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Chance Constrained Quadratic Bi-level Programming Problem

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ABSTRACT: *This paper deals with fuzzy goal programming approach to solve chance constrained quadratic bi-level programming problem. Chance constraints are converted into equivalent deterministic constraints by the prescribed distribution functions. In the model formulation, the quadratic membership functions are formulated by using the individual best solution of the quadratic objective functions subject to the equivalent deterministic constraints. Using first order Taylor's series, the quadratic membership functions are approximated to linear membership functions expanding about the individual best solution points. For avoiding decision deadlock, each level decision maker provides a relaxation of bounds on the decision variables controlled by him. We use two fuzzy goal programming models to reach the highest degree of membership goals by minimizing negative deviational variables. Euclidean distance function is used to identify the most compromise optimal solution. To demonstrate the proposed approach, two numerical examples are solved.*

Keyword: *Bi-level programming, chance constraints, fuzzy goal programming, quadratic programming problem, Taylor's series.*

I. Introduction

In bi-level programming problem (BLPP), there are two types of decision makers (DMs). One is first level decision maker (FLDM) and another is second level decision maker (SLDM). The execution of decision is sequential from first level to second level and each level DM independently controls only a set of decision variables. The FLDM makes his decision first. But SLDM may not be satisfied with the decision of FLDM. Consequently, decision deadlock occurs frequently in the hierarchical decision making context.

Candler and Townsley [1] as well as Fortuny –Amart and McCarl [2] developed the formal bi-level programming problem (BLPP). In 1991, Edmund and Bard [3] studied non linear bi-level programming problems. Malhotra and Arora [4] discussed fractional bi-level programming problem using preemptive goal programming. Sakawa and Nishizaki [5, 6] studied linear fractional BLPP based on interactive fuzzy programming. Using analytical hierarchy process due to Saaty [7], Mishra [8] developed weighting method for linear fractional BLPP. Pramanik and Dey [9] presented linear fractional BLPP based on fuzzy goal programming (FGP) using first order Taylor polynomial series.

Anandalingam [10] discussed multi level programming problem (MLPP) as well as bi-level decentralized programming problem by using Stackelberg solution approach. The concept of fuzzy set theory in MLPP was first introduced by Lai [11]. Shih et al. [12] and Shih and Lee [13] extended Lai's ideas by introducing non-compensatory max min aggregation operator and compensatory fuzzy operator respectively. Fuzzy goal programming for MLPP was studied by Pramanik and Roy [14].

Quadratic bi-level programming problem (QBLPP) is a special type of non-linear bi-level programming problem. In this paper, we consider the objective function of each level DM is quadratic function and the constraints are linear functions. There are many research fields where QBLPP arise such as robust data fitting, traffic assignment problems, portfolio optimizations, transportations. In 1994, Faustino and Judice [15] developed the linear QBLPP. Vicente et al. [16] presented descent method for QBLPP. Optimality conditions and algorithm for solving QBLPP were developed by Wang et al. [17]. QBLPP for integer variables was presented by Thirwani and Arora [18]. They used linearization method and obtained integer solution for QBLPP by using Gomory cut and dual simplex method. Using Karush-Kuhn-Tucker conditions and duality theory, Calvete and Gale [19] discussed optimality conditions for the linear fractional / quadratic BLPP. Pal and Moitra [20] developed FGP approach for solving QBLPP in 2003. Recently, Pramanik and Dey [21] studied multi objective quadratic programming problem. They [22] also developed priority based FGP approach to multi objective quadratic programming problem. They [23] also extended their ideas for solving QBLPP based on FGP.

Uncertainties may occur in the decision making situations. Generally, uncertainties can be fuzzily or stochastically described. Using probability theory, Dantzig [24] introduced stochastic programming. There are two main approaches of stochastic programming, namely, chance constrained programming (CCP) and two- stage programming. Charnes and Cooper [25] developed the CCP.

In the present paper, we present QBLPP with chance constraints which is called chance constrained QBLPP. We first convert the chance constraints into equivalent deterministic constraints with prescribed distribution functions and confidence levels. We form quadratic membership function by using individual best solution. Using first order Taylor's series, the quadratic membership function are approximated into linear membership functions by expanding about the respective individual best solution point. For avoiding decision deadlock, each decision maker prefers some bounds on the decision variables controlled by him. Two FGP models are formulated and Euclidean distance function is used to determine the most compromise solution. Two numerical examples are solved to demonstrate the efficiency of the proposed approach.

