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Static and Dynamic Analysis of Thin-Walled Cyclic Shells

S. N. Krivoshapko, DSc[1], Christian A. Bock Hyeng, Ph.D.[2]
[1](Professor, Peoples' Friendship University of Russia, 6, Miklukho-Maklaya Str, Moscow, 117198, Russia)
[2](North Carolina Agricultural and Technical State University, 1601 E. Market St., Greensboro, NC 27411, USA)

ABSTRACT: *The principal achievements of science and engineering in the sphere of static, vibrational, and buckling analysis of thin-walled structures and buildings in the shape of cyclic surfaces with circular generators are summarized in this review paper. These shells are useful as fragments of pipelines, spiral chambers of refrigerating units, as well as in spiral chambers of turbines in hydroelectric power stations, in high pressure units, in public and commercial buildings, for example, as coverings of stadiums, and so on. This review paper contains 62 references, and these are practically all original sources dealing with static and dynamic analysis of thin-walled cyclic shells.*

Keywords: *Canal surfaces of Joachimsthal, Cyclic surfaces with circular generators, Dupin's cyclides, epitrochoidal shell, spiral chamber, static and dynamic analysis, strength, tubular shell*

I. Introduction

To date great progress has been achieved in the strength analysis of thin elastic shells. Thin-walled shell structures combining lightness with considerable strength find widespread use in modern engineering and building. Many have noted the rapid progress of the practice and the theory of the application of thin shells and thin-walled shell structures in the last 10 years. But shells used in constructions belong to limited classes of surfaces. An accurate tendency in world practice is the application of spatial structures of arbitrary form, giving expressive architectural images and solving functional problems. Today we have a new generation of young architects and engineers who have also shown interest in designing wide-span spatial coverings. This process amplifies occurrence of new materials, such as fibrous reinforced polymeric composites, which can be used in covers due to its ability to curve. One family of generating curves in cyclic surfaces is represented by circles of constant or variable radii that considerably reduces the cost price and simplifies the process of manufacturing thin shells in the form of these surfaces without a decrease in operational possibilities. However, until the middle of 20th century the analytical method for calculation of cyclic shells was replaced by the approached calculation concerning simple systems on which it was possible to dismember a cyclic structure. Design research on models was widely applied. Engineers, mechanics, and architects, using only the rough methods of analysis, basically spent on the basis of intuitive reasons, have created

many interesting structures and buildings in the form of cyclic surfaces.

In connection with inquiries of practice spanning 60 years to the present time, attention of many scientists attracts the problem of the determination of the stress-strain state of shells of complex geometry including cyclic shells; therefore, in the presented review considerable attention will be paid to this problem. Such cyclic surfaces as surfaces of revolution, circular helical surfaces, and tubular surfaces are the most known and widely used. In this review, surfaces of revolution won't be considered, as hundreds of publications are devoted to them, and these surfaces are usually allocated in a separate class.

II. The review of works on research of strength of tubular shells

A paper [1] and a dissertation [2] were the first works on strength analysis of tubular shells. A. Bantlin [1] has experimentally established that a pipe with the curvilinear axis under bending is more flexible than a pipe with a rectilinear axis of the same cross-section [3]. The effect of T. Karman consisting of flattening of the cross-section of a pipe under bending raises flexibility of curvilinear pipes. I.V. Stasenko [4] determined that distortion of a contour of a cross-section is accompanied by the occurrence of the circular bending moments and normal stresses which can be several times more than that of nominal stresses of bending calculated without the effect of flattening. Let b be the radius of the cross-section of a pipe, a the radius of a centerline, and h the thickness of the pipe wall. When the parameter b^2/ah is small in comparison with unity then the influence of flattening does not matter. Using a trigonometric series for reception of quantitative results it is necessary to take the larger number of members, as it has more value than the parameter b^2/ah. T. Karman [3] considered a problem about a pipe of round cross-section using a principle of a minimum of potential energy in the form of the Rayleigh – Ritz. G. Lorentz has applied a principle of the least work of Al. Castigliano. R. Clark and E. Reissner [5] for the first time having applied a method of asymptotic integration have shown that G. Lorentz's theoretical results are less exact.

In the monograph [3], the short review of results of strength research of tubular shells with the references [1, 2, 5-8] is presented. Without repeating the above-stated review, we will specify only that in a work [6] the scheme of a semi-analytical method of final elements of calculation of curvilinear tubular shells with arbitrary spatial axis is investigated. Here approximation of fields of displacements in a shell in the form of the final sum of Fourier's series for one variable and oblique-polynomial approximation of type of final element method for another variable was applied. In a work [8], a thin-walled pipe is modeled by toroid and described by the equations of semi-momentless shell theories.

