# Market power Analysis Based on Relaxation Algorithm and the Nikaido-Isoda Function in Electricity Markets

Davoud Abbasi[1], Mohsen Assili[2], Seyed Alireza Razavian[3]
* *Department of Electrical Engineering, Shahrood University of technology, Iran*
** *Department of Electrical Engineering, Islamic Azad University, Shahrood branch, Iran*

***Abstract:*** *The issue of evaluating market power is an important challenge in power system planning. This paper develops an analysis based on a relaxation algorithm and Nikaido–Isoda function for calculation of Nash equilibrium and evolution of market structure and performance. This is done through the development of a generation company trading game that, via Nikaido–Isoda function, simulates how players coordinate their behavior in generation to maximize their profit. In this paper, with changing type of power plants belonging to generation company and demand elasticity, market performance is evaluated. The results for pool-based markets shows, generation companies coordination is a very important factor and able to decrease competition and efficiency in market.*

***Keywords:*** *Market power, Nikaido–Isoda, relaxation algorithm, Nash equilibrium, electricity markets, demand elasticity*

## I. INTRODUCTION

All competitive markets are free markets, but not all free markets are competitive. Markets where one or more generation companies (Gen Co) have the ability to raise price and profit are not perfectly competitive.

All capital intensive industries manifest a co-evolution of market structure and performance, but because of the instantaneous, non-storable nature of electricity, low demand elasticity, high requirements for security of supply and wide seasonal variations this co-evolution is not deterministic. This means that electricity is provided from an economic and technical mix of base load, mid-merit and peaking plant is not a certain parameter. This raises the strategic issue for competing companies to evolve towards with a mix of different kinds of generations or which players are more dominant in the base, mid or peaking segments of the market [1]. The traditional assessment of market power has focused on the supplier's ability to profitably alter prices away from competitive levels [2] and how major players manage its contribution of peak, mid and base load plant, in order to set market prices with their marginal plant and thereby reap higher profit contributions [1]. [3] suggested in the liberalized markets, different segments would emerge at least for base load and peak plants. [4] is looked at generator bid and cost data to analyze market power for two largest generating companies in the England and Wales electricity pool.

Capacity withholding is analyzed as an important parameter in market power in [5] and has shown electric generating firms whose market shares range between 10 percent and 40 percent may be profitable.

In [6] this mechanism is used as an effective way to exercise market power in the electricity spot market of England and Wales.

## II. DEFINITIONS AND CONCEPTS

### 1.1 Nikaido-Isoda Methodology

An N-person game can be used as a mathematical model of electricity market. In this game a number of players (electricity companies) interact in a setting of strategies. This means that the profit of each player depends on his own actions and on the actions of the other participants in the game.

Let N be the number of players. The $i_{th}$ player has a set of strategies $X_i$ and $\phi_i$ is the profit function of $i_{th}$ player. The collective action set $X = X_1 \times X_2 \times ..........X_N$ be the vector formed by all these decision variables. Each player makes a choice according to his own strategy for maximizing its profit. In real electricity market each Gen Co (player) able to have enough information about its own and other players past actions. This is called the information set.

Assume that there are players participating in a game. Each player can take an individual action represented by vector $x_i$. All players, when acting together, can take a vector $x = x_1 \times x_2 \times ..........x_N$ that is a subset of X.

The vector x is defined as the joint action vector [7] formed by the strategies of each player. Also $\left( y_i / x \right) = \left( x_1, ..., x_{i-1}, y_i, x_{i+1}, ..., x_n \right)$ is defined as the vector of strategies that player i can take, while the strategies of the other players $x_j \; j \in \{1, 2, ..., i-1, i+1, ......, N\}$ remaining constant.

Nash equilibrium point can be expressed as $x^* = \left( x_1^*, x_2^*, ..., x_n^* \right)$ if for each i it is true that

$$\phi_i\left(x^*\right) = \max_{(x_i|x^*) \in X} \phi_i\left(x_i | x^*\right) \tag{1}$$

Notice that at x* no player can improve his individual profit by his own action. For finding Nash equilibrium point a general way is using the Nikaido–Isoda function. The Nikaido-Isoda function is defined as [8]

$$\Psi_{(x,y)} = \sum_{i=1}^{N} \left[ \phi_i\left( y_i / x \right) - \phi_i\left(x\right) \right] \tag{2}$$

It follows from the definition of the Nikaido–Isoda function that in Nash point $\Psi_{(x,y)} = 0$ . Each summand of the Nikaido–Isoda function can be thought of as the change in the profit of a player when his action changes from $x_i$ to $y_i$ while all other players continue to play according to x. The function thus represents the sum of these changes in profit functions. Maximum value that this function can take by changing y, for a given x, is always greater than zero except in Nash equilibrium point. At equilibrium point, no player can make a unilateral improvement to their profit, and so in this case maximum value of Nikaido–Isoda function can be zero.

