A new form of weaker separation axioms via pgr □- closed sets

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Abstract: *The aim of this paper is to introduce and characterize pgrα- regular spaces and pgrα- normal spaces via the concept of pgrα- closed sets. It also focuses on some of its basic properties and discusses on separation axioms between pgrα-T_0 and pgrα-T_1. An attempt has been made to make a comparative study with other usual separation axioms.*
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I. Introduction

Separation axioms in topological spaces play a dominant role in analysis and are usually denoted with the letter "T" after the German "Training" which mean separation. The separation axioms that were studied together in this way were the axioms for Hausdorff spaces, regular spaces and normal spaces. Separation Axioms and closed sets in topological spaces have been very useful in the study of certain objects in digital topology [3,5]. Khalimsky, Kopperman and Meyer [4] proved that the digital line is a typical example of $T_{1/2}$ spaces. The first step of generalized closed set was done by Levine [6] in general topology which was properly placed between T_0- space and T_1-space.After the works of Levine on semi open sets, several mathematicians turned their attention to the generalization of various concepts of topology. Consequently, many separation axioms has been defined and studied. We introduce a weaker form of separation axioms called pgrα- separation axioms using the concept of pgrα- open sets introduced in [1]. In this paper the concepts of pgrα-T_0, pgrα-T_1, pgrα- regular and pgrα-normal are introduced and basic properties are discussed.

II. Preliminaries

Throughout this paper (X,τ) represents nonempty topological spaces on which no separation axioms is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. In this section, some definitions and theorems are further investigated which are used in this work.

Definition 2.1 A subset A of a space (X,τ) is called
(i) a pre open set [7] if $A\subset int(cl(A))$ and a pre closed set if $cl(int(A))\subset A$.
(ii) a α-open set[8] if $A\subset int(cl(int(A)))$ and α-closed set if $cl(int(cl(A)))\subset A$.
(iii) a regular open set if $A = int(cl(A))$ and a regular closed set if $A= cl(int(A))$.
(iv) a regular α- open set (briefly rα- open)[10] if there is a regular open set U such that $U\subset A\subset \alpha cl(U)$.

The union of all pre open sets of X contained in A is called pre- interior of A and is denoted pint(A). Also the intersection of all pre- closed subsets of X containing A is called pre- closure of A and is denoted by pcl(A). Note that $pcl(A)= A\cup cl(int(A))$ and $pint(A) =A\cap int(cl(A))$.

Definition 2.2 A subset A of a space (X,τ) is called
(i) a generalized closed set (briefly g-closed) [6] if $cl(A)\subset U$ whenever $A \subset U$ and U is open.
(ii) a generalized α-closed set (briefly gα- closed)[6] if $\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is α- open in X.
(iii) a generalized pre regular closed set (briefly gpr- closed)[2] if $pcl(A) \subseteq U$ whenever $A\subseteq U$ and U is regular open in X
(iv) a pgrα- closed set[1] if $pcl(A)\subset U$ whenever $A\subset U$ and U is regular α-open.

The complement of the above mentioned closed sets are their respective open sets.

Definition 2.3 A function $f : X \to Y$ is called
(i) pgrα- continuous [1]if for every closed set V of Y then $f^{-1}(V)$ is pgrα- closed set in X.
(ii) pre- continuous [7] if for every closed set V of Y then $f^{-1}(V)$ is pre-closed set in X.
(iii) regular- continuous [9] if for every closed set V of Y then $f^{-1}(V)$ is regular closed set in X.
(iv) gpr-continuous [2] if for every closed set V of Y then $f^{-1}(V)$ is gpr-closed set in X.
(v) gα- continuous [6] if for every closed set V of Y then $f^{-1}(V)$ is gα- closed set in X.

Definition 2.4 [1] A function $f:X\to Y$ is pgrα- irresolute if for every pgrα- open set V of Y then $f^{-1}(V)$ is pgrα-open set in X.
Definition 2.5 A space (X,τ) is called a $T_{1/2}$ space [6] (pgrα- $T_{1/2}$ space[1]) if every g-closed(resp. pgrα- closed) is closed (resp.pre closed).
Theorem 2.6 [1] A space X is pgrα- $T_{1/2}$ if and only if every singleton set is regular closed or pre open.