A series of works [9 – 1 2] devoted to strength analysis of tubular shells with application of approximate momentless theory was published by V.N. Ivanov. He showed that a system of three equilibrium equations for a momentless shell referred to lines of principal curvatures may be reduced to one resolving differential equation in two ways. Introduction of function of stresses φ (α, β) [9, 10] is the first way. An exception of two inner forces of momentless state and reception of the resolving equation concerning the one remaining force is the idea of the second method [9, 13]. The analysis of the resolving equation with function φ (α, β) has shown that a method of separation of variables cannot be applied. Exceptions make torus and circular cylindrical shells. The tubular shell of arch type with directing flat curve is considered in a work [11]. For solution of the problem, a difference method of straight lines was used. Derivatives on the coordinate coinciding with the cross-section of a pipe are replaced by difference relations. Thus, resolving equations of momentless theory are reduced to systems of ordinary differential equations with variable coefficients. The tubular shell with a line of centers in the shape of evolvent of the circle subjected to dead weight has been calculated with the help of momentless theory in a work [10]. As a result, resolving the difference equation became the Airu equation and its decision was received in Bessel functions.

The new scheme of discretization (a method of curvilinear grids) which is a generalization of a finite difference method is offered in [14]. It completely excludes an error of approximation of the covariant derivative of functions of rigid displacements. An analysis of stress-strain state of shells in the form of tubular surface with the flat sinusoidal centerline used for the connection of two parallel pipelines of identical radius is fulfilled in the monograph [3] with application of a method of curvilinear grid. Gulyaev and his colleagues put finite difference netting of dimensions 17×17 and 21×21 on the chosen area for control of convergence of calculations. The analysis of results of calculations has shown that the stress-strain state of shells under uniform internal pressure appreciably depends on geometrical parameters, and the maximum values of displacements, membrane forces, and bending moments take place in the joining of a tubular shell to parallel cylindrical shells.

Results of a series of experiments carried out on a breadboard model of the heat exchange tubes for the determination of fatigue failure in the dryout zone over frequent heat changes are reviewed in a work [15]. The technique of carrying out of modeling tests in welded helical pipes of the heat exchanger is described. A.S. Pystogov [16] has presented governing formulas for definition of tangent stresses and deformations of helical springs of tension-and-compression with coils of tubular section from thick-walled pipes. Simmonds James G. [17] has derived geometrically nonlinear parities defining deformations, turns and changes of curvatures of coordinate lines, the equations of compatibility of deformations, the equilibrium equations, and Hooke’s law with reference to analysis of a helicoidal circular cylinder (a tubular helical surface, see Fig. 1) with an inextensible contour subjected to axial tension and torsion. A method of separation of variables for analysis of the stress-strain state of tubular helical shells was realized for a two-dimensional case in papers [18, 19] where properties of this shell depending on values of its parameters are also investigated. It should be noted that stresses and moments under small face pitch do not surpass corresponding levels in toroidal shells [19].

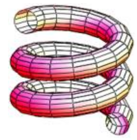

Fig. 1

Attempts at making analytical decision of concerned problems with the stress-strain state of tubular shells meet great difficulties of a calculating nature. The various rough methods are applied to the decision of the resolving differential equations, for example, momentless shell theory and simplifications.

The additional information on static behavior of tubular shells can be found in [20-28] and in paper [21], research in which a modified finite difference method has been allowed to replace the system of differential equations in private derivatives by a system of algebraic equations. The extensive bibliography of works devoted to analysis of pipes with a curvilinear axis resulted in R. Clark and E. Reissner’s manuscript [5].

Elastic equilibrium of tubular shells was considered widely enough but the dynamic behavior of these shells are examined only in [3, 29-32]. In the monograph [3], it is noticed that today’s research on vibrations of pipelines and their connecting sites have a special significance due to the growth of capacities of machines and velocities of movement of working liquids and gases. Frequencies and forms of natural vibrations are the basic dynamic characteristics of any engineering structure. Calculation of frequencies and forms of natural vibrations of a connecting element of two parallel circular pipelines is made in [3]. The rigid fixation was taken on the two opposite circular sides. On the allocated area of calculation, the finite difference net by dimension 17×17 was put and it provided sufficient accuracy of calculations. Analyzing the received results, the authors [3] notice that the curvature of a centerline appreciably has an influence on character of change of frequencies of its natural vibrations and on distribution of forms corresponding to them. Sokolov [30] has defined frequencies of natural vibrations of the curvilinear pipeline with a flowing liquid on a basis of momentless theory of thin-walled shells of “mean bending.” Vibrations of pipes with a flat centerline with taking into account flowing viscous incompressible liquid are also considered by Ya. F. Kayuk in [31]. The account of effect of flowing liquid in a helical pipe was investigated in a work [33]. Influence of filling of a helical pipe by a saturated porous medium was examined in [34]. Only one work [35] devoted to experimental study of helical pipes has been found.

