T_j^0\right)e^{-\left(\frac{(\hat{Q}_{24})^{(4)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{24})^{(4)}\right] \leq (\hat{Q}_{24})^{(4)} \quad 335$$

In order that the operator $\mathcal{A}^{(4)}$ transforms the space of sextuples of functions G_i , T_i satisfying IN to itself 336

The operator $\mathcal{A}^{(4)}$ is a contraction with respect to the metric 337

$$d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}\right), \left((G_{27})^{(2)}, (T_{27})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\hat{M}_{24})^{(4)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{27})}, \widetilde{(T_{27})}$: $\left(\widetilde{(G_{27})}, \widetilde{(T_{27})}\right) = \mathcal{A}^{(4)}((G_{27}), (T_{27}))$

It results

$$\left|\tilde{G}_{24}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{24})^{(4)}\left|G_{25}^{(1)} - G_{25}^{(2)}\right|e^{-(\hat{M}_{24})^{(4)}s_{(24)}}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\,ds_{(24)} +$$

$$\int_0^t\{(a_{24}')^{(4)}\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\hat{M}_{24})^{(4)}s_{(24)}}e^{-(\hat{M}_{24})^{(4)}s_{(24)}} +$$

$$(a_{24}'')^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right)\left|G_{24}^{(1)} - G_{24}^{(2)}\right|e^{-(\hat{M}_{24})^{(4)}s_{(24)}}e^{(\hat{M}_{24})^{(4)}s_{(24)}} +$$

$$G_{24}^{(2)}\left|(a_{24}'')^{(4)}\left(T_{25}^{(1)}, s_{(24)}\right) - (a_{24}'')^{(4)}\left(T_{25}^{(2)}, s_{(24)}\right)\right|\; e^{-(\hat{M}_{24})^{(4)}s_{(24)}}e^{(\hat{M}_{24})^{(4)}s_{(24)}}\}ds_{(24)}$$

Where $s_{(24)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses it follows

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$\left|(G_{27})^{(1)} - (G_{27})^{(2)}\right| e^{-(\widehat{M}_{24})^{(4)}t} \leq$ 339

$\frac{1}{(\widehat{M}_{24})^{(4)}}\left((a_{24})^{(4)} + (a_{24}^{'})^{(4)} + (\widehat{A}_{24})^{(4)} + (\widehat{P}_{24})^{(4)}(\widehat{k}_{24})^{(4)}\right) d\left(\left((G_{27})^{(1)}, (T_{27})^{(1)}; (G_{27})^{(2)}, (T_{27})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{24}^{''})^{(4)}$ and $(b_{24}^{''})^{(4)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ and $(\widehat{Q}_{24})^{(4)} e^{(\widehat{M}_{24})^{(4)}t}$ respectively of $\mathbb{R}_+$. 340

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(4)}$ and $(b_i^{''})^{(4)}, i = 24,25,26$ depend only on T_{25} and respectively on $(G_{27})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t) = 0$ and $T_i\,(t) = 0$ 341

From GLOBAL EQUATIONS it results

$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t \{(a_i^{'})^{(4)} - (a_i^{''})^{(4)}(T_{25}(s_{(24)}), s_{(24)})\} ds_{(24)}\right]} \geq 0$

$T_i\,(t) \geq T_i^0 e^{\left(-(b_i^{'})^{(4)}t\right)} > 0$ for t > 0

Definition of $\left((\widehat{M}_{24})^{(4)}\right)_1, \left((\widehat{M}_{24})^{(4)}\right)_2$ and $\left((\widehat{M}_{24})^{(4)}\right)_3$: 342

Remark 3: if G_{24} is bounded, the same property have also G_{25} and G_{26} . indeed if

$G_{24} < (\widehat{M}_{24})^{(4)}$ it follows $\frac{dG_{25}}{dt} \leq \left((\widehat{M}_{24})^{(4)}\right)_1 - (a_{25}^{'})^{(4)} G_{25}$ and by integrating

$G_{25} \leq \left((\widehat{M}_{24})^{(4)}\right)_2 = G_{25}^0 + 2(a_{25})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_1 / (a_{25}^{'})^{(4)}$

In the same way , one can obtain

$G_{26} \leq \left((\widehat{M}_{24})^{(4)}\right)_3 = G_{26}^0 + 2(a_{26})^{(4)}\left((\widehat{M}_{24})^{(4)}\right)_2 / (a_{26}^{'})^{(4)}$

If G_{25} or G_{26} is bounded, the same property follows for G_{24} , G_{26} and G_{24} , G_{25} respectively.

Remark 4: If G_{24} is bounded, from below, the same property holds for G_{25} and G_{26} . The proof is analogous with the preceding one. An analogous property is true if G_{25} is bounded from below. 343

Remark 5: If T_{24} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(4)}((G_{27})(t), t)) = (b_{25}^{'})^{(4)}$ then $T_{25} \to \infty$. 344

Definition of $(m)^{(4)}$ and ε_4 :

Indeed let t_4 be so that for $t > t_4$

$(b_{25})^{(4)} - (b_i^{''})^{(4)}((G_{27})(t), t) < \varepsilon_4, T_{24}\,(t) > (m)^{(4)}$

Then $\frac{dT_{25}}{dt} \geq (a_{25})^{(4)}(m)^{(4)} - \varepsilon_4 T_{25}$ which leads to 345

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{\varepsilon_4}\right)(1 - e^{-\varepsilon_4 t}) + T_{25}^0 e^{-\varepsilon_4 t}$ If we take t such that $e^{-\varepsilon_4 t} = \frac{1}{2}$ it results

$T_{25} \geq \left(\frac{(a_{25})^{(4)}(m)^{(4)}}{2}\right), \quad t = log\frac{2}{\varepsilon_4}$ By taking now ε_4 sufficiently small one sees that T_{25} is unbounded. The same property holds for T_{26} if $\lim_{t\to\infty}(b''_{26})^{(4)}\left((G_{27})(t),t\right)=(b'_{26})^{(4)}$

We now state a more precise theorem about the behaviors at infinity of the solutions ANALOGOUS inequalities hold also for G_{29} , G_{30}, T_{28}, T_{29}, T_{30}

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It is now sufficient to take $\frac{(a_i)^{(5)}}{(\mathcal{M}_{28})^{(5)}}$, $\frac{(b_i)^{(5)}}{(\mathcal{M}_{28})^{(5)}}<1$ and to choose 347

$(\widehat{P}_{28})^{(5)}$ and $(\widehat{Q}_{28})^{(5)}$ large to have

$$\frac{(a_i)^{(5)}}{(\mathcal{M}_{28})^{(5)}}\left[(\widehat{P}_{28})^{(5)}+\left((\widehat{P}_{28})^{(5)}+G_j^0\right)e^{-\left(\frac{(\widehat{P}_{28})^{(5)}+G_j^0}{G_j^0}\right)}\right]\leq(\widehat{P}_{28})^{(5)}$$ 348

$$\frac{(b_i)^{(5)}}{(\mathcal{M}_{28})^{(5)}}\left[\left((\widehat{Q}_{28})^{(5)}+T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{28})^{(5)}+T_j^0}{T_j^0}\right)}+(\widehat{Q}_{28})^{(5)}\right]\leq(\widehat{Q}_{28})^{(5)}$$ 349

In order that the operator $\mathcal{A}^{(5)}$ transforms the space of sextuples of functions G_i , T_i into itself 350

The operator $\mathcal{A}^{(5)}$ is a contraction with respect to the metric 351

$$d\left(\left((G_{31})^{(1)},(T_{31})^{(1)}\right),\left((G_{31})^{(2)},(T_{31})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{28})^{(5)}t}\}$$

Indeed if we denote

<u>**Definition of**</u> $\widetilde{(G_{31})},\widetilde{(T_{31})}$: $\left(\widetilde{(G_{31})},\widetilde{(T_{31})}\right)=\mathcal{A}^{(5)}\left((G_{31}),(T_{31})\right)$

It results

$$\left|\tilde{G}_{28}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{28})^{(5)}\left|G_{29}^{(1)}-G_{29}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\,ds_{(28)}+$$

$$\int_0^t\{(a'_{28})^{(5)}\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)\left|G_{28}^{(1)}-G_{28}^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}+$$

$$G_{28}^{(2)}|(a''_{28})^{(5)}\left(T_{29}^{(1)},s_{(28)}\right)-(a''_{28})^{(5)}\left(T_{29}^{(2)},s_{(28)}\right)|\;e^{-(\widehat{M}_{28})^{(5)}s_{(28)}}e^{(\widehat{M}_{28})^{(5)}s_{(28)}}\}ds_{(28)}$$

Where $s_{(28)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|(G_{31})^{(1)}-(G_{31})^{(2)}\right|e^{-(\widehat{M}_{28})^{(5)}t}\leq$$ 353

$$\frac{1}{(\widehat{M}_{28})^{(5)}}\left((a_{28})^{(5)}+(a'_{28})^{(5)}+(\widehat{A}_{28})^{(5)}+(\widehat{P}_{28})^{(5)}(\widehat{k}_{28})^{(5)}\right)d\left(\left((G_{31})^{(1)},(T_{31})^{(1)};\;(G_{31})^{(2)},(T_{31})^{(2)}\right)\right)$$

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (35,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{28})^{(5)}$ and $(b''_{28})^{(5)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ and $(\hat{Q}_{28})^{(5)}e^{(\hat{M}_{28})^{(5)}t}$ respectively of $\mathbb{R}_+$. 354

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(5)}$ and $(b''_i)^{(5)}, i = 28,29,30$ depend only on T_{29} and respectively on $(G_{31})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t) = 0$ and $T_i(t) = 0$ 355

From GLOBAL EQUATIONS it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(5)} - (a''_i)^{(5)}(T_{29}(s_{(28)}),s_{(28)})\}ds_{(28)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(5)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\hat{M}_{28})^{(5)}\right)_1, \left((\hat{M}_{28})^{(5)}\right)_2$ and $\left((\hat{M}_{28})^{(5)}\right)_3$: 356

Remark 3: if G_{28} is bounded, the same property have also G_{29} and G_{30} . indeed if

$G_{28} < (\hat{M}_{28})^{(5)}$ it follows $\frac{dG_{29}}{dt} \leq \left((\hat{M}_{28})^{(5)}\right)_1 - (a'_{29})^{(5)}G_{29}$ and by integrating

$$G_{29} \leq \left((\hat{M}_{28})^{(5)}\right)_2 = G_{29}^0 + 2(a_{29})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_1/(a'_{29})^{(5)}$$

In the same way , one can obtain

$$G_{30} \leq \left((\hat{M}_{28})^{(5)}\right)_3 = G_{30}^0 + 2(a_{30})^{(5)}\left((\hat{M}_{28})^{(5)}\right)_2/(a'_{30})^{(5)}$$

If G_{29} or G_{30} is bounded, the same property follows for G_{28} , G_{30} and G_{28} , G_{29} respectively.

Remark 4: If G_{28} is bounded, from below, the same property holds for G_{29} and G_{30} . The proof is analogous with the preceding one. An analogous property is true if G_{29} is bounded from below. 357

Remark 5: If T_{28} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(5)}((G_{31})(t),t)) = (b'_{29})^{(5)}$ then $T_{29} \to \infty$. 358

Definition of $(m)^{(5)}$ and ε_5 :

Indeed let t_5 be so that for $t > t_5$

$$(b_{29})^{(5)} - (b''_i)^{(5)}((G_{31})(t),t) < \varepsilon_5, T_{28}(t) > (m)^{(5)}$$

359

Then $\frac{dT_{29}}{dt} \geq (a_{29})^{(5)}(m)^{(5)} - \varepsilon_5 T_{29}$ which leads to 360

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{\varepsilon_5}\right)(1 - e^{-\varepsilon_5 t}) + T_{29}^0 e^{-\varepsilon_5 t}$ If we take t such that $e^{-\varepsilon_5 t} = \frac{1}{2}$ it results

$T_{29} \geq \left(\frac{(a_{29})^{(5)}(m)^{(5)}}{2}\right), \quad t = log\frac{2}{\varepsilon_5}$ By taking now ε_5 sufficiently small one sees that T_{29} is unbounded. The same property holds for T_{30} if $\lim_{t\to\infty}(b''_{30})^{(5)}\left((G_{31})(t),t\right) = (b'_{30})^{(5)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

Analogous inequalities hold also for G_{33} , $G_{34}, T_{32}, T_{33}, T_{34}$

361

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}$, $\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose 362

$(\widehat{P}_{32})^{(6)}$ and $(\widehat{Q}_{32})^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[(\widehat{P}_{32})^{(6)} + \left((\widehat{P}_{32})^{(6)} + G_j^0\right)e^{-\left(\frac{(\widehat{P}_{32})^{(6)}+G_j^0}{G_j^0}\right)}\right] \leq (\widehat{P}_{32})^{(6)}$$ 363

$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[\left((\widehat{Q}_{32})^{(6)} + T_j^0\right)e^{-\left(\frac{(\widehat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)} + (\widehat{Q}_{32})^{(6)}\right] \leq (\widehat{Q}_{32})^{(6)}$$ 364

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i , T_i into itself 365

The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric 366

$$d\left(\left((G_{35})^{(1)}, (T_{35})^{(1)}\right), \left((G_{35})^{(2)}, (T_{35})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}, \max_{t\in\mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{35})}, \widetilde{(T_{35})}$: $\left(\widetilde{(G_{35})}, \widetilde{(T_{35})}\right) = \mathcal{A}^{(6)}\left((G_{35}), (T_{35})\right)$

It results

$$\left|\tilde{G}_{32}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{32})^{(6)} \left|G_{33}^{(1)} - G_{33}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\, ds_{(32)} +$$

$$\int_0^t \{(a_{32}^{'})^{(6)}\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} +$$

$$(a_{32}^{''})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right)\left|G_{32}^{(1)} - G_{32}^{(2)}\right| e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}} +$$

$$G_{32}^{(2)}|(a_{32}^{''})^{(6)}\left(T_{33}^{(1)}, s_{(32)}\right) - (a_{32}^{''})^{(6)}\left(T_{33}^{(2)}, s_{(32)}\right)|\ e^{-(\widehat{M}_{32})^{(6)}s_{(32)}} e^{(\widehat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$ 367

From the hypotheses it follows

(1) $(a_i^{'})^{(1)}, (a_i^{''})^{(1)}, (b_i)^{(1)}, (b_i^{'})^{(1)}, (b_i^{''})^{(1)} > 0$,
$i, j = 13,14,15$

(2)The functions $(a_i^{''})^{(1)}, (b_i^{''})^{(1)}$ are positive continuous increasing and bounded.

Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$(a_i^{''})^{(1)}(T_{14}, t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)}$

$(b_i^{''})^{(1)}(G, t) \leq (r_i)^{(1)} \leq (b_i^{'})^{(1)} \leq (\hat{B}_{13})^{(1)}$

(3) $\lim_{T_2\to\infty}(a_i^{''})^{(1)}(T_{14}, t) = (p_i)^{(1)}$
$\lim_{G\to\infty}(b_i^{''})^{(1)}(G, t) = (r_i)^{(1)}$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$:

Where $\boxed{(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}}$ are positive constants
and $\boxed{i = 13,14,15}$

They satisfy Lipschitz condition:

$$|(a_i'')^{(1)}(T_{14}', t) - (a_i'')^{(1)}(T_{14}, t)| \leq (\hat{k}_{13})^{(1)} |T_{14} - T_{14}'| e^{-(\hat{M}_{13})^{(1)} t}$$

$$|(b_i'')^{(1)}(G', t) - (b_i'')^{(1)}(G, T)| < (\hat{k}_{13})^{(1)} ||G - G'|| e^{-(\hat{M}_{13})^{(1)} t}$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}', t)$ and $(a_i'')^{(1)}(T_{14}, t)$. (T_{14}', t) and (T_{14}, t) are points belonging to the interval $[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}]$. It is to be noted that $(a_i'')^{(1)}(T_{14}, t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14}, t)$, the **first augmentation coefficient** attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$:

(Z) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \ , \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$:

(AA) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}, \ (r_i)^{(1)}, i = 13,14,15,$
satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)} + (\hat{P}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

$$\frac{1}{(\hat{M}_{13})^{(1)}} [(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} + (\hat{Q}_{13})^{(1)} (\hat{k}_{13})^{(1)}] < 1$$

Analogous inequalities hold also for $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$ 368

It is now sufficient to take $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \ , \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 7$ and to choose

$(\hat{P}_{36})^{(7)}$ and $(\hat{Q}_{36})^{(7)}$ large to have

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$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[(\hat{P}_{36})^{(7)} + ((\hat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_j^0}{G_j^0} \right)} \right] \leq (\hat{P}_{36})^{(7)}$$

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$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[((\hat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{36})^{(7)} + T_j^0}{T_j^0} \right)} + (\hat{Q}_{36})^{(7)} \right] \leq (\hat{Q}_{36})^{(7)}$$

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The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 372

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right)=$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t)-G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t},\max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t)-T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})},\widetilde{(T_{39})}$:

$$\left(\widetilde{(G_{39})},\widetilde{(T_{39})}\right)=\mathcal{A}^{(7)}\left((G_{39}),(T_{39})\right)$$

It results

$$\left|\tilde{G}_{36}^{(1)}-\tilde{G}_i^{(2)}\right|\leq\int_0^t(a_{36})^{(7)}\left|G_{37}^{(1)}-G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)}+$$

$$\int_0^t\{(a_{36}')^{(7)}\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}+$$

$$(a_{36}'')^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)\left|G_{36}^{(1)}-G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}+$$

$$G_{36}^{(2)}\left|(a_{36}'')^{(7)}\left(T_{37}^{(1)},s_{(36)}\right)-(a_{36}'')^{(7)}\left(T_{37}^{(2)},s_{(36)}\right)\right|\;e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0,\mathrm{t}]$

From the hypotheses it follows

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$$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t}\leq$$
$$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a_{36}')^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(\left((G_{39})^{(1)},(T_{39})^{(1)};\;(G_{39})^{(2)},(T_{39})^{(2)}\right)\right)$$

And analogous inequalities for $G_i\;and\;T_i$. Taking into account the hypothesis (37,35,36) the result follows

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Remark 1: The fact that we supposed $(a_{36}'')^{(7)}$ and $(b_{36}'')^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}\;and\;(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 375

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i'')^{(7)}$ and $(b_i'')^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\;not\;on\;t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0\;and\;T_i\,(t)=0$ 376

From 79 to 36 it results

$$G_i\,(t)\geq G_i^0e^{\left[-\int_0^t\{(a_i')^{(7)}-(a_i'')^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]}\geq0$$

$$T_i\,(t)\geq T_i^0e^{\left(-(b_i')^{(7)}t\right)}>0\quad\text{for t}>0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1,\left((\widehat{M}_{36})^{(7)}\right)_2\;and\;\left((\widehat{M}_{36})^{(7)}\right)_3$: 377

Remark 3: if G_{36} is bounded, the same property have also $G_{37}\;and\;G_{38}$. indeed if

$G_{36}<(\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt}\leq\left((\widehat{M}_{36})^{(7)}\right)_1-(a_{37}')^{(7)}G_{37}$ and by integrating

$$G_{37}\leq\left((\widehat{M}_{36})^{(7)}\right)_2=G_{37}^0+2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}')^{(7)}$$

In the same way , one can obtain

$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}^{'})^{(7)}$

If G_{37} *or* G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If G_{36} *is* bounded, from below, the same property holds for G_{37} *and* G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 378

Remark 5: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(7)}((G_{39})(t),t)) = (b_{37}^{'})^{(7)}$ then $T_{37} \to \infty$. 379

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 380

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $\lim_{t\to\infty}(b_{38}^{''})^{(7)}\left((G_{39})(t),t\right) = (b_{38}^{'})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 72

In order that the operator $\mathcal{A}^{(7)}$ transforms the space of sextuples of functions G_i, T_i satisfying GLOBAL EQUATIONS AND ITS CONCOMITANT CONDITIONALITIES into itself 381

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The operator $\mathcal{A}^{(7)}$ is a contraction with respect to the metric 383

$$d\left(\left((G_{39})^{(1)},(T_{39})^{(1)}\right),\left((G_{39})^{(2)},(T_{39})^{(2)}\right)\right) =$$

$$\sup_i\{\max_{t\in\mathbb{R}_+}\left|G_i^{(1)}(t) - G_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}, \max_{t\in\mathbb{R}_+}\left|T_i^{(1)}(t) - T_i^{(2)}(t)\right|e^{-(\widehat{M}_{36})^{(7)}t}\}$$

Indeed if we denote

Definition of $\widetilde{(G_{39})}, \widetilde{(T_{39})}$:

$$\left(\widetilde{(G_{39})}, \widetilde{(T_{39})}\right) = \mathcal{A}^{(7)}((G_{39}),(T_{39}))$$

It results

$$\left|\tilde{G}_{36}^{(1)} - \tilde{G}_i^{(2)}\right| \leq \int_0^t (a_{36})^{(7)}\left|G_{37}^{(1)} - G_{37}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\,ds_{(36)} +$$

$$\int_0^t\{(a_{36}^{'})^{(7)}\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{-(\widehat{M}_{36})^{(7)}s_{(36)}} +$$

$$(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right)\left|G_{36}^{(1)} - G_{36}^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}} +$$

$$G_{36}^{(2)}\left|(a_{36}^{''})^{(7)}\left(T_{37}^{(1)}, s_{(36)}\right) - (a_{36}^{''})^{(7)}\left(T_{37}^{(2)}, s_{(36)}\right)\right| e^{-(\widehat{M}_{36})^{(7)}s_{(36)}}e^{(\widehat{M}_{36})^{(7)}s_{(36)}}\}ds_{(36)}$$

Where $s_{(36)}$ represents integrand that is integrated over the interval $[0, \mathrm{t}]$

From the hypotheses it follows

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$\left|(G_{39})^{(1)}-(G_{39})^{(2)}\right|e^{-(\widehat{M}_{36})^{(7)}t} \leq$
$\frac{1}{(\widehat{M}_{36})^{(7)}}\left((a_{36})^{(7)}+(a_{36}^{'})^{(7)}+(\widehat{A}_{36})^{(7)}+(\widehat{P}_{36})^{(7)}(\widehat{k}_{36})^{(7)}\right)d\left(\left((G_{39})^{(1)},(T_{39})^{(1)};\ (G_{39})^{(2)},(T_{39})^{(2)}\right)\right)$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis the result follows

Remark 1: The fact that we supposed $(a_{36}^{''})^{(7)}$ and $(b_{36}^{''})^{(7)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ and $(\widehat{Q}_{36})^{(7)}e^{(\widehat{M}_{36})^{(7)}t}$ respectively of $\mathbb{R}_+$. 385

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a_i^{''})^{(7)}$ and $(b_i^{''})^{(7)}, i=36,37,38$ depend only on T_{37} and respectively on $(G_{39})(and\ not\ on\ t)$ and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i\,(t)=0$ and $T_i\,(t)=0$ 386

From CONCATENATED GLOBAL EQUATIONS it results

$$G_i\,(t) \geq G_i^0 e^{\left[-\int_0^t\{(a_i^{'})^{(7)}-(a_i^{''})^{(7)}(T_{37}(s_{(36)}),s_{(36)})\}ds_{(36)}\right]} \geq 0$$

$$T_i\,(t) \geq T_i^0 e^{(-(b_i^{'})^{(7)}t)} > 0 \quad \text{for } t > 0$$

Definition of $\left((\widehat{M}_{36})^{(7)}\right)_1, \left((\widehat{M}_{36})^{(7)}\right)_2$ and $\left((\widehat{M}_{36})^{(7)}\right)_3$: 387

Remark 3: if G_{36} is bounded, the same property have also G_{37} and G_{38} . indeed if

$G_{36} < (\widehat{M}_{36})^{(7)}$ it follows $\frac{dG_{37}}{dt} \leq \left((\widehat{M}_{36})^{(7)}\right)_1 - (a_{37}^{'})^{(7)}G_{37}$ and by integrating

$$G_{37} \leq \left((\widehat{M}_{36})^{(7)}\right)_2 = G_{37}^0 + 2(a_{37})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_1/(a_{37}^{'})^{(7)}$$

In the same way , one can obtain

$$G_{38} \leq \left((\widehat{M}_{36})^{(7)}\right)_3 = G_{38}^0 + 2(a_{38})^{(7)}\left((\widehat{M}_{36})^{(7)}\right)_2/(a_{38}^{'})^{(7)}$$

If G_{37} or G_{38} is bounded, the same property follows for G_{36} , G_{38} and G_{36} , G_{37} respectively.

Remark 7: If G_{36} is bounded, from below, the same property holds for G_{37} and G_{38} . The proof is analogous with the preceding one. An analogous property is true if G_{37} is bounded from below. 388

Remark 5: If T_{36} is bounded from below and $\lim_{t\to\infty}((b_i^{''})^{(7)}((G_{39})(t),t)) = (b_{37}^{'})^{(7)}$ then $T_{37} \to \infty$. 389

Definition of $(m)^{(7)}$ and ε_7 :

Indeed let t_7 be so that for $t > t_7$

$$(b_{37})^{(7)} - (b_i^{''})^{(7)}((G_{39})(t),t) < \varepsilon_7, T_{36}\,(t) > (m)^{(7)}$$

Then $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$ which leads to 390

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7}\right)(1-e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t}$ If we take t such that $e^{-\varepsilon_7 t} = \frac{1}{2}$ it results

$T_{37} \geq \left(\frac{(a_{37})^{(7)}(m)^{(7)}}{2}\right), \quad t = log\frac{2}{\varepsilon_7}$ By taking now ε_7 sufficiently small one sees that T_{37} is unbounded. The same property holds for T_{38} if $\lim_{t\to\infty}(b''_{38})^{(7)}\left((G_{39})(t),t\right)=(b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions

$-(\sigma_2)^{(2)} \le -(a'_{16})^{(2)}+(a'_{17})^{(2)}-(a''_{16})^{(2)}(T_{17},t)+(a''_{17})^{(2)}(T_{17},t) \le -(\sigma_1)^{(2)}$ 391

$-(\tau_2)^{(2)} \le -(b'_{16})^{(2)}+(b'_{17})^{(2)}-(b''_{16})^{(2)}\left((G_{19}),t\right)-(b''_{17})^{(2)}\left((G_{19}),t\right) \le -(\tau_1)^{(2)}$ 392

Definition of $(\nu_1)^{(2)},(\nu_2)^{(2)},(u_1)^{(2)},(u_2)^{(2)}$: 393

By $(\nu_1)^{(2)}>0$, $(\nu_2)^{(2)}<0$ and respectively $(u_1)^{(2)}>0$, $(u_2)^{(2)}<0$ the roots 394

(a) of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_1)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 395

and $(b_{14})^{(2)}\left(u^{(2)}\right)^2+(\tau_1)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ and 396

Definition of $(\bar{\nu}_1)^{(2)},,(\bar{\nu}_2)^{(2)},(\bar{u}_1)^{(2)},(\bar{u}_2)^{(2)}$: 397

By $(\bar{\nu}_1)^{(2)}>0$, $(\bar{\nu}_2)^{(2)}<0$ and respectively $(\bar{u}_1)^{(2)}>0$, $(\bar{u}_2)^{(2)}<0$ the 398

roots of the equations $(a_{17})^{(2)}\left(\nu^{(2)}\right)^2+(\sigma_2)^{(2)}\nu^{(2)}-(a_{16})^{(2)}=0$ 399

and $(b_{17})^{(2)}\left(u^{(2)}\right)^2+(\tau_2)^{(2)}u^{(2)}-(b_{16})^{(2)}=0$ 400

Definition of $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$:- 401

(b) If we define $(m_1)^{(2)},(m_2)^{(2)},(\mu_1)^{(2)},(\mu_2)^{(2)}$ by 402

$(m_2)^{(2)}=(\nu_0)^{(2)},(m_1)^{(2)}=(\nu_1)^{(2)},\ \textit{if}\ (\nu_0)^{(2)}<(\nu_1)^{(2)}$ 403

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\bar{\nu}_1)^{(2)},\textit{if}\ (\nu_1)^{(2)}<(\nu_0)^{(2)}<(\bar{\nu}_1)^{(2)},$ 404

and $\boxed{(\nu_0)^{(2)}=\frac{G_{16}^0}{G_{17}^0}}$

$(m_2)^{(2)}=(\nu_1)^{(2)},(m_1)^{(2)}=(\nu_0)^{(2)},\ \textit{if}\ (\bar{\nu}_1)^{(2)}<(\nu_0)^{(2)}$ 405

and analogously 406

$(\mu_2)^{(2)}=(u_0)^{(2)},(\mu_1)^{(2)}=(u_1)^{(2)},\ \textit{if}\ (u_0)^{(2)}<(u_1)^{(2)}$

$(\mu_2)^{(2)}=(u_1)^{(2)},(\mu_1)^{(2)}=(\bar{u}_1)^{(2)},\textit{if}\ (u_1)^{(2)}<(u_0)^{(2)}<(\bar{u}_1)^{(2)},$

and $\boxed{(u_0)^{(2)}=\frac{T_{16}^0}{T_{17}^0}}$

$(\mu_2)^{(2)}=(u_1)^{(2)},(\mu_1)^{(2)}=(u_0)^{(2)},\ \textit{if}\ (\bar{u}_1)^{(2)}<(u_0)^{(2)}$ 407

Then the solution satisfies the inequalities 408

$G_{16}^0 e^{\left((S_1)^{(2)}-(p_{16})^{(2)}\right)t} \le G_{16}(t) \le G_{16}^0 e^{(S_1)^{(2)}t}$

$(p_i)^{(2)}$ is defined 409

$\frac{1}{(m_1)^{(2)}}G_{16}^0 e^{\left((S_1)^{(2)}-(p_{16})^{(2)}\right)t} \le G_{17}(t) \le \frac{1}{(m_2)^{(2)}}G_{16}^0 e^{(S_1)^{(2)}t}$ 410

$\left(\frac{(a_{18})^{(2)}G_{16}^0}{(m_1)^{(2)}\left((S_1)^{(2)}-(p_{16})^{(2)}-(S_2)^{(2)}\right)}\left[e^{\left((S_1)^{(2)}-(p_{16})^{(2)}\right)t}-e^{-(S_2)^{(2)}t}\right]+G_{18}^0 e^{-(S_2)^{(2)}t} \le G_{18}(t) \le\right.$ 411

$\frac{(a_{18})^{(2)}G_{16}^{0}}{(m_2)^{(2)}((S_1)^{(2)}-(a_{18}^{'})^{(2)})}[e^{(S_1)^{(2)}t} - e^{-(a_{18}^{'})^{(2)}t}] + G_{18}^{0}e^{-(a_{18}^{'})^{(2)}t})$

$\boxed{T_{16}^{0}e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq T_{16}^{0}e^{((R_1)^{(2)}+(r_{16})^{(2)})t}}$ 412

$\frac{1}{(\mu_1)^{(2)}}T_{16}^{0}e^{(R_1)^{(2)}t} \leq T_{16}(t) \leq \frac{1}{(\mu_2)^{(2)}}T_{16}^{0}e^{((R_1)^{(2)}+(r_{16})^{(2)})t}$ 413

$\frac{(b_{18})^{(2)}T_{16}^{0}}{(\mu_1)^{(2)}((R_1)^{(2)}-(b_{18}^{'})^{(2)})}\left[e^{(R_1)^{(2)}t} - e^{-(b_{18}^{'})^{(2)}t}\right] + T_{18}^{0}e^{-(b_{18}^{'})^{(2)}t} \leq T_{18}(t) \leq$ 414

$\frac{(a_{18})^{(2)}T_{16}^{0}}{(\mu_2)^{(2)}((R_1)^{(2)}+(r_{16})^{(2)}+(R_2)^{(2)})}\left[e^{((R_1)^{(2)}+(r_{16})^{(2)})t} - e^{-(R_2)^{(2)}t}\right] + T_{18}^{0}e^{-(R_2)^{(2)}t}$

