Magneto hydro dynamic Non-Similar solutions of a Visco-elastic fluid with Slip flow and Heat Transfer over a Non–linearly Stretching Porous Sheet

K. Ashok Kumar, P. H. Veena, V. K. Pravin,

Dept. of Mathematics, Rural Engineering College, Bhalki , Karnataka, India
Dept. of Mathematics, Smt. V.G.College for Women, Gulbarga, Karnataka, India
Dept. of Mech. Engg., P.D.A. College of Engg., Gulbarga, Karnataka, India

Abstract: *The study of boundary layer flow and heat transfer of an electrically conducting visco-elastic second grade fluid in a porous medium past a stretching sheet is conducting with power law surface temperature or power law surface heat flux. The flow in boundary layer is considered to be generated solely by the linear stretching of the boundary sheet adjacent to porous medium in a magnetic field with boundary wall slip condition. The governing partial differential equations are converted into non linear ordinary differential equations by similarity transformations. The solutions of dimensionless surface temperature as well as non-similar flow and heat transfer characteristics with the governing dimensionless parameters of the problem which include a non-linear stretching sheet , viscous dissipation , internal heat generation /absorption and temperature gradient dependent heat sink, power-law index of wall temperature parameters are obtained in terms of Confluent Hypergeometric Functions (CHF) and tabulated.*

The skin friction at the wall is also derived. It is observed that the suction (S), Slip parameter (L), the permeability of the medium (k_2) and the magnetic parameter (M), visco-elastic parameter (k_1) depress the longitudinal velocity magnitudes but influences positively the transverse velocity while the suction, wall temperature parameter, temperature gradient dependent heat sink (Q), lowers temperature and heat transfer distribution aiding in controlling momentum and heat transfer during material processing.

Key Words: *MHD flow, stretching sheet, slip parameter, non-similer solutions, visco-elastic parameter*

I. Introduction

Study of heat transfer and visco-elastic flow induced by heated stretching surfaces is often encountered in many engineering applications, such as materials manufactured by extrusion process, polymer processing, wire and fiber coating , cooling of metallic sheets or electronic chips, crystal growing.

It is well known that the flow in a boundary layer separates in the regions of adverse pressure gradient and the concurrence of separation has several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and adversely affects the heat transfer from the surface of the body.

In context to the well-known Blasius[1] flow problem (Cortell 2005) which involves the boundary layer flow passing through a stationary flat plate. (Sakiadis, 1961) considered the boundary layer flow on a moving flat plate in quiescent ambient fluid. The afore mentioned problems are two special cases of more general studies(I-Shak et al 2007, Cortell 2007) in which flow an heat transfer of a moving sheet in the presence of a co-flowing fluid were analysed.

Very recently (Sadeghy et al 2005) studied the boundary layer of an upper convicted Maxwell fluid , and the role played by the fluids elasticity on flow characteristics were analysed. Above mentioned works were with respect to the linear stretching sheet and it may be noted the stretching of the sheet may not necessarily be linear on view of this, the flow influenced by a non-linearly stretching sheet was investigated by Vajravelu (2001)and power-law or exponentially stretching sheet was studied by Ali(1995) and Elbashesly (2001) respectively. Further momentum , heat and mass transfer over an exponentially stretching surface were considered by Sanjayanand and Khan(2006). They also enclosed the effects of viscous dissipation and work done by deformation in the energy equation.

Now days the stretching sheet fluid flow is also one of the important flow fields in real world. Therefore the problem of uniqueness of a visco-elastic fluid flow over a stretching sheet has been discussed by Troy et al[1987] and Chang[1989]. Some of these visco-elastic fluids are termed as second grade fluids.. The visco-elastic property of these fluids has found in some dilute polymer solutions or in polymer fluids as mentioned by Markovitz and Colealan (1964).

For an incompressible homogeneous second grade fluid, the constitutive equation based on the postulate of fading memory suggested by Rivlin-Erickson (1955) is expressed as

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \qquad \text{………(1)}$$

Where T is the stress tensor, p is the indeterminate considered by the incompressibility pressure, μ is the dynamic visco-sity , α_1 and α_2 are first and second normal stress co-efficient that are related to the materials modulus. The second grade fluid is compatible with thermo dynamics if the Helmoltz free energy of the fluid is a minimum when it is locally at rest, and further if the second grade fluid is to satisfy the classius-Dehum in equality then the co-efficient μ, $\alpha_{1,}$ α_2 must satisfy the following requirements:

$$\mu \geq 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \qquad \text{………(2)}$$

The kinemetics tensors A_1 and A_2 are defined as

$$A_1 = \nabla V + (\nabla V)^T$$

$$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1 \qquad \text{…………..(3)}$$

where V is the velocity and $\frac{d}{dt}$ is material time derivative

Recently, the interest in that transfer for non-Newtonian fluid flows through MHD and porous medium has grown considerably due to their industrial applications such as in petroleum extrusion, enhanced oil recovery, filtration processes, in nuclear reactors packed bed reactions and many others. In this point of view Chauhan and Takhar(2002) investigated non-Newtonian coupled flow in a Chanel bounded by a highly porous layer.

In all the above analyse, the common feature is the assumption that the flow field satisfies the conventional no-slip condition at the stretching sheet, this assumption of no-slip is not valid and must be replaced by partial slip boundary condition , following Navier(1827) and Gad-el-Hak(1999)

$$u_i = L\frac{\partial u_i}{\partial \eta} \qquad \text{…….(4)}$$

Where $L = \frac{2-F}{F}\lambda,$ u_i is the tangential velocity , η is the normal direction to the wall, F is the momentum accommodation co-efficient, λ is the mean free path and L is the slip length. Most of the researchers investigated about the studies with slip condition of Newtonian fluid flows and very few have worked on non-Newtonian flows with slip condition. On real aspects non-newtonian fluids such as polymer melts which often exhibit boundary wall slip and such fluids are very important from technological point of view ; for example, polymer processing , artificial heart valves polishing. Labropulu et al(2004), Hayat et al(2007) and Ajadi et al(2009) discussed about the non-Newtonian fluid flows with the effects of slip condition. Ariel et al(2006) examined non-Newtonian fluid flow past a stretching sheet with partial slip. Khan et al (2008) investigated effects of slip parameters on shearing non-Newtonian fluid MHD flow through porous medium and obtained numerical solution for these typical shearing flows i.e Couette flow and generalized Couette flow with non linear slip boundary conditions.

In all the above studies the effect of temperature gradient dependent heat sink/source parameter and non-similar term parameters un heat transfer flow have not been which specially find applications in material processing industries.

In the present century, a century of technological advancement, exploration of industries using latest technologies in extrusions in manufacturing processes and melt spinning processes is taking place. In these industries the extradite is stretched into a filament when it is drawn from the dye and solidifies in the desired shape through a controlled cooling system coupled flow in a channel bounded by a stretching sheet and a highly porous medium. Garg and Rajgopal (1991), Singh et al (2006) examined the non-newtonian fluid past a wedge.