The rest of the paper is organized in the following way. In Section II, we formulate chance constrained QBLPP. In Section III, chance constraints are transformed into equivalent deterministic constraints. Quadratic membership functions are constructed in Section IV. In Section V, technique of linearization of quadratic membership function is discussed by using first order Taylor's series. In Section VI, preference bounds on the decision variables are defined. Section VII is devoted to develop two FGP models for solving chance constrained QBLPP for maximization type objective functions. Section VIII discusses FGP model formulation for solving chance constrained QBLPP for minimization type objective functions. The Euclidean distance function is described in the next Section IX. The step wise descriptions of the whole paper are summarized in the Section X. Section XI presents two numerical examples. Finally, Section XII concludes the paper with final conclusion and future work.

II. FORMULATION OF CHANCE CONSTRAINED QUADRATIC BI-LEVEL PROGRAMMING PROBLEM

The generic form of chance constrained QBLPP is

$$\text{[FLDM]}\ \underset{\overline{X}_1}{\text{Max}}\ Z_1(\overline{X}) = \overline{A}_1\overline{X} + \frac{1}{2}\overline{X}^T\overline{B}_1\overline{X} \quad (1)$$

$$\text{[SLDM]}\ \underset{\overline{X}_2}{\text{Max}}\ Z_2(\overline{X}) = \overline{A}_2\overline{X} + \frac{1}{2}\overline{X}^T\overline{B}_2\overline{X} \quad (2)$$

subject to

$\overline{X} \in X$`=

$$\{\overline{X} \in \overline{R}^n : \Pr(\overline{C}\overline{X} \ \substack{\le \\ \ge}\ \overline{d}) > \overline{I} - \overline{m},\ \overline{X} \ge \overline{0}\} \quad (3)$$

Here, the decision vector $\overline{X}_1 = (x_{11}, x_{12}, x_{13},\ldots, x_{1n1})$ is controlled by FLDM and $\overline{X}_2 = (x_{21}, x_{22}, x_{23},\ldots, x_{2n2})$ is controlled by SLDM. $\overline{X}_1 \cup \overline{X}_2 = \overline{X} \in \overline{R}^n$, $n_1 + n_2 = n$, 'T' means transposition of vector. $\overline{A}_1, \overline{B}_1, \overline{A}_2, \overline{B}_2, \overline{I}, \overline{m}$ are given vectors. The order of $\overline{A}_1, \overline{A}_2$ are $1 \times n$, the order of symmetric matrices $\overline{B}_1, \overline{B}_2$ are $n \times n$, $\overline{I}, \overline{d}, \overline{m}$ are vectors of order $p \times 1$, every elements of $\overline{I}$ is unity. $\overline{C}$ is the given matrix of order $p \times n$. The polyhedron X' is assumed to be non-empty and bounded.

III. CONVERSION OF STOCHASTIC CONSTRAINTS INTO DETERMINISTIC CONSTRAINTS

First, we consider the chance constraints of the form:

$$\Pr\left(\sum_{j=1}^{n} c_{ij}x_j \le d_i\right) \ge 1 - m_i,\quad i = 1, 2, \ldots, p_1. \quad (4)$$

$$\Rightarrow \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \le \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \ge 1 - m_i,\ i = 1, 2,\ldots, p_1$$

$$\Rightarrow m_i \ge 1 - \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \le \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right)$$

$$\Rightarrow m_i \ge \Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} > \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right)$$

$$\Rightarrow \Psi^{-1}(m_i) \ge \frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}$$

$$\Rightarrow \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)} \ge \sum_{j=1}^{n} c_{ij}x_j - E(d_i)$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \le E(d_i) + \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)},$$

$i = 1, 2, \ldots, p_1$ (5)

Here Ψ(.) and Ψ^{-1}(.) represent the distribution function and inverse of distribution function of standard normal variable respectively.

