# **A Review on Finite Element Simulations in Metal Forming**

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**Abstract:** Finite element simulations are often required to reduce the experimental cost and time by reducing number of trials in the product development cycle. Metal forming is one of such area where a lot of trials are required to arrive at the die design to produce defect free parts. Hence in this paper the authors review the literature on finite element analysis in the area of metal forming

#### Key Words: Metal Forming, Finite Element Analysis, Simulations

The term simulation is derived from the Latin word "simulare" what means "to pretend". However, the technical meaning of simulation is the description and reproduction of physical and technical processes by use of mathematical and physical models. In comparison with practical tests, the simulation often is cheaper and not so dangerous. Combined with modern methods of computation, the simulation is a powerful tool which gains more and more importance for describing and developing new processing methods. Because of higher requirements on the quality of products and narrow tolerances of measures, optimizing, planning and simulating of forming processes becomes more and more important. As the computational power has increased during the last years, numerical methods play an outstanding roll. The most important numerical method is the method of finite elements (FEM). Numerous finite element programmes have been developed which are able to solve linear, non linear, static, dynamic, elastic, plastic, elastic – plastic, steady state, transient, isothermal as well as non isothermal problems [1].

The deep drawing process is applied with the intention of manufacturing a product with a desired shape and no failures. The final product shape after deep drawing is defined by the tools, the blank and the process parameters. An incorrect design of the tools and blank shape or an incorrect choice of material and process parameters can yield a product with a deviating shape or with failures. A deviating shape is caused by elastic springback after forming and retracting the tools. The most frequent types of failure are wrinkling, necking (and subsequently tearing), scratching and orange peel. Wrinkling may occur in areas with high compressive strains, necking may occur in areas with high tensile strains, scratching is caused by defects of the tool surface and orange peel may occur after excessive deformations, depending on the grain size of the material. The deformation patterns of the sheet material are influenced by the material properties and the processing and tooling variables. Generally, sheet material behaves anisotropically which means that the material shows a different deformation behavior in different directions because of the rolling process. An example of anisotropy is the development of 'ears' in cylindrical cup drawing. The friction conditions during forming depend on the lubricant, the presence of coatings on the blank, surface roughness of the tools and the blank, blank holder pressure and process velocity. Without extensive knowledge of the influences of all these variables on the deep drawing process, it is hardly possible to design the tools adequately and make a proper choice of blank material and lubricant to manufacture a product with the desired shape and performance. As a result, after the first design of the tools and choice of blank material and lubricant, an extensive and time consuming trial and error process is started to determine the proper tool design and all other variables, leading to the desired product. This trial and error process can yield an unnecessary number of deep drawing strokes, or may even require redesigning the expensive tools. To reduce this waste of time and cost, process modeling for computer simulation can be used to replace the experimental trial and error process by a virtual trial and error process. The prime objective of an analysis is to assist in the design of a product. To design or select the tools and the equipment, such design essentially consists of predicting the material flow, determining whether it is possible to form the part without surface or internal defects, predicting the forces necessary to execute the forming operation and stresses induced during the operation.

Analytical study of Metal forming processes was started in the mid of 20<sup>th</sup> century [2,3]. Later a number of analyzing methods have been developed and applied to various forming processes. Some of these methods are the slab method, the slip-line field method, the viscoplasticity method, upper and lower bound techniques and Hill's general method. These methods have been useful in qualitatively predicting forming loads, overall geometry changes of the deformed blank and material flow and approximate optimum process conditions. Numerical procedures (finite difference method) were applied to analyze axisymetric deep drawing process in 1960s[4] Althouh the work contributed to greatly to the development of theory of sheet metal forming analysis, that could not be applicable to the industrial components. However, a more accurate determination of the effects of various process parameters on the deep drawing process has become possible only, when the non linear finite element method was developed for these analyses [5-7]. Later, three dimensional auto body panel forming process of and left window outer and binder wrapping process of deck lid. But they were in the state of testing and evaluation. Because finite element analysis by that time is was still extremely time consuming and unreliable tool to the engineers in the press shop.