Therefore many authors including Veena et al (2010) , Rafia et al (2007) have analysed problems on boundary layer flow caused by a stretching sheet with temperature gradient heat sink effects for different flow models. Similarly Veena et al (2006), Shahjahan et al(2007), Pravin et al (2006) have investigated the non similar solutions on boundary layer flow past a stretching sheet with different heat transfer parameters. Rajgopal and Gupta (1984) and Garg and Rajgopal (1991) discussed that an additional boundary condition is required since the order of momentum equation of a second grade fluid is one order higher than that for a Navier-Stokes fluid and $\frac{\partial u}{\partial y} = 0 \ as \ y \to \infty$ may be taken as the augmented condition for the flow in an unbounded domain.

Thus in the present paper we are concerned not only with the natural convection over a stretching sheet but also the non-similar solutions of heat transfer flow of second grade fluid in the presence magnetic field and permeability parameter with temperature gradient dependent heat sink effects with power-law surface temperature (PST) or power-law surface heat flux (PHF). Slip flow boundary condition has been applied at the stretching sheet. In the heat equation viscous dissipation, internal heat generation or absorption, are also considered. Effects of slip parameter, permeability parameter (k_2), magnetic parameter (Mn), visco-elastic parameter (k_1), suction parameter (S), wall temperature parameter (T) temperature gradient dependent heat sink parameter (Q). Longitudinal and transverse velocity distribution for both PST and PHF cases are investigated and the results obtained are depicted graphically.

II. FORMULATION OF THE PROBLEM

In Cartesian co-ordinate system (x, y) consider two – dimensional non-similar solutions of free convection steady laminar boundary layer flow of an incompressible visco-elastic fluid caused by moving porous sheet embedded in a porous medium in presence of a temperature gradient dependent heat sink. The porous sheet is subjected to a constant suction velocity normal to the wall and uniform magnetic field. The x-axis is taken along the wall in the direction of motion of the flow and y-axis perpendicular to it. Let the components of velocity be u and v along x and y directions respectively. It is envisaged that the sheet issues from a thin slit at the origin (0, 0) and the speed at a point on the plate is proportional to its distance from the plate but the boundary layer approximations holds true.

Under the above mentioned assumptions and following Vafai and Tien`s [28] model, the steady state boundary layer equations of mass momentum and energy are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots (5)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial x}.\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] - \frac{\upsilon}{K'}u - \frac{\sigma B_0^{\ 2} u}{\rho} \qquad \dots (6)$$

and

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) + Q'\frac{\partial T}{\partial y} + \alpha_1\frac{\partial u}{\partial y}.\frac{\partial}{\partial y}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad \dots .(7)$$

The supplementary terms in the momentum equations namely the Darcian body force term

$-\frac{\upsilon}{K'}u$ and magnetic conductivity term $-\frac{\sigma B_0^{\ 2} u}{\rho}$ are linear in terms of the x-direction velocity u i.e they are parallel to the direction of the stretching motion. T-the temperature, ρ- the density , $\nu = \frac{\mu}{\rho}$- the kinematics' viscosity, k_2 the permeability parameter, Mn be the magnetic parameter , α_1, the non-Newtonian parameter, C_p , the specific heat , Q' - the uniform specific heat source or sink, k- the thermal conductivity and T_∞- the temperature at infinity.

The supplementary term in the energy equation namely the temperature gradient dependent heat sink $-Q'\frac{\partial T}{\partial y}$ and $Q'(= -CQG)$ which is the volumetric rate is a linear function of the temperature field.

It is assumed that the contribution due to the normal stress is of the same order of magnitude as that due to the shear stress since the flow is driven solely by stretching the sheet and the pressure gradient is assumed to be absent.

The appropriate boundary conditions for the momentum problem are

$$u(x, y) - bx = L\frac{\partial u}{\partial y}, \quad v = -V_0 \quad \text{at} \quad y = 0$$

$$u \to 0, \qquad \frac{\partial u}{\partial y} \to 0 \qquad as \qquad y \to \infty \qquad \dots ..(8)$$

where b>0 is a constant.

III. SOLUTION OF MOMENTUM PROBLEM

To solve equation (6), we postulate a solution by introducing the following similarity transformations:

$$u = bx\, G'(\eta), \quad v = -\sqrt{b\upsilon}\; G(\eta) \quad \text{and} \quad \eta = \sqrt{\frac{b}{\upsilon}}\; y \qquad \dots (9)$$

Obviously with this choice of velocity variables u and v , equation (1) of continuity is identically satisfied.

Substituting (9) in equation (6) we obtain

$$G'^{2}(\eta)-GG''=G'''+\left[2GG'''-G''^{2}-GG^{IV}\right]-(K_2+Mn)G'=0 \qquad \ldots.(10)$$

where $k_2 = \dfrac{\nu}{K'b}$ permeability parameter, $Mn=\dfrac{\sigma B_0^2}{\rho b}$ - Magnetic parameter,

$k_1 = \dfrac{\alpha_1 b}{\mu} = visco-elastic\ parametr$

By use of (9), the boundary conditions (8) corresponding to equation (6) reduce to:

$$G(\eta)=S, \qquad G'(\eta)=1+\gamma G''(\eta) \qquad at \quad \eta=0$$

$$\ldots(11)$$

$$G'(\eta)\to 0, \qquad G''(\eta)\to 0 \qquad as \quad \eta\to\infty$$

where $\gamma = L\left(\dfrac{b}{\nu}\right)^{1/2}$ is the slip parameter

and $S=\dfrac{V_0}{\sqrt{\nu b}}$ is the suction parameter.

The solution of equation (10)subjected to the boundary conditions (11) is

$$G(\eta)=A_1+B_1 e^{-\alpha\eta} \qquad \ldots(12)$$

And hence $G'(\eta)=-\alpha B_1 e^{-\alpha\eta}$

where A_1 and B_1 are constants to be determined such that it satisfied (10) under the boundary conditions (11). Thus

$$G(\eta)=\frac{1}{\alpha(1+\alpha\gamma)}\left(1-e^{-\alpha\eta}\right) \qquad \ldots(13)$$

and here $\alpha \neq 1/\gamma$

It satisfies all the boundary conditions (11) and it is an exact solution provided one root α is real of the following cubic equation

$$\gamma\alpha^3+(1+k_1)\alpha^2-(\gamma(k_2+Mn))\alpha-(1+k_2+Mn)=0 \qquad \ldots..(14)$$

SKIN FRICTION

The co-efficient of skin friction at the stretching sheet (η=0) is obtained as

$$\tau=\frac{\tau_{yx}(0)}{\rho u_w^2/2}=\frac{-2\alpha[1+\alpha\gamma+3k_1]}{Re_x^{1/2}} \qquad \ldots..(15)$$

where $Re_x=\dfrac{bx^2}{\nu}$

IV. SOLUTION OF HEAT TRANSFER PROBLEM

The boundary conditions for temperature field depend on the type of heating process under consideration.

The prescribed surface temperature (PST) case:

In this case the boundary conditions are

$$T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2 \qquad at \qquad y=0$$

$$T\to T_\infty \qquad as \qquad y\to\infty \qquad \ldots..(16)$$

where A is a constant and $T_w = T_\infty + A\left(\frac{x}{l}\right)^2$ is the wall temperature function defining non-isothermal behavior interms of quadratic power law. At the leading edge of the boundary layer x = 0 the wall temperature reduces to an isothermal law i.e. $T_w \to T_\infty$.