III. The review of works on research of epitrochoidal shell strength

Geometry and working with methods of analysis of epitrochoid shells using momentless theory have been considered for the first time in a master’s thesis [36] by Mahmud Hussein Suleiman. Geometrical characteristics of epitrochoid shell give an opportunity to reduce a system of equations of momentless theory of shells due to a function of stress $\varphi(\alpha,\beta)$ to one equation

$$\frac{2}{\tilde{R}(\alpha)}\frac{\partial}{\partial\alpha}\left[\tilde{R}^2(\alpha)\frac{\partial\varphi}{\partial\alpha}\right]+\frac{1}{\omega(\alpha,\beta)\dot{f}}\frac{\partial}{\partial\beta}\left[\frac{\omega^2(\alpha,\beta)}{\dot{f}}\frac{\partial\varphi}{\partial\beta}\right]=G(\alpha,\beta), \quad \omega(\alpha,\beta)=\xi(\alpha)-f^2(\beta);$$

$$\xi(\alpha)=\left[2\sigma^2(\alpha)-\tilde{R}(\alpha)\right]/\tilde{R}^3(\alpha); \ \tilde{R}(\alpha)=R(\alpha)/a=1+\mu\cos\alpha; \ \sigma=\sqrt{\tilde{R}^2+\tilde{R}'^2};$$

where G is a function of load. Dependence of the function ω on parameters α, β does not generally allow for application of a method of separation of variables for the resolving equation. The analysis of dependence of function $\xi(\alpha)$ from parameter μ has shown that for each value μ it is possible to define an interval of change of coordinate parameter α in which $\xi(\alpha)$ will be a slowly changing function and it is possible to replace its value with an average value of this function in the set interval. So, it gives an opportunity to use a method of separation of variables [37]. Calculation of fragments of the epitrochoid shell with $\alpha=0$, $\alpha_0=\pi/4$, $\beta=\pm1$ and with parameters $\mu=0.1$; 0.3; 0.5 has been carried out (Fig. 2). The shell is loaded by own weight q. Conditions of supporting of the fragment provides equality to zero of tangent forces at the edges $\beta=\pm1$ and $\alpha_0=\pi/4$. Normal forces N_α are equal to zero on the edge $\alpha=0$.

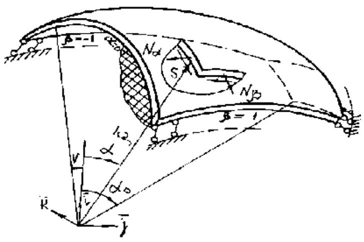

Fig. 2

The function of stresses was accepted in the form

$$\varphi(\alpha,\beta)=qa^3\left[\varphi_0(\alpha,\beta)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\Phi_m(\alpha)F_n(\beta)\right],$$

$$\Phi_m(\alpha)=sin(\tilde{m}t\alpha); \ F_n(\beta)=G^{\varsigma}_{2n-1}(\beta)-C_nG^{\varsigma}_{2n+1}(\beta);$$

$G^{\varsigma}_n(\beta)$ are orthogonal polynomials of Gegenbauer which are functions of the resolving equation. Similar calculations have been made for a compartment symmetrical on coordinate $\alpha(-\pi/4\le\alpha\le\pi/4)$. Results of calculation are presented in [38] in graphical representation.

Research of stress-strain state of shells in the shape of epitrochoidal surfaces have been continued in the dissertation [39]. It was shown that epitrochoidal surfaces belong to a class of canal of surfaces of Joachimsthal. Using approaches to analysis of epitrochoidal shells offered in [36, 37], Gil Oulbe Mathieu has carried out a strength analysis of the part of epitrochoidal shell, limited by edges $\alpha=\pm\alpha_0$, $\alpha_0=\pi/4$, $\beta=0$ and $\beta=1$. The line of centers lies in a horizontal plane (Fig. 3). Boundary conditions provide equality of tangent forces on the contour. Research of the convergence process of calculation was realized with various number of members of a series. Researchers have shown that in the main part of a shell, accuracy of calculation is provided in the 10th approach. For the support zone $\beta=0$, convergence has been reached in the 18th approach [40]. The results of calculation are presented in graphical representation.

