

Unsteady Mixed Convective Heat and Mass Transfer flow through a porous medium in a vertical channel with Soret and Dissipation effects

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Abstract: Unsteady Hydromagnetic Mixed Convection flow of a viscous, electrically conducting fluid through a porous medium confined in a vertical channel bounded by flat walls. The unsteadiness in the flow is due to the travelling thermal wave is imposed on the bounding walls. The concentration on the walls is maintained constant. A uniform magnetic field of strength H_0 is applied transverse to the boundaries. The coupled equations governing the flow, heat and mass transfer are solved by using the perturbation technique with δ , the aspect ratio as a perturbation parameter. The combined influence of the Soret and dissipation effects on the velocity, temperature, concentration, stress and rate of heat and mass transfer are discussed in detail.

Keywords: CFD, Mixed Convection, Heat Transfer, Mass Transfer, Dissipation

I. Introduction

The time dependent thermal convection flows have applications in chemical engineering, space technology, etc. These flows can be achieved by either time dependent movement of the boundary or unsteady temperature of the boundary. The unsteady temperature may be attributed to the free stream oscillations or oscillatory flux or temperature oscillations. The oscillatory convection problems are important from the technological point of view as the effect of surface temperature oscillations on skin friction and heat transfer from surface to the surrounding fluid has special interest in heat transfer engineering.

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. Obviously, the understanding of this transport process is desirable in order to effectively control the overall transport characteristics. The problem of combined buoyancy driven thermal and mass diffusion has been studied in parallel plate geometries by a few authors, notably, Lai[1], Chen et al.,[2], Mehta and Nandakumar[3] and Angirasa et al.,[4].

Adrian Postelnicu [5], Emmanuel Osalusi et al.,[6], Mohammed Abd-El-Aziz[7] have studied thermo-diffusion and diffusion thermo effects on combined heat and mass transfer through a porous medium under different conditions.

Theoretical study of free convection in a horizontal porous annulus, including possible three dimensional and transient effects. Similar studies for fluid filled annuli are available in the literature [8]. In view of this, several authors, notably Tunc et al [9],Oliveira et al.,[10]. Martin Ostoja [11], El – Hakein [12], and Bulent Yesilata [13] have studied the effect of viscous dissipation on convective flows past an infinite vertical plates and through vertical channels and ducts.

II. The Problem formulation

We consider the motion of viscous, incompressible, electrically conducting fluid through a porous medium in a vertical channel bounded by flat walls. The thermal buoyancy in the flow field is created by a traveling thermal wave imposed on the boundary wall at $y = L$ while the boundary at $y = -L$ is maintained at constant temperature T_1 . The walls are maintained at constant concentrations. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous and Darcy dissipations are taken into account to the transport of heat by conduction and convection in the energy equation. We take Soret effect into account in the diffusion equation. Also the kinematic viscosity ν , the thermal conductivity k are treated as constants. We choose a rectangular Cartesian system $O(x,y)$ with x -axis in the vertical direction and y -axis normal to the walls. The walls of the channel are at $y = \pm L$. The equations governing the unsteady flow and heat transfer are

Equation of linear momentum

$$\rho_e \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \left(\frac{\partial p}{\partial x} \right) + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - \left(\frac{\mu}{k} \right) u - \left(\sigma \mu_e^2 H_0^2 \right) u \quad (2.1)$$

$$\rho_e \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \left(\frac{\partial p}{\partial y} \right) + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left(\frac{\mu}{k} \right) v \quad (2.2)$$

Equation of continuity

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{2.3}$$

Equation of energy

$$\rho_e C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q + \mu \left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2\right] + \left(\frac{\mu}{\lambda k} + \sigma \mu_e^2 H_0^2\right)(u^2 + v^2) \tag{2.4}$$

Equation of

Diffusion

$$\left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y}\right) = D_1 \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + k_{11} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.5}$$

Equation of state

$$\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \tag{2.6}$$

where ρ_e is the density of the fluid in the equilibrium state, T_e, C_e are the temperature and concentration in the equilibrium state, (u, v) are the velocity components along $O(x, y)$ directions, p is the pressure, T, C are the temperature and concentration in the flow region, ρ is the density of the fluid, μ is the constant coefficient of viscosity, C_p is the specific heat at constant pressure, λ is the coefficient of thermal conductivity, k is the permeability of the porous medium, D_1 is the molecular diffusivity, k_{11} is the cross diffusivity, β is the coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with mass fraction and Q is the strength of the constant internal heat source.

