International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol. 2, Issue. 5, Sept.-Oct. 2012 pp-2986-2991 ISSN: 2249-6645

Structural complex configuration plate mathematical modeling and optimization

Sh. A. Nazirov, S. H. Yakubov
Tashkent University of information technologies, Karshi State University

Abstract: *This paper examines the optimization problem of lamellar complex configuration structures. The results of a complex configuration plate structures' weight optimization calculation.*
Keywords: *Mathematical modeling, optimization, plate design, a complex form, methods of solution, R-function, various methods, numerical experiments, software package.*

I. Introduction

In the design of various engineering structures, namely the construction of facilities, aircraft, missile, ship, etc. - there are problems of complex configuration (not rectangular plate shape, with cut-outs, multiply, etc.) lamellar structural elements calculation and optimization. The mathematical complexity of the calculation of these arbitrary shape plate elements, especially their optimization, resulted in a significant research and publications backlog on these issues from the calculation and optimization of the "tra

II. Statement of the Problem

The problem of engineering design optimizing will be put as mathematical programming problem: it is necessary to determine the vector $X(x_1, x_2, \dots, x_n)$ optimized parameters x_i $(i=1,\overline{n})$, giving the objective function F(x) extreme (for definiteness, we take min), keeping restrictions on the parameters $a_i \le x_i \le b_i$, $(i=1,\overline{n})$ and functional limitations $f_j(x) \le 0$ $(j=1,\overline{m})$. This problem can be written

$$\left.\begin{aligned} &F(X) \to \min, \\ &f_j(X) \le 0 \quad (j=1,\overline{m}), \\ &a_i \le x \le b_i \quad (i=1,\overline{n}). \end{aligned}\right\} \qquad (1)$$

We shall consider equation (1) in details. The most commonly accepted parameters to be optimized in the structural elements are geometric (plate thickness h, curvature radius R_1, external and internal edges, cutouts, etc.) and physical (elastic modulus E, etc.). The lower a_i and upper b_i meanings of x_i limits of parameters are defined on the basis of design and technological, operational, etc. requirements. For the objective function F(X) the most commonly accepted parameters are: weight, materials consumption, the cost of construction.

The main functional constraints $f_j(X) \le 0$ $(j=1,\overline{m})$ for engineering structures, subjected to various external influences, optimization, are the following.

1. Stress state restrictions :

$$\max \sigma^{(\psi)}_{экв}(X) \le [\sigma]^{\psi} \qquad (\psi=1,n). \qquad (2$$

Here ψ is the number of variants of the design impact; $\max \sigma^{(\psi)}_{eq}(X)$ - the maximum equivalent structural stress, defined according to the accepted hypothesis, or theory of strength, with ψ - м version of the impact, $[\sigma]^{\psi}$ - the allowable stress for the material of construction in the - m option exposure.

To the canonical form (1) restrictions (2) come as follows:

$$F_1(X) = \max \sigma^{(\psi)}_{eq}(X) - [\sigma]_{\psi} \le 0.$$

2. Deformed state restrictions :

$$\max \left| u^{(\psi)}(X) \right| \le [u]^{\psi},$$

where $\max \left| u^{(\psi)}(X) \right|$ - the maximum the surface structure displacement with m -option impact, $[u]^{\psi}$ - allowable surface structure displacement.

In the canonical form:

$$F_2(X) = \max \left| u^{(\psi)}(X) \right| - [u]^{\psi} \le 0.$$

3. Stability conditions:

$$P\psi \le P_{cr},$$

where $P\psi$ - compressive force with ψ -effect, P_{cr} - the critical force on the structure.
In the canonical form:
$F_3(X) = P\psi - P_c \le 0$.

4. Restrictions on the natural oscillations frequency. The variable (periodic) loads activity at a certain frequency demands to analyze natural frequency constraints :

$$\min\{\ w_i^{(\psi)}(X)\ \} \ge [w]^{\psi} \qquad (i = 1,2,\ldots),$$

where the $\min\{w_i^{(\psi)}(X)\}$ is the lowest natural ψ -x oscillation frequency, $[w]^{\psi}$ -the lowest allowed natural ψ -x oscillation frequency, appointed as the calculated value of the compelled ψ vibrations.
In the canonical form:
$F_4(X)=[w]\psi - \min\{w_i^{(\psi)}(X)\} \le 0$