Considering the case when $\Pr\left(\sum_{j=1}^{n} c_{ij}x_j \ge d_i\right) \ge 1 - m_i$,

$i = p_1 + 1, p_1 + 2, \ldots, p.$ (6)

The constraints can be rewritten as:

$$\Pr\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}} \ge \frac{d_i - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \ge 1 - m_i,\ i = p_1+1, p_1+2, \ldots, p.$$

$$\Rightarrow \Psi\left(\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \ge 1 - m_i$$

$$\Rightarrow 1 - \Psi\left(-\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}\right) \ge 1 - m_i$$

$$\Rightarrow \Psi^{-1}(m_i) \ge -\frac{\sum_{j=1}^{n} c_{ij}x_j - E(d_i)}{\sqrt{\text{var}(d_i)}}$$

$$\Rightarrow \sum_{j=1}^{n} c_{ij}x_j \ge E(d_i) - \Psi^{-1}(m_i)\sqrt{\text{var}(d_i)}, \quad (7)$$

$i = p_1 + 1, p_1 + 2, \ldots, p.$

$$\overline{X} \ge \overline{0} \quad (8)$$

Let us denote the equivalent deterministic system constraints (5), (7) and (8) by X. Here, X` and X are equivalent set of constraints.

IV. CONSTRUCTION OF MEMBERSHIP FUNCTION

In order to construct quadratic membership function subject to the equivalent deterministic system constraints, the quadratic objective functions are maximized separately.

Let $\overline{X}_i^B = (x_{i1}^B, x_{i2}^B, x_{i3}^B, \ldots, x_{in_i}^B, x_{in_i+1}^B, \ldots, x_{in}^B)$, $i = 1, 2$ be the individual best solution for the objective function $Z_i(\overline{X})$.

Let $\max_{\overline{X} \in X} Z_1(\overline{X}) = Z_1^B = Z_1(\overline{X}_1^B)$ and $\max_{\overline{X} \in X} Z_2(\overline{X}) = Z_2^B = Z_2(\overline{X}_2^B)$.

Considering the individual best solution as the aspiration level, the fuzzy goal appears as:

$$Z_i(\bar{X}) \underset{\sim}{\geq} Z_i^B, \; i = 1, 2 \tag{9}$$

We consider $\min_{\bar{X}\in X} Z_1(\bar{X}) = Z_1^W$ and $\min_{\bar{X}\in X} Z_2(\bar{X}) = Z_2^W$ as the lower tolerance limits of the fuzzy objective goals of FLDM and SLDM.

Now, the membership function for the objective function $Z_1(\bar{X})$ of FLDM can be written as:

$$\mu_1(\bar{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_1(\bar{X}) \geq Z_1^B, \\ \dfrac{Z_1(\bar{X}) - Z_1^W}{Z_1^B - Z_1^W}, & \text{if } Z_1^W \leq Z_1(\bar{X}) \leq Z_1^B, \\ 0, & \text{if } Z_1(\bar{X}) \leq Z_1^W \end{array} \right\rangle \tag{10}$$

and the membership function for the objective function $Z_2(\bar{X})$ of SLDM can be formulated as:

$$\mu_2(\bar{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_2(\bar{X}) \geq Z_2^B, \\ \dfrac{Z_2(\bar{X}) - Z_2^W}{Z_2^B - Z_2^W}, & \text{if } Z_2^W \leq Z_2(\bar{X}) \leq Z_2^B, \\ 0, & \text{if } Z_2(\bar{X}) \leq Z_2^W \end{array} \right\rangle \tag{11}$$

Now, the chance constrained QBLP reduces to

$\max \mu_1(\bar{X})$,

$\max \mu_2(\bar{X})$,

subject to

$$\bar{X} \in X. \tag{12}$$

V. Linearization Of Quadratic Membership Functions By Using Taylor's Series Approximation

Let $\bar{X}_i^* = (x_{i1}^*, x_{i2}^*, x_{i3}^*, \ldots, x_{in_i}^*, x_{in_i+1}^*, \ldots, x_{in}^*)$, $i = 1, 2$ be the individual best solution of $\mu_i(\bar{X})$ subject to the equivalent deterministic system constraints. Then we transform the quadratic membership function $\mu_i(\bar{X})$ into an equivalent linear membership function $\mu_i^*(\bar{X})$ at the point $\bar{X}_i^*$ by using first order Taylor's series as follows:

$$\mu_1(\bar{X}) \cong \mu_1(\bar{X}_1^*) + (x_1 - x_{11}^*) \frac{\partial}{\partial x_1}\mu_1(\bar{X}_1^*) + (x_2 - x_{12}^*) \frac{\partial}{\partial x_2}\mu_1(\bar{X}_1^*) + \ldots + (x_{n_1} - x_{1n_1}^*) \frac{\partial}{\partial x_{n_1}}\mu_1(\bar{X}_1^*) + (x_{n_1+1} - x_{1n_1+1}^*) \frac{\partial}{\partial x_{n_1+1}}\mu_1(\bar{X}_1^*) + \ldots + (x_n - x_{1n}^*) \frac{\partial}{\partial x_n}\mu_1(\bar{X}_1^*) = \mu_1^*(\bar{X}) \tag{13}$$

$$\mu_2(\bar{X}) \cong \mu_2(\bar{X}_2^*) + (x_1 - x_{21}^*) \frac{\partial}{\partial x_1}\mu_2(\bar{X}_2^*) + (x_2 - x_{22}^*) \frac{\partial}{\partial x_2}\mu_2(\bar{X}_2^*) + \ldots + (x_{n_2} - x_{2n_2}^*) \frac{\partial}{\partial x_{n_2}}\mu_2(\bar{X}_2^*) + (x_{n_2+1} - x_{2n_2+1}^*) \frac{\partial}{\partial x_{n_2+1}}\mu_2(\bar{X}_2^*) + \ldots + (x_n - x_{2n}^*) \frac{\partial}{\partial x_n}\mu_2(\bar{X}_2^*) = \mu_2^*(\bar{X}) \tag{14}$$

VI. Characterization Of Preference Bounds On The Decision Variables For Both Level DECISION MAKERS

Since the objectives of level DMs are conflicting, cooperation between the level DMs is necessary in order to reach compromise optimal solution. Each DM tries to reach maximum profit with the consideration of benefit of other. Here, the relaxations on both decision variables are considered for overall benefit.

Let $(x_{1j}^* - r_{1j}^-)$ and $(x_{1j}^* + r_{1j}^+)$ (j = 1, 2, …, n_1) be the lower and upper bounds of decision variable x_{1j} (j = 1, 2, …, n_1) provided by the FLDM. Here, $\bar{X}_1^* = (x_{11}^*, x_{12}^*, \ldots, x_{1n_1}^*, x_{1n_1+1}^*, \ldots, x_{1n}^*)$ is the individual best solution of the quadratic membership function $\mu_1(\bar{x})$ of FDLM when calculated in isolation subject to the equivalent deterministic system constraints.

Similarly, $(x_{2j}^* - r_{2j}^-)$ and $(x_{2j}^* + r_{2j}^+)$ (j = 1, 2, …, n_2) be the lower and upper bounds of decision variables x_{2j} (j = 1, 2, …, n_2) provided by the SLDM. $\bar{X}_2^* = (x_{21}^*, x_{22}^*, \ldots, x_{2n_2}^*, x_{2n_2}^*)$ is the individual best solution of the quadratic membership function $\mu_2(\bar{x})$ of SLDM when calculated in isolation subject to the equivalent deterministic system constraints. Therefore, preference bounds on the decision variable can presented as follows:

$$(x_{1j}^* - r_{1j}^-) \leq x_{1j} \leq (x_{1j}^* + r_{1j}^+) \; (j = 1, 2, \ldots, n_1) \tag{15}$$

$$(x_{2j}^* - r_{2j}^-) \leq x_{2j} \leq (x_{2j}^* + r_{2j}^+) \; (j = 1, 2, \ldots, n_2) \tag{16}$$

Here, r_{1j}^- and r_{1j}^+ (j = 1, 2, …, n_1) and are the negative and positive tolerance values, which are not necessarily same. Similarly, r_{2j}^- and r_{2j}^+ (j = 1, 2, …, n_2) are negative and positive tolerance values that may be not be necessarily same.