In the equilibrium state

$$-\left(\frac{\partial p_e}{\partial x}\right) - \rho_e g = 0 \tag{2.7}$$

where $p = p_e + p_D$, p_D being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q = \frac{1}{2L} \int_{-L}^L u \, dy \tag{2.8}$$

The boundary conditions for the velocity and temperature fields are

$$\begin{aligned} u = 0, v = 0, T = T_1, C = C_1 & \quad \text{on } y = -L \\ u = 0, v = 0, T = T_2 + \Delta T_e \sin(mx + nt), C = C_2 & \quad \text{on } y = L \end{aligned} \tag{2.9}$$

where $\Delta T_e = T_2 - T_1$ and $\sin(mx + nt)$ is the imposed traveling thermal wave.

In view of the continuity equation we define the stream function ψ as

$$u = -\psi_y, v = \psi_x \tag{2.10}$$

Eliminating pressure p from equations (2.1) & (2.2), the equations governing the flow in terms of ψ are

$$\begin{aligned} \left((\nabla^2 \psi)_t + \psi_x (\nabla^2 \psi)_y - \psi_y (\nabla^2 \psi)_x\right) = \nu \nabla^4 \psi - \beta g (T - T_0)_y \\ - \beta^* g (C - C_0)_y - \left(\frac{\nu}{k}\right) \nabla^2 \psi - \left(\frac{\sigma \mu_e^2 H_0^2}{\rho_0}\right) \left(\frac{\partial^2 \psi}{\partial y^2}\right) \end{aligned} \tag{2.11}$$

$$\begin{aligned} \rho_e C_p \left(\frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y}\right) = \lambda \nabla^2 \theta + Q + \mu \left[\left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 + \left(\frac{\partial^2 \psi}{\partial x^2}\right)^2\right] \\ + \left(\frac{\mu}{k} + \sigma \mu_e^2 H_0^2\right) \left[\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right] \end{aligned} \tag{2.12}$$

$$\left(\frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D_1 \nabla^2 C + \left(\frac{ScS_0}{N} \right) \nabla^2 \theta \quad (2.13)$$

Introducing the non-dimensional variables in (2.11) - (2.13) as

$$x' = mx, \quad y' = \frac{y}{L}, \quad t' = t v m^2, \quad \Psi' = \frac{\Psi}{v}, \quad \theta = \left(\frac{T - T_e}{\Delta T_e} \right), \quad C' = \left(\frac{C - C_1}{C_2 - C_1} \right) \quad (2.14)$$

(under the equilibrium state $\Delta T_e = T_e(L) - T_e(-L) = \frac{QL^2}{\lambda}$)

the governing equations in the non-dimensional form (after dropping the dashes) are

$$\delta R \left(\delta (\nabla_1^2 \psi)_t + \frac{\partial(\psi, \nabla_1^2 \psi)}{\partial(x, y)} \right) = \nabla_1^4 \psi + \left(\frac{G}{R} \right) (\theta_y + NC_y) - D^{-1} (\nabla_1^2 \psi) - M^2 \left(\frac{\partial^2 \psi}{\partial y^2} \right) \quad (2.15)$$

The energy equation in the non-dimensional form is

$$\begin{aligned} \delta P \left(\delta \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) &= \nabla_1^2 \theta + \alpha + \left(\frac{PR^2 E_c}{G} \right) \left(\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 + \delta^2 \left(\frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) \\ &+ (D^{-1} + M^2) \left(\delta^2 \left(\frac{\partial \psi}{\partial x} \right)^2 + \left(\frac{\partial \psi}{\partial y} \right)^2 \right) \end{aligned} \quad (2.16)$$