In conclusion, when the Nikaido–Isoda function satisfies certain concavity conditions [7] and cannot be made positive for a given y, the Nash equilibrium point is reached. This is used to construct a termination condition for the relaxation algorithm that explained in next section.

Finally, the optimum response function at point x can be defined as

$$Z(x) = \arg\max_{y\in X} \Psi(x, y),\ x, Z(x) \in X \tag{3}$$

It is the result of maximizing the Nikaido–Isoda function, where all players try to improve their profits. This function returns the set of players' actions whereby they all try to unilaterally maximize their respective profits with respect to actions $y_i$ and so, by "playing" actions Z(x) rather than x, the players approach the equilibrium [9]. Note that, by doing that, a player maximizes its profit assuming that the competitors are fixed in their actions, which is the definition of Nash-Cournot equilibrium. In the next section, a relaxation algorithm that uses the Nikaido–Isoda function to compute Nash equilibrium is presented. In simple Nikaido–Isoda function the players wish to move to a point that represents an improvement in compare with current player situations and these movements may cause system instability. The relaxation algorithm adds a ratio of previous value of x to the new response Z(x). Technical definitions that are used in the convergence theorem of the algorithm are expressed in [7].

### 1.2 Relaxation Algorithm

In order to find a Nash equilibrium of a game, having an initial estimate x0, the relaxation algorithm of the optimum response function, when Z(x) is single-valued (every input is associated with one output only) and the concavity conditions are satisfied (the Nikaido-Isoda function is weakly convex-concave [8]) is

$$x(j+1) = (1-\alpha(j))x(j) + \alpha(j)Z(x(j)) \quad j = 0,1,2,..... \tag{4}$$

where $0 < \alpha(j) < 1$. An iterative algorithm is constructed as a convex combination of the improvement point $Z(x(j))$ and the current point $x(j)$. The optimum response function $Z(x(j))$ is calculated after solving an optimization problem as seen in (3). This averaging ensures convergence of the algorithm under certain conditions [7].

It is interesting to note that we can consider the algorithm as either performing a static optimization or calculating successive actions of players in convergence to equilibrium in a real time process. If all profits are known to us, we can directly find the Nash equilibrium using the relaxation algorithm. However, if we only have access to one player's profit function and all players past actions, then at each stage in the real time process the optimum response should be chosen for that player, assuming that the other players will play as they had in the previous period. In this way, convergence to the Nash normalized equilibrium will occur at $j \to \infty$

Thus, by taking a sufficient number of iterations, the algorithm converges to the Nash equilibrium. The problem can be either considered a centralized optimization model or a calculation of the succession of actions by the players at each stage, where players choose their optimum response given the actions of the opponents in the previous period. The theorem that ensures convergence of the relaxation algorithm is presented in full detail in [7].

$\alpha(j)$ is an important factor to converge and optimize the convergence rate of algorithm. In [7] $\alpha(j)$ is supposed constant until convergence conditions are reached, and thereafter decaying with factors $\frac{1}{2},\frac{1}{3},\frac{1}{4},........$. We have found that using a constant value lower than 0.4 leads to a good and approximately quick convergence but values higher than 0.4 in most of our experiments leads to algorithm divergence. At last suitable $\alpha(j)$ values in any applications may be obtained by trial and error.

### 1.3 HHI index

The most widely used measure of concentration in a market is the Herfindahl–Hirschman Index (HHI). HHI is linked directly to market power in one theoretical model of competition [10]. This parameter widely applied in competition law and also technology management. It is defined as the sum of the squares of the market shares, where the market shares are expressed as fractions. The result is proportional to the average market share, weighted by market share.

$$HHI = \sum S_i^2 \tag{5}$$

Where $S_i$ is market share of each generation company [10].

If the market share of each company is expressed in percentage terms, the HHI lies between 0 and 10,000.

The maximum value of the HHI occurs when market is monopoly and without any competition. The minimum value of the HHI occurs in the limit that the generation companies comprise a very large number of companies, each with negligible market shares.

Increases in the HHI index generally indicate a decrease in competition and an increase of market power, whereas decreases indicate the opposite. The major benefit of the HHI is in relationship to such measures as the concentration ratio is that it gives more weight to larger companies.