Definition of $(S_1)^{(2)}, (S_2)^{(2)}, (R_1)^{(2)}, (R_2)^{(2)}$:- 415

Where $(S_1)^{(2)} = (a_{16})^{(2)}(m_2)^{(2)} - (a_{16}^{'})^{(2)}$ 416

$(S_2)^{(2)} = (a_{18})^{(2)} - (p_{18})^{(2)}$

$(R_1)^{(2)} = (b_{16})^{(2)}(\mu_2)^{(1)} - (b_{16}^{'})^{(2)}$ 417

$(R_2)^{(2)} = (b_{18}^{'})^{(2)} - (r_{18})^{(2)}$

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Behavior of the solutions 419

If we denote and define

Definition of $(\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$:

(a) $\sigma_1)^{(3)}, (\sigma_2)^{(3)}, (\tau_1)^{(3)}, (\tau_2)^{(3)}$ four constants satisfying

$-(\sigma_2)^{(3)} \leq -(a_{20}^{'})^{(3)} + (a_{21}^{'})^{(3)} - (a_{20}^{''})^{(3)}(T_{21}, t) + (a_{21}^{''})^{(3)}(T_{21}, t) \leq -(\sigma_1)^{(3)}$

$-(\tau_2)^{(3)} \leq -(b_{20}^{'})^{(3)} + (b_{21}^{'})^{(3)} - (b_{20}^{''})^{(3)}(G, t) - (b_{21}^{''})^{(3)}((G_{23}), t) \leq -(\tau_1)^{(3)}$

Definition of $(\nu_1)^{(3)}, (\nu_2)^{(3)}, (u_1)^{(3)}, (u_2)^{(3)}$: 420

(b) By $(\nu_1)^{(3)} > 0, (\nu_2)^{(3)} < 0$ and respectively $(u_1)^{(3)} > 0, (u_2)^{(3)} < 0$ the roots of the equations $(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_1)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_1)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$ and

By $(\bar{\nu}_1)^{(3)} > 0, (\bar{\nu}_2)^{(3)} < 0$ and respectively $(\bar{u}_1)^{(3)} > 0, (\bar{u}_2)^{(3)} < 0$ the

roots of the equations $(a_{21})^{(3)}(\nu^{(3)})^2 + (\sigma_2)^{(3)}\nu^{(3)} - (a_{20})^{(3)} = 0$

and $(b_{21})^{(3)}(u^{(3)})^2 + (\tau_2)^{(3)}u^{(3)} - (b_{20})^{(3)} = 0$

Definition of $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$:- 421

(c) If we define $(m_1)^{(3)}, (m_2)^{(3)}, (\mu_1)^{(3)}, (\mu_2)^{(3)}$ by

$(m_2)^{(3)} = (\nu_0)^{(3)}, (m_1)^{(3)} = (\nu_1)^{(3)}, \ \boldsymbol{if} \ (\nu_0)^{(3)} < (\nu_1)^{(3)}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\bar{\nu}_1)^{(3)}, \boldsymbol{if} \ (\nu_1)^{(3)} < (\nu_0)^{(3)} < (\bar{\nu}_1)^{(3)}$,

and $\boxed{(\nu_0)^{(3)} = \frac{G_{20}^{0}}{G_{21}^{0}}}$

$(m_2)^{(3)} = (\nu_1)^{(3)}, (m_1)^{(3)} = (\nu_0)^{(3)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(3)} < (\nu_0)^{(3)}$

and analogously 422

$(\mu_2)^{(3)} = (u_0)^{(3)}, (\mu_1)^{(3)} = (u_1)^{(3)},\ \boldsymbol{if}\ (u_0)^{(3)} < (u_1)^{(3)}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (\bar{u}_1)^{(3)}, \boldsymbol{if}\ (u_1)^{(3)} < (u_0)^{(3)} < (\bar{u}_1)^{(3)},$ and $\boxed{(u_0)^{(3)} = \frac{T_{20}^0}{T_{21}^0}}$

$(\mu_2)^{(3)} = (u_1)^{(3)}, (\mu_1)^{(3)} = (u_0)^{(3)},\ \boldsymbol{if}\ (\bar{u}_1)^{(3)} < (u_0)^{(3)}$

Then the solution satisfies the inequalities

$G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{20}(t) \leq G_{20}^0 e^{(S_1)^{(3)}t}$

$(p_i)^{(3)}$ is defined 423

$\frac{1}{(m_1)^{(3)}} G_{20}^0 e^{((S_1)^{(3)}-(p_{20})^{(3)})t} \leq G_{21}(t) \leq \frac{1}{(m_2)^{(3)}} G_{20}^0 e^{(S_1)^{(3)}t}$ 424

$\left(\frac{(a_{22})^{(3)}G_{20}^0}{(m_1)^{(3)}((S_1)^{(3)}-(p_{20})^{(3)}-(S_2)^{(3)})}\left[e^{((S_1)^{(3)}-(p_{20})^{(3)})t} - e^{-(S_2)^{(3)}t}\right] + G_{22}^0 e^{-(S_2)^{(3)}t} \leq G_{22}(t) \leq\right.$ 425
$\left.\frac{(a_{22})^{(3)}G_{20}^0}{(m_2)^{(3)}((S_1)^{(3)}-(a_{22}')^{(3)})}[e^{(S_1)^{(3)}t} - e^{-(a_{22}')^{(3)}t}] + G_{22}^0 e^{-(a_{22}')^{(3)}t}\right)$

$\boxed{T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}}$ 426

$\frac{1}{(\mu_1)^{(3)}} T_{20}^0 e^{(R_1)^{(3)}t} \leq T_{20}(t) \leq \frac{1}{(\mu_2)^{(3)}} T_{20}^0 e^{((R_1)^{(3)}+(r_{20})^{(3)})t}$ 427

$\frac{(b_{22})^{(3)}T_{20}^0}{(\mu_1)^{(3)}((R_1)^{(3)}-(b_{22}')^{(3)})}\left[e^{(R_1)^{(3)}t} - e^{-(b_{22}')^{(3)}t}\right] + T_{22}^0 e^{-(b_{22}')^{(3)}t} \leq T_{22}(t) \leq$ 428

$\frac{(a_{22})^{(3)}T_{20}^0}{(\mu_2)^{(3)}((R_1)^{(3)}+(r_{20})^{(3)}+(R_2)^{(3)})}\left[e^{((R_1)^{(3)}+(r_{20})^{(3)})t} - e^{-(R_2)^{(3)}t}\right] + T_{22}^0 e^{-(R_2)^{(3)}t}$

Definition of $(S_1)^{(3)}, (S_2)^{(3)}, (R_1)^{(3)}, (R_2)^{(3)}$:- 429

Where $(S_1)^{(3)} = (a_{20})^{(3)}(m_2)^{(3)} - (a_{20}')^{(3)}$

$(S_2)^{(3)} = (a_{22})^{(3)} - (p_{22})^{(3)}$

$(R_1)^{(3)} = (b_{20})^{(3)}(\mu_2)^{(3)} - (b_{20}')^{(3)}$

$(R_2)^{(3)} = (b_{22}')^{(3)} - (r_{22})^{(3)}$

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If we denote and define

Definition of $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$:

(d) $(\sigma_1)^{(4)}, (\sigma_2)^{(4)}, (\tau_1)^{(4)}, (\tau_2)^{(4)}$ four constants satisfying

$-(\sigma_2)^{(4)} \leq -(a_{24}')^{(4)} + (a_{25}')^{(4)} - (a_{24}'')^{(4)}(T_{25}, t) + (a_{25}'')^{(4)}(T_{25}, t) \leq -(\sigma_1)^{(4)}$

$-(\tau_2)^{(4)} \leq -(b_{24}')^{(4)} + (b_{25}')^{(4)} - (b_{24}'')^{(4)}\big((G_{27}), t\big) - (b_{25}'')^{(4)}\big((G_{27}), t\big) \leq -(\tau_1)^{(4)}$

Definition of $(\nu_1)^{(4)}, (\nu_2)^{(4)}, (u_1)^{(4)}, (u_2)^{(4)}, \nu^{(4)}, u^{(4)}$: 433

(e) By $(\nu_1)^{(4)} > 0, (\nu_2)^{(4)} < 0$ and respectively $(u_1)^{(4)} > 0, (u_2)^{(4)} < 0$ the roots of the equations

$(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_1)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_1)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ and

Definition of $(\bar{\nu}_1)^{(4)},, (\bar{\nu}_2)^{(4)}, (\bar{u}_1)^{(4)}, (\bar{u}_2)^{(4)}$: 434
435

By $(\bar{\nu}_1)^{(4)} > 0$, $(\bar{\nu}_2)^{(4)} < 0$ and respectively $(\bar{u}_1)^{(4)} > 0$, $(\bar{u}_2)^{(4)} < 0$ the roots of the equations $(a_{25})^{(4)}\left(\nu^{(4)}\right)^2 + (\sigma_2)^{(4)}\nu^{(4)} - (a_{24})^{(4)} = 0$
and $(b_{25})^{(4)}\left(u^{(4)}\right)^2 + (\tau_2)^{(4)}u^{(4)} - (b_{24})^{(4)} = 0$ 436

Definition of $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}, (\nu_0)^{(4)}$:-

(f) If we define $(m_1)^{(4)}$, $(m_2)^{(4)}$, $(\mu_1)^{(4)}, (\mu_2)^{(4)}$ by

$$(m_2)^{(4)} = (\nu_0)^{(4)}, (m_1)^{(4)} = (\nu_1)^{(4)}, \; \boldsymbol{if} \; (\nu_0)^{(4)} < (\nu_1)^{(4)}$$

$$(m_2)^{(4)} = (\nu_1)^{(4)}, (m_1)^{(4)} = (\bar{\nu}_1)^{(4)} \,, \boldsymbol{if} \; (\nu_4)^{(4)} < (\nu_0)^{(4)} < (\bar{\nu}_1)^{(4)},$$

and $\boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$

$$(m_2)^{(4)} = (\nu_4)^{(4)}, (m_1)^{(4)} = (\nu_0)^{(4)}, \; \boldsymbol{if} \; (\bar{\nu}_4)^{(4)} < (\nu_0)^{(4)}$$

and analogously 437
438

$$(\mu_2)^{(4)} = (u_0)^{(4)}, (\mu_1)^{(4)} = (u_1)^{(4)}, \; \boldsymbol{if} \; (u_0)^{(4)} < (u_1)^{(4)}$$

$$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (\bar{u}_1)^{(4)} \,, \boldsymbol{if} \; (u_1)^{(4)} < (u_0)^{(4)} < (\bar{u}_1)^{(4)},$$

and $\boxed{(u_0)^{(4)} = \frac{T_{24}^0}{T_{25}^0}}$

$(\mu_2)^{(4)} = (u_1)^{(4)}, (\mu_1)^{(4)} = (u_0)^{(4)}, \boldsymbol{if} \; (\bar{u}_1)^{(4)} < (u_0)^{(4)}$ where $(u_1)^{(4)}, (\bar{u}_1)^{(4)}$ are defined respectively

Then the solution satisfies the inequalities 439
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$$G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \leq G_{24}(t) \leq G_{24}^0 e^{(S_1)^{(4)}t}$$ 441
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where $(p_i)^{(4)}$ is defined 443
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$$\frac{1}{(m_1)^{(4)}} G_{24}^0 e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} \leq G_{25}(t) \leq \frac{1}{(m_2)^{(4)}} G_{24}^0 e^{(S_1)^{(4)}t}$$ 446
447

$$\left(\frac{(a_{26})^{(4)} G_{24}^0}{(m_1)^{(4)}\left((S_1)^{(4)} - (p_{24})^{(4)} - (S_2)^{(4)}\right)}\left[e^{\left((S_1)^{(4)} - (p_{24})^{(4)}\right)t} - e^{-(S_2)^{(4)}t}\right] + G_{26}^0 e^{-(S_2)^{(4)}t} \leq G_{26}(t) \leq\right.$$ 448

(a26)4G240(m2)4(S1)4−(a26′)4e(S1)4t−e−(a26′)4t+ G260e−(a26′)4t

$$\boxed{T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}}$$ 449

$$\frac{1}{(\mu_1)^{(4)}} T_{24}^0 e^{(R_1)^{(4)}t} \leq T_{24}(t) \leq \frac{1}{(\mu_2)^{(4)}} T_{24}^0 e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t}$$ 450

$$\frac{(b_{26})^{(4)} T_{24}^0}{(\mu_1)^{(4)}\left((R_1)^{(4)} - (b_{26}')^{(4)}\right)}\left[e^{(R_1)^{(4)}t} - e^{-(b_{26}')^{(4)}t}\right] + T_{26}^0 e^{-(b_{26}')^{(4)}t} \leq T_{26}(t) \leq$$ 451

$$\frac{(a_{26})^{(4)} T_{24}^0}{(\mu_2)^{(4)}\left((R_1)^{(4)} + (r_{24})^{(4)} + (R_2)^{(4)}\right)}\left[e^{\left((R_1)^{(4)} + (r_{24})^{(4)}\right)t} - e^{-(R_2)^{(4)}t}\right] + T_{26}^0 e^{-(R_2)^{(4)}t}$$

Definition of $(S_1)^{(4)}, (S_2)^{(4)}, (R_1)^{(4)}, (R_2)^{(4)}$:- 452

Where $(S_1)^{(4)} = (a_{24})^{(4)}(m_2)^{(4)} - (a'_{24})^{(4)}$

$(S_2)^{(4)} = (a_{26})^{(4)} - (p_{26})^{(4)}$

$(R_1)^{(4)} = (b_{24})^{(4)}(\mu_2)^{(4)} - (b'_{24})^{(4)}$

$(R_2)^{(4)} = (b'_{26})^{(4)} - (r_{26})^{(4)}$ 453

Behavior of the solutions 454

If we denote and define

Definition of $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$:

(g) $(\sigma_1)^{(5)}, (\sigma_2)^{(5)}, (\tau_1)^{(5)}, (\tau_2)^{(5)}$ four constants satisfying

$-(\sigma_2)^{(5)} \le -(a'_{28})^{(5)} + (a'_{29})^{(5)} - (a''_{28})^{(5)}(T_{29}, t) + (a''_{29})^{(5)}(T_{29}, t) \le -(\sigma_1)^{(5)}$

$-(\tau_2)^{(5)} \le -(b'_{28})^{(5)} + (b'_{29})^{(5)} - (b''_{28})^{(5)}\big((G_{31}), t\big) - (b''_{29})^{(5)}\big((G_{31}), t\big) \le -(\tau_1)^{(5)}$

Definition of $(\nu_1)^{(5)}, (\nu_2)^{(5)}, (u_1)^{(5)}, (u_2)^{(5)}, \nu^{(5)}, u^{(5)}$: 455

(h) By $(\nu_1)^{(5)} > 0, (\nu_2)^{(5)} < 0$ and respectively $(u_1)^{(5)} > 0, (u_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_1)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$ and

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(5)}, (\bar{u}_1)^{(5)}, (\bar{u}_2)^{(5)}$: 456

By $(\bar{\nu}_1)^{(5)} > 0, (\bar{\nu}_2)^{(5)} < 0$ and respectively $(\bar{u}_1)^{(5)} > 0, (\bar{u}_2)^{(5)} < 0$ the roots of the equations $(a_{29})^{(5)}\big(\nu^{(5)}\big)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)} = 0$ and $(b_{29})^{(5)}\big(u^{(5)}\big)^2 + (\tau_2)^{(5)}u^{(5)} - (b_{28})^{(5)} = 0$

Definition of $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}, (\nu_0)^{(5)}$:-