The Diffusion equation is

$$\delta Sc \left(\delta \frac{\partial C}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \nabla_1^2 C + \left(\frac{ScS_0}{N} \right) \nabla_1^2 \theta \quad (2.17)$$

where

$$R = \frac{UL}{v} \quad (\text{Reynolds number})$$

$$G = \frac{\beta g \Delta T_e L^3}{v^2} \quad (\text{Grashof number})$$

$$P = \frac{\mu c_p}{k_1} \quad (\text{Prandtl number}),$$

$$D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}),$$

$$E_c = \frac{\beta g L^3}{C_p} \quad (\text{Eckert number})$$

$$\delta = mL \quad (\text{Aspect ratio})$$

$$\gamma = \frac{n}{v m^2} \quad (\text{Non-dimensional thermal wave velocity})$$

$$Sc = \frac{v}{D_1} \quad (\text{Schmidt Number})$$

$$N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad (\text{Buoyancy ratio})$$

$$So = \frac{\beta^* k_{11}}{v \beta} \quad (\text{Soret Parameter})$$

$$M^2 = \frac{\sigma \mu_e^2 H_o^2 L^2}{v^2} \quad (\text{Hartman number})$$

$$\nabla_1^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = 1 \tag{2.18}$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \tag{2.19}$$

$$\theta(x, y) = 1, \quad C(x, y) = 0 \quad \text{on } y = -1 \tag{2.20}$$

$$\theta(x, y) = \sin(x + \gamma t), \quad C(x, y) = 1 \quad \text{on } y = 1 \tag{2.21}$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \tag{2.22}$$

The value of ψ on the boundary assumes the constant volumetric flow consistent with the hypothesis(2.8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function t .

III. Shear Stress, Nusselt Number And Sherwood Number

The Shear Stress on the channel walls is given by

$$\tau = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=\pm L} \tag{3.1}$$

Which in the non- dimensional form reduces to

$$\tau = \left(\frac{\tau}{\mu U} \right) = (\psi_{yy} - \delta^2 \psi_{xx}) \tag{3.2}$$

$$= [\psi_{00,yy} + Ec\psi_{01,yy} + \delta(\psi_{10,yy} + Ec\psi_{11,yy} + O(\delta^2))]_{y=\pm 1}$$

And the corresponding expressions are

$$(\tau)_{y=+1} = b_{90} + \delta b_{91} + O(\delta^2) \tag{3.3}$$

$$(\tau)_{y=-1} = b_{92} + \delta b_{93} + O(\delta^2) \tag{3.4}$$

The local rate of heat transfer coefficient (Nusselt number Nu) on the walls has been calculated using the formula

$$Nu = \frac{1}{\theta_m - \theta_w} \left(\frac{\partial \theta}{\partial y} \right)_{y=\pm 1} \tag{3.5}$$

and the corresponding expressions are

$$(Nu)_{y=+1} = \frac{(b_{51} + \delta b_{52})}{(b_{44} - \sin(D_1) + \delta b_{45})} \tag{3.6}$$

$$(Nu)_{y=-1} = \frac{(b_{53} + \delta b_{54})}{(b_{44} - 1 + \delta b_{45})} \tag{3.7}$$

The local rate of mass transfer coefficient (Sherwood number Sh) on the walls has been calculated using the formula

$$Sh = \frac{1}{C_m - C_w} \left(\frac{\partial C}{\partial y} \right)_{y=\pm 1} \tag{3.8}$$

and the corresponding expressions are

$$(Sh)_{y=+1} = \frac{(b_{65} + \delta b_{63})}{(b_{58} - 1 + \delta b_{57})} \tag{3.9} \quad (Sh)_{y=-1} = \frac{(-b_{65} + \delta b_{63})}{(b_{58} + \delta b_{57})}$$