## III. CASE STUDIES

The most sensible method of calculating market power impacts in an electricity market is to simulate the operation of that electricity market and, thereby, directly measure the price and revenue impacts of generation company strategies.

The methodology presented above is used to simulate the market using Nash-Cournot equilibrium computed with Nikaido-Isoda function and relaxation algorithm, of some case studies, assuming a pool-type market.

In all case studies, it is assumed that there are three generating companies and each of them possesses several generating unit, as shown in Table I.

Proposed model has 19 units. The cost of a generating unit is defined as:

$$C_i(P_{gi}) = \frac{C_i}{2}P_{gi}^2 + d_iP_{gi} + e_i \tag{6}$$

$C_i$, $d_i$ and $e_i$ coefficients are shown in table 1 and $P_{gi}$ is generation of unit.

Table 1: system data of proposed model

| No of unit | Case A Company type | Case B Company type | Case C Company type | Case D Company type | Minimum power (MW) | Maximum power (MW) | $C_i$ ($/MW$^2$ h) | $d_i$ ($/MWh) | $e_i$ ($/h) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 736 | 0.00021 | 0 | 0 |
| 2 | 2 | 1 | 1 | 1 | 0 | 501.8 | 0.0003 | 0 | 0 |
| 3 | 3 | 1 | 1 | 1 | 0 | 354.8 | 0.00046 | 0 | 0 |
| 4 | 3 | 1 | 1 | 2 | 0 | 261 | 0.0006 | 0 | 0 |
| 5 | 1 | 1 | 2 | 1 | 0 | 340 | 0.00466 | 31.68 | 0 |
| 6 | 1 | 1 | 3 | 1 | 0 | 379 | 0.00194 | 14.68 | 0 |
| 7 | 2 | 1 | 2 | 1 | 0 | 379 | 0.00171 | 12.97 | 0 |
| 8 | 3 | 1 | 3 | 1 | 0 | 368.4 | 0.00183 | 13.52 | 0 |
| 9 | 2 | 1 | 2 | 2 | 0 | 304 | 0.00377 | 22.91 | 0 |
| 10 | 3 | 1 | 3 | 3 | 0 | 250 | 0.00471 | 23.57 | 0 |
| 11 | 3 | 1 | 3 | 3 | 0 | 244.9 | 0.00347 | 16.99 | 0 |
| 12 | 1 | 1 | 2 | 2 | 0 | 128 | 0.01099 | 28.13 | 0 |
| 13 | 2 | 1 | 3 | 3 | 0 | 108 | 0.04202 | 90.76 | 0 |
| 14 | 3 | 1 | 2 | 2 | 0 | 97 | 0.06649 | 129 | 0 |
| 15 | 3 | 1 | 3 | 3 | 0 | 58 | 0.11197 | 129.88 | 0 |
| 16 | 2 | 1 | 2 | 2 | 0 | 49 | 0.07562 | 74.11 | 0 |
| 17 | 1 | 1 | 3 | 3 | 0 | 23.8 | 0.26658 | 126.89 | 0 |
| 18 | 1 | 1 | 2 | 2 | 0 | 16 | 0.05076 | 58.88 | 0 |
| 19 | 3 | 1 | 3 | 3 | 0 | 12 | 0.41042 | 98.5 | 0 |

Units with number 1 to 4 are baseload power plants and have low cost coefficients in compare with other units. Units with numbers 5 ta 11 have a few higher cost coefficients and is classified as mid merit units. Remaining units are peak.

Other details of each power plat including minimum power, maximum power and owner of unit in each case study are given in table I.

In case A, all Gen Cos have approximately similar portion of baseload, mid merit and peak units. In case B, the market is monopoly and one company is owner of total units and for case C, all baseload units are owned by company no 1 and mid merit and peak units are divided equally between other generation companies.

In D, company no 1 is dominant in baseload and mid merit units and other companies have only a few portion of units in this two parts.

To determine each company's profit function $\phi_i$ as used in (1), price is assumed a strictly decreasing function of the electricity demand [7].

$$p = \alpha - \beta P_{load} \qquad (7)$$

The income function of each company is equal to its generation multiplied by price. Now profit function of each company is defined as [7]:

$$\varphi_i\left(P_{gi}\right) = pP_{gi} - C_i(P_{gi}) = pP_{gi} - \frac{C_i}{2}P_{gi}^2 + d_i P_{gi} + e_i \qquad (8)$$

That should be computed for all units.

