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Stochastic Model for Expected Time to Green House Effect

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ABSTRACT: *One of the important aspects in the study of global warming is relating to increase of temperature. The factors like CO2, CO and Nitrogen etc., plays a vital role to hasten the process of increase in global temperature. The only source of global warming is CO2 emission. The interarrival times between two successive CO2 emissions in this identified is a potential cause.[3] have obtained the expected time to Seroconversion and its varience when the interarrival times are identically independent random variables and also the case where they are correlated. In this paper the expected time to Green House effect is derived assuming the interarrival times are not independent.*

Key words: *Greenhouse effect, Global warming, threshold. The AMS classification number is 92C60.*

I. INTRODUCTION

Stochastic models are widely used in the study of global warming and its consequences.There are many aspects which are taken for study the time to green house effect depending upon the increase of global temperature.If the global temperature crosses the threshold level which in turn leads to greenhouse effect.In developing such stochastic models the authors have used the concept of shock models and cumulative damage process discussed by[1]. [3] have obtained the expected time to Seroconversion under the assumption that the interarrival time between contacts are not independent but constantly correlated. In this paper it is assumed that the interarrival times which form a sample of observations that form order statistics and so they are not independent.

II. ASSUMPTION OF THE MODEL

1. By burning of fossil and other fuels, a random amount of CO2 emmission occurs.
2. CO2 emission is the only source of global warming.
3. CO2 emission is a damage process which is linear and cumulative
4. Increase of global temperature is caused by CO2 emission are assumed to be identically independent random variable
5. If the global temperature exeeds threshold level Y which is itself is a random variable, then greenhouse effect takes place.
6. The process which generate the CO2 emission, the sequence of increase in global temperature and threshold are mutually independent.
7. From the large number of CO2 emissions between successive events, a random sample of K observations are taken

III. NOTATIONS

X_i - A random variable representing the increase of the global temperature due to CO2 emission in the i^{th} event. X_i's are i.i.d with p.d.f g(.) and distribution function G(.).
Y - A random variable representing the global warming threshold with p.d.f h(.) and distribution function H(.).
U_i - A random variable representing the interarrival times between successive events, i = 1, 2, ... k, with p.d.f f(.).
$g_k(.)$ - The p.d.f. of the random variable $\sum x_i$, i= 1, 2, . . .k.
$F_k(.)$ - The kth convolution of F(.)
T - The continuous random variable denoting the time to Green house effect.
$U_{(i)}$ - Smallest order statistic with p.d.f $f_{u(1)}(t)$.
$U_{(k)}$ - The largest order statistic with p.d.f $f_{u(k)}(t)$
f*(s) - Laplace transform of f(.)
$f_{u(1)}*(s), f*_{u(k)}(s)$- Laplace transform of $f_{u(1)}(.)$ and $f_{u(k)}(.)$ respectively.

IV. RESULTS

If X_i i = 1, 2, . . . k are the contributions to the global warming in k events during the period (0, t), the time to cross the global warming threshold level can be obtained as follows.

S(t) - Survivor function = P(T>t)

$$P(T>t) - \sum_{K=o}^{\infty} \Pr[\text{there are exactly } k \text{ events on } (0, t)] * \Pr[\text{the cumulative total of global warming} < Y]$$

It can be shown that

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^{\infty} g_k(x)\, \overline{H(x)}\, dx$$

Assuming that y ~ exp (θ), we have

$$P\left[\sum_{i=1}^{k} X_i < Y\right] = \int_0^\infty g_k(x)\, e^{-\theta x} dx = g_k^*(\theta) = [g^*(\theta)]^k \qquad \text{--------(4.1)}$$

since x_i are all i.i.d,

Also P_r [exactly k shocks in(0,t)] $= F_k(t) - F_{k+1}(t)$
[by renewal theory]

$$\text{Hence } P(T>t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\, [g^*(\theta)]^k$$

$$= 1 - [1 - g^*(\theta)] \sum_{k=1}^{\infty} F_k(t)\, (g^*(\theta))^{k-1} \quad \text{on simplification}$$

$$\text{Hence } L(t) = P[T<t] = 1 - S(t)$$

$$= [1 - g^*(\theta)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(\theta)]^{k-1} \qquad \text{------ (4.2)}$$