(3.10) where b_4, \dots, b_{90} are constants

IV. Discussion of the Numerical results

The aim of the analysis is to discuss the flow, heat and mass transfer of a viscous electrically conducting fluid through a porous medium in a vertical channel bounded by flat walls on which a travelling thermal wave is imposed. In this analysis, the viscous Darcy dissipation, Joule heating and Soret effect are taken into account. For computational purpose, we

take $P = 0.71$ and $\delta = 0.01$. It is observed that the temperature variation on the boundary, dissipative and Soret effects contribute substantially to the flow field. This contribution may be represented as perturbations over the mixed convective flow generated. These perturbations not only depend on the wall temperature, M , Ec and So but also on the nature of the mixed convective flow. In general, we note that the creation of the reversal flow in the flow field depends on whether the free convection effects dominates over the forced flow or vice versa. If the free convection effects are sufficiently large as to create reversal flow, the variation in the wall temperature, M , Ec and So affects the flow remarkably.

The variation of u with Soret parameter So shows that the reversal flow which appears in the vicinity of the left boundary disappears for higher $So > 0$ and $So < 0$. Also, $|u|$ depreciates with increase in $So > 0$ and an increase in $|So| < 0$, enhances $|u|$ in the left region and depreciates in the right region (Fig.1)

Fig.2 shows the an increase in $|So| > 0$ depreciates v in the entire flow region while in $|So| < 0$ enhances v in the left region and depreciates in the right region

An increase in $So > 0$ depreciates Rt in the flow region and an increase in $|So| < 0$ enhances Rt in the left region and reduces it in the right region (Fig. 3).

The non-dimensional temperature θ is shown in Fig.4 An increase in Sc or $So > 0$ enhances θ , while an increase in $|So| < 0$ depreciates the actual temperature .

The behaviour of C with Soret parameter So shows that an increase in $So > 0$ enhances the actual concentration and depreciates with $|So| < 0$ (Fig.5).

The shear stress on the boundary walls have been evaluated numerically for different G , Sc , and So , are shown in (Tables 1- 6) . Lesser the molecular diffusivity, lesser τ at $y = 1$ and larger τ at $y = -1$. An increase in $So > 0$ enhances τ in the heating case and depreciates it in the cooling case at $y = 1$ while enhances τ in both the heating and cooling cases with increase in $|So| (< 0)$. At $y = -1$, the stress enhances with $So > 0$ and depreciates with $|So| (< 0)$ for all $G (>, < 0)$ (Tables.1 and 2)

The average Nusselt number Nu which measures the rate of heat transfer has been exhibited in Tables. 3 and 4. The variation of Nu with the Soret parameter So reveals that $|Nu|$ at $y = 1$ enhances with increase in $|So| (> 0)$ and depreciates with $|So| (< 0)$ while at $y = -1$, it enhances with increase in $|So| (> < 0)$.

The Sherwood number Sh which measures the rate of mass transfer is shown in Tables.5 and 6 for different parametric values. The variation of Sh with Sc shows that lesser the molecular diffusivity, higher $|Sh|$ at $y = 1$ and lesser $|Sh|$ at $y = -1$ and lesser $|Sh|$ at $y = -1$. An increase in $|So| (> 0)$ depreciates $|Sh|$ at both the walls while an increase in $|So| (< 0)$ increases for $|G| = 10^3$ and depreciates for $|G| \geq 3 \times 10^3$ (Tables.5 and 6).