(i) If we define $(m_1)^{(5)}, (m_2)^{(5)}, (\mu_1)^{(5)}, (\mu_2)^{(5)}$ by

$(m_2)^{(5)} = (\nu_0)^{(5)}, (m_1)^{(5)} = (\nu_1)^{(5)}, \; \boldsymbol{if} \; (\nu_0)^{(5)} < (\nu_1)^{(5)}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\bar{\nu}_1)^{(5)}, \boldsymbol{if} \; (\nu_1)^{(5)} < (\nu_0)^{(5)} < (\bar{\nu}_1)^{(5)},$

and $\boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$

$(m_2)^{(5)} = (\nu_1)^{(5)}, (m_1)^{(5)} = (\nu_0)^{(5)}, \; \boldsymbol{if} \; (\bar{\nu}_1)^{(5)} < (\nu_0)^{(5)}$

and analogously 457

$(\mu_2)^{(5)} = (u_0)^{(5)}, (\mu_1)^{(5)} = (u_1)^{(5)}, \; \boldsymbol{if} \; (u_0)^{(5)} < (u_1)^{(5)}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (\bar{u}_1)^{(5)}, \boldsymbol{if} \; (u_1)^{(5)} < (u_0)^{(5)} < (\bar{u}_1)^{(5)},$

and $\boxed{(u_0)^{(5)} = \frac{T_{28}^0}{T_{29}^0}}$

$(\mu_2)^{(5)} = (u_1)^{(5)}, (\mu_1)^{(5)} = (u_0)^{(5)}, \boldsymbol{if} \; (\bar{u}_1)^{(5)} < (u_0)^{(5)}$ where $(u_1)^{(5)}, (\bar{u}_1)^{(5)}$ are defined respectively

Then the solution satisfies the inequalities 458

$G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \leq G_{28}(t) \leq G_{28}^0 e^{(S_1)^{(5)}t}$

where $(p_i)^{(5)}$ is defined

$\frac{1}{(m_5)^{(5)}} G_{28}^0 e^{((S_1)^{(5)}-(p_{28})^{(5)})t} \leq G_{29}(t) \leq \frac{1}{(m_2)^{(5)}} G_{28}^0 e^{(S_1)^{(5)}t}$ 459

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$\left(\frac{(a_{30})^{(5)} G_{28}^0}{(m_1)^{(5)}((S_1)^{(5)}-(p_{28})^{(5)}-(S_2)^{(5)})}\left[e^{((S_1)^{(5)}-(p_{28})^{(5)})t} - e^{-(S_2)^{(5)}t}\right] + G_{30}^0 e^{-(S_2)^{(5)}t} \leq G_{30}(t) \leq\right.$ 461

(a30)5G280(m2)5(S1)5−(a30′)5e(S1)5t−e−(a30′)5t+ G300e−(a30′)5t

$\boxed{T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}}$ 462

$\frac{1}{(\mu_1)^{(5)}} T_{28}^0 e^{(R_1)^{(5)}t} \leq T_{28}(t) \leq \frac{1}{(\mu_2)^{(5)}} T_{28}^0 e^{((R_1)^{(5)}+(r_{28})^{(5)})t}$ 463

$\frac{(b_{30})^{(5)} T_{28}^0}{(\mu_1)^{(5)}((R_1)^{(5)}-(b_{30}')^{(5)})}\left[e^{(R_1)^{(5)}t} - e^{-(b_{30}')^{(5)}t}\right] + T_{30}^0 e^{-(b_{30}')^{(5)}t} \leq T_{30}(t) \leq$ 464

$\frac{(a_{30})^{(5)} T_{28}^0}{(\mu_2)^{(5)}((R_1)^{(5)}+(r_{28})^{(5)}+(R_2)^{(5)})}\left[e^{((R_1)^{(5)}+(r_{28})^{(5)})t} - e^{-(R_2)^{(5)}t}\right] + T_{30}^0 e^{-(R_2)^{(5)}t}$

Definition of $(S_1)^{(5)}, (S_2)^{(5)}, (R_1)^{(5)}, (R_2)^{(5)}$:- 465

Where $(S_1)^{(5)} = (a_{28})^{(5)}(m_2)^{(5)} - (a_{28}')^{(5)}$

$(S_2)^{(5)} = (a_{30})^{(5)} - (p_{30})^{(5)}$

$(R_1)^{(5)} = (b_{28})^{(5)}(\mu_2)^{(5)} - (b_{28}')^{(5)}$

$(R_2)^{(5)} = (b_{30}')^{(5)} - (r_{30})^{(5)}$

Behavior of the solutions 466

If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(j) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying

$-(\sigma_2)^{(6)} \leq -(a_{32}')^{(6)} + (a_{33}')^{(6)} - (a_{32}'')^{(6)}(T_{33}, t) + (a_{33}'')^{(6)}(T_{33}, t) \leq -(\sigma_1)^{(6)}$

$-(\tau_2)^{(6)} \leq -(b_{32}')^{(6)} + (b_{33}')^{(6)} - (b_{32}'')^{(6)}((G_{35}), t) - (b_{33}'')^{(6)}((G_{35}), t) \leq -(\tau_1)^{(6)}$

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 467

(k) By $(\nu_1)^{(6)} > 0, (\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and

Definition of $(\bar{\nu}_1)^{(6)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 468

By $(\bar{\nu}_1)^{(6)} > 0, (\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the roots of the equations $(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:-

(l) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by

$(m_2)^{(6)} = (\nu_0)^{(6)}, (m_1)^{(6)} = (\nu_1)^{(6)},\ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)}\ ,\boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$ 470

and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$

and analogously 471

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)}\ ,\boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$

and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$ are defined respectively

Then the solution satisfies the inequalities 472

$$G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{32}(t) \le G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{((S_1)^{(6)} - (p_{32})^{(6)})t} \le G_{33}(t) \le \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$ 473

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)})}\left[e^{((S_1)^{(6)} - (p_{32})^{(6)})t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \le G_{34}(t) \le\right.$$ 474

(a34)6G320(m2)6(S1)6−(a34′)6e(S1)6t−e−(a34′)6t+ G340e−(a34′)6t

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}}$$ 475

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)} + (r_{32})^{(6)})t}$$ 476

$$\frac{(b_{34})^{(6)} T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)} - (b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le$$ 477

$$\frac{(a_{34})^{(6)} T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)} + (r_{32})^{(6)} + (R_2)^{(6)})}\left[e^{((R_1)^{(6)} + (r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:- 478

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$

$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$

$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$

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If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(m) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$

$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\big((G_{39}), t\big) - (b''_{37})^{(7)}\big((G_{39}), t\big) \leq -(\tau_1)^{(7)}$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: 480

(n) By $(\nu_1)^{(7)} > 0$, $(\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0$, $(u_2)^{(7)} < 0$ the roots of the equations

$(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$ 481

and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 482

By $(\bar{\nu}_1)^{(7)} > 0$, $(\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0$, $(\bar{u}_2)^{(7)} < 0$ the

roots of the equations $(a_{37})^{(7)}\big(\nu^{(7)}\big)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\big(u^{(7)}\big)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

(o) If we define $(m_1)^{(7)}$, $(m_2)^{(7)}$, $(\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)}, \; \boldsymbol{if} \; (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}, \boldsymbol{if} \; (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)}, \; \boldsymbol{if} \; (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 483

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \; \boldsymbol{if} \; (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \boldsymbol{if} \; (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if} \; (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$

are defined respectively

Then the solution satisfies the inequalities 484

$G_{36}^0 e^{\left((S_1)^{(7)} - (p_{36})^{(7)}\right)t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$

where $(p_i)^{(7)}$ is defined

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$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t}$$ 486

(487

$$\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq$$

$$\frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)}-(a_{38}')^{(7)})}[e^{(S_1)^{(7)}t} - e^{-(a_{38}')^{(7)}t}] + G_{38}^0 e^{-(a_{38}')^{(7)}t})$$

$$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}}$$ 488

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}$$ 489

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)}-(b_{38}')^{(7)})}\left[e^{(R_1)^{(7)}t} - e^{-(b_{38}')^{(7)}t}\right] + T_{38}^0 e^{-(b_{38}')^{(7)}t} \leq T_{38}(t) \leq$$ 490

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

Definition of $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 491

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}')^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}')^{(7)}$

$(R_2)^{(7)} = (b_{38}')^{(7)} - (r_{38})^{(7)}$

From GLOBAL EQUATIONS we obtain 492

$$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}')^{(7)} - (a_{37}')^{(7)} + (a_{36}'')^{(7)}(T_{37},t)\right) - (a_{37}'')^{(7)}(T_{37},t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq -\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

(a) For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}}{1+(C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 493

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)}-(\nu_0)^{(7)}}{(\nu_0)^{(7)}-(\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

(b) If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 494

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)}+(C)^{(7)}(\nu_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}}{1+(C)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)}-(\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

(c) If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 495

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)}+(\bar{C})^{(7)}(\bar{\nu}_2)^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}}{1+(\bar{C})^{(7)}e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)}-(\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)} G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then $(u_1)^{(7)} = (\bar{u}_1)^{(7)}$ if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

We can prove the following 496

If $(a''_i)^{(7)} and\ (b''_i)^{(7)}$ are independent on t , and the conditions 496A

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$ 496B

$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$ 496C

497C

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0$, 497D

497E

$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$ 497F

497G

with $(p_{36})^{(7)}, (r_{37})^{(7)}$ as defined are satisfied , then the system WITH THE SATISFACTION OF THE FOLLOWING PROPERTIES HAS A SOLUTION AS DERIVED BELOW.

Particular case : 498

If $(a''_{16})^{(2)} = (a''_{17})^{(2)}, then\ (\sigma_1)^{(2)} = (\sigma_2)^{(2)}$ and in this case $(\nu_1)^{(2)} = (\bar{\nu}_1)^{(2)}$ if in addition $(\nu_0)^{(2)} = (\nu_1)^{(2)}$ then $\nu^{(2)}(t) = (\nu_0)^{(2)}$ and as a consequence $G_{16}(t) = (\nu_0)^{(2)} G_{17}(t)$

Analogously if $(b''_{16})^{(2)} = (b''_{17})^{(2)}, then\ (\tau_1)^{(2)} = (\tau_2)^{(2)}$ and then

$(u_1)^{(2)} = (\bar{u}_1)^{(2)}$ if in addition $(u_0)^{(2)} = (u_1)^{(2)}$ then $T_{16}(t) = (u_0)^{(2)} T_{17}(t)$ This is an important consequence of the relation between $(\nu_1)^{(2)}$ and $(\bar{\nu}_1)^{(2)}$

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From GLOBAL EQUATIONS we obtain 500

$\frac{dv^{(3)}}{dt} = (a_{20})^{(3)} - \left((a'_{20})^{(3)} - (a'_{21})^{(3)} + (a''_{20})^{(3)}(T_{21},t)\right) - (a''_{21})^{(3)}(T_{21},t)v^{(3)} - (a_{21})^{(3)}v^{(3)}$

Definition of $v^{(3)}$:- $\boxed{v^{(3)} = \frac{G_{20}}{G_{21}}}$ 501

It follows

$-\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_2)^{(3)}v^{(3)} - (a_{20})^{(3)}\right) \leq \frac{dv^{(3)}}{dt} \leq -\left((a_{21})^{(3)}\left(v^{(3)}\right)^2 + (\sigma_1)^{(3)}v^{(3)} - (a_{20})^{(3)}\right)$

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From which one obtains

(a) For $0 < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (v_1)^{(3)} < (\bar{v}_1)^{(3)}$

$v^{(3)}(t) \geq \frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_0)^{(3)}\right)t\right]}}$, $\boxed{(C)^{(3)} = \frac{(v_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(v_2)^{(3)}}}$

it follows $(v_0)^{(3)} \leq v^{(3)}(t) \leq (v_1)^{(3)}$

In the same manner , we get 503

$v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}$, $\boxed{(\bar{C})^{(3)} = \frac{(\bar{v}_1)^{(3)}-(v_0)^{(3)}}{(v_0)^{(3)}-(\bar{v}_2)^{(3)}}}$

Definition of $(\bar{v}_1)^{(3)}$:-

From which we deduce $(v_0)^{(3)} \leq v^{(3)}(t) \leq (\bar{v}_1)^{(3)}$

(b) If $0 < (v_1)^{(3)} < (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0} < (\bar{v}_1)^{(3)}$ we find like in the previous case, 504

$(v_1)^{(3)} \leq \frac{(v_1)^{(3)}+(C)^{(3)}(v_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}}{1+(C)^{(3)}e^{\left[-(a_{21})^{(3)}\left((v_1)^{(3)}-(v_2)^{(3)}\right)t\right]}} \leq v^{(3)}(t) \leq$

$\frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}} \leq (\bar{v}_1)^{(3)}$

(c) If $0 < (v_1)^{(3)} \leq (\bar{v}_1)^{(3)} \leq (v_0)^{(3)} = \frac{G_{20}^0}{G_{21}^0}$, we obtain 505

$(v_1)^{(3)} \leq v^{(3)}(t) \leq \frac{(\bar{v}_1)^{(3)}+(\bar{C})^{(3)}(\bar{v}_2)^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}}{1+(\bar{C})^{(3)}e^{\left[-(a_{21})^{(3)}\left((\bar{v}_1)^{(3)}-(\bar{v}_2)^{(3)}\right)t\right]}} \leq (v_0)^{(3)}$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(3)}(t)$:-

$(m_2)^{(3)} \leq v^{(3)}(t) \leq (m_1)^{(3)}$, $\boxed{v^{(3)}(t) = \frac{G_{20}(t)}{G_{21}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(3)}(t)$:-

$(\mu_2)^{(3)} \leq u^{(3)}(t) \leq (\mu_1)^{(3)}$, $\boxed{u^{(3)}(t) = \frac{T_{20}(t)}{T_{21}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{20})^{(3)} = (a''_{21})^{(3)}, then\ (\sigma_1)^{(3)} = (\sigma_2)^{(3)}$ and in this case $(v_1)^{(3)} = (\bar{v}_1)^{(3)}$ if in addition $(v_0)^{(3)} = (v_1)^{(3)}$ then $v^{(3)}(t) = (v_0)^{(3)}$ and as a consequence $G_{20}(t) = (v_0)^{(3)}G_{21}(t)$

Analogously if $(b''_{20})^{(3)} = (b''_{21})^{(3)}, then\ (\tau_1)^{(3)} = (\tau_2)^{(3)}$ and then $(u_1)^{(3)} = (\bar{u}_1)^{(3)}$if in addition $(u_0)^{(3)} = (u_1)^{(3)}$ then $T_{20}(t) = (u_0)^{(3)}T_{21}(t)$ This is an important consequence of the relation between $(v_1)^{(3)}$ and $(\bar{v}_1)^{(3)}$

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: From GLOBAL EQUATIONS we obtain 507

$\frac{dv^{(4)}}{dt} = (a_{24})^{(4)} - \left((a'_{24})^{(4)} - (a'_{25})^{(4)} + (a''_{24})^{(4)}(T_{25},t)\right) - (a''_{25})^{(4)}(T_{25},t)v^{(4)} - (a_{25})^{(4)}v^{(4)}$

Definition of $v^{(4)}$:- $\boxed{v^{(4)} = \frac{G_{24}}{G_{25}}}$ 508

It follows

$-\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_2)^{(4)}v^{(4)} - (a_{24})^{(4)}\right) \leq \frac{dv^{(4)}}{dt} \leq -\left((a_{25})^{(4)}\left(v^{(4)}\right)^2 + (\sigma_4)^{(4)}v^{(4)} - (a_{24})^{(4)}\right)$

From which one obtains

Definition of $(\bar{\nu}_1)^{(4)}, (\nu_0)^{(4)}$:-

(d) For $0 < \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}} < (\nu_1)^{(4)} < (\bar{\nu}_1)^{(4)}$

$$\nu^{(4)}(t) \geq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}}{4+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_0)^{(4)}\right)t\right]}} \quad , \quad \boxed{(C)^{(4)} = \frac{(\nu_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\nu_2)^{(4)}}}$$

it follows $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\nu_1)^{(4)}$

In the same manner , we get 509

$$\nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{4+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(4)} = \frac{(\bar{\nu}_1)^{(4)}-(\nu_0)^{(4)}}{(\nu_0)^{(4)}-(\bar{\nu}_2)^{(4)}}}$$