To solve heat equation (7), we introduce the following non-dimensional variable θ :

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad \text{....(17)}$$

Further introducing (16) and (17), equation (7) reduces to

$$\begin{aligned}&\theta'' + \Pr\left(1+Q^*\right)G\theta' - \Pr\left(2G' - \beta\right)\theta = -Ec \\ &\Pr\left[-k_1 G''\left(G'G'' - GG''' + G''^2\right)\right]\end{aligned} \qquad \text{....(18)}$$

where $\Pr = \frac{\mu Cp}{k}$ - the Prandtl number ; $\beta = \frac{Q}{b\rho Cp}$ - viscous dissipation

$Ec = \frac{b^2 l^2}{ACp}\sqrt{\frac{b}{\nu}}$ - Eckert number ; $Q^* = Q\sqrt{\frac{b}{\nu}}$ - Internal heat generation

and the boundary conditions (16) transform to

$$\begin{aligned}&\theta(\eta) = 1 && at && \eta = 0 \\ &\theta(\eta) \to 0 && as && \eta \to \infty\end{aligned} \qquad \text{......(19)}$$

To obtain the solution of equation (18), we introduce a change of variable ξ defined as

$$\xi^{-1} = \frac{-\alpha^2 e^{\alpha\eta}}{\Pr(1+Q)} \qquad \text{....(20)}$$

Hence with the help of (20) equation (18) transforms to

$$\xi\theta'' + [1 - a_0 - \xi]\theta' + \left[S_1 + \frac{\Pr}{\alpha^2}\beta\xi^{-1}\right]\theta = \frac{-Ec}{\Pr}\left[1 - k_1\left\{1 - \frac{\nu_0}{\sqrt{b\upsilon}}\alpha\right\}\right]\frac{\alpha^4}{\left(1+Q^*\right)^2}\xi \qquad \text{...(21)}$$

where $a_0 = \frac{\Pr\left(1+Q^*\right)}{\alpha^2}\left\{1 - \frac{\nu_0}{\sqrt{b\upsilon}}\alpha\right\}$, $S_1 = \frac{2}{1+Q^*}$ and the corresponding boundary conditions are

$$\theta\left(\xi = \frac{\Pr(1+Q)}{\alpha^2}\right) = 1, \quad \theta(\xi = 0) = 0 \qquad \text{.... (22)}$$

Equation (21) is in standard confluent hypergeometric equation form that is kummer`s equation [see Sanyal and DasGupta[2003] and Abramowitz and Stegun (1965) and the solution of equation (21) with respect to boundary conditions (22) is obtained as

$$\theta(\xi) = C_1 \xi^k M^{\Gamma}\left[K - S_1, 1 + b_0,\ \xi\right] - \frac{Ec\left[1 - k_1\left(1 - \frac{V_0}{\sqrt{b\upsilon}}\alpha\right)\right]\alpha^4}{\Pr\left[4 - 2a_0 + \frac{\Pr}{\alpha^2}\beta\right]}\xi^2 \qquad \text{.....(23)}$$

where $K = \frac{a_0 + b_0}{2}$, $a_0 = \frac{\Pr(1+Q^*)}{\alpha^2}\left\{1 - \frac{v_0}{\sqrt{b\upsilon}}\alpha\right\}$, $b_0 = \sqrt{a_0^2 - \frac{4\Pr\beta}{\alpha^2}}$

Solution (23) interms of the similarity variable η is expressed as

$$\theta(\eta) = \xi e^{-\alpha k\eta}\frac{M\left[K - S_1, 1 + b_0,\ \frac{-\Pr(1+Q^*)}{\alpha^2}e^{-\alpha\eta}\right]}{\left[K - S_1, 1 + b_0,\ \frac{-\Pr(1+Q^*)}{\alpha^2}\right]} + B_1 e^{-2\alpha\eta} \qquad \text{.....(24)}$$

where $\xi = 1 - B_1$, $B_1 = \frac{-Ec\alpha^2\left[1 - k_1\left(1 - \frac{v_0\alpha}{\sqrt{c\upsilon}}\right)\right]\Pr(1+Q^*)^2}{\left[4\alpha^2 - 2a_0\alpha^2 + \beta\Pr\right]}$

The dimensionless temperature gradient $\theta'(0)$ derived from equation (24) is as follows

$$\theta'(0) = A_2\left[k_3\alpha\left(\frac{K - S_1}{1 + b_0}\right)M\left[K - S_1 + 1, b_0 + 2,\ -k_3\right] - K\alpha M\left[K - S_1, 1 + b_0,\ -k_3\right]\right] - 2B_1\alpha$$

....(25)

where $k_3 = \frac{\Pr(1+Q^*)}{\alpha^2}$, $A_2 = \frac{A_1}{M\left[K - S_1, 1 + b_0,\ -k_3\right]}$

The dimensionless rate of heat transfer at the stretching sheet (η = 0), characterized by the Nusselt number is given by

$$Nu = \frac{-k\left(\frac{\partial T}{\partial y}\right)_{y=0}}{k(T_w - T_\infty)}x = \mathrm{Re}_x^{1/2}\theta'(0) \qquad \text{.......(26)}$$

The prescribed power law surface heat flux (PHF)

In this case we take the following boundary conditions.

$$-K\frac{\partial T}{\partial y} = q_w = \frac{B}{K}\left(\frac{x}{l}\right)^2 \quad at \quad y = 0$$

$$and \quad T \to T_\infty \qquad as \quad y \to \infty \qquad \text{....(27)}$$

we define $T - T_\infty = \frac{B}{K}\left(\frac{x}{l}\right)^2\left(\frac{\upsilon}{c}\right)^{1/2} g(\eta)$ ...(28)

on substituting (28) and (17) in to equation (7), we obtain

$$g'' + \Pr(1+Q^*)G\,g' - \Pr(2G' - \beta)g = -Ec\Pr\left[-k_1 G''\left(G'G'' - GG''' + G''^2\right)\right] \quad \text{.....(29)}$$

where $Ec = \dfrac{K b^2 l^2}{ACp}\sqrt{\dfrac{b}{\nu}}$, $k_1 = \dfrac{k_0 b}{\upsilon}$ - and all other parameters are the same as before

Using the transformation

$$\xi = \frac{-\Pr(1+Q^*)}{\alpha^2 e^{\alpha\eta}} \quad \text{.....(30)}$$

Equation (29) takes the form which reduces to the following confluent hypergeometric equation

$$\xi g'' + [1 - a_0 - \xi]g' + \left[S_1 + \frac{\Pr}{\alpha^2}\beta\xi^{-1}\right]g = \frac{-Ec}{\Pr}\left[1 - k_1\left\{1 - \frac{v_0}{\sqrt{c\upsilon}}\alpha\right\}\right]\frac{\alpha^4}{(1+Q^*)^2}\xi \quad \text{...(31)}$$

If we take

$$g(\xi) = g_c(\xi) + g_p(\xi) \quad \text{......(32)}$$

The corresponding boundary conditions (27) reduce to

$g'(0) = -1$ and $g(0) = 0$ ....(33)

Solving equation (31) under the boundary conditions (33) and using (32), we obtain the solution 0f (31) interms of η as

$$g(\eta) = B_1 e^{-2\alpha\eta} + B_2 e^{-\alpha K\eta} M\left[K - S_1\,,\ 1 + b_0\,,\ -k_3\, e^{-\alpha\eta}\right] \quad \text{.......(34)}$$

where

$$B_2 = \frac{(-1 + 2\alpha B_1)}{\{-\alpha K M[K - S_1,\ 1 + b_0\ ,-k_3] + B_3 M[K - S_1 + 1\ ,\ 2 + b_0\ ,-k_3]\}}$$

$$B_3 = \frac{\Pr(1+Q^*)}{\alpha}\left(\frac{K - S_1}{1 + b_0}\right)$$

The dimensionless wall temperature is derived as

$$T_w - T_\infty = \frac{B}{k}\left(\frac{x}{l}\right)^2\left(\frac{\upsilon}{c}\right)^{1/2} g(0)$$

V. Results and Discussion

The study of boundary layer flow behavior and heat transfers of a visco-elastic fluid (Walter liquid B') is considered in the presence magnetic field and porous medium adjacent to the stretching sheet with two different types of heating processes namely power-law surface temperature (PST) or power-law temperature gradient and power-law surface wall heat flux(PHF) or power-law wall temperature are considered. In addition the account of viscous dissipation, internal heat generation or absorption effect and temperature dependent gradient heat sink/source term are also considered. Several closed form solutions for the velocity and temperature fields are obtained.