All case studies, with above assumption are simulated in three different demand elasticity for covering baseload, mid merit and peak zones and nash-cournot equilibrium is find with combination of relaxation algorithm and nikaido-isoda function. Final results are presented in table 2 ta 9.

**Table 2: Case A system Nash equilibrium results**

| | Company no 1 | Company no 2 | Company no 3 | Total profit | Price |
|---|---|---|---|---|---|
| profit Case 1 | 516448.72 | 508797.93 | 513686.67 | 1538933.32 | 1023.4 |
| profit Case 2 | 1264799.64 | 1214044.41 | 1241125.49 | 3719969.54 | 1673.89 |
| profit Case 3 | 4389293.47 | 3702676.98 | 4479941.43 | 12571911.88 | 3229.17 |

**Table 3: the generation details for Case A and HHI index**

| | Total | Company no 1 | Company no 2 | Company no 3 | HHI |
|---|---|---|---|---|---|
| Generation Case 1 | 1513.3 | 33.52% | 33.1% | 33.38% | 3333.43 |
| Generation Case 2 | 2268.05 | 33.56% | 33.06% | 33.37% | 3333.46 |
| Generation Case 3 | 4055.41 | 34.9% | 29.51% | 35.59% | 3355.51 |

Table 2 shows Nash equilibrium results for case A (balanced market). Case 1 is for elasticity in base load, case 2 for mid merit and case 3 for peak zone. Corresponding to each zone, the price and generation is increased in table 2.

Table 3 shows that HHI index is at its minimum value for 3 company and portion of companies in market is approximately constant in all demand situations and market is clearly competitive.

**Table 4: Case B system Nash equilibrium results**

| | Company no 1 | Total profit | Price |
|---|---|---|---|
| profit Case 1 | 2046027.02 | 2046027.02 | 2048.82 |
| profit Case 2 | 4810436.86 | 4810436.86 | 3141.53 |
| profit Case 3 | 15902321.57 | 15902321.57 | 5785.97 |

**Table 5: the generation details for Case B and HHI index**

| | Total | Company no 1 | HHI |
|---|---|---|---|
| Generation Case 1 | 1000.59 | 100 | 10000 |
| Generation Case 2 | 1534.23 | 100 | 10000 |
| Generation Case 3 | 2777.01 | 100 | 10000 |

Table 4 shows Nash equilibrium results for case b (monopoly market). Similar to case A, Corresponding to each zone, the price and generation is increased in table 4.

It is obvious, that price and total profit in each case is higher than table 2 (about 100%), because the market is monopoly and not competitive. In fact in a monopoly market, market owner (dominant generation company) constrains its strategy to customers and set market price to its marginal price and for this reason will obtain maximum profit. Table 5 shows that HHI index is at its maximum value and company no 1, with a generation about 30% lower, earn a higher profit than case A.

**Table 6: Case C system Nash equilibrium results**

| | Company no 1 | Company no 2 | Company no 3 | Total profit | Price |
|---|---|---|---|---|---|
| profit Case 1 | 620793.95 | 415518.76 | 453096.35 | 1489409.06 | 1122.1 |
| profit Case 2 | 1407778.09 | 1005773.82 | 1121793.74 | 3535345.65 | 1686.11 |
| profit Case 3 | 5264842.32 | 3488842.61 | 3772183.04 | 12525867.97 | 3248.43 |

**Table 7: the generation details for Case C and HHI index**

| | Total | Company no 1 | Company no 2 | Company no 3 | HHI |
|---|---|---|---|---|---|
| Generation Case 1 | 1463.95 | 37.87 | 29.89 | 32.24 | 3366.95 |
| Generation Case 2 | 2261.94 | 36.99 | 30.24 | 32.78 | 3356.6 |
| Generation Case 3 | 4045.78 | 40.14 | 29.21 | 30.65 | 3403.94 |

Nash equilibrium results for case C are given table 6. In this case, company 1 has all base load plants and market power belongs to this company. Company 1 has higher generation in all zones and due to its low cost plants, earns more profits than others. Price in this case is a few higher than fully competitive market but is so lower than monopoly market. Table 7 shows that market concentration is a few higher than case A. generation details given in table 7 apparently explain company 1 imposed its generation strategy to market.

Nash equilibrium and HHI results for case D are given table 8 and 9. In this case, company 1 has a large number of baseload and mid merit plants and market power belongs to this company more than case C.