V. References

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Figures-Captions

Fig.1 u with $S_0, Sc=1.3, N=1, M=2$

	I	II	III	IV
S_0	0.5	1.0	-0.5	-1.0

Fig.2 v with $S_0, Sc=1.3, N=1, M=2$

	I	II	III	IV
S_0	0.5	1.0	-0.5	-1.0

Fig.3 Rt with $S_0, Sc=1.3, N=1, M=2$

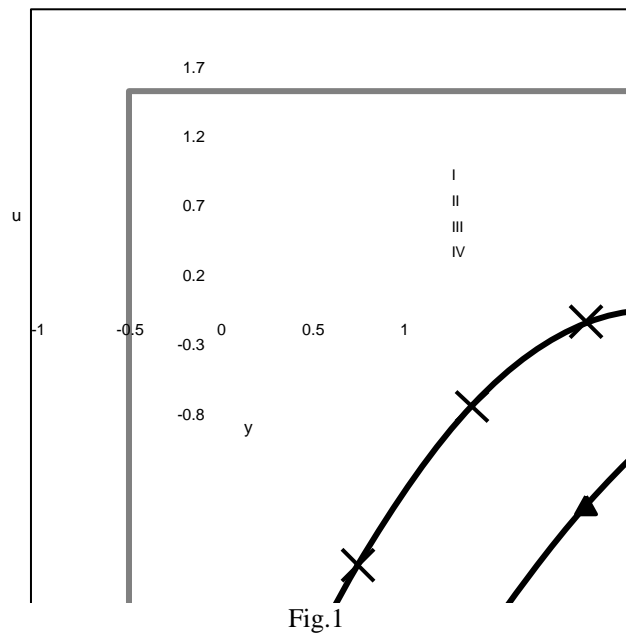
	I	II	III	IV
S_0	0.5	1.0	-0.5	-1.0

Fig 4 θ with Sc & $SoG=2 \times 10^3 m, D^{-1}=2 \times 10^3, M=2, N=1$

	I	II	III	IV	V	VI	VII
Sc	1.3	2.01	0.24	0.6	1.3	1.3	1.3
So	0.5	0.5	0.5	0.5	1.0	-0.5	-1

Fig.5 C with So

	I	II	III	IV
So	0.5	1.0	-0.5	-1.0