Figures (1) and (2) show the variation of G(η) and G'(η) with the similarity variable η for different values of various parameters such as non-dimensional permeability parameter k_2 magnetic parameter Mn visco-elastic (k_1) and slip parameter (γ). It is clear from these figures that the flow velocity increases as expected. That is G and G' increase with decrease in permeability parameter $k_2 = \left(\dfrac{\upsilon/c}{k_0}\right)$ implying increasing permeability k_0 of the porous medium causes faster flow. Flow also increases with the increase of magnetic parameter Mn and non-Newtonian parameter k_1 however Slip parameter causes decrease in the values of both G and G' for all values of η I the boundary layer.

Fig(3) depicts $|f''(0)|$ against the slip parameter γ for various values of all the other parameters k_2 , Mn and k_1. It is observed from the figure that the magnitude of Skin friction i.e. the magnitude of dimensionless surface velocity gradient $|f''(0)|$ decreases with slip parameter (γ) or non-Newtonian parameter k_1 which further implies that the effect of γ or k_1 is to decrease the power needed to stretch the sheet. Further the power needed to stretch the sheet is also reduced by increasing the permeability k_2 and effect of magnetic field Mn of the flow medium. The magnitude of slip, that is $(1- G'(0))$ increases with the increase in the values of γ, because the frictional resistance between the stretching of the sheet imposes less motion of the fluid as slip parameter γ increases.

Figures (4a) and (4b) shows the variation of temperature profiles both in PST case and PHF case respectively with η for various values of the parameters. From these figures it is clear that for a given position η, the temperature decreases with an increase in the values of Prandtl number P_r with fixed values of Ec , k_1 and β in the both the cases of PST and PHF. On the other hand, the slip parameter γ has opposite effects on the temperature profiles. It may be described physically that the thermal characteristics are more influenced by the slip factor than by those other flow parameters. We see that as the value of slip parameter γ increases, the temperature distribution in boundary layer gets increased. Further both $\theta(\eta)$ and g(η) decrease at all values of η with an increase in the values of Prandtl number P_r and thus the thermal boundary layer thickness decreases.

Fig (5a) and (5b) depict the variations in the temperature field with η for different values of P_r effects of different combinations of suction parameter S , Prandtl number P_r, temperature gradient dependent heat sink parameter Q' and with fixed values of Eckert number Ec, visco-elastic parameter k_1. Maximum temperature corresponds to the curve- I for which S = 0.5, Pr = 0.5 and Q = 0.5. This physically implies that the union of weak suction with low wall temperature and weak heat sink. As expected the temperature is lowered for rise in S, Pr , Q and γ – slip parameter. However for the second curve II the S values are higher than for curve –III and curve IV. This shows that S impinges relatively less effects on decaying the temperature in comparison with the values of Pr. And Q. Such a scenario explains that a stronger heat sink plays more dominant role for lowering the temperature. These present results in general are well in agreement with the earlier studies of Sanyal and Das Gupta [2003], and Elbashbeshy and Bazid [2004] in both the cases of PST and PHF.

Fig (6a) and (6b) shows the variations in temperature profiles for different values k_1 for various combinations of the parameters S, Pr, Q, γ , k_2 . From both the figures it may be concluded that the thermal characteristics are more impressed by the slip parameter. We see that for increasing values of non-Newtonian parameter k_1 and slip parameter γ, the temperature in the thermal boundary layer gets increased, resulting in a increase in the thermal boundary layer thickness. Further the heat generation due to viscous dissipation and heat source is characterized by it is also observed that there is a temperature overshoot near the stretching surface with an increase in the Eckert number, in fact it means that there is significant heat generation due to fluid friction near the sheet.

Fig (7) represents the results of the rate of heat transfer- $\theta'(0)$ and g(0) Vs γ at the sheet for various values of Eckert number Ec. The effect of Eckert number Ec is to reduce the rate of heat transfer in both the case of PST in absolute sense it may be explained that for small values of Ec the arte of heat transferred is occurred from the stretching sheet to the fluid. Further it is reduced with the increase in the value of slip parameter γ. However again if the Eckert number values are allowed to be large , the rate of heat transfer decreases to its minimum value and then changes sign and finally its magnitude increases. In this scenario heat transfer takes place from the fluid region to the sheet because Ec is large enough to generate heat in the fluid is large. It is also seen that for higher values of Ec this change of sign occurs at greater value of the slip factor γ.

Figure (8) is drawn to explain the observation for the dimensionless temperature distribution g(0) for fixed values of γ, β and Ec. g(η) increases at all the values of η in the boundary layer however it reduces by the visco-elastic parameter k_1, Prandtl number Pr or the porosity and magnetic field of he medium. Physically it is apparent that the surface temperature g(0) in PHF case increase with the values of Eckert number Ec and internal heat generation β but g(0) decreases with increasing values of Pr, k_1 or permeability k_2 and Mn. Further slip factor γ also causes an increase in the value of g(0) as seen from the figure (8).

VI. Conclusions

The main conclusions of the study are as follows.

i. The slip parameter γ, suction parameter S, have substantial effect on the flow and heat transfer process. The longitudinal velocity is maximum at η = 0 for all values of γ and S and decreases rapidly with increase in η and far away from the surface of the sheet.
ii. The longitudinal velocity u increases with the decrease in the values of permeability and magnetic parameters
iii. The transverse velocity normal to the stretching sheet tends to a constant negative value and this inflow towards the sheet from the ambient fluid decays with increasing the values of γ and S.
iv. The transverse velocity is minimum at η = 0 for values of γ , S , k_2 and Mn and slowly start increasing with increase in values of η.
v. The magnitude of skin friction decreases with increasing slip parameter γ, non-Newtonian parameter k_1 and increases by the permeability and magnetic parameters that is the flow in the boundary layer increases with increase in permeability of the porous medium and magnetic effect or viscid-elasticity but decreases by the slip parameter γ.

However an increase in suction parameter S or distance in the direction of the longitudinal velocity decreases the skin friction .

vi. An increase in in slip parameter γ , suction parameter S, wall temperature parameter or heat skin parameter or temperature gradient dependent heat sink parameter Q^* results in lowering the temperature field steadily.

vii. The effect of the slip parameter γ is to reduce the heat transfer rate. The visco-elasticity of the Walter's liquid ($B^{'}$) model enhances the rate of heat transfer in PST takes place from the fluid to the stretching sheet when Ec is large enough rather than from the sheet the fluid when Ec is small.