VII. Formulation of Fuzzy Goal Programming Model of Chance Constrained Quadratic Bi-level Programming Problem

The chance constrained QBLPP reduces to the following problem

Max $\mu_1^*(\overline{X})$,

Max $\mu_2^*(\overline{X})$

subject to

$(x_{1j}^* - r_{1j}^-) \le x_{1j} \le (x_{1j}^* + r_{1j}^+)$, (j = 1, 2, …, n_1)

$(x_{2j}^* - r_{2j}^-) \le x_{2j} \le (x_{2j}^* + r_{2j}^+)$, (j = 1, 2, …,$n_2$)

$\overline{X} \in X$ (17)

According to Pramanik and Dey, it can be written [23] as:

$\mu_i^* + d_i^- = 1$, i = 1, 2. (18)

d_1^-, d_2^- are the negative deviational variables. Now, two FGP models are formulated as follows:

Model-I

min λ (19)

subject to

$\mu_1^*(\overline{X}) + d_1^- = 1$,

$\mu_2^*(\overline{X}) + d_2^- = 1$,

$\lambda \ge d_1^-$,

$\lambda \ge d_2^-$,

$0 \le d_1^- \le 1$,

$0 \le d_2^- \le 1$,

$(x_{1j}^* - r_{1j}^-) \le x_{1j} \le (x_{1j}^* + r_{1j}^+)$, (j = 1, 2, …, n_1)

$(x_{2j}^* - r_{2j}^-) \le x_{2j} \le (x_{2j}^* + r_{2j}^+)$, (j = 1, 2, …,$n_2$)

$\overline{X} \in X$

Model –II (20)

Min $\xi = \sum_{i=1}^{2} d_i^-$

subject to

$\mu_1^*(\overline{X}) + d_1^- = 1$,

$\mu_2^*(\overline{X}) + d_2^- = 1$,

$0 \le d_1^- \le 1$,

$0 \le d_2^- \le 1$,

$(x_{1j}^* - r_{1j}^-) \le x_{1j} \le (x_{1j}^* + r_{1j}^+)$, j = 1, 2, …, n_1

$(x_{2j}^* - r_{2j}^-) \le x_{2j} \le (x_{2j}^* + r_{2j}^+)$, j = 1, 2, …,$n_2$

$\overline{X} \in X$

VIII. Model Formulation for Quadratic Bi-level Programming Problem with Chance Constraints for Minimization Type Objective Functions

Let us consider the following problem of chance constrained QBLPP with minimization type objective functions.

[FLDM] $\underset{X_1}{\text{Min}}\, Z_1(\overline{X}) = \overline{A}_1\overline{X} + \frac{1}{2}\overline{X}^T\overline{B}_1\overline{X}$ (21)

[SLDM] $\underset{X_2}{\text{Min}}\, Z_2(\overline{X}) = \overline{A}_2\overline{X} + \frac{1}{2}\overline{X}^T\overline{B}_2\overline{X}$ (22)

subject to

$\overline{X} \in X' =$

$\{\overline{X} \in \overline{R}^n : \Pr(\overline{C}\overline{X} \begin{smallmatrix}\le\\ \ge\end{smallmatrix} \overline{d}) > \overline{I} - \overline{m},\ \overline{X} \ge \overline{0}\}$ (23)

The descriptions of the coefficients, matrices, and vectors of the problem are already provided in section 2.

The chance constraints are converted into equivalent deterministic constraints as described in (5) and (7). Then, the objective functions are solved separately subject to equivalent deterministic constraints. Let $\overline{X}_1^B, \overline{X}_2^B$ be the individual best solutions for the quadratic objective functions $Z_1(\overline{X})$, $Z_2(\overline{X})$ and $\min_{\overline{X}\in X} Z_1(\overline{X}) = Z_1^B = Z_1(\overline{X}_1^B)$ and $\min_{\overline{X}\in X} Z_2(\overline{X}) = Z_2^B = Z_2(\overline{X}_2^B)$.

Considering the individual best solution as the aspiration level, the fuzzy goal appears as:

$Z_i(\overline{X}) \lesssim Z_i^B$, i = 1, 2 (24)

We consider $\max_{\overline{X}\in X} Z_1(\overline{X}) = Z_1^W$ and $\max_{\overline{X}\in X} Z_2(\overline{X}) = Z_2^W$ as the upper tolerance limits of the fuzzy objective goals of FLDM and SLDM.