Gil Oulbe Mathieu [39] devised an algorithm for the calculation of displacements of an epitrochoidal shell. For the determination of displacements, the geometrical equations of tangential deformations of a middle surface of the shell were used. This calculation is possible after defining the internal forces which are used further for the determination of displacements. Three geometrical equations are reduced to one resolving equation by introduction of the generalized function of displacements [41]. Though the resolving equation in displacements differs from the resolving equation of equilibrium in forces, types of the equations coincide and for the determination of displacements, orthogonal polynomials of Gegenbauer were also used. Results of the calculation are presented as diagrams of normal and tangential displacements of points of sections of the shell fragment (Fig. 3) [12, 42].

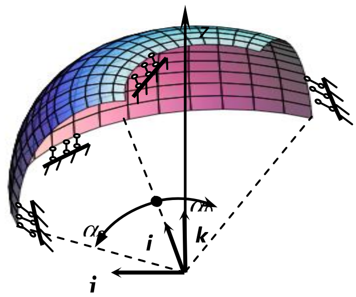

Fig. 3

IV. The review of works on research of strength of canal surfaces of Joachimsthal

Research of geometry and stress-strain state of shells in the form of canal surfaces of Joachimsthal are discussed in a dissertational work [43]. The equation of canal surfaces of Joachimsthal with arbitrary directrix in lines of principal curvatures, coefficients of the first and second fundamental forms in the theory of surfaces and principal curvatures have been received in [44]. The investigation of methods of analysis of shells has shown that analytical methods of analysis are not applicable for analysis of shells of complex forms even if one uses momentless theory of shells. Different numerical methods are used for moment analysis of shells of complex forms. For analysis of thin shells having the shape of the canal surfaces of Joachimsthal, a finite difference energy method has been chosen. The choice has been made on the basis of the analysis of possibilities of three methods: a difference method of netting, a finite element method, and a finite difference energy method. Systems of the equations describing stress-strain state of shells in the form of Joachimsthal's surfaces are used in a difference method of netting directly. The order of derivatives can reach the 4th – 8th order. Necessity to satisfy to all boundary conditions of a shell structure extremely complicates the realization of this problem, especially near curvilinear edges, in a zone of openings and on free shell edges. A finite element method and a finite difference energy method are based on the minimization of functional of total energy of deformations, the derivatives in which not above the 2nd order. The methods allow satisfying only the kinematic boundary conditions, realization of which usually does not represent any difficulties. The FEM is convenient for calculation of shells with openings and with curvilinear contour of the shell not coinciding with a coordinate net. Complex contours and openings are easily approximated by selection of special final elements.

This problem becomes complicated if one uses a finite difference energy method, but it is quite realistic. A FEM has entered into settlement practice in the 2nd half of the 20th century and has won great popularity and it is widely applied for analysis of thin-walled spatial structures. Program complexes realizing calculations of various shell structures are created. However, the equations of middle surfaces in FEM are used usually only for creation of a net of knots of finite elements on the middle surface of the shell meant for analysis. The geometrical characteristics reflecting features of internal and external geometry of a surface usually are not used in FEM. It brings additional errors in results of calculation of thin shells of complex geometry. For analysis of shells of complex geometry by finite difference energy method, an algorithm considering geometrical characteristics of middle surface of shells had been developed [45-48]. The program complex using FORTRAN algorithmic language has been developed in Peoples' Friendship University of Russia for realization of the developed algorithm. The library of curves and surfaces is connected to the program module calculating values of geometrical characteristics of the middle surfaces of shells used for the determination of their stress-strain state. The program complex allows analyzing shells of canonical geometry and shells of complex geometry, including shells in the shape of canal surfaces of Joachimsthal. At debugging of the program complex, analyses of plates, spherical and cylindrical shells have been carried out. The results of calculation were compared to the known exact decisions. The calculations have shown good convergence at calculation of shell structures of cylindrical form on a net 8×8 and spherical form on a net 10×10 [49].