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III. On pgr □- T_k Spaces (K= 0, 1, 2, B, 1/2, 1/3, 3/4)

Definition 3.1 A **topological** space X is called

(i) a pgrα-T_0 if for each pair of distinct points x, y of X, there exists a pgrα- open sets G in X containing one of them and not the other.

(ii) a pgrα- T_1 if for each pair of distinct points x, y of X there exists two pgrα- open sets G_1, G_2 in X such that $x \in G_1$, $y \notin G_1$, and $y \in G_2$, $x \notin G_2$.

(iii) a pgrα- T_2 (pgrα- Hausdorff) if for each pair of distinct points x, y of X there exists distinct pgrα- open sets H_1 and H_2 such that H_1 containing x but not y and H_2 containing y but not x.

Theorem 3.2

(i) Every T_0–space is pgrα- T_0 space.

(ii) Every T_1- space is pgrα- T_0 space.

(iii) Every T_1 space is pgrα- T_1 space.

(iv) Every T_2 space is pgrα-T_2 space.

(v) Every pgrα- T_1 space is pgrα- T_0 space.

(vi) Every pgrα-T_2 space is pgrα-T_1 space.

Proof: Straight forward.

The converse of the theorem need not be true as in the examples.

Example 3.3 Let X= {a, b, c} and τ= {X,ϕ, {a},{b},{a, b}}.

Then PGRαC(X) = {ϕ, X, {a, b}, {c}, {a, c}, {b, c}}.

Here (X,τ) is pgrα-T_0 space but not T_0 space and pgrα-T_1space.

Example 3.4 Let X= {a, b, c} and τ= {X, ϕ, {a, b}}.

Then PGRαC(X) = {X,ϕ,{a}, {b},{c},{a, b},{b, c},{a, c}. Here (X, τ) is pgrα-T_1 space but not T_1 space and pgrα-T_2 space.

Example 3.5 Let X= {a, b, c} and τ is indiscrete topology on X, then (X, τ) is pgrα-T_2 but not T_2 space.

The following diagram shows the relation between usual separation axiom and pgrα- separation axiom.

$$\begin{array}{ccccc} T_0 \leftarrow & & T_1 & \leftarrow & T_2 \\ \downarrow & & \downarrow & & \downarrow \\ \text{pgr}\alpha\text{-}T_0 \leftarrow & & \text{pgr}\alpha\text{-}T_1 & \leftarrow & \text{pgr}\alpha\text{-}T_2 \end{array}$$

Theorem 3.6 Let X be a topological space and Y is an pgrα- T_2 space. If f: X → Y is injective and pgrα- irresolute then X is pgrα- T_2 space.

Proof: Suppose x, y∈X such that x≠y. Since f is injective then f(x)≠f(y).

Since Y is pgrα-T_2 space then there are two pgrα- open sets U and V in Y such that f(x)∈U , f(y)∈V and U∩V=ϕ.

Since f is pgrα- irresolute then $f^{-1}(U)$, $f^{-1}(V)$ are two pgrα- open sets in X, $x \in f^{-1}(U)$, $y \in f^{-1}(V)$,$f^{-1}(U) \cap f^{-1}(V)$ =ϕ. Hence X is pgrα- T_2 space.

Theorem 3.7 Let X be a topological space and Y is an T_2 space .If f: X → Yis injective and pgrα- continuous then X is pgrα-T_2 space.

Proof: Suppose x, y∈X such that x≠ y. Since f is injective, then f(x)≠ f(y).

Since Y is an T_2 space, then there are two open sets U and V in Y such that f(x)∈U, f(y)∈V and U∩V =ϕ. Since f is pgrα-continuous then $f^{-1}(U)$, $f^{-1}(V)$ are two pgrα- open sets in X. Then $x \in f^{-1}(u)$, $y \in f^{-1}(V)$, $f^{-1}(U) \cap f^{-1}(V)$ =ϕ. Hence X is pgrα-T_2 space.