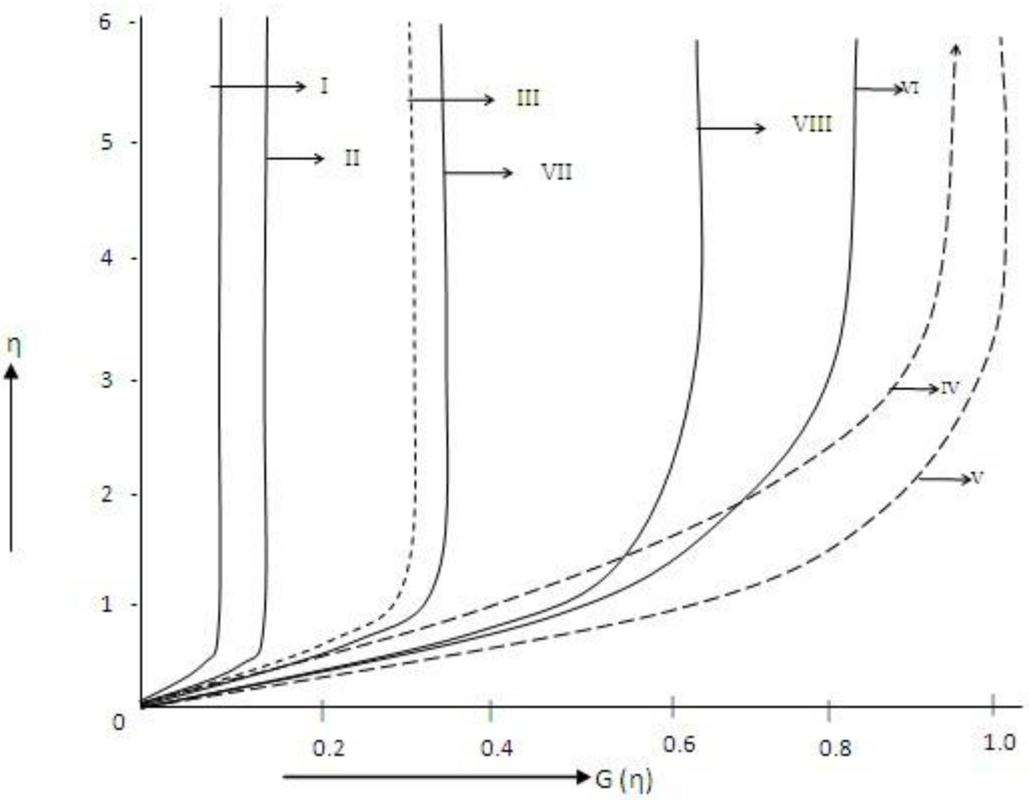
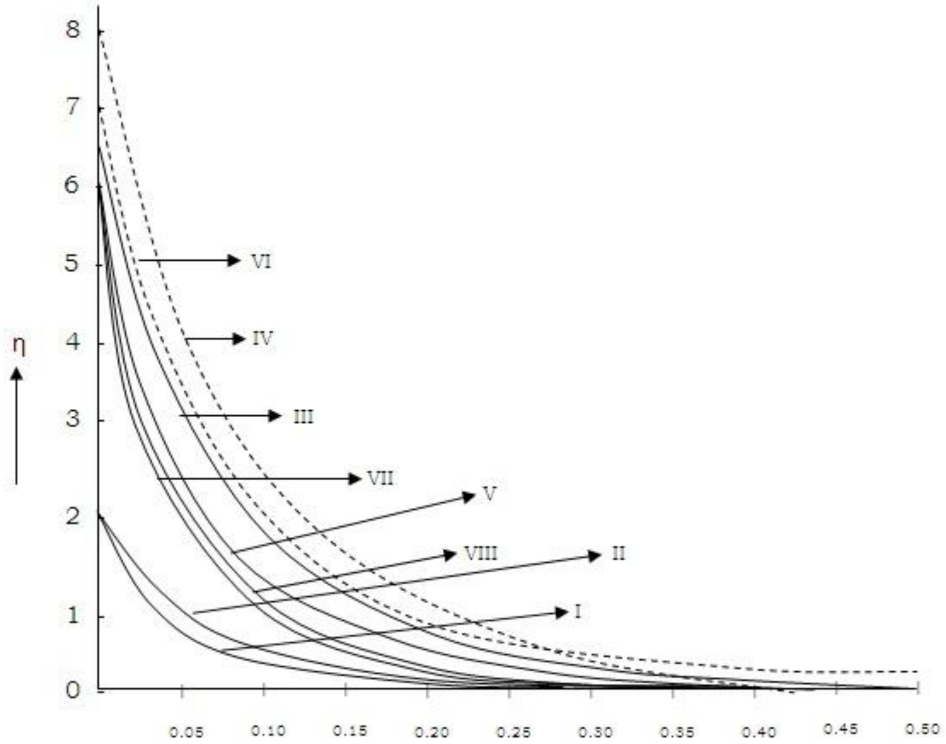


Fig1 : Longitudinal velocity profiles G(η) Vs η for various combinations of values of k_1, k_2 , Mn and γ

Curve	k_1	k_2	Mn	γ
I	1	10	10	1
II	1	5	5	1
III	1	1	1	1
IV	1	0	0	1
V	1	1	1	0
VI	1	1	1	0.3
VII	1	1	1	0.5
VIII	0	1	1	1

Fig(2): Transverse velocity profiles $G^{'}$ (η) Vs. η for various combinations of the values of k_1, k_2 , Mn, and γ



Curve	k_1	K_2	Mn
I	1	10	10
II	0	1	1
III	0.1	1	1
IV	1	1	1
V	1	0	0

Fig (3) Skin friction co-efficient $|f''(0)|$ Vs slip parameter γ

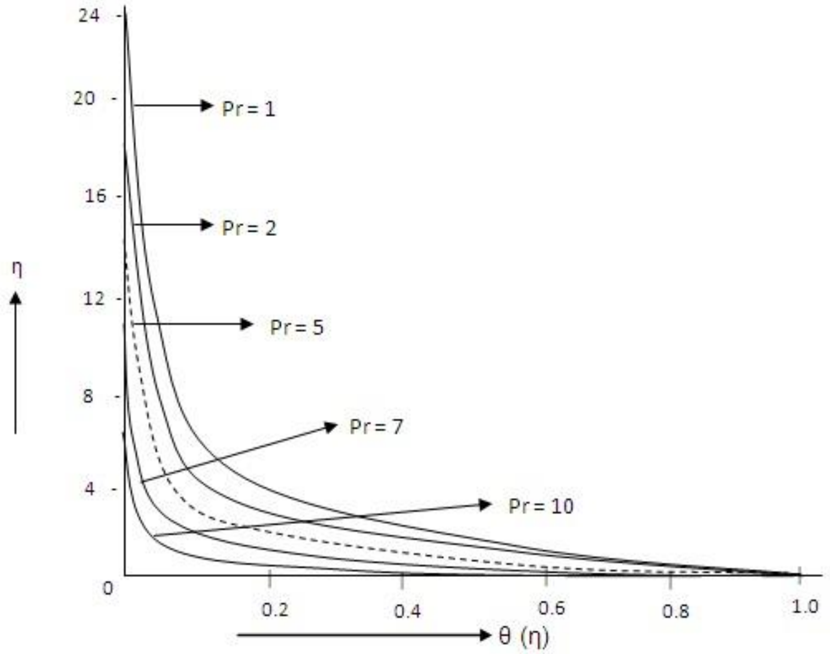


Fig (4a) Temperature profiles θ (η) Vs. η for different values of Prandtl number Pr and for fixed values of $k_1 = 1$, $k_2 = 10$, Mn = 10, γ = 1 (PST case) and Ec = 0.1