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Rapid developments in computer hardware make the finite element analysis of complex deformation responses increasingly applicable. The finite element method is used worldwide to simulate the deep drawing process and has become a reliable numerical simulation technology. For an accurate simulation of a real-life deep drawing process an accurate numerical description of the tools is necessary, as well as an accurate description of material behavior, contact behavior and other process variables. The numerical description of the tools is provided by CAD packages which are generally used by tool designers. The description of material behavior, contact behavior and other process variables evolved from rather simple models in the earlier days to more and more sophisticated models nowadays. Developments have been made in the field of finite element types, mesh adaptivity, material laws, failure criteria, wrinkling and surface defects, springback, contact algorithms, friction, and simulation of new processes (optimization and process design). The conventional finite element codes are based on implicit time integration. This involves repeated solutions of large systems of equations. Furthermore, equilibrium must be fulfilled after each incremental step. As a result, implicit codes are computational time and memory consuming. Hence, a new class of finite element codes based on explicit time integration was developed, resulting in a drastic decrease of computational time. Honecker et.al [10] first demonstrated the deep drawing of an oil pan and a radiator part by explicit method, obtaining deeply drawn shapes including wrinkle on the flange. After this several dynamic explicit codes specialized to the sheet metal forming were developed and many automotive industries started to develop these codes. In an explicit code no system of equations needs to be solved and static equilibrium is not checked after each incremental step, as the algorithm assumes an inertia dominated process. The explicit procedure is conditionally stable with a critical time step, which is proportional to the smallest element in the mesh [11]. However, in most sheet metal forming processes inertia effects can be neglected. In order to apply the explicit algorithms in these processes, it is necessary to assume artificially high velocities and accelerations or artificially high mass density, which seems rather unrealistic [12].

On the other hand one step method proposed by Batoz et.al [13] was developed based on the idea of Chung and Lee [14] in which single time step was used, deforming the sheet inversely from final part configuration to initial blank configuration. A major advantage of this method is very short computation time. Mean while there were several activities to develop codes based on static implicit incremental approach [15-19]. But convergence is the basic problem of this approach. To avoid the convergence problem static explicit codes were developed [20,21].

In a nutshell all these codes may be classified into five categories based on the formulation and solution strategy used. These are dynamic explicit codes, static explicit codes, Static implicit incremental codes, Static implicit large step code and Static implicit one step code.

The dynamic explicit approach was originally developed for the problems in which dynamic effects are important, such as impact problems and crash simulation, and includes inertia in the equilibrium equations. The reasons for using a method like this in metal forming are two fold. The method is extremely robust and it is very efficient for large scale problems. In this approach the central difference explicit scheme is used to integrate the equations of motion. Lumped mass matrices are used, which implies that the mass matrix is diagonal, and no system equations has to be solved. A typical time step is of the order of a micro seconds and the number of time steps in typical sheet forming simulation are normally several tens of thousands. In spite of its success for industrial applications, it has also some intrinsic drawbacks i.e. in order to achieve significant computational advantage several numerical artifacts have to be introduced into the explicit solution procedure. In particular the parameters like mass density, punch velocity, loading history etc are to be modified. Since the maximum permissible time step, as defined by the current stability limit is directly proportional to the square root of the material density, this parameter is increased, usually by at least one order of magnitude. In order to reduce the total number of time steps necessary to model the sheet metal forming process, the punch velocity is increased, again by at least one order of magnitude. Since increase in both the material density and punch velocity results in increased inertia forces, the punch travel must be suitably controlled so as to minimize the inertia effects. Thus, the very nature of the dynamic explicit method, the simulation of forming defects requires a considerable experience on the user side for adequately designing the finite element mesh and choosing the scaling parameters of mass, velocity and damping. Other issues that must be given attention in the dynamic explicit analysis is the simulation of the spring back. One way of improving reliability of spring back is to combine the dynamic explicit analysis with quasi -static implicit simulation.

In the static explicit method, the system of equations representing the rate of equilibrium is integrated with a simple forward Euler scheme, involving no iterations. This implies that equilibrium equations are satisfied only in rate form and the obtained solution can gradually drift away from the true one. Inorder to reduce the error involved vary small incremental steps have to be taken. An ordinary simulations normally involves several thousand steps. The main advantage of this approach is the robustness, since there are no iterative processes.

The Static implicit incremental approach may seem ideally suited for metal forming problems, since the static equilibrium equations are solved iteratively, ensuring that the equilibrium conditions are full filled in every step. However, in practice complex nonlinear problems involving many contacts may results in slow or even lack of convergence. The method is also inefficient for solving large scale problems, since time taken for solving the system of equations increases approximately quadratically with the number of degrees of freedom.

Static implicit large step codes employ large incremental time step under special contact treatment, uncoupled bending and stretching solution algorithm, and adoptive mesh refinement of refinement levels. These features, specialized for the simulations of thin sheet metal forming renders code extremely efficient ,but unfortunately also make the results approximate in respects. For instance, the contact and discontact process are not accurately simulated and there fore wrinkling and buckling are poorly predicted.

In static implicit one step codes use a single time step, usually taking the deformation process from the final part configuration to the initial flat blank configuration, assuming a linear strain path and neglecting the history of contact.

Although, this method incorporates significantly drastic simplifications, its major advantage is a very short computation time and less input data. These features enable the use of these codes at the product design stage, in the absence of information of the stamping tools.