Definition 3.8 The intersection(resp. union) of all pgrα- closed (resp.pgrα-open) sets each containing in (resp. contained) a set A in a space X is called pgrα- closure (resp.pgrα-interior)of A and it is denoted by pgrα-cl(A) (resp. pgrα-int(A)).

Remark 3.9 Let X be a topological space such that A⊂X then pgrα-cl(A) is contained in every pgrα-closed set containing A.

Theorem 3.10 Let X be a topological space and A⊂B⊂X then

(i) pgrα-cl(A) is the smallest pgrα-closed set which contains A.

(ii) pgrα-cl(A) ⊂ pgrα-cl(B).

(iii) A is an pgrα- closed set if and only if pgrα-cl(A) = A.

(iv) pgrα-cl(pgrα-cl(A)) = pgrα-cl(A)

Theorem 3.11(X, τ) is pgrα- T_0 space if and only if for each pair of distinct x, y of X, pgrα-cl({x}) ≠ pgrα-cl({y}) .

Proof: Let (X,τ) be a pgrα-T_0 space. Let x, y ∈ X such that x≠y, then there exists a pgrα- open set V containing one of the points but not the other, say x∈V and y∉ V. Then V^c is a pgrα- closed containing y but not x. But pgrα-cl({y}) is the smallest pgrα- closed set containing y.

Therefore pgrα-cl({y}) ⊂V^c and hence x∉ pgrα- cl({y}).

Thus pgrα-cl({x}) ≠pgrα-cl({y}).

Conversely, suppose x, y ∈ X, x≠ y and pgrα-cl({x}) ≠ pgrα-cl({y}). Let z∈ X such that z ∈pgrα-cl({x}) but z∉ pgrα-cl({y}). If x∈ pgrα-cl({y}) then pgrα-cl({x}) ⊂ pgrα-cl({y}) and hence z∈pgrα-cl({y}). This is a contradiction. Therefore x∉ pgrα-cl({y}). That is x∈ $(\text{pgr}\alpha\text{-cl}(y))^c$.

Therefore $(\text{pgr}\alpha\text{-cl}(\{y\}))^c$ is a pgrα- open set containing x but not y. Hence (X,τ) is pgrα-T_0 space.

Theorem 3.12 A topological space X is pgrα- T_1 space if and only if for every x∈ X singleton {x} is pgrα- closed set in X.

Proof: Let X be pgrα- T_1 space and let x∈X, to prove that {x} is pgrα- closed set. We will prove X- {x} is pgrα- open set in X. Let y ∈ X-{x}, implies x≠y ∈ X and since X is pgrα-T_1 space then their exit two pgrα- open sets G_1, G_2 such that x∉G_1, y∈ G_2 ⊆ X-{x}.
Since y∈ G_2 ⊆ X-{x} then X-{x} is pgrα- open set. Hence {x} is pgrα- closed set.
Conversely, Let x≠y ∈X then {x}, {y} are pgrα- closed sets. That is X-{x} is pgrα- open set.
Clearly, x∉ X-{x} and y∈ X-{x}.Similarly X-{y} is pgrα- open set, y∉X-{y} and x∈X-{y}. Hence X is pgrα- T_1 space.
Theorem 3.13 For a topological space (X, τ), the following are equivalent
(i) (X, τ) is pgrα- T_2 space.
(ii) If x∈X , then for each y≠ x , there is a pgrα- open set U containing x such that y∉pgrα-cl(U)

Proof: (i)⟹(ii) Let x∈X. If y∈X is such that y≠x there exists disjoint pgrα-open sets U and V such that x∈U and y∈V. Then x∈U ⊂X-V which implies X-V is pgrα- open and y∉X-V.Therefore y∉pgrα-cl(U).
(ii) ⟹(i) Let x, y∈ X and x ≠y. By (ii) ,there exists a pgrα- open U containing x such that y∉pgrα-cl(U).
Therefore y∈ X- (pgrα-cl(U)). X- (pgrα-cl(U)) is pgrα-open and x∉ X- (pgrα-cl(U)). Also U∩X-(pgrα- cl(U))= ϕ.
Hence (X,τ) is pgrα-T_2 space.