5. Mechanical vibrations amplitude constraints:

$$a_0^{(\psi)}(X,w_i) \le [a_0(w_i)^{\psi}],$$

where $a_0^{(\psi)}(X,w_i)$ is the maximum forced ψ x- oscillations amplitude with the w_i frequency; and $[a_0(w_i)^{\psi}]$ is the permissible amplitude.
The above mentioned restrictions are most common in the structures ' optimization, but the certain structures optimization solving tasks may require additional structural, technological, operational and other constraints.
Problem (1) of complex configuration structures engineering optimization is non-linear programming problem, which has a number of specific features. First, the calculation of the objective function (weight, cost) needs much less time than to check the restrictions, which require the construction calculation direct task solving, Second, the global minimum will always be at a border or at their junction, otherwise we will have a stockpile of material, that can be removed without violating the conditions of strength, stiffness, stability, etc. Third, the form of the σ (X), u(X), P_c(X) etc. functions are a priori unknown and can only be determined numerically. Thus, to solve the problem (1) we shall apply the algorithms described in [2-15] taking into account the above features. The algorithm has high convergence speed and reliability.

III. Calculation methods

Let us consider the methods for solving the direct problem of calculation.
It is known that the equations of equilibrium, fluctuations and the stability of anisotropic plates, according to the moments are [1]:

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} = q_1(x,y), \qquad (3)$$

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + h(\sigma_x \frac{\partial^2 W}{\partial x^2} + \sigma_y \frac{\partial^2 W}{\partial y^2} + 2\sigma_{xy}\frac{\partial^2 W}{\partial x \partial y}) = 0 \qquad (4),$$

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + m\frac{\partial^2 W}{\partial t^2} = q_2(x,y,t). \qquad (5)$$

Here, W- plate deflection, M1, M12, M2 - the bending and tensional moments, $m = \gamma\ h / g$, γ - the volume weight per unit, g - plate gravity acceleration, h - thickness.
Relations for the M1, M12, M2, when the plate is isotropic, orthotropic and anisotropic, are given in [1].
Substituting in (3), (4) the ratio of M1, M12, M2, when the plate is isotropic, orthotropic, or in other cases of anisotropy, it is possible to obtain the corresponding equations. These equations are given in many textbooks on the theory of elasticity [1-2].
Equations (3), (4) are supplied with the boundary conditions, and equation (5) –with both boundary and initial conditions.
Here are the types of encountered boundary conditions frequencies [1]:

a) rigidly clamped-edge

$$W|_{\Gamma=0}\ _{\Gamma=0}, \frac{\partial W}{\partial n}|_{\Gamma} = 0$$

b) free-simply supported edge

$$W|_{\Gamma=0},\ M_n|_{\Gamma} = (M_1\cos^2\alpha + M_{12}\cos\alpha\sin\alpha + M_2\sin^2\alpha)|_{\Gamma} = 0$$

where $\alpha = (n \wedge ox)$ and $\beta = (n \wedge oy)$ are the angles between the normals, relatively the axis Ox, Oy;

c) the free edge

$M_n\big|_{\Gamma=0}$, $(\theta_n + \frac{\partial}{\partial S} M_{n\tau})\big|_{\Gamma} = 0$

Where $\theta_n = M_1 \cos\alpha + M_2 \cos\beta$, $M_\tau = (M_2 - M_1)\cos\alpha\cos\beta + M_{12}(\cos^2\alpha - \cos^2\beta)$.

In addition, there are possible combinations of these boundary conditions, depending on the plates edges fixing method.
The initial conditions for equations (5) have the form

$$W(x,y,t)\big|_{t=t_0} = W_0(x,y), \quad \dot{W}(x,y,t)\big|_{i=t_0} = \dot{W}_0(x,y).$$

The formation of the matrix to solve the above problems is carried out by V.L.Rvachev's R function [12] and Bubnov-Galerkin 's [3-4] method s combination.
It should be noted that the direct application of the Bubnov-Galerkin method to solve equations (3), (4), (5) leads to computational difficulties. In this work further for the formation of resolving equations elements we shall use the method proposed in [12].
Here the application of the R – functions method is associated with the coordinate sequences construction, that will satisfy the boundary conditions without any approximations.
Coordinate sequences that satisfy the boundary conditions can be represented as an expansion

$$W = \sum_{i=1}^{n} T_i(t) B(\omega, \phi_i) = \sum_{i=1}^{n} T_i(t) W_i(x,y), \qquad (6)$$