Now, the quadratic membership functions are formulated as:

$$\mu_i(\overline{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_i(\overline{X}) \le Z_i^B, \\ \dfrac{Z_i^W - Z_i(\overline{X})}{Z_i^W - Z_i^B}, & \text{if } Z_i^W \ge Z_i(\overline{X}) \ge Z_i^B, \\ 0, & \text{if } Z_i(\overline{X}) \ge Z_i^W \end{array} \right\rangle \quad (25)$$

i = 1,2.

The theoretical development of chance constrained QBLPP with minimization type objective functions is remain the same as developed for chance constrained QBLPP with maximization type objective functions.

IX. DISTANCE FUNCTION FOR DETERMINATION OF COMPROMISE SOLUTION

For multi objective programming, the objectives are incommensurable and conflicting in nature. The aim of decision makers is to find out the compromise solution which is as near as possible to the ideal solution points in the decision making context. Here, we use the Euclidean distance function [26] of the type

$$S_2 = \left[\sum_{i=1}^{2}(1-\mu_i^*)^2\right]^{1/2}$$

The solution with the minimum distance is considered as the best compromise optimal solution.

X. SUMMARIZATION OF THE PROCESS FOR SOLVING CHANCE CONSTRAINTS QUADRATIC BI-LEVEL PROGRAMMING PROBLEM

To solve chance constrained QBLPP we use the following steps.

S-1. Transform the chance constraints into equivalent deterministic constraints.

S-2. Calculate individual best solution for each quadratic objective function of the level DM subject to the equivalent deterministic constraints.

S-3. Lower and upper tolerance limits are determined for each quadratic objective function by minimizing and maximizing separately subject to the equivalent deterministic constraints.

S-4. Quadratic membership functions are formulated by using individual best solutions subject to the equivalent deterministic system constraints.

S-5. Find out the individual best solution for each of the quadratic membership functions subject to the equivalent deterministic constraints.

S-6. Using first order Taylor's series, the quadratic membership functions are approximated into linear functions at the individual best solution point.

S-7. Both level DMs express their choices for the upper and lower preference bounds on the decision variables controlled by them.

S-8. Two FGP models are formulated and solved.

S-9. Determine the Euclidean distance for two optimal compromise solutions obtained from two FGP Models.

S-10. Select the solution with the minimum Euclidean distance as the best compromise optimal solution.

XI. ILLUSTRATIVE EXAMPLES OF CHANCE CONSTRAINED QUADRATIC BI-LEVEL PROGRAMMING PROBLEM

11.1 Example 1.

To illustrate the proposed FGP approach, the following chance constrained QBLPP with maximization type objective function at each level is considered.

$$\max_{x_2} Z_1(\overline{X}) = x_1^2 - 5x_2^2 + 2x_1x_2 \quad (26)$$

$$\max_{x_1} Z_2(\overline{X}) = 3x_1 + 12x_2 - x_1x_2 + 45 \quad (27)$$

subject to

$$\Pr(x_1 + x_2 \le d_1) \ge 1 - m_1 \quad (28)$$

$$\Pr(-2x_1 + 5x_2 \le d_2) \ge 1 - m_2 \quad (29)$$

$$\Pr(3x_1 - 4x_2 \ge d_3) \ge 1 - m_3 \quad (30)$$

$$x_1 \ge 0,\ x_2 \ge 0 \quad (31)$$

The mean, variance and the confidence levels are prescribed as follows:

$$E(d_1) = 3,\ var(d_1) = 2,\ m_1 = 0.03 \quad (32)$$

$$E(d_2) = 12,\ var(d_2) = 8,\ m_2 = 0.01 \quad (33)$$

$$E(d_3) = 10,\ var(d_3) = 18,\ m_3 = 0.05 \quad (34)$$

Using (5) and (7), the chance constraints defined in (28), (29) and (30) can be converted into equivalent deterministic constraints as:

$$x_1 + x_2 \le 5.66579 \quad (35)$$

$$-2x_1 + 5x_2 \le 18.57609 \quad (36)$$

$$3x_1 - 4x_2 \ge 3.020856 \quad (37)$$

The individual solution for each quadratic objective function of level DM subject to the equivalent deterministic constraints is obtained as $Z_1^B = 32.10118$, at $\overline{X}_1^B = (5.66579, 0)$, and $Z_2^B = 72.64119$, at $\overline{X}_2^B = (3.669145, 1.996645)$.