As application of pgrα- closed sets, four spaces namely, pgrα-$T_{1/2}$ spaces, pgrα-$T_{1/3}$ spaces, pgrα-T_b spaces, pgrα-$T_{3/4}$ spaces are introduced. The following implication diagram will be useful in this paper.
Diagram 3.14
regular closed⟶pre closed→pgrα- closed →gpr- closed
↑
gα- closed
Examples can be constructed to show that the reverse implications are not true. This motivates us to introduce the following spaces.
Definition 3.15 A space (X,τ) is called pgrα-$T_{1/3}$ if every gpr- closed set is pgrα- closed.
Definition 3.16 A space (X,τ) is called pgrα-T_b if every pgrα- closed set is regular closed.
Definition 3.17 A space (X,τ) is called pgrα-$T_{3/4}$ if every pgrα-closed set is gα- closed.
Theorem 3.18
(i) Every pgrα-T_b space is pgrα-$T_{1/2}$ space.
(ii) Every pgrα-T_b space is pre regular $T_{1/2}$ space.
(iii) Every pre regular $T_{1/2}$ space is pgrα- $T_{1/3}$ space.
(iv) Every pgrα- $T_{3/4}$ space is pgrα- $T_{1/2}$ space.
Proof: Straight forward.

The converse of the theorem need not be true as in the examples
Example 3.19 Let X= {a, b, c} and τ= {ϕ,X,{c},{a, b}}.Then (X,τ) is a pgrα-$T_{1/2}$ and pre regular-$T_{1/2}$but not pgrα-T_b space.
Example 3.20 Let X={a, b, c} and τ= {X,ϕ,{a, b}}. Then (X,τ) is pgrα-$T_{1/3}$ but not pre-regular $T_{1/2}$ space.
Example 3.21 Let X= {a, b, c, d} and τ={ϕ,X,{a, b},{b, c, d}}.Then (X, τ) is pgrα- $T_{1/2}$ but not pgrα- $T_{3/4}$ space.
Theorem 3.22 Let X be a pgrα- $T_{1/3}$ space .Then X is pgrα-$T_{1/2}$ if and only if it is pre regular $T_{1/2}$ space.
Proof: Suppose X is pgrα- $T_{1/2}$ and pgrα- $T_{1/3}$ space. Let A be gpr- closed set in X. then A is pgrα- closed set.
Since X is pgrα- $T_{1/2}$ space, then A is pre closed. Therefore X is pre regular $T_{1/2}$ space.
Conversely, we assume that X is pre regular $T_{1/2}$ space.
Suppose A is pgrα- closed set. Since every pgrα- closed set is gpr- closed set, and then A is gpr- closed set.
Since X is pre regular $T_{1/2}$ space then A is pre closed. This proves that X is pgrα- $T_{1/2}$space.

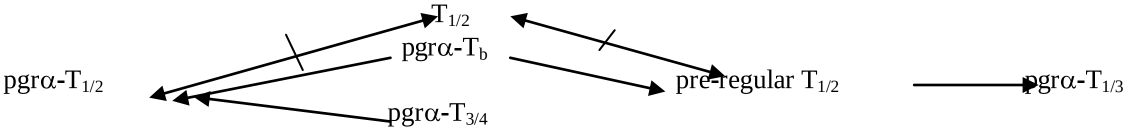
Theorem 3.23
(i) If (X, τ) is an pgrα-$T_{1/3}$ space then for each x∈X , {x} is either regular closed or pgrα- open.
(ii) If (X, τ) is an pgrα- T_b space then for each x∈X, {x} is either regular closed or regular open.
(iii) If (X, τ) is an pgrα- $T_{3/4}$ space then for each x∈X, {x} is either regular closed or gα- open.
Proof: Straight forward.

Theorem 3.24
(i) **I**f X is pgrα-$T_{1/2}$ then every pgrα- continuous functions is pre continuous.
(ii) If X is pgrα- $T_{1/3}$ then every gpr-continuous function is pgrα- continuous.
(iii) If X is pgrα-T_b then every pgrα- continuous function is regular- continuous.
(iv) If X is pgrα- $T_{3/4}$ then every pgrα- continuous function is gα- continuous.
Proof: Straight forward.