From which we deduce $(\nu_0)^{(4)} \leq \nu^{(4)}(t) \leq (\bar{\nu}_1)^{(4)}$

(e) If $0 < (\nu_1)^{(4)} < (\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0} < (\bar{\nu}_1)^{(4)}$ we find like in the previous case, 510

$$(\nu_1)^{(4)} \leq \frac{(\nu_1)^{(4)}+(C)^{(4)}(\nu_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}}{1+(C)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\nu_1)^{(4)}-(\nu_2)^{(4)}\right)t\right]}} \leq \nu^{(4)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\bar{\nu}_1)^{(4)}$$

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(f) If $0 < (\nu_1)^{(4)} \leq (\bar{\nu}_1)^{(4)} \leq \boxed{(\nu_0)^{(4)} = \frac{G_{24}^0}{G_{25}^0}}$, we obtain

$$(\nu_1)^{(4)} \leq \nu^{(4)}(t) \leq \frac{(\bar{\nu}_1)^{(4)}+(\bar{C})^{(4)}(\bar{\nu}_2)^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}}{1+(\bar{C})^{(4)}e^{\left[-(a_{25})^{(4)}\left((\bar{\nu}_1)^{(4)}-(\bar{\nu}_2)^{(4)}\right)t\right]}} \leq (\nu_0)^{(4)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(4)}(t)$:-

$(m_2)^{(4)} \leq \nu^{(4)}(t) \leq (m_1)^{(4)}, \quad \boxed{\nu^{(4)}(t) = \frac{G_{24}(t)}{G_{25}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(4)}(t)$:-

$(\mu_2)^{(4)} \leq u^{(4)}(t) \leq (\mu_1)^{(4)}, \quad \boxed{u^{(4)}(t) = \frac{T_{24}(t)}{T_{25}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{24})^{(4)} = (a''_{25})^{(4)}, then\ (\sigma_1)^{(4)} = (\sigma_2)^{(4)}$ and in this case $(\nu_1)^{(4)} = (\bar{\nu}_1)^{(4)}$ if in addition $(\nu_0)^{(4)} = (\nu_1)^{(4)}$ then $\nu^{(4)}(t) = (\nu_0)^{(4)}$ and as a consequence $G_{24}(t) = (\nu_0)^{(4)}G_{25}(t)$ **this also defines** $(\nu_0)^{(4)}$ **for the special case** . 513

Analogously if $(b''_{24})^{(4)} = (b''_{25})^{(4)}, then\ (\tau_1)^{(4)} = (\tau_2)^{(4)}$ and then $(u_1)^{(4)} = (\bar{u}_4)^{(4)}$if in addition $(u_0)^{(4)} = (u_1)^{(4)}$ then $T_{24}(t) = (u_0)^{(4)}T_{25}(t)$ This is an important consequence of the relation between $(\nu_1)^{(4)}$ and $(\bar{\nu}_1)^{(4)}$, **and definition of** $(u_0)^{(4)}$**.**

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From GLOBAL EQUATIONS we obtain 515

$$\frac{d\nu^{(5)}}{dt} = (a_{28})^{(5)} - \left((a_{28}')^{(5)} - (a_{29}')^{(5)} + (a_{28}'')^{(5)}(T_{29}, t)\right) - (a_{29}'')^{(5)}(T_{29}, t)\nu^{(5)} - (a_{29})^{(5)}\nu^{(5)}$$

Definition of $\nu^{(5)}$:- $\boxed{\nu^{(5)} = \frac{G_{28}}{G_{29}}}$

It follows

$$-\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_2)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right) \leq \frac{d\nu^{(5)}}{dt} \leq -\left((a_{29})^{(5)}\left(\nu^{(5)}\right)^2 + (\sigma_1)^{(5)}\nu^{(5)} - (a_{28})^{(5)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(5)}, (\nu_0)^{(5)}$:-

(g) For $0 < \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}} < (\nu_1)^{(5)} < (\bar{\nu}_1)^{(5)}$

$$\nu^{(5)}(t) \geq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}}{5 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_0)^{(5)}\right)t\right]}} \quad , \quad \boxed{(C)^{(5)} = \frac{(\nu_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\nu_2)^{(5)}}}$$

it follows $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\nu_1)^{(5)}$

In the same manner , we get 516

$$\nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{5 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(5)} = \frac{(\bar{\nu}_1)^{(5)} - (\nu_0)^{(5)}}{(\nu_0)^{(5)} - (\bar{\nu}_2)^{(5)}}}$$

From which we deduce $(\nu_0)^{(5)} \leq \nu^{(5)}(t) \leq (\bar{\nu}_5)^{(5)}$

(h) If $0 < (\nu_1)^{(5)} < (\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0} < (\bar{\nu}_1)^{(5)}$ we find like in the previous case, 517

$$(\nu_1)^{(5)} \leq \frac{(\nu_1)^{(5)} + (C)^{(5)}(\nu_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}}{1 + (C)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\nu_1)^{(5)} - (\nu_2)^{(5)}\right)t\right]}} \leq \nu^{(5)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\bar{\nu}_1)^{(5)}$$

(i) If $0 < (\nu_1)^{(5)} \leq (\bar{\nu}_1)^{(5)} \leq \boxed{(\nu_0)^{(5)} = \frac{G_{28}^0}{G_{29}^0}}$, we obtain 518

$$(\nu_1)^{(5)} \leq \nu^{(5)}(t) \leq \frac{(\bar{\nu}_1)^{(5)} + (\bar{C})^{(5)}(\bar{\nu}_2)^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}}{1 + (\bar{C})^{(5)} e^{\left[-(a_{29})^{(5)}\left((\bar{\nu}_1)^{(5)} - (\bar{\nu}_2)^{(5)}\right)t\right]}} \leq (\nu_0)^{(5)}$$

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And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(5)}(t)$:-

$(m_2)^{(5)} \leq \nu^{(5)}(t) \leq (m_1)^{(5)}$, $\boxed{\nu^{(5)}(t) = \frac{G_{28}(t)}{G_{29}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(5)}(t)$:-

$(\mu_2)^{(5)} \leq u^{(5)}(t) \leq (\mu_1)^{(5)}$, $\boxed{u^{(5)}(t) = \frac{T_{28}(t)}{T_{29}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{28})^{(5)} = (a''_{29})^{(5)}, then\ (\sigma_1)^{(5)} = (\sigma_2)^{(5)}$ and in this case $(\nu_1)^{(5)} = (\bar{\nu}_1)^{(5)}$ if in addition $(\nu_0)^{(5)} = (\nu_5)^{(5)}$ then $\nu^{(5)}(t) = (\nu_0)^{(5)}$ and as a consequence $G_{28}(t) = (\nu_0)^{(5)} G_{29}(t)$ **this also defines** $(\nu_0)^{(5)}$ **for the special case** .

Analogously if $(b''_{28})^{(5)} = (b''_{29})^{(5)}, then\ (\tau_1)^{(5)} = (\tau_2)^{(5)}$ and then $(u_1)^{(5)} = (\bar{u}_1)^{(5)}$if in addition $(u_0)^{(5)} = (u_1)^{(5)}$ then $T_{28}(t) = (u_0)^{(5)} T_{29}(t)$ This is an important consequence of the relation between $(\nu_1)^{(5)}$ and $(\bar{\nu}_1)^{(5)}$, **and definition of** $(u_0)^{(5)}$.

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we obtain 521

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33}, t)\right) - (a''_{33})^{(6)}(T_{33}, t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$

<u>Definition of</u> $\nu^{(6)}$:- $\boxed{\nu^{(6)} = \frac{G_{32}}{G_{33}}}$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \leq \frac{d\nu^{(6)}}{dt} \leq -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$

From which one obtains

<u>Definition of</u> $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(j) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

$$\nu^{(6)}(t) \geq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\nu_1)^{(6)}$

In the same manner , we get 522

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$$\nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)} - (\nu_0)^{(6)}}{(\nu_0)^{(6)} - (\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(k) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 524

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)} + (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}}{1 + (C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)} - (\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}}{1 + (\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)} - (\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(l) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 525

$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\mathcal{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\mathcal{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}$, $\boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}$, $\boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$

Now, using this result and replacing it in GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{32})^{(6)} = (a''_{33})^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b''_{32})^{(6)} = (b''_{33})^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then $(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$.

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Behavior of the solutions 527

If we denote and define

Definition of $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$:

(p) $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$ four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}\left((G_{39}), t\right) - (b''_{37})^{(7)}\left((G_{39}), t\right) \leq -(\tau_1)^{(7)}$$

Definition of $(\nu_1)^{(7)}, (\nu_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, \nu^{(7)}, u^{(7)}$: **528**

(q) By $(\nu_1)^{(7)} > 0, (\nu_2)^{(7)} < 0$ and respectively $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$ the roots of the equations

$(a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$ **529**

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$ and

Definition of $(\bar{\nu}_1)^{(7)},, (\bar{\nu}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$: 530.

By $(\bar{\nu}_1)^{(7)} > 0, (\bar{\nu}_2)^{(7)} < 0$ and respectively $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$ the

roots of the equations $(a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)} = 0$

and $(b_{37})^{(7)}\left(u^{(7)}\right)^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

Definition of $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (\nu_0)^{(7)}$:-

(r) If we define $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$ by

$(m_2)^{(7)} = (\nu_0)^{(7)}, (m_1)^{(7)} = (\nu_1)^{(7)},\ \boldsymbol{if}\ (\nu_0)^{(7)} < (\nu_1)^{(7)}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\bar{\nu}_1)^{(7)}\ , \boldsymbol{if}\ (\nu_1)^{(7)} < (\nu_0)^{(7)} < (\bar{\nu}_1)^{(7)},$

and $\boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$

$(m_2)^{(7)} = (\nu_1)^{(7)}, (m_1)^{(7)} = (\nu_0)^{(7)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(7)} < (\nu_0)^{(7)}$

and analogously 531

$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)},\ \boldsymbol{if}\ (u_0)^{(7)} < (u_1)^{(7)}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}\ , \boldsymbol{if}\ (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$

and $\boxed{(u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}}$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}, \boldsymbol{if}\ (\bar{u}_1)^{(7)} < (u_0)^{(7)}$ where $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$ are defined by 59 and 67 respectively

Then the solution of GLOBAL EQUATIONS satisfies the inequalities 532

$G_{36}^0 e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{36}(t) \le G_{36}^0 e^{(S_1)^{(7)}t}$

where $(p_i)^{(7)}$ is defined

$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)}-(p_{36})^{(7)})t} \le G_{37}(t) \le \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t}$ 533

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$\frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)}-(p_{36})^{(7)}-(S_2)^{(7)})}\left[e^{((S_1)^{(7)}-(p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t}\ \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \le G_{38}(t) \le$

$\frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)}-(a_{38}')^{(7)})}[e^{(S_1)^{(7)}t} - e^{-(a_{38}')^{(7)}t}] +\ G_{38}^0 e^{-(a_{38}')^{(7)}t})$

$\boxed{T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}}$ 535

$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \le T_{36}(t) \le \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)}+(r_{36})^{(7)})t}$ 536

$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)}-(b_{38}')^{(7)})}\left[e^{(R_1)^{(7)}t} - e^{-(b_{38}')^{(7)}t}\right] + T_{38}^0 e^{-(b_{38}')^{(7)}t} \le T_{38}(t) \le$ 537

$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)}+(r_{36})^{(7)}+(R_2)^{(7)})}\left[e^{((R_1)^{(7)}+(r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t}\right] + T_{38}^0 e^{-(R_2)^{(7)}t}$

<u>Definition of</u> $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$:- 538

Where $(S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a_{36}')^{(7)}$

$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$

$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b_{36}')^{(7)}$

$(R_2)^{(7)} = (b_{38}')^{(7)} - (r_{38})^{(7)}$ 539

From CONCATENATED GLOBAL EQUATIONS we obtain 540

$\frac{d\nu^{(7)}}{dt} = (a_{36})^{(7)} - \left((a_{36}')^{(7)} - (a_{37}')^{(7)} + (a_{36}'')^{(7)}(T_{37}, t)\right) -$

$(a_{37}'')^{(7)}(T_{37}, t)\nu^{(7)} - (a_{37})^{(7)}\nu^{(7)}$

Definition of $\nu^{(7)}$:- $\boxed{\nu^{(7)} = \frac{G_{36}}{G_{37}}}$

It follows

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_2)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right) \leq \frac{d\nu^{(7)}}{dt} \leq$$

$$-\left((a_{37})^{(7)}\left(\nu^{(7)}\right)^2 + (\sigma_1)^{(7)}\nu^{(7)} - (a_{36})^{(7)}\right)$$

From which one obtains

Definition of $(\bar{\nu}_1)^{(7)}, (\nu_0)^{(7)}$:-

(m) For $0 < \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (\nu_1)^{(7)} < (\bar{\nu}_1)^{(7)}$

$$\nu^{(7)}(t) \geq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_0)^{(7)}\right)t\right]}} \quad , \quad \boxed{(C)^{(7)} = \frac{(\nu_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\nu_2)^{(7)}}}$$

it follows $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\nu_1)^{(7)}$

In the same manner , we get 541

$$\nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{\nu}_1)^{(7)} - (\nu_0)^{(7)}}{(\nu_0)^{(7)} - (\bar{\nu}_2)^{(7)}}}$$

From which we deduce $(\nu_0)^{(7)} \leq \nu^{(7)}(t) \leq (\bar{\nu}_1)^{(7)}$

(n) If $0 < (\nu_1)^{(7)} < (\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{\nu}_1)^{(7)}$ we find like in the previous case, 542

$$(\nu_1)^{(7)} \leq \frac{(\nu_1)^{(7)} + (C)^{(7)}(\nu_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}}{1 + (C)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\nu_1)^{(7)} - (\nu_2)^{(7)}\right)t\right]}} \leq \nu^{(7)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\bar{\nu}_1)^{(7)}$$

(o) If $0 < (\nu_1)^{(7)} \leq (\bar{\nu}_1)^{(7)} \leq \boxed{(\nu_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$, we obtain 543

$$(\nu_1)^{(7)} \leq \nu^{(7)}(t) \leq \frac{(\bar{\nu}_1)^{(7)} + (\bar{C})^{(7)}(\bar{\nu}_2)^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}}{1 + (\bar{C})^{(7)} e^{\left[-(a_{37})^{(7)}\left((\bar{\nu}_1)^{(7)} - (\bar{\nu}_2)^{(7)}\right)t\right]}} \leq (\nu_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(7)}(t)$:-

$$(m_2)^{(7)} \leq \nu^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{\nu^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(7)}(t)$:-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in CONCATENATED GLOBAL EQUATIONS we get easily the result stated in the theorem.

Particular case :

If $(a''_{36})^{(7)} = (a''_{37})^{(7)}, then\ (\sigma_1)^{(7)} = (\sigma_2)^{(7)}$ and in this case $(\nu_1)^{(7)} = (\bar{\nu}_1)^{(7)}$ if in addition $(\nu_0)^{(7)} = (\nu_1)^{(7)}$ then $\nu^{(7)}(t) = (\nu_0)^{(7)}$ and as a consequence $G_{36}(t) = (\nu_0)^{(7)}G_{37}(t)$ **this also defines** $(\nu_0)^{(7)}$ **for the special case** .

Analogously if $(b''_{36})^{(7)} = (b''_{37})^{(7)}, then\ (\tau_1)^{(7)} = (\tau_2)^{(7)}$ and then
$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$if in addition $(u_0)^{(7)} = (u_1)^{(7)}$ then $T_{36}(t) = (u_0)^{(7)}T_{37}(t)$ This is an important consequence of the relation between $(\nu_1)^{(7)}$ and $(\bar{\nu}_1)^{(7)}$, **and definition of** $(u_0)^{(7)}$.