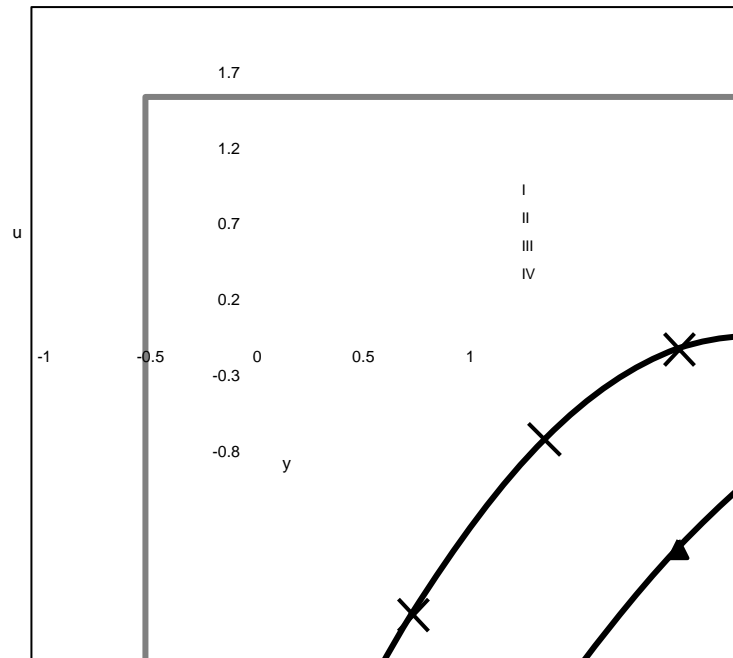


Fig.2

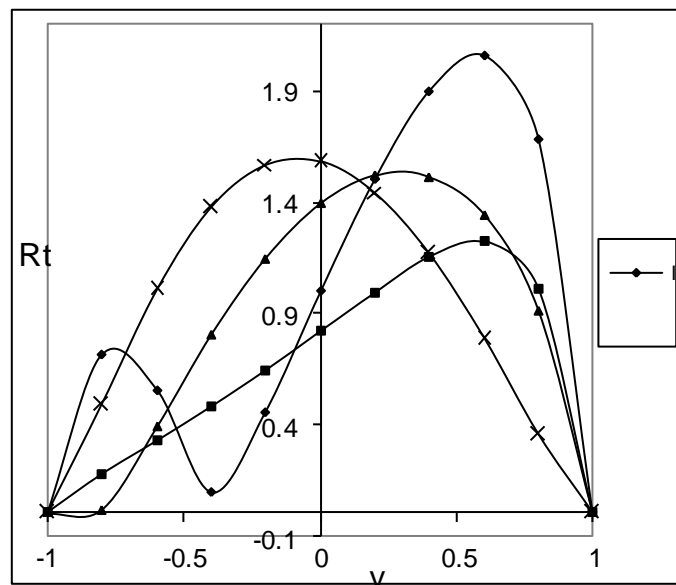


Fig.3

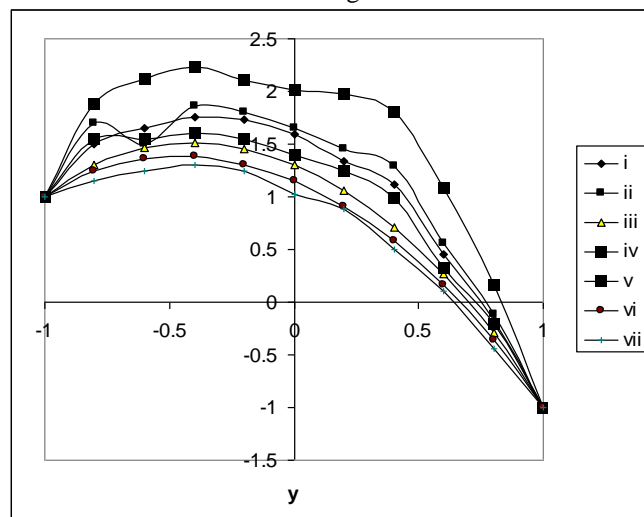


Fig.4

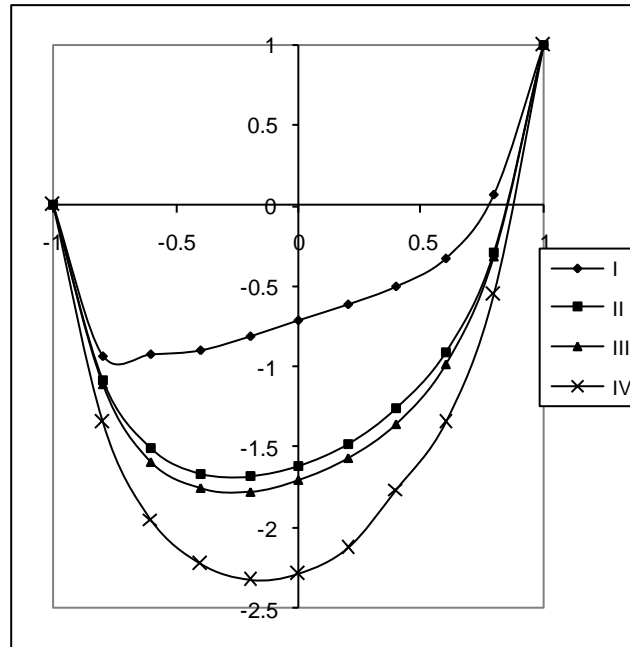


Fig.5

Table.1 Shear Stress (τ) at $y=1, P=0.71, x + \gamma t = \frac{\pi}{4}, D^{-1}=10^3, N=1, M=2$

G/ τ	I	II	III	IV	V	VI	VII
10^3	-9.6761	15.5287	-42.709	-27.539	-14.205	-151.32	-299.69
3×10^3	-10.132	11.7261	-265.16	-143.74	-104.85	-167.11	-312.17
-10^3	-11.038	19.0067	-49.949	-32.934	-6.8686	-162.09	-207.23
-3×10^3	-119.93	23.7171	-232.19	-362.41	-7.7614	-189.56	-286.78