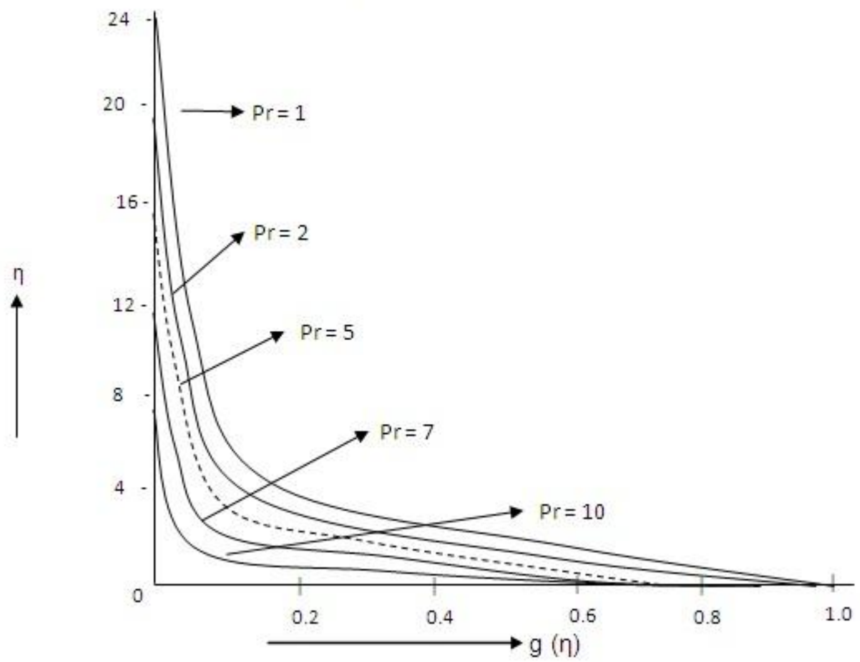


Fig (4b) Dimensionless temperature profiles g (η) Vs. η for different values of Prandtl number Pr. (PHF-Case)

Curve	S	Pr	Q
I	0.5	0.5	0.5
II	1.0	0.5	0.5
III	0.5	1.0	0.5
IV	0.5	0.5	1.0

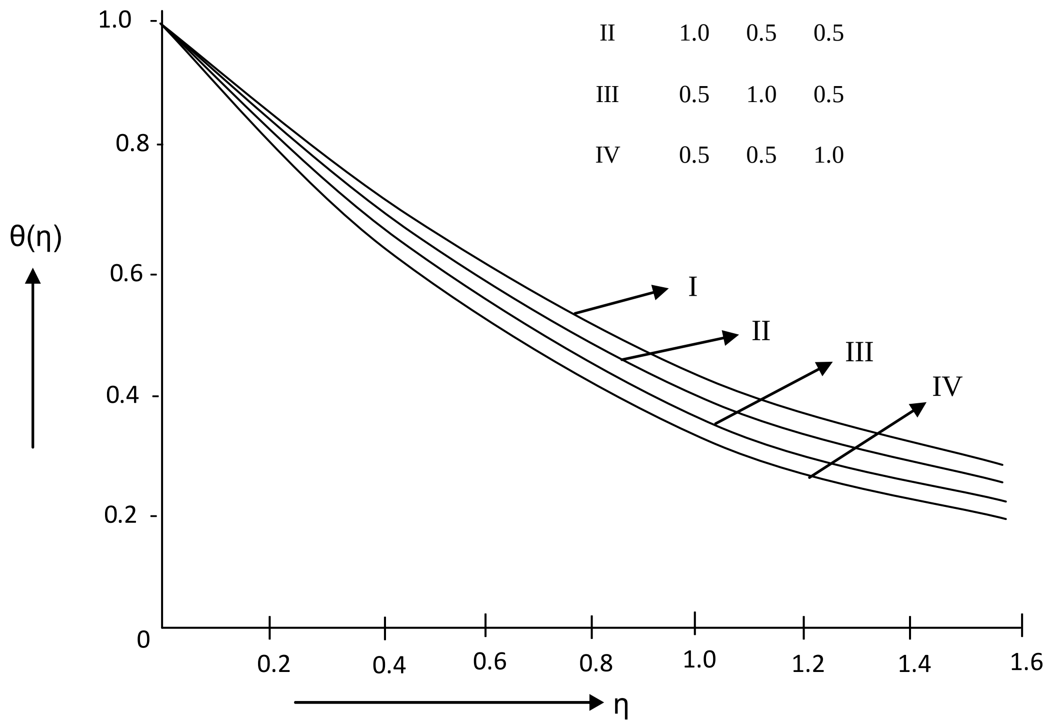
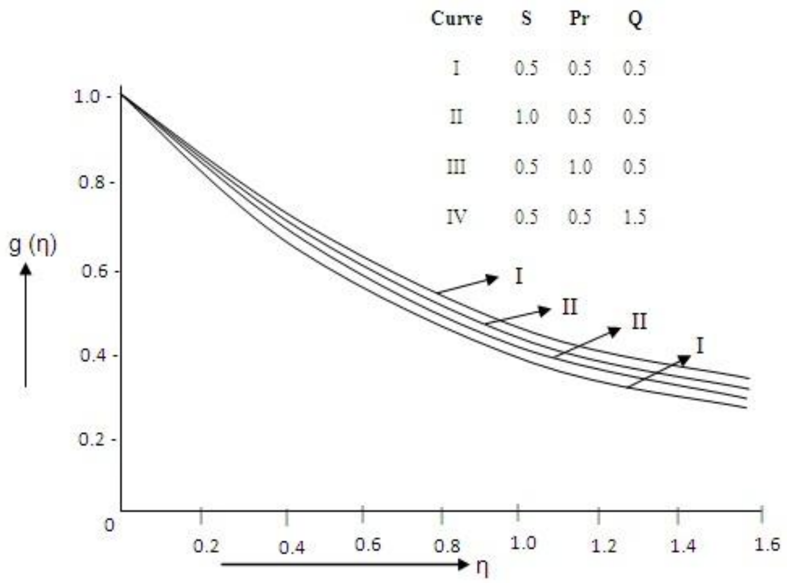


Fig (5a) Dimensionless temperature profiles θ(η) Vs. η for various combinations of S, Pr and Q and fixed values of $k_1 = 1$, $k_2 = 10$, Mn = 10, and Ec= 0.1 (PST – case)



Fig(5b) Dimensionless temperature profiles g(η) Vs. η for different combinations of S, Pr and Q and fixed values of $k_1 = 1$, $k_2 = 10$, Mn = 10, and Ec= 0.1 (PHF – case)