$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)]T_{14} = 0$ 544

$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)]T_{15} = 0$ 545

has a unique positive solution , which is an equilibrium solution for the system 546

$(a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)} + (a''_{16})^{(2)}(T_{17})\right]G_{16} = 0$ 547

$(a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)} + (a''_{17})^{(2)}(T_{17})\right]G_{17} = 0$ 548

$(a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)} + (a''_{18})^{(2)}(T_{17})\right]G_{18} = 0$ 549

$(b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19})]T_{16} = 0$ 550

$(b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19})]T_{17} = 0$ 551

$(b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19})]T_{18} = 0$ 552

has a unique positive solution , which is an equilibrium solution for 553

$(a_{20})^{(3)}G_{21} - \left[(a'_{20})^{(3)} + (a''_{20})^{(3)}(T_{21})\right]G_{20} = 0$ 554

$(a_{21})^{(3)}G_{20} - \left[(a'_{21})^{(3)} + (a''_{21})^{(3)}(T_{21})\right]G_{21} = 0$ 555

$(a_{22})^{(3)}G_{21} - \left[(a'_{22})^{(3)} + (a''_{22})^{(3)}(T_{21})\right]G_{22} = 0$ 556

$(b_{20})^{(3)}T_{21} - [(b'_{20})^{(3)} - (b''_{20})^{(3)}(G_{23})]T_{20} = 0$ 557

$(b_{21})^{(3)}T_{20} - [(b'_{21})^{(3)} - (b''_{21})^{(3)}(G_{23})]T_{21} = 0$ 558

$(b_{22})^{(3)}T_{21} - [(b'_{22})^{(3)} - (b''_{22})^{(3)}(G_{23})]T_{22} = 0$ 559

has a unique positive solution , which is an equilibrium solution 560

$(a_{24})^{(4)}G_{25} - \left[(a'_{24})^{(4)} + (a''_{24})^{(4)}(T_{25})\right]G_{24} = 0$ 561

$(a_{25})^{(4)}G_{24} - \left[(a'_{25})^{(4)} + (a''_{25})^{(4)}(T_{25})\right]G_{25} = 0$ 563

$(a_{26})^{(4)}G_{25} - \left[(a'_{26})^{(4)} + (a''_{26})^{(4)}(T_{25})\right]G_{26} = 0$ 564

$(b_{24})^{(4)}T_{25} - [(b'_{24})^{(4)} - (b''_{24})^{(4)}\left((G_{27})\right)]T_{24} = 0$ 565

$(b_{25})^{(4)}T_{24} - [(b'_{25})^{(4)} - (b''_{25})^{(4)}\left((G_{27})\right)]T_{25} = 0$ 566

$(b_{26})^{(4)}T_{25} - [(b'_{26})^{(4)} - (b''_{26})^{(4)}\left((G_{27})\right)]T_{26} = 0$ 567

has a unique positive solution , which is an equilibrium solution for the system 568

$(a_{28})^{(5)}G_{29} - \left[(a'_{28})^{(5)} + (a''_{28})^{(5)}(T_{29})\right]G_{28} = 0$ 569

$(a_{29})^{(5)}G_{28} - \left[(a'_{29})^{(5)} + (a''_{29})^{(5)}(T_{29})\right]G_{29} = 0$ 570

$(a_{30})^{(5)}G_{29} - \left[(a'_{30})^{(5)} + (a''_{30})^{(5)}(T_{29})\right]G_{30} = 0$ 571

$(b_{28})^{(5)}T_{29} - [(b'_{28})^{(5)} - (b''_{28})^{(5)}(G_{31})]T_{28} = 0$ 572

$(b_{29})^{(5)}T_{28} - [(b'_{29})^{(5)} - (b''_{29})^{(5)}(G_{31})]T_{29} = 0$ 573

$(b_{30})^{(5)}T_{29} - [(b'_{30})^{(5)} - (b''_{30})^{(5)}(G_{31})\,]T_{30} = 0$ 574

has a unique positive solution , which is an equilibrium solution for the system 575

$(a_{32})^{(6)}G_{33} - \left[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T_{33})\right]G_{32} = 0$ 576

$(a_{33})^{(6)}G_{32} - \left[(a'_{33})^{(6)} + (a''_{33})^{(6)}(T_{33})\right]G_{33} = 0$ 577

$(a_{34})^{(6)}G_{33} - \left[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T_{33})\right]G_{34} = 0$ 578

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})\,]T_{32} = 0$ 579

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})\,]T_{33} = 0$ 580

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})\,]T_{34} = 0$ 584

has a unique positive solution , which is an equilibrium solution for the system 582

$(a_{36})^{(7)}G_{37} - \left[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})\right]G_{36} = 0$ 583

$(a_{37})^{(7)}G_{36} - \left[(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})\right]G_{37} = 0$ 584

$(a_{38})^{(7)}G_{37} - \left[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})\right]G_{38} = 0$ 585

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$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0$ 587

$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})\,]T_{37} = 0$ 588

$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})\,]T_{38} = 0$ 589

has a unique positive solution , which is an equilibrium solution for the system 560

(a) Indeed the first two equations have a nontrivial solution G_{36}, G_{37} if

$$F(T_{39}) = (a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

<u>Definition and uniqueness of</u> T^*_{37} :- 561

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(7)}(T_{37})$ being increasing, it follows that there exists a unique T^*_{37} for which $f(T^*_{37}) = 0$. With this value , we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T^*_{37})]} \quad , \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T^*_{37})]}$$

(e) By the same argument, the equations(SOLUTIONAL) admit solutions G_{36}, G_{37} if

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} -$$

$$\left[(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})\right] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0$$

Where in $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{37} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{37}^* such that $\varphi(G^*) = 0$ 562

Finally we obtain the unique solution OF THE SYSTEM

G_{37}^* given by $\varphi((G_{39})^*) = 0$, T_{37}^* given by $f(T_{37}^*) = 0$ and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a_{36}^{'})^{(7)} + (a_{36}^{''})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a_{38}^{'})^{(7)} + (a_{38}^{''})^{(7)}(T_{37}^*)]}$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b_{36}^{'})^{(7)} - (b_{36}^{''})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b_{38}^{'})^{(7)} - (b_{38}^{''})^{(7)}((G_{39})^*)]}$$ 563

Definition and uniqueness of T_{21}^* :- 564

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(1)}(T_{21})$ being increasing, it follows that there exists a unique T_{21}^* for which $f(T_{21}^*) = 0$. With this value , we obtain from the three first equations

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$$G_{20} = \frac{(a_{20})^{(3)} G_{21}}{[(a_{20}^{'})^{(3)} + (a_{20}^{''})^{(3)}(T_{21}^*)]} \quad , \quad G_{22} = \frac{(a_{22})^{(3)} G_{21}}{[(a_{22}^{'})^{(3)} + (a_{22}^{''})^{(3)}(T_{21}^*)]}$$

Definition and uniqueness of T_{25}^* :- 566

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(4)}(T_{25})$ being increasing, it follows that there exists a unique T_{25}^* for which $f(T_{25}^*) = 0$. With this value , we obtain from the three first equations

$$G_{24} = \frac{(a_{24})^{(4)} G_{25}}{[(a_{24}^{'})^{(4)} + (a_{24}^{''})^{(4)}(T_{25}^*)]} \quad , \quad G_{26} = \frac{(a_{26})^{(4)} G_{25}}{[(a_{26}^{'})^{(4)} + (a_{26}^{''})^{(4)}(T_{25}^*)]}$$

Definition and uniqueness of T_{29}^* :- 567

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(5)}(T_{29})$ being increasing, it follows that there exists a unique T_{29}^* for which $f(T_{29}^*) = 0$. With this value , we obtain from the three first equations

$$G_{28} = \frac{(a_{28})^{(5)} G_{29}}{[(a_{28}^{'})^{(5)} + (a_{28}^{''})^{(5)}(T_{29}^*)]} \quad , \quad G_{30} = \frac{(a_{30})^{(5)} G_{29}}{[(a_{30}^{'})^{(5)} + (a_{30}^{''})^{(5)}(T_{29}^*)]}$$

Definition and uniqueness of T_{33}^* :- 568

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a_i^{''})^{(6)}(T_{33})$ being increasing, it follows that there exists a unique T_{33}^* for which $f(T_{33}^*) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)} G_{33}}{[(a_{32}^{'})^{(6)} + (a_{32}^{''})^{(6)}(T_{33}^*)]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)} G_{33}}{[(a_{34}^{'})^{(6)} + (a_{34}^{''})^{(6)}(T_{33}^*)]}$$

(f) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if 569

$$\varphi(G) = (b_{13}^{'})^{(1)}(b_{14}^{'})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} -$$

$$\left[(b_{13}^{'})^{(1)}(b_{14}^{''})^{(1)}(G) + (b_{14}^{'})^{(1)}(b_{13}^{''})^{(1)}(G)\right] + (b_{13}^{''})^{(1)}(G)(b_{14}^{''})^{(1)}(G) = 0$$

Where in $G(G_{13}, G_{14}, G_{15}), G_{13}, G_{15}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*) = 0$

(g) By the same argument, the equations 92,93 admit solutions G_{16}, G_{17} if 570

$$\varphi(G_{19}) = (b_{16}^{'})^{(2)}(b_{17}^{'})^{(2)} - (b_{16})^{(2)}(b_{17})^{(2)} -$$

$\left[(b_{16}^{'})^{(2)}(b_{17}^{''})^{(2)}(G_{19}) + (b_{17}^{'})^{(2)}(b_{16}^{''})^{(2)}(G_{19})\right] + (b_{16}^{''})^{(2)}(G_{19})(b_{17}^{''})^{(2)}(G_{19}) = 0$

Where in $(G_{19})(G_{16}, G_{17}, G_{18}), G_{16}, G_{18}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{17} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{14}^{*} such that $\varphi((G_{19})^{*}) = 0$ 571

(a) By the same argument, the concatenated equations admit solutions G_{20}, G_{21} if 572

$\varphi(G_{23}) = (b_{20}^{'})^{(3)}(b_{21}^{'})^{(3)} - (b_{20})^{(3)}(b_{21})^{(3)} -$

$\left[(b_{20}^{'})^{(3)}(b_{21}^{''})^{(3)}(G_{23}) + (b_{21}^{'})^{(3)}(b_{20}^{''})^{(3)}(G_{23})\right] + (b_{20}^{''})^{(3)}(G_{23})(b_{21}^{''})^{(3)}(G_{23}) = 0$

Where in $G_{23}(G_{20}, G_{21}, G_{22}), G_{20}, G_{22}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{21} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{21}^{*} such that $\varphi((G_{23})^{*}) = 0$

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(b) By the same argument, the equations of modules admit solutions G_{24}, G_{25} if 574

$\varphi(G_{27}) = (b_{24}^{'})^{(4)}(b_{25}^{'})^{(4)} - (b_{24})^{(4)}(b_{25})^{(4)} -$

$\left[(b_{24}^{'})^{(4)}(b_{25}^{''})^{(4)}(G_{27}) + (b_{25}^{'})^{(4)}(b_{24}^{''})^{(4)}(G_{27})\right] + (b_{24}^{''})^{(4)}(G_{27})(b_{25}^{''})^{(4)}(G_{27}) = 0$

Where in $(G_{27})(G_{24}, G_{25}, G_{26}), G_{24}, G_{26}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{25} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{25}^{*} such that $\varphi((G_{27})^{*}) = 0$

(c) By the same argument, the equations (modules) admit solutions G_{28}, G_{29} if 575

$\varphi(G_{31}) = (b_{28}^{'})^{(5)}(b_{29}^{'})^{(5)} - (b_{28})^{(5)}(b_{29})^{(5)} -$

$\left[(b_{28}^{'})^{(5)}(b_{29}^{''})^{(5)}(G_{31}) + (b_{29}^{'})^{(5)}(b_{28}^{''})^{(5)}(G_{31})\right] + (b_{28}^{''})^{(5)}(G_{31})(b_{29}^{''})^{(5)}(G_{31}) = 0$

Where in $(G_{31})(G_{28}, G_{29}, G_{30}), G_{28}, G_{30}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{29} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{29}^{*} such that $\varphi((G_{31})^{*}) = 0$

(d) By the same argument, the equations (modules) admit solutions G_{32}, G_{33} if 578

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$\varphi(G_{35}) = (b_{32}^{'})^{(6)}(b_{33}^{'})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} -$

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$\left[(b_{32}^{'})^{(6)}(b_{33}^{''})^{(6)}(G_{35}) + (b_{33}^{'})^{(6)}(b_{32}^{''})^{(6)}(G_{35})\right] + (b_{32}^{''})^{(6)}(G_{35})(b_{33}^{''})^{(6)}(G_{35}) = 0$

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Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0, \varphi(\infty) < 0$ it follows that there exists a unique G_{33}^{*} such that $\varphi(G^{*}) = 0$

Finally we obtain the unique solution of 89 to 94 582

G_{14}^{*} given by $\varphi(G^{*}) = 0$, T_{14}^{*} given by $f(T_{14}^{*}) = 0$ and

$G_{13}^{*} = \frac{(a_{13})^{(1)}G_{14}^{*}}{[(a_{13}^{'})^{(1)} + (a_{13}^{''})^{(1)}(T_{14}^{*})]}$, $G_{15}^{*} = \frac{(a_{15})^{(1)}G_{14}^{*}}{[(a_{15}^{'})^{(1)} + (a_{15}^{''})^{(1)}(T_{14}^{*})]}$

$T_{13}^{*} = \frac{(b_{13})^{(1)}T_{14}^{*}}{[(b_{13}^{'})^{(1)} - (b_{13}^{''})^{(1)}(G^{*})]}$, $T_{15}^{*} = \frac{(b_{15})^{(1)}T_{14}^{*}}{[(b_{15}^{'})^{(1)} - (b_{15}^{''})^{(1)}(G^{*})]}$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 583

G_{17}^* given by $\varphi((G_{19})^*) = 0$, T_{17}^* given by $f(T_{17}^*) = 0$ and 584

$$G_{16}^* = \frac{(a_{16})^{(2)} G_{17}^*}{[(a_{16}')^{(2)} + (a_{16}'')^{(2)}(T_{17}^*)]} \quad , \quad G_{18}^* = \frac{(a_{18})^{(2)} G_{17}^*}{[(a_{18}')^{(2)} + (a_{18}'')^{(2)}(T_{17}^*)]}$$ 585

$$T_{16}^* = \frac{(b_{16})^{(2)} T_{17}^*}{[(b_{16}')^{(2)} - (b_{16}'')^{(2)}((G_{19})^*)]} \quad , \quad T_{18}^* = \frac{(b_{18})^{(2)} T_{17}^*}{[(b_{18}')^{(2)} - (b_{18}'')^{(2)}((G_{19})^*)]}$$ 586

Obviously, these values represent an equilibrium solution 587

Finally we obtain the unique solution 588

G_{21}^* given by $\varphi((G_{23})^*) = 0$, T_{21}^* given by $f(T_{21}^*) = 0$ and

$$G_{20}^* = \frac{(a_{20})^{(3)} G_{21}^*}{[(a_{20}')^{(3)} + (a_{20}'')^{(3)}(T_{21}^*)]} \quad , \quad G_{22}^* = \frac{(a_{22})^{(3)} G_{21}^*}{[(a_{22}')^{(3)} + (a_{22}'')^{(3)}(T_{21}^*)]}$$

$$T_{20}^* = \frac{(b_{20})^{(3)} T_{21}^*}{[(b_{20}')^{(3)} - (b_{20}'')^{(3)}(G_{23}{}^*)]} \quad , \quad T_{22}^* = \frac{(b_{22})^{(3)} T_{21}^*}{[(b_{22}')^{(3)} - (b_{22}'')^{(3)}(G_{23}{}^*)]}$$