**Table 8: Case D system Nash equilibrium results**

| | Company no 1 | Company no 2 | Company no 3 | Total profit | Price |
|---|---|---|---|---|---|
| profit Case 1 | 674989.85 | 464005.19 | 391417.16 | 1530412.2 | 1166.86 |
| profit Case 2 | 1827767.46 | 1193932.73 | 826664.13 | 3848364.32 | 1917.45 |
| profit Case 3 | 8403715.89 | 3242221.7 | 2529087.17 | 14175024.76 | 4197.44 |

**Table 9: the generation details for Case D and HHI index**

| | Total | Company no 1 | Company no 2 | Company no 3 | HHI |
|---|---|---|---|---|---|
| Generation Case 1 | 1441.57 | 40.22 | 30.49 | 29.29 | 3405.3 |
| Generation Case 2 | 2146.27 | 44.52 | 32.26 | 23.22 | 3562.02 |
| Generation Case 3 | 3571.28 | 56.57 | 23.93 | 19.5 | 4152.95 |

Price is increased in all demand situations in compare with case A and C. Company 1 has more generation in all zones and due to its plants, earns more profits than others. Table 9 shows that market concentration is higher than case A and C. In this case company 1 has the ability to raise market price and its profit due to its portion of generation.

## IV. CONCLUSION

The issue of evaluating market power is an important challenge in restructured electricity markets. The model presented in this paper carries out an iterative Nash-Cournot equilibrium game that considers a generating pool market model in some practical case to evaluate market power. For solving game, an algorithm based on the Nikaido–Isoda function and a relaxation algorithm is used. It allows for the incorporation of the network constraints and different type of plants. It is possible to add independent demand curves for each time and thereby assigning different elasticity and consumption values and evaluate market power separately. To evaluate market power, HHI index as an economic parameter for showing concentration in markets is used. Our model is a useful tool to analyze the market power and strategic behavior of the agents in a competitive electricity market and can be used by independent system operator to enhance market efficiency.

Four case studies of electricity markets are presented. The first case study shows a balanced market. In this case HHI index is near to ideal value for all demand situations. In case B, a monopoly market is simulated and results, showed a very high price increases in compare with case A. HHI index in this situation is at its maximum value. In case C and D, Changes in generation company properties increase market power and decrease competition in market. Results shows that in different elasticity, different sets of coalitions among generating companies can be change significantly market power. As a final conclusion the results for pool based markets shows, generation companies coordination according to demand elasticity is a very important factor and able to decrease competition and efficiency in market.

## REFERENCES

[1] F. Oliveira , D. W. Bunn Agent-Based Analysis of Technological Diversification and Specialisation in Electricity Markets, *European Journal of Operational Research, Jan 2007, Issue 181*, p. 1265-1278

[2] Stoft, S. *Power System Economics: Designing Markets for Electricity*, IEEE Press, 2002.

[3] Borenstein, S., J. Bushnell, E. Kahn, And S. Stoft, Market power in California electricity markets, *Utilities Policy, 1995, 5 (3-4)*, 219-236.

[4] Von der Fehr, N-H and Harbord, D. Spot Market Competition in the UK Electricity Industry, *The Economic Journal, 1993, Vol. 103*, pp. 531 – 546.

[5] Newbery, D.M., Regulatory Policies and Reform in the Electricity Supply Industry, *Cambridge Working*

*Papers in Economics 9421*, *1995*, Faculty of Economics, University of Cambridge.

[6] Wolak, F.A., Patrick, R.H. Industry structure and regulation in the England and Wales electricity market. 1997. *Crew, M. (Ed.), Pricing and Regulatory Innovation under Increasing Competition. Kluwer Academic Publishers, Dordrecht, Netherlands.*

[7] J. Contreras, M. Klusch, and J. B. Krawczyk, Numerical solutions to Nash-Cournot equilibria in coupled constraint electricity markets, *IEEE Trans. Power Syst. , Feb. 2004, vol. 19, no. 1*, pp. 195–206,.

[8] J. B. Krawczyk and S. Uryasev, Relaxation algorithms to find Nash equilibria with economic applications, *Environment. Model Assess. Jan. 2000, vol. 5, no. 1*, pp. 63–73,. [Online]. Available: http://www.ise.ufl.edu/uryasev/relaxation_algorithms_for_Nash_equilibrium.pdf.

[9] Molina, J.P. ,Zolezzi, J.M. , Contreras, J. , Rudnick, H. , Reveco, M.J. Nash-Cournot Equilibria in Hydrothermal Electricity Markets , *IEEE Trans. Power Syst. , Aug. 2011, vol. 26, no. 3*, pp. 1089-1101.

[10] Williams, Edward and Richard Rosen,. A Better Approach to Market Power Analysis, *1999, Working Paper,* available at http://www.tellus.org/general/publications.html.