Theorem 3.25 If X is pre-regular $T_{1/2}$ and f: X→Y then the following are equivalent
(i) f is gpr- continuous.
(ii) f is pre- continuous.
(iii) f is pgrα- continuous.

Proof: Suppose f is gpr-continuous. Let A⊆Y be closed. Since f is gpr-continuous then $f^{-1}(A)$ is gpr- closed in X. Since X is pre regular $T_{1/2}$ space then $f^{-1}(A)$ is pre-closed.
Therefore f is pre-continuous. This proves (i) implies (ii).
Suppose f is pre-continuous. Let A⊆Y be closed. Since f is pre- continuous then
f^{-1} (A) is pre-closed in X. We have $f^{-1}(A)$ is pgrα- closed. Therefore f is pgrα-continuous. This proves (ii) implies (iii).
Suppose f is pgrα- continuous. Let A⊆Y be closed. Since f is pgrα- continuous then $f^{-1}(A)$ is pgrα-closed. Since every pgrα-closed set is gpr- closed then $f^{-1}(A)$ is gpr-closed. Therefore f is gpr-continuous. This proves (iii) implies (i).

A→B we mean A implies B but not conversely and A↮B means A and B are independent of each other.
Diagram 3.26



IV. pgr □- regular spaces and pgr □-normal spaces

Definition 4.1 A topological space X is said to be an pgrα- regular space if for every pgrα-closed set F and each point x of X which is not in F, there exists disjoint pre-open sets U and V such that x∈U , F⊆V and U∩V=ϕ.
Definition 4.2 A topological space X is said to be an pgrα- normal space if for every pair of disjoint pgrα-closed sets F_1 and F_2 in X , there exist disjoint pre- open sets U and V such that F_1⊂U, F_2⊂V, U∩V=ϕ.
Theorem4.3 Let (X, τ) be a topological space .Then the following statements are equivalent
(i) (X,τ) is pgrα- regular space.
(ii) For each point x∈X and for each pgrα-open neighborhood W of x , there exists a pre- open set of x such that pcl(V)⊆W.
(iii) For each point x∈X and for each pgrα-closed not containing x, there exists a pre-open set V of x such that pcl(V) ∩F=ϕ

Proof: Let W be a pgrα-open neighborhood of x. Then there exists a pgrα-open set G such that x∈G⊆W. Since G^c is pgrα-closed set and x∉G^c, by hypothesis there exists pre-open sets U and V such that G^c⊆U, x∈V and U∩V =ϕ and so V⊂U^c. Now pcl(V)⊆pcl(U^c)= U^c and G^c ⊆U implies U^c⊆G⊆W. Therefore pcl(V)⊆W. Hence (i) implies (ii).
Let F be any pgrα-closed set and x∉F .Then x∈F^c and F^c is pgrα-open and so F^c is an pgrα- open neighborhood of x .By hypothesis ,there exists a pre-open set V of x such that x∈V and pcl(V)⊆F^c, which implies F⊆$(pcl(V))^c$. Then $(pcl(V))^c$ is a pre-open set containing F and V∩$(pcl(V))^c$ = ϕ. Therefore X is pgrα- regular space. Hence (ii) implies (i).
Let x∈X and F be an pgrα-closed set such that x∉F. Then F^c is an pgrα-open open neighborhood of x and by hypothesis there exists a pre-open set V of x such that pcl(V)⊆F^c and therefore pcl(V)∩F=ϕ. Hence (ii) implies (iii).
Let x∈X and W be an pgrα-open neighborhood of x. Then there exists an pgrα-open set G such that x∈G⊆ W. Since G^c is pgrα-closed and x∉G^c, by hypothesis there exists a pre-open set V of x such that pcl (V)∩G^c=ϕ .Therefore pcl(V)⊆G⊆W. Hence (iii) implies (ii).