Now taking the Laplace transform of L(t), it can shown that

$$L^*(s) = \frac{[1 - g^*(\theta)]\, f^*(s)}{} \qquad \text{--------(4.3)} \qquad 1 - g^*(\theta)\, f^*(s)$$

on simplification

The interarrival times $U_1, U_2, \ldots U_k$ are i.i.d random variables and $U_{(1)} < U_{(2)} < \ldots < U_{(k)}$ form k order statistics which are also random variables which are not independent.

Now the p.d.f of $U_{(k)}$ is

$$F_{U_{(k)}}(t) = k[F(t)]^{k-1} f(t) \qquad \text{--------(4.5)}$$

Assuming that $f(t) \sim \exp(\lambda)$, it can be shown that

$$F^*_{u(k)}(s) = \frac{k!\, \lambda^k}{(\lambda+S)(2\lambda+S) \ldots\ldots (k\lambda+S)} \qquad \text{------ (4.6)}$$

Substituting (4.6) in (4.3) and assuming $g(.) \sim \exp(\theta)$ it can be shown that

$$\square^*(s) = \frac{\theta\, k!\, \lambda^k}{(c+\theta)(\lambda+s)(2\lambda+s) \ldots (k\lambda+s) - ck!\lambda^k} \qquad \text{------(4.7)}$$

on simplification.

Now the expected time to greenhouse effect is given by $E(T) = \dfrac{-d\square^*(s)}{ds}$ at s=0.

$$= \frac{c+\theta}{c\lambda} \left(\sum_{n=1}^{k} 1/n \right) \qquad \text{-------(4.8)}$$

It implies that E(T) becomes larger as k increases since $\sum 1/n$ increases with K. Hence it may be concluded that the maximum value $U_{(k)}$ increases with increase in k and also the interarrival time $U_{(k)}$ becomes longer. This results in the larger value of E(T).

It may be shown that

$$E(T^2) = \frac{d^2 \square^*(s)}{ds^2}\Big|_{s=0}$$

$$= \frac{2(c+\theta)^2}{\theta^2 \lambda^2} \left(\sum_{n=1}^{k} 1/n \right)^2 - \frac{c+\theta}{\lambda^2\theta} \left((\sum 1/n)^2 + \frac{c+\theta}{\lambda^2\theta} \sum_{n=1}^{k} 1/n^2 \right) \qquad \text{-----(4.9)}$$

We have

$$V(T) = \text{Variance of } T$$

$$= E(T^2) - [E(T)]^2$$

$$= \frac{c(c+\theta)}{\theta^2 \lambda^2} \sum_{n=1}^{k} 1/n + \left(\frac{c+\theta}{\lambda^2\theta} \right)^2 \sum_{n=1}^{k} 1/n^2 > 0 \qquad \text{------(4.10)}$$

This implies that for fixed C,□λ, θ the variance of T increases as k increases.

Now considering the first order statistics $U_{(1)}$ it can be shown that

$$f^*_{U(1)}(s) = \frac{k\lambda}{\lambda k+s} \quad \text{-----(4.11)}$$

Substituting (4.11) in (4.3) for $f^*(s)$ it is seen that

$$\square^*(s) = \frac{c\,\lambda\,k}{((c+\theta)(\lambda k+s)-(\theta\lambda k))} \quad \text{-----(4.12)}$$