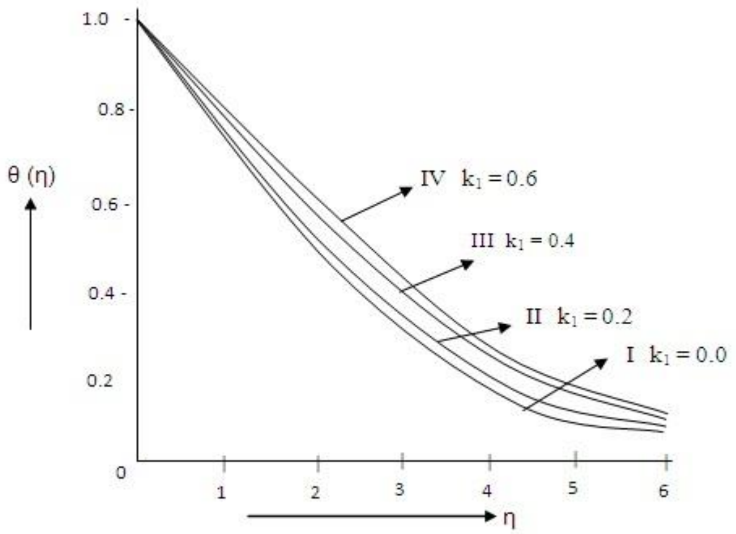
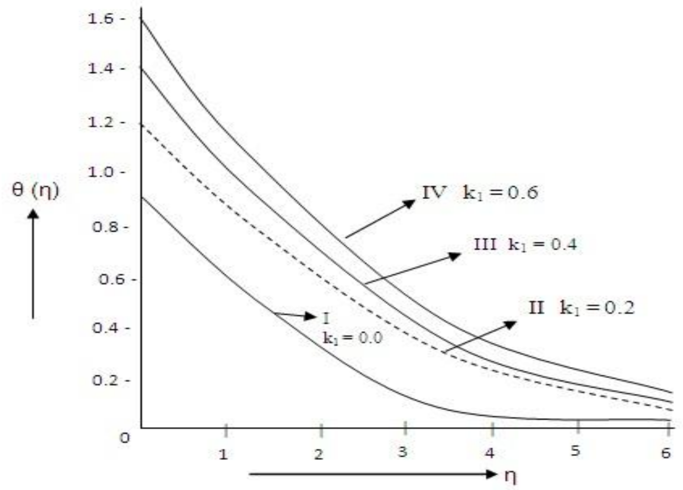


Fig (6a): Temperature profiles θ(η) Vs. η for various values of visco-elastic parameter k_1 with Ec=0.5, Pr=1.0, β=-0.03 in PST - case



Fig(6b): Temperature profiles g(η) Vs. η for various values of k_1 with all other parameters as in Fig(6a). PHF - case

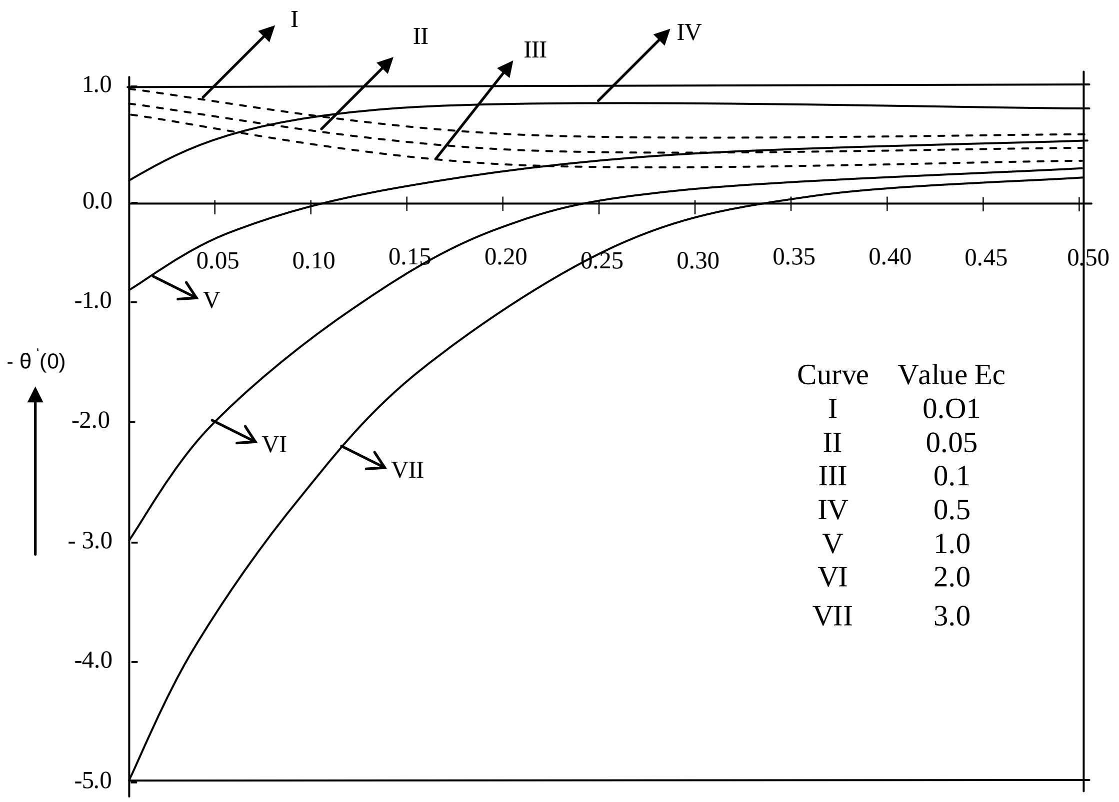
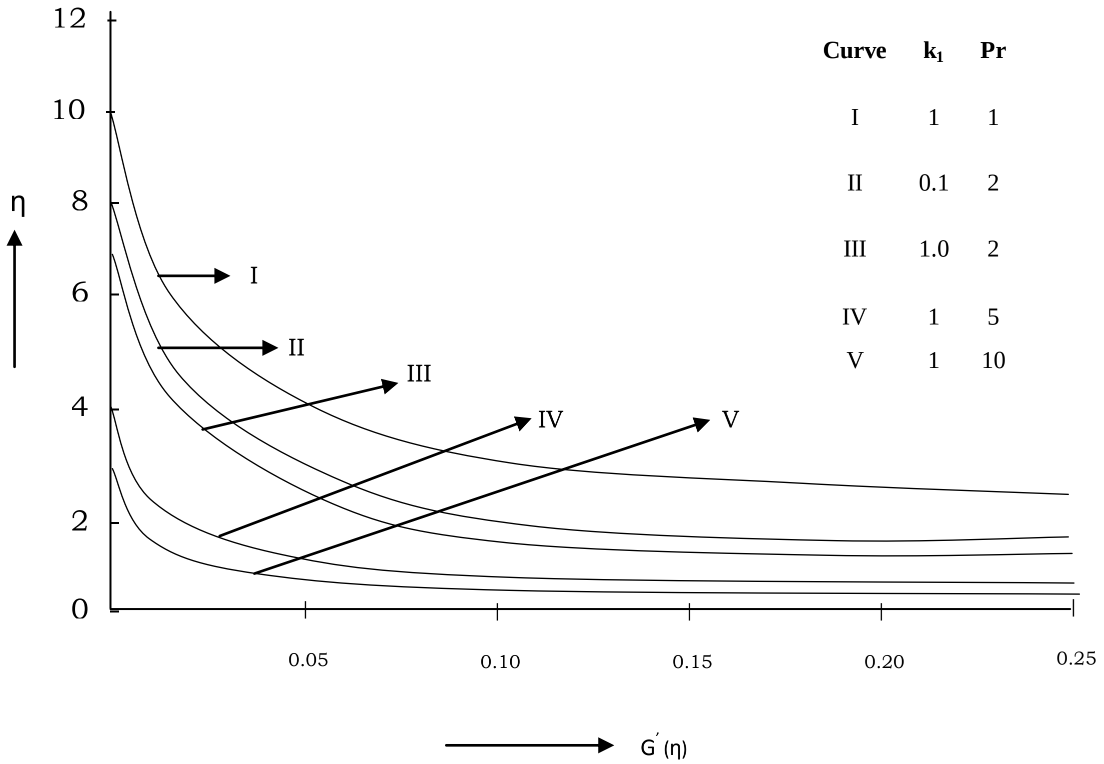