Table.2 Shear Stress (τ) at $y=-1, P=0.71, x + \gamma t = \frac{\pi}{4}, D^{-1}=10^3, N=1, M=2$

G/ τ	I	II	III	IV	V	VI	VII
10^3	-1.0491	40.1131	0.3942	0.5227	-5.4294	-1.4265	-4.6217
3×10^3	-3.1121	34.1477	-4.5043	-3.2796	-9.6707	-18.304	-41.315
-10^3	5.2573	46.9346	6.7127	6.8696	11.5307	8.5845	1.6457
-3×10^3	9.3025	54.7654	-2.3945	2.2412	15.9049	-29.015	-59.501
	I	II	III	IV	V	VI	VII
Sc	1.3	2.01	0.24	0.6	1.3	1.3	1.3
S ₀	0.5	0.5	0.5	0.5	1.0	-0.5	-1.0

Table.3 Average Nusselt Number (Nu) at $y=1, P=0.71, x + \gamma t = \frac{\pi}{4}, N=1, M=2$

G/Nu	I	II	III	IV	V	VI	VII
10^3	-2.1681	-3.1421	-1.5071	-1.6211	-2.3371	-1.3321	-1.2846
3×10^3	-1.4014	-3.1502	-1.2211	-1.2351	-1.4641	-1.1055	-0.9654
-10^3	-2.1906	-3.1509	-1.5061	-1.6278	-2.3111	-1.3327	-1.2836
-3×10^3	-1.4036	-3.1479	0.12197	-1.2509	-1.4602	-1.1025	-0.9608

Table.4 Average Nusselt Number (Nu) at $y=-1, P=0.71, x + \gamma t = \frac{\pi}{4}, N=1, M=2$

G/Nu	I	II	III	IV	V	VI	VII
10^3	3.1504	2.9837	3.5057	3.4061	3.7321	3.1669	3.8022
3×10^3	3.5277	2.9858	3.7679	3.7241	3.9155	3.4642	4.1889
-10^3	3.1438	2.9839	3.5012	3.4006	3.7227	3.1131	3.8004
-3×10^3	3.5243	2.9863	3.7659	3.7224	3.9025	2.4685	4.1848
	I	II	III	IV	V	VI	VII
Sc	1.3	2.01	0.24	0.6	1.3	1.3	1.3
S ₀	0.5	0.5	0.5	0.5	1.0	-0.5	-1.0

Table.5 Sherwood Number(Sh) at $y=1$ $P=0.71$, $x + \gamma t = \frac{\pi}{4}$, $N=1, M=2$

G/Sh	I	II	III	IV	V	VI	VII
10^3	1.0619	1.5431	0.6162	0.5469	1.2133	2.2015	3.7554
3×10^3	0.0853	1.5356	0.5545	0.2526	-0.0693	1.8437	2.5289
-10^3	1.0805	1.5428	0.6179	0.5538	1.2019	2.1436	3.4643
-3×10^3	0.0854	1.5347	0.5548	0.2477	-0.0756	1.7516	2.2699

Table.6 Sherwood Number(Sh) at $y = -1$ $P=0.71$, $x + \gamma t = \frac{\pi}{4}$, $N=1, M=2$

G/Sh	I	II	III	IV	V	VI	VII
10^3	-1.3532	-1.5193	-6.7056	-1.9113	-1.3316	12.2447	24.1939
3×10^3	-0.4354	-1.5122	29.3535	9.8665	0.1615	-9.7093	-5.9196
-10^3	-1.3622	-1.5191	-6.7909	-1.9218	-1.3198	4.5883	67.6452
-3×10^3	-0.4562	-1.5119	19.4073	5.5267	0.1745	-7.0936	-4.5248
	I	II	III	IV	V	VI	VII
Sc	1.3	2.01	0.24	0.6	1.3	1.3	1.3
So	0.5	0.5	0.5	0.5	1.0	-0.5	-1.0