where Ti (t) are unknown function of time, to be determined;$\{W_i(x,y)\}$ - a complete, linearly independent system of functions, which we will build, using V.L.Rvachev's R - functions method [12].
Note that in the case of static's in the representation (5) instead of $T_i(t)$ function the unknown coefficients C_i will occure.
Substituting (6) to (3) - (5) and performing the usual procedure of the Bubnov-Galerkin method, we obtain the following equation:

$$AC=B, \qquad (7)$$

$$A-\lambda B=0, \qquad (8)$$

$$M\ddot{T} + AT = F, \qquad (9)$$

$$T(t_0) = T_0, \quad \dot{T}(t_0) = \dot{T}_0,$$

where

$$b = \{b_{ij}\} = \frac{1}{S}\left\{\iint_\Omega q_1 W_i d\Omega\right\}, \quad \{a_{ij}\} = \frac{1}{S}\left\{\iint_\Omega f_{ij} d\Omega\right\}, \text{A=}$$

$$\text{B=}\{b_{ij}\} = \frac{1}{S}\left\{\iint_\Omega \varphi_{ij} d\Omega\right\}, \quad F = \{f_i\} = \frac{1}{S}\left\{\iint_\Omega q_2 W_i d\Omega\right\},$$

$$M = \{m_{ij}\} = \frac{1}{S}\left\{\iint_\Omega m W_i W_j d\Omega\right\}, \quad T(0) = \mu^{-1} T_1(t_0), \dot{T}(0) = \mu^{-1} T_2(t_0)$$

$$T_1(t_0) = \frac{1}{S}\left\{\iint_\Omega W_0 W_i d\Omega\right\}, \quad T_2(t_0) = \frac{1}{S}\left\{\iint_\Omega \dot{W}_0 W_i d\Omega\right\},$$

$$f_{ij} = \left(\frac{\partial^2 W_i}{\partial x^2} + \nu \frac{\partial^2 W_i}{\partial y^2}\right)\frac{\partial^2 W_j}{\partial x^2} + \left(\frac{\partial^2 W_i}{\partial y^2} + \nu \frac{\partial^2 W_i}{\partial x^2}\right)\frac{\partial^2 W_j}{\partial y^2} + \frac{\partial^2 W_i}{\partial x \partial y} \cdot \frac{\partial^2 W_j}{\partial x \partial y}$$

$$\varphi_{ij} = \left(\sigma_x \frac{\partial^2 W_j}{\partial x^2} + \sigma_{xy} \frac{\partial^2 W_j}{\partial x \partial y} + \sigma_y \frac{\partial^2 W_j}{\partial y^2}\right) W_i,$$

To solve the system of equations (7) Gaussian elimination or the method of least squares and other methods depending on the properties of the matrix are applied. To determine the critical load the QL – method is applied.
We find the solution of equation (8) under condition (9) with the help of a variety of numerical methods: for example, by the central difference method or the Newmark method, or the method of quadrature sums or,others [4].
It should be noted that in the formation of the matrix, computation of the coordinate functions and their the n-th order derivatives' values, is carried out by the card operations [5, 12]. Here the integrals are computed by the n-point Gauss formula [4].

The above-described numerical algorithm allows to optimize the plate-like structure s of both constant and variable thickness.

Thus, the computational algorithm of plate structures optimization consists of the following steps:
1. Objective function formation.
2. Functional limitations formation.
3. The parameters restrictions formation.
4. The direct calculation.
5. Strength, stiffness, stability and other conditions checking.

In its turn, the direct phase calculation consists of:
- Constructing a sequence of coordinate functions, satisfying the boundary conditions of the problem;
- The solving equation matrix elements formation ;
- The equation calculation.

It should be noted that the resolving equations can be algebraic or differential, depending on the problems considered in the static or dynamic formulation.
As mentioned above, the problem of engineering design optimizing will be put as a problem of mathematical programming. Starting from the equation (1), we consider the optimization of weight plates, where $F(X)$ is the weight of a plate of isotropic material under the action of the external load *q*. Functional limitations, taking into account in the engineering designs optimization , as well as a numerical optimization algorithm of complex configuration lamellar structures are described in detail in [4-9,11,14,15]. As the optimized option we take the plate thickness, constant in the plate range.