Fig (7): Dimensionless Temperature gradient θ´(0) at the stretching sheet for various values of Ec and fixed values of k_1=1, k_2 = 10, Mn = 10, Pr=1, β= -0.1



Fig(8): Dimensionless wall temperature g(0) Vs. η in PHF case for γ=1, k_2 = 1, Mn=1, β= -0.1 and Ec = 0.1 for various values of combinations of k_1 and Pr

REFERENCES

1. Cortell, R (2005): Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing Fluid Dynamics : Res., 37, 231-245.
2. Sakiadis, B.C (1961) : Boundary layer behaviour on continuous solid surfaces: A. I. Ch. E. J., 7, 26 - 28.
3. Ishak , A. , Nazar , R., pop, : (2007) : Boundary layer on a moving wall with suction and injection. Chin. Phys. Lett. 24 (8) , 2274-2276.
4. Garg, V.K. and Rajgoal, K.R. (1991). Flow of non-Newtonian fluid past a wedge. Acta Mech., 88, 113-123.
5. Sadeghy, K., Najafi , A. H ., Saffaripour, M., (2005): Sakiadis flow of an upper- convicted Maxwell – fluid. Int. J. Non-linear Mech. 40, 1220 - 1228
6. Vajravelu , K. , (2001) : Viscous flow over a non-linearly stretching sheet. Appl. Math. Comput. 124, 281-288
7. Ali , M.E.(1995) : On thermal boundary on a porous law stretched surface with suction or injection. Int. J. Heat Mass Flow. 16, 280-290
8. Elbashbeshy, E.M.A.(2001) : Heat transfer over an exponentially stretching continuous surface with suction Arch. Mech. 53, 643-651
9. Sanjayanand, E., Khan S.K., (2006) : On heat and mass transfer in a visco-elastic boundary layer flow over an exponentially stretching sheet *Int. J. Thermal. Sciences. 45 , 819-828*
10. Troy , W. C. , Overman , E. A., Ermentrout, G.B., and Keener, J. P., (1987) :Uniqueness of flow of a second order fluid past a stretching sheet. *Quart. Appl. Math., 44 , 753-755*
11. Chang, W. D., (1989) : The non- uniqueness of the flow of a visco-elastic fluid over a stretching sheet. *Quart. Appl. Math. , 47 , 365 – 366.*
12. Markvitz, H and Coleman, B. D. (1964): Advances in Applied Mechanics.*Vol 8. Academic press, New York.*
13. Rivlin, R. S and Erickson, J. L. (1955): Stress deformation relation for isotropic materials J. Rat. Mech. Anal. , 4, 323-325.
14. Chauhan , D.S. and Takhar, P.K. (2002) : Non-Newtonian stretching coupled flow in a channel bounded by a highly porous layer. Modeling Measurement and control, B , 71 (4), 33-40.
15. Navier, C.L.M.H. (1827). Sur les lois du movement des fluids, Mem, Acad. R.Sci. Inst. Fr., 6, 389-440.
16. Gad-el-Hak, M. (1999). The fluid mechanics of micro devices-The Free Scholar Lecture, J. Fluid Eng- T.ASME, 121, 5-33.
17. Labropulu, F., Husain, I. and Chinichain, M. (2004). Stagnation-point flow of the Walters' B'fluid with slip. IJMMS, 61, 3249-3258.
18. Hayat, T., Khan, M. and Ayub, M. (2007). The effect of the slip condition on flows of an Oldroyd 6-constant fluid. J. Comput. Appl. Math., 202, 402-413.
19. Ajadi, S.O., Adegoke, A. and Aziz, A. (2009). Slip boundary layer flow of non-Newtonian fluid over a flat plate with convective thermal boundary conditions. Internationa J. of Non-Linear Science, 8(3), 300-306.
20. Ariel, P.D., Hayat, T. and Asghar, S. (2006). The flow of an elastico- viscous fluid past a stretching sheet with slip. Acta Mech., 187, 29-35.
21. Khan, M., Hayat, T. and Wang, Y. (2008). Slip effects on shearing flows in a porous medium. Acta Mech. Sin., 24, 51-89.
22. Singh. A. K : Heat Transfer and Boundary Layer Flow past a stretching porous Wall with Temperature gradient Dependent Heat sink
JEHMT, 28 (2006), 109-225.
23. Veena, P.H., Pravin. K., and J.K.Nagbhooshan : MHD Visco-elastic Flow and Heat Transfer with Temperature Gradient Dependent Heat Sink in a Porous Medium past a stretching Sheet. Int.J.Math. & Applications 3, (2010), 65-81
24. Veena,P.H., V.K.Pravin and Rafia Begum : Free and Forced MHD Convective Mass Transfer Flow with Heat Source and Thermal Diffusion embedded in a porous medium with Variable Thermal Conductivity. Int. J. Mathematics and Analysis , 5, (2007) 267-280
25. Veena,P.H., Pravin, V.K., and Shahjahan, M : Non-Similar Solutions for Heat and Mass Transfer Flow in an Electrically conducting Visco-elastic fluid over a Stretching Sheet embedded in a porous Medium. *Int.J.Modern Mathematics, 2* (2007) 9-26.
26. Veena,P.H., V.K.Pravin., and Padashetty, S. : Non-Similar Solutions in Visco-Elastic MHD Flow and Heat Transfer over a porous Stretching Sheet with Internal Heat Generation and Stress Work. IJMM, 2 (2007), 267-281
27. Veena,P.H., Abel.S., RajGopal, K and Pravin ,V.K: Heat transfer in a visco-elastic fluid past a stretching sheet with viscous dissipation and internal Heat Generation. ZAMP , 57 (2006) 447-463.
28. Vafai, K and Tien, C.L.: Boundary and Inertia Effects on Flow and Heat Transfer in Porous Media.
Int. J. of Heat Mass Transfer, 24(1981); 195-203
29. Sanyal, D.C., and Dasgupta ; S., : steady Flow and Heat Transfer of a conducting fluid by stretching porous wall.
Ind.J.Theo. Phys., 51, (2003) 47-58.
30. Elbaashbeshy, E.M.A., and Bazid, M.A.A: Heat transfer over a steady stretching Surface.
J. Heat and Mass Transfer, 41. (2004) 1-4.