Having used a complex program for analysis of shells with the help of a finite difference energy method, N.Yu. Abbushi [43] carried out an analysis of a shell in the shape of Joachimsthal's canal surfaces with circular sinusoidal directrix $r_2(\alpha_1) = a[1+\mu(1+\cos k\alpha_1)]$ (Fig. 4). Shell calculation was fulfilled on the action of dead weight with c = 0.707; μ = 0.1; k = 8; $f(\alpha_2) = \mathrm{tg}\alpha_2$. The shell is closed along the coordinate α_1 ($0 \le \alpha_1 \le 2\pi$) but $0 \le \alpha_2 \le 2/3\pi$. The contour edge α_2 = 0 is rigidly fixed and the upper edge $\alpha_2 = 2/3\pi$ is free. Values of internal forces and moments were received and resulted in the diagrams [12, 43]. An analysis of the closed shell in the shape of canal surfaces of Joachimsthal with oval directrix $r_2(\alpha_1) = \sqrt{l_1^2\cos^2\alpha_1 + l_2^2\sin^2\alpha_1}$ was also performed. The form of an oval curve depends on the relation of dimensional parameters $\lambda = l_1/l_2$ (Fig. 5). It was assumed that a = 20 m, $f(\alpha_2) = \tan\alpha_2$, that the material of the shell was homogeneous with $E = 2 \cdot 10^4$ MPa, and that the Poisson coefficient of the material was ν = 0.15; a thickness of the shell was h = 0.10 m. The shell was loaded by the inner constant pressure P_{in} =10 KH/m^2 (Fig. 6). Tables of values of the internal tangential normal forces and the bending moments are presented in [12, 43].

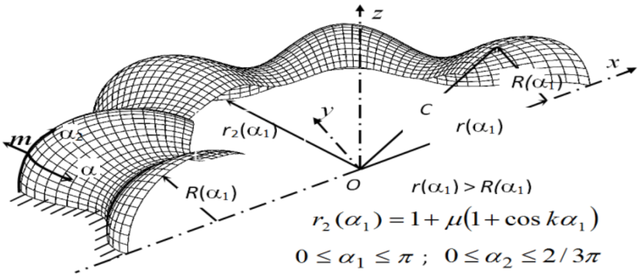

Fig. 4

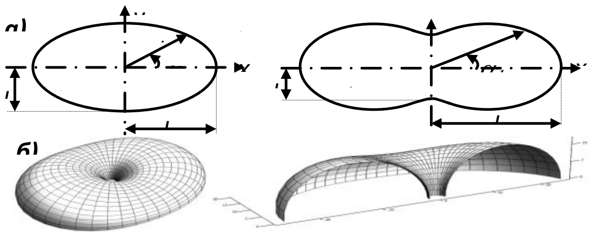
Fig. 5

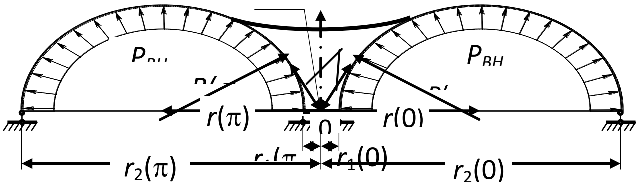

Fig. 6

V. The review of works on research of strength of shells in the shape of Dupin's cyclides

The fullest materials closely associated with definition of the stress-strain state of shells in the shape of Dupin's cyclides loaded by a uniform surface load are presented in [12, 50]. First, Krishna Reddi in his dissertation [51] and papers [52-54] presented numerical results of analysis of shells in the shape of Dupin's cyclides. He used a momentless shell theory. A momentless theory of shells in the shape of Dupin's cyclides of the forth order of the first type (Fig. 7) was offered in [52]. An article [54] devoted to momentless theory of shells in the form of the third order cyclides of the second type, Fig. 8. Methods of analysis of cyclides presented by Krishna Reddi [51-54] are based on the representation of coefficients of the first fundamental form and principal radii as

$$A = \frac{f_3(\alpha)f_4(\beta)}{f(\alpha,\beta)},\quad B = \frac{f_5(\alpha)f_6(\beta)}{f(\alpha,\beta)},\quad R_1 = f_1(\alpha)f_2(\beta),\quad R_2 = R_2(\alpha),$$

where

$$f_1(\alpha),\ f_2(\beta),\ f_3(\alpha),\ f_4(\beta),\ f_5(\alpha),\ f_6(\beta),\ f(\alpha,\beta)$$

are any functions. In this case, the system of equations of equilibrium is reduced to one resolving differential equation of the second order concerning the put function with private derivatives. This differential equation is solved in double trigonometric series for both types of Dupin's cyclides.