IV. Experimental calculations.

Task 1. The tightly clamped round the whole contour plate under uniform external pressure q = 10 kg. weight optimization. The radius of the plate is R = 100 cm, elastic modulus is $E = 2^{10} \cdot 6\,кг/см^2$, Poisson's ratio is $\nu = 0.3$,permitted deflection is $[W] = 1\,\text{см}$ cm and equivalent stress is $[\sigma_{\acute{y}\hat{e}\hat{a}}] = 2550\ \hat{e}\tilde{a}/\tilde{n}\grave{i}^{\ 2}$, the gravity (specific weight) is $\gamma = 7 \cdot 8\,г/см^3$, $G(h) = \pi R^2 \gamma\, h \to \min,\ W_{\max} \le [W],\ \sigma_{\max}^{экв} \le [\sigma_{экв}],\ 1см \le h \le 10\,см.$

Optimization was carried out up to $\varepsilon = 0.01$. We obtained the following results:

$$G_{\min} = 1141.19\,кг,\quad h = 4.6571\,см,$$

$$W_{\max} = 0.8206\,см,\quad \sigma_{\max}^{экв} = 2549.99\,кг/см^2.$$

This problem has an exact solution:

$$W_{\max} = \frac{qR^4 12(1-\nu^2)}{64Eh^3}.$$

With the calculated value of h, we have :

$$W_{\max} = 0.8446\ см.$$

The accuracy of the obtained approximate solution is satisfactory.
Task 2. Optimization of the entire ring rigidly clamped at both the contours plate under uniform external pressure intensity $q = 10\ кг/см^2$.
The outer radius of the plate R = 100 cm, inner - r = 50 cm. The other parameters are the same as in Task 1:

$$G(h) = \pi\,\gamma\,h\left(R^2 - r^2\right) \to \min$$

The results are :

$$G_{\min} = 382.95\,\hat{e}\tilde{a},\ h = 2.0837\ \tilde{n}\grave{i}\ ,\ \sigma_{\max}^{\acute{y}\hat{e}\hat{a}} = 2549.82\ \hat{e}\tilde{a}/\tilde{n}\grave{i}^{\ 2}$$

Task 3. Optimization round the whole ring (Fig. 1). All parameters are the same as in Task 2. We obtained the following results:

. $G_{\min} = 134.48\,кг,\ h = 1.126\ см,\ \sigma_{\max}^{экв} = 2549.99\,кг/см^2$

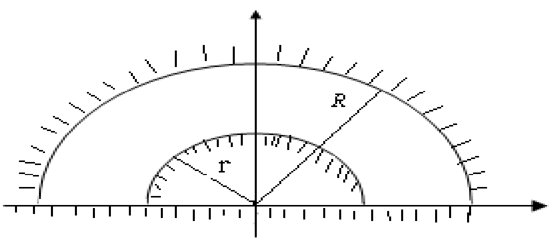

Fig. 1

Task 4. A square with round neck weight optimization (Fig. 2).
Initial data for the square with a round neck weight optimization calculation:

$$G(h)=(ab-\pi r^2)\gamma h,\ a=200\,см,\ b=200\,см,\ r=50\ см,\ q=10\ кг/см^2,$$

$$W_{\max}=[W],\ \sigma_{\max}^{экв}\le[\sigma_{экв}],1см\le h\le 10\,см.$$

The results of the calculation: $G_{\min}=296.07\,кг,\ h=3.9767\ см,\ \sigma_{\max}^{экв}=2546.07\ кг/см^2.$

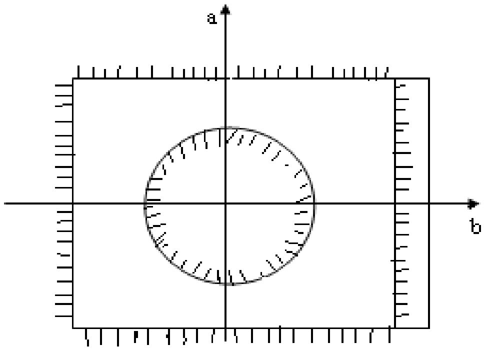

Fig.2

Task 5. Weight optimization of the figure shown in Fig. 3 Baseline data:

$$G(h)=(ab-3\pi r^2)\gamma h,\ a=200\text{ñì},\ b=200\text{ñì},\ r=20\text{ñì},$$

$$W_{\max}\le[W],\ \sigma_{\max}^{\text{ýêâ}}\le[\sigma_{\text{ýêâ}}],1\text{ñì}\le h\le 10\text{ñì}.$$

We obtained the following results:

$$G_{\min}=725.96\,кг,\ h=2.5689\ см,\ W_{\max}=0.1262\ см,\ \sigma_{\max}^{экв}=2549.9998\ кг/см^2.$$