Currently, the accuracy and reliability of numerical simulations of sheet metal forming processes do not yet satisfy the industrial requirements. One of the limitations of numerical simulations is still the high computational time for complex deep drawing parts, despite the development of iterative solvers, fast contact algorithms and the ever ongoing progress in computer hardware. Another limitation is the lack of detailed knowledge of material physics such as material behavior at high deformations and contact behavior. Therefore extensive research in the field of sheet metal forming is and will be necessary to decrease the existing gap between the real-life deep drawing process and the predictions obtained from deep drawing simulations.

Deep drawing, even though is one of the most basic processes in sheet metal working, it involves very complicated deformation mechanics. The numerical difficulty in the finite element analysis of the deep drawing processes arises due to the existence of compressive stress in the sheet plane and the occurrence of unloading. The drawing load increases with the punch displacement. As the punch moves, the flange part of the sheet is drawn into the die cavity. The punch load decreases after a critical point because less resisting force to drawing is developed in the flange. In the range of decreasing punch load, unloading occurs at the wall of a drawn cup. Therefore, in analyzing the bending-dominant processes like deep drawing, the effect of unloading should be also considered. The state of stress at the wall and at the flange is basically tensile stress in axial or radial direction and compressive stress in the circumferential direction. As the sheet metal has relatively a small dimension in the thickness direction, the compressive stress may cause wrinkling in the actual process or numerical buckling is stress like actual buckling phenomenon occurring in the finite element analysis at the region of high compressive stress like actual buckling.

The modeling of the blank elements can be done by two alternative approaches [23] namely a structural based elasto-plastic / elasto-viscoplastic approach or rigid-plastic/ rigid-visco- plastic approach. Elastic-plastic analysis of sheet metal forming can be broadly classified into three categories according to the element types; membrane analysis, continuum analysis and shell analysis. Membrane analysis done by Wang [24] Arlinghaus [25], Mattiason [26], Massoni (27), Saran (28) and Batoz (29), has been widely applied to various sheet metal forming problems because of small computation time and small memory size. However, it provides insufficient information when treating the bending-dominant processes. Continuum analysis by Wifi (30), Anderson (31), Stalman (32), Makinouchi (33,34), Keck (35) has several merits; e.g. the bending effect can be considered and the formulation is much simpler than other methods of analysis using different element types. The continuum analysis, however, involves extremely large computation time and enormous memory size when threedimensional problems are to be treated for any practical purposes. The shell analysis by Tatenami (36), Wang (37, 38), Gelin (39), Honnor (40), Batoz (29) and Honecker (41) may be regarded as a compromise between the continuum analysis and the membrane analysis. It is possible to consider the effect of bending with much less computation time and less memory size than the continuum analysis. However, most of the applications using shell elements are limited to the axisymmetric and plane strain problems because of the difficulty in treating kinematics of three-dimensional shells, computation time and memory size etc. Recently, the effect of bending has been studied through the comparison between the membrane analysis, the continuum analysis and the shell analysis. Wang and Tang (38) analyzed stretching and deep drawing with axisymmetric membrane elements and axisymmetric shell elements. In the analysis of stretching, both theories do not show any appreciable discrepancy, while the discrepancy between two theories becomes apparent in the analysis of deep drawing. Hambrecht et al. (42) studied the effect of bending in the plane strain punch stretching and axisymmetric stretching by the comparison of above-mentioned three kinds of approaches. Through the study, the continuum or shell theory is necessary in analyzing deep drawing and above three kinds of approaches do not give any difference in analyzing stretching. Yang et al. (22) analyzed stretching of a square plate as well as deep drawing of a cylindrical cup with the rigid-plastic finite element analyses using membrane and then they have investigated the effect of bending in the stretch dominant process and drawdominant processes. Now, the effect of bending has become significant in the modelling of sheet metal forming process in order to obtain more accurate information for the die design of deep drawing. Shim and Yang [43] analysed deep drawing of cylindrical and square cups using membrane and shell elements. They found that both theories are in good agreement with each other in case of load-displacement curve. How ever in the neighborhood of punch round, the thinning appearing in the experiment can not be predicted by membrane analysis. In the shell analysis the thickness strain distribution is better predicted over whole range of sheet. It is due to the fact that bending of sheet effects considerably the thickness strain distribution and bending has no significant affect in load-displacement curve. So shell element is considered to be most suitable for the analysis in deep drawing process.

Finite Element models for rigid visco-plastic thin sheet problems was presented by Bellet et.al.[44]. A viscous shell formulation was introduced by Onate et.al [28], who considered visco-plastic deformation of the work material as well as frictional affects of punch and die. Onate et.al [45]discussed viscous shell approach based on bending and membrane shell elements.

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