The fuzzy goals appear as:

$$Z_1(\overline{X}) \underset{\sim}{\ge} 32.10118,\ Z_2(\overline{X}) \underset{\sim}{\ge} 72.64119 \quad (38)$$

The lower tolerance limits are obtained as $Z_1^W = 1.013952$ and $Z_2^W = 48.02086$.

Now, the quadratic membership function for FLDM and SLDM are constructed as follows:

$\mu_1(\bar{X}) =$

$$\left\langle \begin{array}{ll} 1, & \text{if } Z_1(\bar{X}) \geq 32.10118, \\ \dfrac{Z_1(\bar{X}) - 1.013952}{32.10118 - 1.013952}, & \text{if } 1.013952 \leq Z_1(\bar{X}) \leq 32.10118, \\ 0, & \text{if } Z_1(\bar{X}) \leq 1.013952 \end{array} \right\rangle \quad (39)$$

$\mu_2(\bar{X}) =$

$$\left\langle \begin{array}{ll} 1, & \text{if } Z_2(\bar{X}) \geq 72.64119, \\ \dfrac{Z_2(\bar{X}) - 48.02086}{72.64119 - 48.02086}, & \text{if } 48.02086 \leq Z_2(\bar{X}) \leq 72.64119, \\ 0, & \text{if } Z_2(\bar{X}) \leq 48.02086 \end{array} \right\rangle \quad (40)$$

The quadratic membership functions are linearized at their individual best solution point at $\bar{X}_1^B = (5.66579, 0)$, $\bar{X}_2^B =$ (3.669145, 1.996645) and we obtained equivalent linear membership functions as follows:

$$\mu_1^*(\bar{X}) = 1 + (x_1 - 5.66579) \times (2\times5.66579 / 31.087228) + (x_2 - 0) \times (2\times5.66579 / 31.087228), \quad (41)$$

$$\mu_2^*(\bar{X}) = 1 + (x_1 - 3.669145) \times ((3 - 1.996645) / 24.62033) + (x_2 - 1.996645) \times ((12 - 3.669145) / 24.62033) \quad (42)$$

Let $0 \leq x_2 \leq 2$ and $3 \leq x_1 \leq 6$ be the preference bounds provided by the level DMs.

Proposed two FGP models (see Table 1) in (19) and (20) offer the same solution at $x_1 = 3.669145$, $x_2 = 1.996645$, with $Z_1 = 8.181629$ and $Z_2 = 72.64119$.

TABLE1. COMPARISON OF DISTANCES FOR THE OPTIMAL SOLUTIONS OBTAINED FROM TWO FGP MODELS OF THE PROBLEM 11.1

MODEL NUMBER	MEMBERSHIP FUNCTION	DISTANCE FUNCTION
Model I, Model II	$\mu_1^* = 0.2305666$ $\mu_2^* = 1$	0.7694334

It is clear from the table that two FGP Models offer the same result.

1.2 Example 2.

To illustrate the proposed FGP approach, the following chance constrained QBLPP with minimization type objective function at each level is considered.

$$\min_{x_1} Z_1(\bar{X}) = 7x_1^2 + 6x_2^2 + 8x_1 + 11x_2 \quad (43)$$

$$\min_{x_2} Z_2(\bar{X}) = (x_1 - 1)^2 + (x_2 + 3)^2 + 7x_1x_2 + 21 \quad (44)$$

subject

$$\Pr(2x_1 + 5x_2 \geq d_1) \geq 1 - m_1 \quad (45)$$

$$\Pr(3x_1 + 6x_2 \leq d_2) \geq 1 - m_2 \quad (46)$$

The means, variances and the confidence levels are prescribed as follows:

$$E(d_1) = 6,\ var(d_1) = 9,\ m_1 = 0.06 \quad (47)$$

$$E(d_2) = 4,\ var(d_2) = 4,\ m_2 = 0.04 \quad (48)$$

Using (5), (7) the chance constraints defined in (45), (46) can be converted into equivalent deterministic constraints as:

$$2x_1 + 5x_2 \geq 1.335 \quad (49)$$

$$3x_1 + 6x_2 \leq 7.51 \quad (50)$$

The individual solution for each quadratic objective function subject to equivalent deterministic system constraints is obtained as: $Z_1^B = 3.364734$, at $\bar{X}_1^B = (0, 0.267)$, $Z_2^B = 30$, at $\bar{X}_2^B = (1, 0)$ (51)

The upper tolerance limits are obtained as $Z_1^W = 63.89341$ and $Z_2^W = 40.77668$.