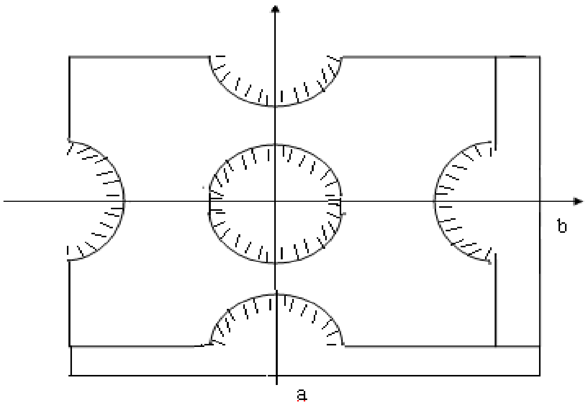

Fig. 3

Tasks 1 and 2 have the exact solutions and are given only for the algorithm [5] performance monitoring possibility. According to the solved problems, the main limitation (with the taken values $[\sigma_{экв}],\ [\sigma]\ u\ [W]$) is the strength limitation, and the algorithm provided a high degree approximation to the boundary. A variety of forms of plates indicates wide opportunity of applying the algorithm to solve optimization problems of plates of complex configuration [4, 15].

V. Conclusion

Thus, the proposed technique allows to optimize on the weight the calculation experiments of complex configuration design plates.

VI. References

[1] Ambartsumyan S.A. Theory of anisotropic plates: strength, stability and fluctuations. // 2nd ed. revised. and add. - Moscow: Nauka, 1987. - 360 p.

[2] Vlasov V.Z . The general theory of shells. -M. Fizmatgiz, 1962.-T. A.

[3] Kabulov V.K. Algorithmic in continuum mechanics Avg.-Fan, Tashkent, 1980.-304 p.

[4] Kabulov V.K., Nazirov SH.A., Yakubov S.H. Algorithm of solving optimization problems. - Fan, Tashkent, 2008. - 204 p.

[5] Nazirov SH.A., Piskorskii L.F. A numerical algorithm for optimization of plate structures with a complex configuration // Questions of High and Appl. Mathematics.": Science works coll., Tashkent, IR AS RU, 1995. - Issue 100. - C. 49-56.

[6] Nazirov SH.A., Piskorskii L.F. Software for calculation and optimization of engineering structures of complex configuration // Mathematical modeling and computational experiment Conf. report. Tashkent, 1994 - C. 215.
[7] Nazirov SH.A., Piskorskii L.F. Software for calculation and optimization of plate structures with complex shape // Algorithms. Science works coll. - Tashkent, IR AS RU, 1995. - Vyp.80. - C. 41-54.
[8] Nazirov SH.A., Piskorskii L.F, Bobokulov SH.O, Ngo Ngoc Hung. Optimization of elastic plates of complex configuration // Issues. Comput. and Appl. Mathematics. Science research, Tashkent, IR AS RU, 1995. - Vyp.99. - C. 41-45.
[9] Nazirov Sh.A, Piskorskii L.F, Bobokulov SH.O. Algorithmic optimization problems in the theory of elasticity // "Problems of Informatics and Energy" magazine. Tashkent, 1997. - № 3.
[10] .Nazirov SH.A., Yakubov S.H. The algorithmic system that automates the process of optimization for the design of engineering structures and buildings // The Rep. Uz. State Patent Office. Certificate, DGU 01422. 13.11. In 2007.
[11]. Nazirov S.A, Yakubov S.H. The development of algorithmic solutions of classes of optimization problems // Strategy and the development of science and technology in the twenty-first century. Proceedings of the science practical conference. Bukhara, 2009. V.2. - S. 91-94.
[12] Rvachev V.L, Kurpa L.V. R - function in the plate theory - Kiev: Naukova Dumka, 1987. - 176 p.
[13] .Samarsky A.A., Nikolaev E.S. Methods for net (grid) equations solving. - Moscow: Nauka, 1978. -592 p
[14] akubov S.H. On the intelligent systems for the optimal design of engineering structures of complex configuration development // Proceedings of the conference. Karshi State University-Karshi: Nasaf, 2009. - S. 159-162.
[15] .Yakubov S.H. The engineering structures and buildings design optimization system analysis // Complex systems' optimization problems The seventh Intern. Asian Summer School report. Tashkent, 2011. - P.154-163.