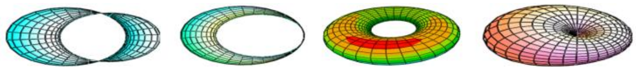
Fig. 7

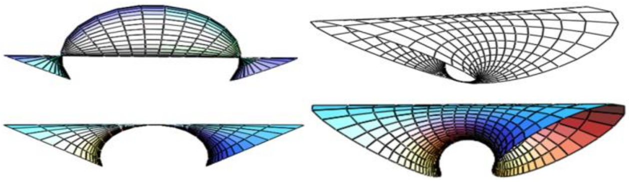

Fig. 8

Having changed coefficients f_i, S.A. Duheisat [55] has solved an additional example for a shell in the form of Dupin's cyclides with the help of momentless theories. For reception of two resolving differential equations of moment shell theory, Krishna Reddi [51] used the equations of a linear theory of shells in the complex form which have been received for the first time by V.V. Novozhilov.

Apparently, the works set forth above [50-56] limit the list of the works devoted to research of the stress-stain state of Dupin's cyclides. Dupin's cyclides of the forth order of the third type, which are circular torus, are not examined in this review.

VI. Additional information on analysis of cyclic shells

The most well-known results devoted to analysis of cyclic shells subjected to static and dynamic actions are stated above in sections 1-4. In the present section, we shall note only several works devoted to some problems of cyclic shells which have not been mentioned elsewhere.

D.V. Vainberg and V.I. Gulyaev [57] have presented analysis of a spiral shell of the circular cross-section loaded by uniform external pressure and by a temperature field. The problem was solved numerically by a method of nets. L.S. Panasyuk [58] has shown the isometric nature of helical circular cylinder and torus. The method of complex limitations, having wide application in dynamic calculations in Western European countries, has been applied in [59] to definition of natural frequencies of spiral heat exchangers.

Analysis of spiral chambers of turbines of HYDROELECTRIC POWER STATION (Fig. 9) [60-62] possesses some specificity in comparison with a moment theory analysis of cyclic shells with generatrix circles of variable radius and with a flat line of centers constructed around the circular cylinder (Fig. 10), therefore strength analysis of shells of this type is not considered in this review.

Fig. 9

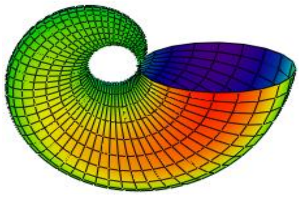

Fig. 10

Vibration of cyclic shells is examined in a monograph [3]. In particular, free vibration frequencies and corresponding mode shapes of the connecting channel for two cylindrical pipes with parallel axes and with different diameters (Fig. 11) are defined. It was established that the curvature of a centerline appreciably influences on character of change of nature frequencies and on the distribution of corresponding mode shapes. Special influence on dynamic behavior of shells renders the relation of initial and final radii of the generatrix circles. The curvature of a centerline leads to increase of value of the lowest frequency but reduction of parameter Rin / Rfi is accompanied by sharp decrease in its value. In the already mentioned monograph [3], frequencies and forms of own vibration of shells in the form of a normal cyclic surface with an elliptic centerline and with a generatrix circle of variable radius (Fig. 11) are investigated also. The deviation of an axial line from a circle leads to decrease in frequencies of own vibration both in shells with constant generatrix circles and in shells with variable radius of cross-sections. But the deviation of an axial line doesn't render essential influence on character of change of the shapes of own vibrations of these shells.

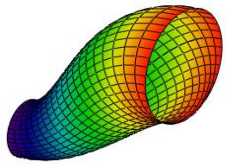

Fig. 11

VII. Conclusion

The materials contained in 62 monographs, dissertational works, scientific papers, proceedings of conferences, and listed in the review include almost all data on static and dynamic analysis of cyclic shells known in the world today.

As to engineering possibilities of application of cyclic shells, they have found wide application in curvilinear pipes (http://www.ttb.com/, Tulsa Tube Bending), basically in the form of torus, in spiral chambers of water-wheels, and in the circular translation shells used as spatial coverings of buildings.

Cyclic surfaces with a straight center line, or shells of revolution, are widely used in architecture. Sometimes architects specially underline the presence of generatrix circles in the design of buildings. As to cyclic structures of the general non-degenerate type, only single examples of their application are known. It is the authors' hope that literature resulting from the review and analysis of tendencies of development of strength analysis of cyclic shells will help architects, civil engineers, and mechanical engineers expand and diversify projected spatial structures using cyclic surfaces and will assist post-graduate students choose themes of scientific research.

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