The fuzzy goals assume the form:

$$Z_1(\bar{X}) \lesssim 3.364734,\ Z_2(\bar{X}) \lesssim 30$$

Now, the quadratic membership functions for FLDM and SLDM are formulated as follows:

$\mu_1(\bar{X}) =$

$$\left\langle \begin{array}{ll} 1, & \text{if } Z_1(\bar{X}) \leq 3.364734, \\ \dfrac{63.89341 - Z_1(\bar{X})}{63.89341 - 3.364734} & \text{if } 63.89341 \geq Z_1(\bar{X}) \geq 3.364734, \\ 0 & \text{if } Z_1(\bar{X}) \geq 63.89341 \end{array} \right\rangle \quad (52)$$

$$\mu_2(\bar{X}) = \left\langle \begin{array}{ll} 1, & \text{if } Z_2(\bar{X}) \leq 30, \\ \dfrac{40.77668 - Z_2(\bar{X})}{40.77668 - 30} & \text{if } 40.77668 \geq Z_2(\bar{X}) \geq 30, \\ 0 & \text{if } Z_2(\bar{X}) \geq 40.77668 \end{array} \right\rangle \quad (53)$$

The quadratic membership functions are linearized at their individual best solution point at $\bar{X}_1^B = (0, 0.267)$, $\bar{X}_2^B =$ (1, 0) by using Taylor's series approximation method as follows:

$$\mu_1^*(\bar{X}) = 1 + (x_1 - 0) \times (-8 / 60.5287) + (x_2 - 0.267) \times (-12\times0.267 + 11) / 60.5287 \quad (54)$$

$$\mu_2^*(\bar{X}) = 1 + (x_2 - 0) \times (-(13 / 10.7767)) \quad (55)$$

Let $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 0.5$ be the preference bounds provided by the level DMs.

FGP model I (see Table 2) offers the optimal solution at $x_1 = 0.6184107$, $x_2 = 0.0196357$ with $Z_1 = 7.842614$ and $Z_2 = 30.34881$.

FGP Model II (See Table 2) offers the optimal solution at $x_1 = 0.6675$, $x_2 = 0$, with $Z_1 = 8.458894$ and $Z_2 = 30.11056$.

TABLE2. COMPARISON OF DISTANCES FOR THE OPTIMAL SOLUTIONS OBTAINED FROM TWO FGP MODELS OF THE PROBLEM 11.2.

MODEL NUMBER	MEMBERSHIP FUNCTION	EUCLIDEAN DISTANCE FUNCTION
Model I	$\mu_1^* = 0.92602$, $\mu_2^* = 0.96763$	0.08075066
Model II	$\mu_1^* = 0.91584$, $\mu_2^* = 0.98974$	0.08478454

It is clear from the Table 2 that the Euclidean distance is minimal for the FGP Model I which implies that Model I provides better compromise solution for this example.

Note 1. Lingo ver.11.0 is used for solution purpose.

XII. CONCLUSION

In this paper, we present chance constrained QBLPP by using FGP approach which is simple to understand and easy to apply. After transforming the chance constraints into equivalent deterministic constraints, we transform quadratic bi-level programming problem into linear bi-level programming problem by using the first order Taylor's series approximation. To avoid decision deadlock, each level DM provides preference bounds on the decision variables controlled by him. Two FGP models are proposed for solution purpose. The proposed approach can be used to deal with chance constrained multi-level quadratic programming problem. The proposed approach can further be used for solving chance constrained linear quadratic fractional programming problem.

For the future study in the hierarchical decision making context, we hope, the proposed approach can be used for chance constrained quadratic decentralized multi-level multi-objective programming problems. We hope further that the proposed approach will be useful for the real decision making problems that arise in industrial belt, supply chain, marketing, IT sector, management sciences.

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