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 589

G_{25}^* given by $\varphi(G_{27}) = 0$, T_{25}^* given by $f(T_{25}^*) = 0$ and

$$G_{24}^* = \frac{(a_{24})^{(4)} G_{25}^*}{[(a_{24}')^{(4)} + (a_{24}'')^{(4)}(T_{25}^*)]} \quad , \quad G_{26}^* = \frac{(a_{26})^{(4)} G_{25}^*}{[(a_{26}')^{(4)} + (a_{26}'')^{(4)}(T_{25}^*)]}$$

$$T_{24}^* = \frac{(b_{24})^{(4)} T_{25}^*}{[(b_{24}')^{(4)} - (b_{24}'')^{(4)}((G_{27})^*)]} \quad , \quad T_{26}^* = \frac{(b_{26})^{(4)} T_{25}^*}{[(b_{26}')^{(4)} - (b_{26}'')^{(4)}((G_{27})^*)]}$$ 590

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 591

G_{29}^* given by $\varphi((G_{31})^*) = 0$, T_{29}^* given by $f(T_{29}^*) = 0$ and

$$G_{28}^* = \frac{(a_{28})^{(5)} G_{29}^*}{[(a_{28}')^{(5)} + (a_{28}'')^{(5)}(T_{29}^*)]} \quad , \quad G_{30}^* = \frac{(a_{30})^{(5)} G_{29}^*}{[(a_{30}')^{(5)} + (a_{30}'')^{(5)}(T_{29}^*)]}$$

$$T_{28}^* = \frac{(b_{28})^{(5)} T_{29}^*}{[(b_{28}')^{(5)} - (b_{28}'')^{(5)}((G_{31})^*)]} \quad , \quad T_{30}^* = \frac{(b_{30})^{(5)} T_{29}^*}{[(b_{30}')^{(5)} - (b_{30}'')^{(5)}((G_{31})^*)]}$$ 592

Obviously, these values represent an equilibrium solution

Finally we obtain the unique solution 593

G_{33}^* given by $\varphi((G_{35})^*) = 0$, T_{33}^* given by $f(T_{33}^*) = 0$ and

$$G_{32}^* = \frac{(a_{32})^{(6)} G_{33}^*}{[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33}^*)]} \quad , \quad G_{34}^* = \frac{(a_{34})^{(6)} G_{33}^*}{[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33}^*)]}$$

$$T_{32}^* = \frac{(b_{32})^{(6)} T_{33}^*}{[(b_{32}')^{(6)} - (b_{32}'')^{(6)}((G_{35})^*)]} \quad , \quad T_{34}^* = \frac{(b_{34})^{(6)} T_{33}^*}{[(b_{34}')^{(6)} - (b_{34}'')^{(6)}((G_{35})^*)]}$$ 594

Obviously, these values represent an equilibrium solution

ASYMPTOTIC STABILITY ANALYSIS 595

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(1)}$ *and* $(b_i^{''})^{(1)}$ Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof:_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$ 596

$$\frac{\partial (a_{14}^{''})^{(1)}}{\partial T_{14}}(T_{14}^*) = (q_{14})^{(1)} \quad , \frac{\partial (b_i^{''})^{(1)}}{\partial G_j}(G^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 597

$$\frac{d\mathbb{G}_{13}}{dt} = -\left((a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\mathbb{G}_{13} + (a_{13})^{(1)}\mathbb{G}_{14} - (q_{13})^{(1)}G_{13}^*\mathbb{T}_{14}$$ 598

$$\frac{d\mathbb{G}_{14}}{dt} = -\left((a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)\mathbb{G}_{14} + (a_{14})^{(1)}\mathbb{G}_{13} - (q_{14})^{(1)}G_{14}^*\mathbb{T}_{14}$$ 599

$$\frac{d\mathbb{G}_{15}}{dt} = -\left((a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)\mathbb{G}_{15} + (a_{15})^{(1)}\mathbb{G}_{14} - (q_{15})^{(1)}G_{15}^*\mathbb{T}_{14}$$ 600

$$\frac{d\mathbb{T}_{13}}{dt} = -\left((b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)\mathbb{T}_{13} + (b_{13})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right)$$ 601

$$\frac{d\mathbb{T}_{14}}{dt} = -\left((b_{14}^{'})^{(1)} - (r_{14})^{(1)}\right)\mathbb{T}_{14} + (b_{14})^{(1)}\mathbb{T}_{13} + \sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right)$$ 602

$$\frac{d\mathbb{T}_{15}}{dt} = -\left((b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\mathbb{T}_{15} + (b_{15})^{(1)}\mathbb{T}_{14} + \sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right)$$ 603

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(2)}$ and $(b_i^{''})^{(2)}$ Belong to $C^{(2)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable 604

Denote 605

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$ 606

$$\frac{\partial (a_{17}^{''})^{(2)}}{\partial T_{17}}(T_{17}^*) = (q_{17})^{(2)} \quad , \frac{\partial (b_i^{''})^{(2)}}{\partial G_j}((G_{19})^*) = s_{ij}$$ 607

taking into account equations (global)and neglecting the terms of power 2, we obtain 608

$$\frac{d\mathbb{G}_{16}}{dt} = -\left((a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)\mathbb{G}_{16} + (a_{16})^{(2)}\mathbb{G}_{17} - (q_{16})^{(2)}G_{16}^*\mathbb{T}_{17}$$ 609

$$\frac{d\mathbb{G}_{17}}{dt} = -\left((a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)\mathbb{G}_{17} + (a_{17})^{(2)}\mathbb{G}_{16} - (q_{17})^{(2)}G_{17}^*\mathbb{T}_{17}$$ 610

$$\frac{d\mathbb{G}_{18}}{dt} = -\left((a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)\mathbb{G}_{18} + (a_{18})^{(2)}\mathbb{G}_{17} - (q_{18})^{(2)}G_{18}^*\mathbb{T}_{17}$$ 611

$$\frac{d\mathbb{T}_{16}}{dt} = -\left((b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)\mathbb{T}_{16} + (b_{16})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(16)(j)}T_{16}^*\mathbb{G}_j\right)$$ 612

$$\frac{d\mathbb{T}_{17}}{dt} = -\left((b_{17}^{'})^{(2)} - (r_{17})^{(2)}\right)\mathbb{T}_{17} + (b_{17})^{(2)}\mathbb{T}_{16} + \sum_{j=16}^{18}\left(s_{(17)(j)}T_{17}^*\mathbb{G}_j\right)$$ 613

$$\frac{d\mathbb{T}_{18}}{dt} = -\left((b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\mathbb{T}_{18} + (b_{18})^{(2)}\mathbb{T}_{17} + \sum_{j=16}^{18}\left(s_{(18)(j)}T_{18}^*\mathbb{G}_j\right)$$ 614

If the conditions of the previous theorem are satisfied and if the functions $(a_i^{''})^{(3)}$ *and* $(b_i^{''})^{(3)}$ Belong to $C^{(3)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 615

_Denote 616

Definition of $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{21}'')^{(3)}}{\partial T_{21}}(T_{21}^*) = (q_{21})^{(3)} \quad , \frac{\partial (b_i'')^{(3)}}{\partial G_j}((G_{23})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 617

$$\frac{d\mathbb{G}_{20}}{dt} = -\left((a_{20}')^{(3)} + (p_{20})^{(3)}\right)\mathbb{G}_{20} + (a_{20})^{(3)}\mathbb{G}_{21} - (q_{20})^{(3)} G_{20}^* \mathbb{T}_{21}$$ 618

$$\frac{d\mathbb{G}_{21}}{dt} = -\left((a_{21}')^{(3)} + (p_{21})^{(3)}\right)\mathbb{G}_{21} + (a_{21})^{(3)}\mathbb{G}_{20} - (q_{21})^{(3)} G_{21}^* \mathbb{T}_{21}$$ 619

$$\frac{d\mathbb{G}_{22}}{dt} = -\left((a_{22}')^{(3)} + (p_{22})^{(3)}\right)\mathbb{G}_{22} + (a_{22})^{(3)}\mathbb{G}_{21} - (q_{22})^{(3)} G_{22}^* \mathbb{T}_{21}$$ 6120

$$\frac{d\mathbb{T}_{20}}{dt} = -\left((b_{20}')^{(3)} - (r_{20})^{(3)}\right)\mathbb{T}_{20} + (b_{20})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(20)(j)} T_{20}^* \mathbb{G}_j\right)$$ 621

$$\frac{d\mathbb{T}_{21}}{dt} = -\left((b_{21}')^{(3)} - (r_{21})^{(3)}\right)\mathbb{T}_{21} + (b_{21})^{(3)}\mathbb{T}_{20} + \sum_{j=20}^{22}\left(s_{(21)(j)} T_{21}^* \mathbb{G}_j\right)$$ 622

$$\frac{d\mathbb{T}_{22}}{dt} = -\left((b_{22}')^{(3)} - (r_{22})^{(3)}\right)\mathbb{T}_{22} + (b_{22})^{(3)}\mathbb{T}_{21} + \sum_{j=20}^{22}\left(s_{(22)(j)} T_{22}^* \mathbb{G}_j\right)$$ 623

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(4)}$ *and* $(b_i'')^{(4)}$ Belong to $C^{(4)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stabl 624

_Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 625

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$

$$\frac{\partial (a_{25}'')^{(4)}}{\partial T_{25}}(T_{25}^*) = (q_{25})^{(4)} \quad , \frac{\partial (b_i'')^{(4)}}{\partial G_j}((G_{27})^*) = s_{ij}$$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 626

$$\frac{d\mathbb{G}_{24}}{dt} = -\left((a_{24}')^{(4)} + (p_{24})^{(4)}\right)\mathbb{G}_{24} + (a_{24})^{(4)}\mathbb{G}_{25} - (q_{24})^{(4)} G_{24}^* \mathbb{T}_{25}$$ 627

$$\frac{d\mathbb{G}_{25}}{dt} = -\left((a_{25}')^{(4)} + (p_{25})^{(4)}\right)\mathbb{G}_{25} + (a_{25})^{(4)}\mathbb{G}_{24} - (q_{25})^{(4)} G_{25}^* \mathbb{T}_{25}$$ 628

$$\frac{d\mathbb{G}_{26}}{dt} = -\left((a_{26}')^{(4)} + (p_{26})^{(4)}\right)\mathbb{G}_{26} + (a_{26})^{(4)}\mathbb{G}_{25} - (q_{26})^{(4)} G_{26}^* \mathbb{T}_{25}$$ 629

$$\frac{d\mathbb{T}_{24}}{dt} = -\left((b_{24}')^{(4)} - (r_{24})^{(4)}\right)\mathbb{T}_{24} + (b_{24})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(24)(j)} T_{24}^* \mathbb{G}_j\right)$$ 630

$$\frac{d\mathbb{T}_{25}}{dt} = -\left((b_{25}')^{(4)} - (r_{25})^{(4)}\right)\mathbb{T}_{25} + (b_{25})^{(4)}\mathbb{T}_{24} + \sum_{j=24}^{26}\left(s_{(25)(j)} T_{25}^* \mathbb{G}_j\right)$$ 631

$$\frac{d\mathbb{T}_{26}}{dt} = -\left((b_{26}')^{(4)} - (r_{26})^{(4)}\right)\mathbb{T}_{26} + (b_{26})^{(4)}\mathbb{T}_{25} + \sum_{j=24}^{26}\left(s_{(26)(j)} T_{26}^* \mathbb{G}_j\right)$$ 632

633

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(5)}$ *and* $(b_i'')^{(5)}$ Belong to $C^{(5)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 634

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{29}'')^{(5)}}{\partial T_{29}}(T_{29}^*) = (q_{29})^{(5)} \quad , \frac{\partial (b_i'')^{(5)}}{\partial G_j}((G_{31})^*) = s_{ij}$

Then taking into account equations (global) and neglecting the terms of power 2, we obtain 635

$\frac{d\mathbb{G}_{28}}{dt} = -\left((a_{28}')^{(5)} + (p_{28})^{(5)}\right)\mathbb{G}_{28} + (a_{28})^{(5)}\mathbb{G}_{29} - (q_{28})^{(5)} G_{28}^* \mathbb{T}_{29}$ 636

$\frac{d\mathbb{G}_{29}}{dt} = -\left((a_{29}')^{(5)} + (p_{29})^{(5)}\right)\mathbb{G}_{29} + (a_{29})^{(5)}\mathbb{G}_{28} - (q_{29})^{(5)} G_{29}^* \mathbb{T}_{29}$ 637

$\frac{d\mathbb{G}_{30}}{dt} = -\left((a_{30}')^{(5)} + (p_{30})^{(5)}\right)\mathbb{G}_{30} + (a_{30})^{(5)}\mathbb{G}_{29} - (q_{30})^{(5)} G_{30}^* \mathbb{T}_{29}$ 638

$\frac{d\mathbb{T}_{28}}{dt} = -\left((b_{28}')^{(5)} - (r_{28})^{(5)}\right)\mathbb{T}_{28} + (b_{28})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(28)(j)} T_{28}^* \mathbb{G}_j\right)$ 639

$\frac{d\mathbb{T}_{29}}{dt} = -\left((b_{29}')^{(5)} - (r_{29})^{(5)}\right)\mathbb{T}_{29} + (b_{29})^{(5)}\mathbb{T}_{28} + \sum_{j=28}^{30}\left(s_{(29)(j)} T_{29}^* \mathbb{G}_j\right)$ 640

$\frac{d\mathbb{T}_{30}}{dt} = -\left((b_{30}')^{(5)} - (r_{30})^{(5)}\right)\mathbb{T}_{30} + (b_{30})^{(5)}\mathbb{T}_{29} + \sum_{j=28}^{30}\left(s_{(30)(j)} T_{30}^* \mathbb{G}_j\right)$ 641

If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(6)}$ *and* $(b_i'')^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable 642

Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 643

$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$

$\frac{\partial (a_{33}'')^{(6)}}{\partial T_{33}}(T_{33}^*) = (q_{33})^{(6)} \quad , \frac{\partial (b_i'')^{(6)}}{\partial G_j}((G_{35})^*) = s_{ij}$

Then taking into account equations(global) and neglecting the terms of power 2, we obtain 644

$\frac{d\mathbb{G}_{32}}{dt} = -\left((a_{32}')^{(6)} + (p_{32})^{(6)}\right)\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)} G_{32}^* \mathbb{T}_{33}$ 645

$\frac{d\mathbb{G}_{33}}{dt} = -\left((a_{33}')^{(6)} + (p_{33})^{(6)}\right)\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)} G_{33}^* \mathbb{T}_{33}$ 646

$\frac{d\mathbb{G}_{34}}{dt} = -\left((a_{34}')^{(6)} + (p_{34})^{(6)}\right)\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)} G_{34}^* \mathbb{T}_{33}$ 647

$\frac{d\mathbb{T}_{32}}{dt} = -\left((b_{32}')^{(6)} - (r_{32})^{(6)}\right)\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(32)(j)} T_{32}^* \mathbb{G}_j\right)$ 648

$\frac{d\mathbb{T}_{33}}{dt} = -\left((b_{33}')^{(6)} - (r_{33})^{(6)}\right)\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} + \sum_{j=32}^{34}\left(s_{(33)(j)} T_{33}^* \mathbb{G}_j\right)$ 649

$\frac{d\mathbb{T}_{34}}{dt} = -\left((b_{34}')^{(6)} - (r_{34})^{(6)}\right)\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} + \sum_{j=32}^{34}\left(s_{(34)(j)} T_{34}^* \mathbb{G}_j\right)$ 650

Obviously, these values represent an equilibrium solution of 79,20,36,22,23, 651
If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(7)}$ *and* $(b_i'')^{(7)}$ Belong to $C^{(7)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i, \mathbb{T}_i$:- 652

653

$$G_i = G_i^* + \mathbb{G}_i \qquad , T_i = T_i^* + \mathbb{T}_i$$
$$\frac{\partial (a_{37}^{''})^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \frac{\partial (b_i^{''})^{(7)}}{\partial G_j}\left((G_{39})^{**} \right) = s_{ij}$$

Then taking into account equations(SOLUTIONAL) and neglecting the terms of power 2, we obtain 654

655

$$\frac{d\mathbb{G}_{36}}{dt} = -\left((a_{36}^{'})^{(7)} + (p_{36})^{(7)}\right)\mathbb{G}_{36} + (a_{36})^{(7)}\mathbb{G}_{37} - (q_{36})^{(7)}G_{36}^*\mathbb{T}_{37}$$ 656

$$\frac{d\mathbb{G}_{37}}{dt} = -\left((a_{37}^{'})^{(7)} + (p_{37})^{(7)}\right)\mathbb{G}_{37} + (a_{37})^{(7)}\mathbb{G}_{36} - (q_{37})^{(7)}G_{37}^*\mathbb{T}_{37}$$ 657

$$\frac{d\mathbb{G}_{38}}{dt} = -\left((a_{38}^{'})^{(7)} + (p_{38})^{(7)}\right)\mathbb{G}_{38} + (a_{38})^{(7)}\mathbb{G}_{37} - (q_{38})^{(7)}G_{38}^*\mathbb{T}_{37}$$ 658

$$\frac{d\mathbb{T}_{36}}{dt} = -\left((b_{36}^{'})^{(7)} - (r_{36})^{(7)}\right)\mathbb{T}_{36} + (b_{36})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(36)(j)}T_{36}^*\mathbb{G}_j\right)$$ 659

$$\frac{d\mathbb{T}_{37}}{dt} = -\left((b_{37}^{'})^{(7)} - (r_{37})^{(7)}\right)\mathbb{T}_{37} + (b_{37})^{(7)}\mathbb{T}_{36} + \sum_{j=36}^{38}\left(s_{(37)(j)}T_{37}^*\mathbb{G}_j\right)$$ 660

$$\frac{d\mathbb{T}_{38}}{dt} = -\left((b_{38}^{'})^{(7)} - (r_{38})^{(7)}\right)\mathbb{T}_{38} + (b_{38})^{(7)}\mathbb{T}_{37} + \sum_{j=36}^{38}\left(s_{(38)(j)}T_{38}^*\mathbb{G}_j\right)$$ 661

2.