So that the mean and variance of T are obtained as

$$E(T) = \frac{-d\, l^*(s)}{ds}\bigg|_{s=0}$$

$$= \frac{C+\theta}{C\lambda k} \quad \text{-----(4.13)}$$

$$V(T) = \text{Variance of } T$$

$$= E(T^2) - [E(T)]^2$$

$$= \frac{2(C+\theta)^2}{C^2\lambda^2k^2} - \frac{(C+\theta)^2}{C^2\lambda^2k^2}$$

$$= \left(\frac{C+\theta}{C\lambda k}\right)^2 > 0 \quad \text{-----(4.14)}$$

V. NUMERICAL ILLUSTRATION

For the case of $U_{(k)}(t)$

C = 1.5 θ = 0.5 λ = 1.0 are fixed

k	E(T)	V(T)
1	4.0	16
2	6.0	32
3	7.32	45.4
4	8.32	57.59
5	9.12	68.21
6	9.76	77.348
7	10.32	85.92
8	10.8	93.56
9	11.24	100.87
10	11.64	107.77

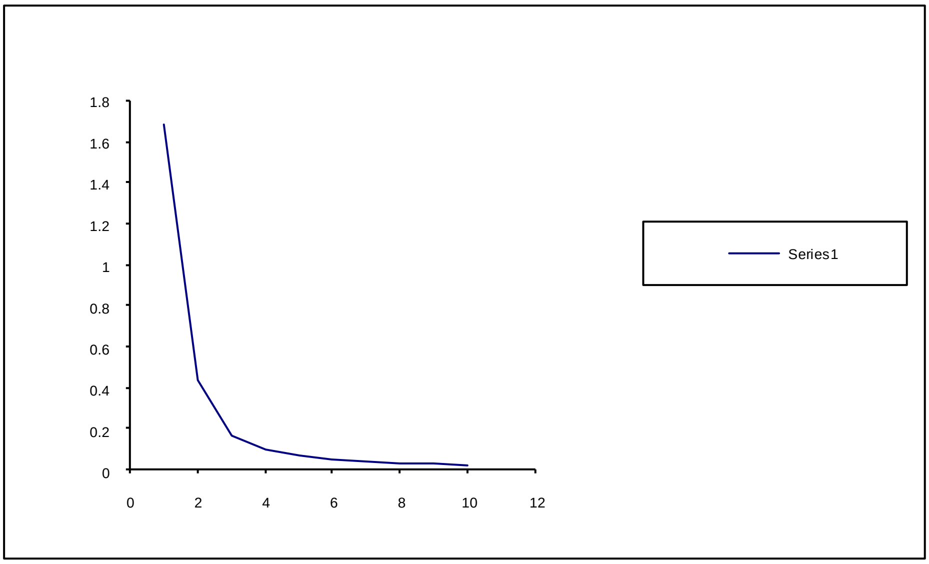
For the case of $U_{(1)}$ (t)

C= 1.5 θ = 0.5 λ = 1.0 are fixed

k	E(T)	V(T)
1	1.3	1.69
2	0.6	0.435
3	0.4	0.16
4	0.3	0.09
5	0.26	0.067
6	0.22	0.048
7	0.19	0.036

C = 1.5 θ = 0.5 λ = 1.0

For the case of $U_{(1)}$ (t) Variance



VI. CONCLUSION

It is very interesting to observe the following from the study of the numerical rate and the respective graphs.

1. The values of E(T) and V(T) both increase with an increase in 'k', namely the number of events. If k becomes larger than the corresponding $U_{(k)}$ also becomes larger thereby implying that it is the largest of the interarrival times. In such a case the inter global warming times are elongated thereby having a delayed time to greenhouse effect. Hence the curves for E(T) and V(T) go upwards in both the cases.
2. The values of E(T) and V(T) both decreases with an increase in 'k' for the case of $U_{(1)}$(t). If the inter global warming time is the smallest random variable, than it implies that if number of events are more, the greenhouse effect will be much earlier and the process gets speeded up resulting in a decline in both of E(T) and V(T) as indicated by the graphs.

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