Theorem4.4 A topological space X is an pgrα- regular space if and only if given any x∈X and any open set U of X there is pgrα-open set V such that x∈V⊂pgrα-cl(V)⊆U.
Proof: Let U be an open set, x∈U.
So U^c is closed set such that x∉U. Since X is a pgrα-regular space then there exist pgrα-open sets V_1 and V_2 such that $V_1∩V_2$ =ϕ, U^c⊂V_2, x∈V_1.Since $V_1∩V_2$ =ϕ, we have pgrα-cl(V_1) ⊂pgrα-cl(V_2^c)=V_2^c.
Since U^c⊂V_2, we have V_2^c⊂ U. Hence we have x∈V_1⊂pgrα-cl(V_1)⊂V_2^c⊆U.
Conversely, let F be a closed set in X and x∈X-F. So F^c is an open set such that x∈F^c.
Hence there exist a pgrα- open set U such that x∈U ⊂pgrα-cl(U)⊆F^c. Let V=X-pgrα-cl(U).So V is a pgrα-open set which contains F and U∩V=ϕ.Hence X is an pgrα-regular space.
Theorem 4.5 Let X and Y be topological spaces and Y is a regular space. If f: X →Y is closed, pgrα- irresolute and one to one then X is an pgrα- regular space.

Proof: Let F be closed set in X, x∉F. Since f is closed mapping, then f(F) is closed set in Y, f(x)= y∉ f(F). But Y is pgrα- regular space then there are two pgrα- open sets U and V in Y such that f(F)⊆V, y∈U , U∩V=ϕ. Since f is pgrα-irresolute mapping and one to one so $f^{-1}(U)$, $f^{-1}(V)$ are two pgrα- open sets in X and x∈$f^{-1}(U)$, F⊂$f^{-1}(V)$, $f^{-1}(U)∩f^{-1}(V)$= ϕ.
Hence X is pgrα- regular space.
Theorem 4.6 A topological space X is said to be an pgrα- normal space if and only if for every closed set F and for every open set G contain F there exists pgrα- open set U such that F⊂U⊂pgrα-cl(U)⊂G.
Proof: Let F be a closed set in X and G be an open set in X such that F⊂G, G^c is a closed set and G^c∩F=ϕ. Since X is pgrα-normal space then there exist pgrα- open sets U and V of X such that U∩V=ϕ, G^c⊂V and F⊂U , U⊂V^c.
We have pgrα-cl(U)⊂pgrα-cl(V^c) = V^c. Hence F⊂U⊂pgrα-cl(U)⊂V^c⊂G.
Theorem 4.7 Let f be a closed and pgrα- irresolute mapping from a topological space X into a topological space Y. If Y is pgrα- normal, so is X.

Proof: Let F_1 and F_2 be closed sets in X such that $F_1 \cap F_2 = \phi$.
Since f is a closed map, we have f (F_1), f (F_2) are two closed sets in Y and $f(F_1) \cap f(F_2) = \phi$.
Since Y is pgrα- normal and f is pgrα- irresolute then there exists two pgrα- open sets U,V in Y such that $f(F_1) \subset U$, $f(F_2) \subset V$, $U \cap V = \phi$, also $f^{-1}(U)$, $f^{-1}(V)$ are pgrα- open sets in X and $F_1 \subset f^{-1}(U)$, $F_2 \subset f^{-1}(V)$, $f^{-1}(U) \cap f^{-1}(V) = \phi$. Hence X is pgrα-normal.

Theorem 4.8 Let X be a topological space. If X is a pgrα- regular and a T_1 space then X is an pgrα-T_2 space.
Proof: Suppose x, y ∈X such that x≠y. Since X is T_1- space then there is an open set U such that x∈U, y∉U.
Since X is pgrα- regular space and U is an open set which contains x, then there is pgrα- open set V such that x∈V⊂pgrα-cl(V) ⊆U.Since y∉U, hence y∉pgrα-cl(V).Therefore y∈X -(pgrα-cl(V)).Hence there are pgrα-open sets V and X- (pgrα-cl(V)) such that (X- (pgrα-cl(V)))∩V = ϕ.Hence X is pgrα-T_2 space.

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