The characteristic equation of this system is

$$\left((\lambda)^{(1)} + (b_{15}^{'})^{(1)} - (r_{15})^{(1)}\right)\{\left((\lambda)^{(1)} + (a_{15}^{'})^{(1)} + (p_{15})^{(1)}\right)$$
$$\left[\left(\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\right]$$
$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^* + (b_{14})^{(1)}s_{(13),(14)}T_{14}^*\right)$$
$$+\left(\left((\lambda)^{(1)} + (a_{14}^{'})^{(1)} + (p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^* + (a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\right)$$
$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^* + (b_{14})^{(1)}s_{(13),(13)}T_{13}^*\right)$$
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$
$$\left(\left((\lambda)^{(1)}\right)^2 + \left((b_{13}^{'})^{(1)} + (b_{14}^{'})^{(1)} - (r_{13})^{(1)} + (r_{14})^{(1)}\right)(\lambda)^{(1)}\right)$$
$$+\left(\left((\lambda)^{(1)}\right)^2 + \left((a_{13}^{'})^{(1)} + (a_{14}^{'})^{(1)} + (p_{13})^{(1)} + (p_{14})^{(1)}\right)(\lambda)^{(1)}\right)(q_{15})^{(1)}G_{15}$$
$$+\left((\lambda)^{(1)} + (a_{13}^{'})^{(1)} + (p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^* + (a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$$
$$\left(\left((\lambda)^{(1)} + (b_{13}^{'})^{(1)} - (r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^* + (b_{14})^{(1)}s_{(13),(15)}T_{13}^*\right)\} = 0$$

$$+$$

$$\left((\lambda)^{(2)} + (b_{18}^{'})^{(2)} - (r_{18})^{(2)}\right)\{\left((\lambda)^{(2)} + (a_{18}^{'})^{(2)} + (p_{18})^{(2)}\right)$$
$$\left[\left(\left((\lambda)^{(2)} + (a_{16}^{'})^{(2)} + (p_{16})^{(2)}\right)(q_{17})^{(2)}G_{17}^* + (a_{17})^{(2)}(q_{16})^{(2)}G_{16}^*\right)\right]$$
$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(17)}T_{17}^* + (b_{17})^{(2)}s_{(16),(17)}T_{17}^*\right)$$
$$+\left(\left((\lambda)^{(2)} + (a_{17}^{'})^{(2)} + (p_{17})^{(2)}\right)(q_{16})^{(2)}G_{16}^* + (a_{16})^{(2)}(q_{17})^{(2)}G_{17}^*\right)$$
$$\left(\left((\lambda)^{(2)} + (b_{16}^{'})^{(2)} - (r_{16})^{(2)}\right)s_{(17),(16)}T_{17}^* + (b_{17})^{(2)}s_{(16),(16)}T_{16}^*\right)$$
$$\left(\left((\lambda)^{(2)}\right)^2 + \left((a_{16}^{'})^{(2)} + (a_{17}^{'})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right)(\lambda)^{(2)}\right)$$

$$\left(\left((\lambda)^{(2)}\right)^2 + \left((b'_{16})^{(2)} + (b'_{17})^{(2)} - (r_{16})^{(2)} + (r_{17})^{(2)}\right) (\lambda)^{(2)}\right)$$
$$+\left(\left((\lambda)^{(2)}\right)^2 + \left((a'_{16})^{(2)} + (a'_{17})^{(2)} + (p_{16})^{(2)} + (p_{17})^{(2)}\right) (\lambda)^{(2)}\right) (q_{18})^{(2)}\mathrm{G}_{18}$$
$$+\left((\lambda)^{(2)} + (a'_{16})^{(2)} + (p_{16})^{(2)}\right) \left((a_{18})^{(2)}(q_{17})^{(2)}\mathrm{G}^*_{17} + (a_{17})^{(2)}(a_{18})^{(2)}(q_{16})^{(2)}\mathrm{G}^*_{16}\right)$$
$$\left(\left((\lambda)^{(2)} + (b'_{16})^{(2)} - (r_{16})^{(2)}\right) s_{(17),(18)}\mathrm{T}^*_{17} + (b_{17})^{(2)} s_{(16),(18)}\mathrm{T}^*_{16}\right)\} = 0$$

$+$

$$\left((\lambda)^{(3)} + (b'_{22})^{(3)} - (r_{22})^{(3)}\right)\{\left((\lambda)^{(3)} + (a'_{22})^{(3)} + (p_{22})^{(3)}\right)$$
$$\left[\left(\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right) (q_{21})^{(3)} G^*_{21} + (a_{21})^{(3)}(q_{20})^{(3)} G^*_{20}\right)\right]$$
$$\left(\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right) s_{(21),(21)}T^*_{21} + (b_{21})^{(3)} s_{(20),(21)}T^*_{21}\right)$$
$$+\left(\left((\lambda)^{(3)} + (a'_{21})^{(3)} + (p_{21})^{(3)}\right)(q_{20})^{(3)}G^*_{20} + (a_{20})^{(3)}(q_{21})^{(1)}G^*_{21}\right)$$
$$\left(\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right) s_{(21),(20)}T^*_{21} + (b_{21})^{(3)} s_{(20),(20)}T^*_{20}\right)$$
$$\left(\left((\lambda)^{(3)}\right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right) (\lambda)^{(3)}\right)$$
$$\left(\left((\lambda)^{(3)}\right)^2 + \left((b'_{20})^{(3)} + (b'_{21})^{(3)} - (r_{20})^{(3)} + (r_{21})^{(3)}\right) (\lambda)^{(3)}\right)$$
$$+\left(\left((\lambda)^{(3)}\right)^2 + \left((a'_{20})^{(3)} + (a'_{21})^{(3)} + (p_{20})^{(3)} + (p_{21})^{(3)}\right) (\lambda)^{(3)}\right) (q_{22})^{(3)}G_{22}$$
$$+\left((\lambda)^{(3)} + (a'_{20})^{(3)} + (p_{20})^{(3)}\right) \left((a_{22})^{(3)}(q_{21})^{(3)}G^*_{21} + (a_{21})^{(3)}(a_{22})^{(3)}(q_{20})^{(3)}G^*_{20}\right)$$
$$\left(\left((\lambda)^{(3)} + (b'_{20})^{(3)} - (r_{20})^{(3)}\right) s_{(21),(22)}T^*_{21} + (b_{21})^{(3)} s_{(20),(22)}T^*_{20}\right)\} = 0$$

$+$

$$\left((\lambda)^{(4)} + (b'_{26})^{(4)} - (r_{26})^{(4)}\right)\{\left((\lambda)^{(4)} + (a'_{26})^{(4)} + (p_{26})^{(4)}\right)$$
$$\left[\left(\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right) (q_{25})^{(4)} G^*_{25} + (a_{25})^{(4)}(q_{24})^{(4)} G^*_{24}\right)\right]$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right) s_{(25),(25)}T^*_{25} + (b_{25})^{(4)} s_{(24),(25)}T^*_{25}\right)$$
$$+\left(\left((\lambda)^{(4)} + (a'_{25})^{(4)} + (p_{25})^{(4)}\right)(q_{24})^{(4)}G^*_{24} + (a_{24})^{(4)}(q_{25})^{(4)}G^*_{25}\right)$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right) s_{(25),(24)}T^*_{25} + (b_{25})^{(4)} s_{(24),(24)}T^*_{24}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right) (\lambda)^{(4)}\right)$$
$$\left(\left((\lambda)^{(4)}\right)^2 + \left((b'_{24})^{(4)} + (b'_{25})^{(4)} - (r_{24})^{(4)} + (r_{25})^{(4)}\right) (\lambda)^{(4)}\right)$$
$$+\left(\left((\lambda)^{(4)}\right)^2 + \left((a'_{24})^{(4)} + (a'_{25})^{(4)} + (p_{24})^{(4)} + (p_{25})^{(4)}\right) (\lambda)^{(4)}\right) (q_{26})^{(4)}G_{26}$$
$$+\left((\lambda)^{(4)} + (a'_{24})^{(4)} + (p_{24})^{(4)}\right) \left((a_{26})^{(4)}(q_{25})^{(4)}G^*_{25} + (a_{25})^{(4)}(a_{26})^{(4)}(q_{24})^{(4)}G^*_{24}\right)$$
$$\left(\left((\lambda)^{(4)} + (b'_{24})^{(4)} - (r_{24})^{(4)}\right) s_{(25),(26)}T^*_{25} + (b_{25})^{(4)} s_{(24),(26)}T^*_{24}\right)\} = 0$$

$+$

$$\left((\lambda)^{(5)} + (b'_{30})^{(5)} - (r_{30})^{(5)}\right)\{\left((\lambda)^{(5)} + (a'_{30})^{(5)} + (p_{30})^{(5)}\right)$$
$$\left[\left(\left((\lambda)^{(5)} + (a'_{28})^{(5)} + (p_{28})^{(5)}\right) (q_{29})^{(5)} G^*_{29} + (a_{29})^{(5)}(q_{28})^{(5)} G^*_{28}\right)\right]$$

$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(29)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(29)}T_{29}^{*}\right)$$
$$+\left(\left((\lambda)^{(5)}+(a_{29}^{'})^{(5)}+(p_{29})^{(5)}\right)(q_{28})^{(5)}G_{28}^{*}+(a_{28})^{(5)}(q_{29})^{(5)}G_{29}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(28)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(28)}T_{28}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$\left(\left((\lambda)^{(5)}\right)^{2}+\left((b_{28}^{'})^{(5)}+(b_{29}^{'})^{(5)}-(r_{28})^{(5)}+(r_{29})^{(5)}\right)(\lambda)^{(5)}\right)$$
$$+\left(\left((\lambda)^{(5)}\right)^{2}+\left((a_{28}^{'})^{(5)}+(a_{29}^{'})^{(5)}+(p_{28})^{(5)}+(p_{29})^{(5)}\right)(\lambda)^{(5)}\right)(q_{30})^{(5)}G_{30}$$
$$+\left((\lambda)^{(5)}+(a_{28}^{'})^{(5)}+(p_{28})^{(5)}\right)\left((a_{30})^{(5)}(q_{29})^{(5)}G_{29}^{*}+(a_{29})^{(5)}(a_{30})^{(5)}(q_{28})^{(5)}G_{28}^{*}\right)$$
$$\left(\left((\lambda)^{(5)}+(b_{28}^{'})^{(5)}-(r_{28})^{(5)}\right)s_{(29),(30)}T_{29}^{*}+(b_{29})^{(5)}s_{(28),(30)}T_{28}^{*}\right)\}=0$$

$+$

$$\left((\lambda)^{(6)}+(b_{34}^{'})^{(6)}-(r_{34})^{(6)}\right)\{\left((\lambda)^{(6)}+(a_{34}^{'})^{(6)}+(p_{34})^{(6)}\right)$$
$$\left[\left(\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(33)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$
$$+\left(\left((\lambda)^{(6)}+(a_{33}^{'})^{(6)}+(p_{33})^{(6)}\right)(q_{32})^{(6)}G_{32}^{*}+(a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(32)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$
$$\left(\left((\lambda)^{(6)}\right)^{2}+\left((b_{32}^{'})^{(6)}+(b_{33}^{'})^{(6)}-(r_{32})^{(6)}+(r_{33})^{(6)}\right)(\lambda)^{(6)}\right)$$
$$+\left(\left((\lambda)^{(6)}\right)^{2}+\left((a_{32}^{'})^{(6)}+(a_{33}^{'})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$
$$+\left((\lambda)^{(6)}+(a_{32}^{'})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G_{33}^{*}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)$$
$$\left(\left((\lambda)^{(6)}+(b_{32}^{'})^{(6)}-(r_{32})^{(6)}\right)s_{(33),(34)}T_{33}^{*}+(b_{33})^{(6)}s_{(32),(34)}T_{32}^{*}\right)\}=0$$

$+$

$$\left((\lambda)^{(7)}+(b_{38}^{'})^{(7)}-(r_{38})^{(7)}\right)\{\left((\lambda)^{(7)}+(a_{38}^{'})^{(7)}+(p_{38})^{(7)}\right)$$
$$\left[\left(\left((\lambda)^{(7)}+(a_{36}^{'})^{(7)}+(p_{36})^{(7)}\right)(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)\right]$$
$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(37)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(37)}T_{37}^{*}\right)$$
$$+\left(\left((\lambda)^{(7)}+(a_{37}^{'})^{(7)}+(p_{37})^{(7)}\right)(q_{36})^{(7)}G_{36}^{*}+(a_{36})^{(7)}(q_{37})^{(7)}G_{37}^{*}\right)$$
$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(36)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(36)}T_{36}^{*}\right)$$
$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((a_{36}^{'})^{(7)}+(a_{37}^{'})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$
$$\left(\left((\lambda)^{(7)}\right)^{2}+\left((b_{36}^{'})^{(7)}+(b_{37}^{'})^{(7)}-(r_{36})^{(7)}+(r_{37})^{(7)}\right)(\lambda)^{(7)}\right)$$
$$+\left(\left((\lambda)^{(7)}\right)^{2}+\left((a_{36}^{'})^{(7)}+(a_{37}^{'})^{(7)}+(p_{36})^{(7)}+(p_{37})^{(7)}\right)(\lambda)^{(7)}\right)(q_{38})^{(7)}G_{38}$$
$$+\left((\lambda)^{(7)}+(a_{36}^{'})^{(7)}+(p_{36})^{(7)}\right)\left((a_{38})^{(7)}(q_{37})^{(7)}G_{37}^{*}+(a_{37})^{(7)}(a_{38})^{(7)}(q_{36})^{(7)}G_{36}^{*}\right)$$
$$\left(\left((\lambda)^{(7)}+(b_{36}^{'})^{(7)}-(r_{36})^{(7)}\right)s_{(37),(38)}T_{37}^{*}+(b_{37})^{(7)}s_{(36),(38)}T_{36}^{*}\right)\}=0$$

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23)^ Assuming a 90/10 alloy of Pt/Ir by weight, a Cp of 25.9 for Pt and 25.1 for Ir, a Pt-dominated average Cp of 25.8, 5.134 moles of metal, and 132 J.K-1 for the prototype. A variation of ±1.5 picograms is of course, much smaller than the actual uncertainty in the mass of the international prototype, which are ±2 micrograms.

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