Cost Optimization Of Doubly Reinforced Rectangular Beam Section

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ABSTRACT : *In this paper, optimum design of simply supported doubly reinforced beams with uniformly distributed and concentrated load has been done by incorporating actual self weight of beam, parabolic stress block, moment-equilibrium and serviceability constraints besides other constraints. The optimization techniques in general enable designers to find the best design for the structure under consideration. In this particular case, the principal design objective is to minimize the total cost of a structure. The resulting structure, however, should not only be marked with a low price but also comply with all strength and serviceability requirements for a given level of the applied load. Total cost includes cost of concrete, cost of steel and cost of formwork are considered. A comparative study between the classical optimization techniques, namely the Generalized Reduced Gradient Method, Interior point algorithm optimization technique using MATLAB and one of the heuristic techniques, namely the Genetic Algorithm was carried out in this research. The initial solution for the optimization procedure has been obtained using limit state design as per IS: 456-2000.*

Keywords: *cost optimization, generalized reduced gradient method, genetic algorithm, interior point method, reinforced concrete.*

I. Introduction

Optimum design of structures has been the topic of many studies in the field of structural design. A designer's goal is to develop an "optimal solution" for the structural design under consideration. An optimal solution normally implies the most economic structure without impairing the functional purposes the structure is supposed to serve.

Structural design requires judgment, intuition and experience, besides the ability to design structures to be safe, serviceable and economical. The design codes do not directly give a design satisfying all of the above conditions. Thus, a designer has to execute a number of designs-analyze cycles before converging on the best solution. The intuitive design experience of an expert designer can give a good initial solution, which can reduce the number of design-analyze cycles. The optimization involves choosing of the design variables in such a way that the cost of the beam is the minimum, subject to the satisfaction of behavioral and geometrical constraints as per recommended method of design codes. Doubly reinforced beams (DRB) are required to be designed when the depth of the beam is restricted by architectural considerations and the beam has to take moment greater than limiting moment of resistance of the corresponding singly reinforced beam (SRB).

Some structure optimization work deals with minimization of cost of the structure using G.A. [1, 2, 3], some of the researchers have worked on cost optimization of the doubly reinforced concrete beam structure [4, 5, 6], most of researchers have worked on cost optimization of reinforced concrete structure [7, 8], geometric programming model which gives the unique least-cost design of a beam, considering the cost of materials and shuttering and the structural requirements [9]. Whereas a natural velocity field method for shape optimization of reinforced concrete (RC) flexural members has been demonstrated [10]. The application of the Lagrangian Multiplier Method (LMM) to the minimum cost design of both singly and doubly reinforced concrete rectangular beams under limit state design conditions [11].

An initial solution for each case is obtained using the limit state method, by including self weight of the beam and considering parabolic stress block. The limit state design and the optimization is performed, subject to satisfaction of moment capacity, actual deflection and durability behavioral constraints, besides other geometrical constraints as recommended in IS: 456-2000 [12].

II. Optimization Technique

2.1 Classical Search and Optimization techniques

Traditional search and optimization methods can be classified into two distinct groups: Direct and gradient-based methods. In direct methods, only objective function and constraints are used to guide the search strategy, whereas gradient-based methods use the first and/or second-order derivatives of the objective function and/or constraints to guide the search process. Since derivative information is not used, the direct search methods are usually slow, requiring many function evaluations for convergence. For the same reason, they can be applied to many problems without a major change of the algorithm. On the other hand, gradient-based methods quickly converge to an optimal solution, but are not efficient in non differentiable or discontinuous problems.

2.1.1 The Generalized Reduced Gradient Method

The Generalized Reduced Gradient (GRG) Methods are algorithms for solving nonlinear programs of general structure.

GRG uses first partial derivatives of each function with respect to each variable. These are automatically computed by finite difference approximation (either forward or central differences). After an initial data entry segment, the program operates in two phases. If the initial values of the variables supplied by the user do not satisfy all the constraints, a Phase I optimization is started. The Phase I objective function is the sum of the constraint violations plus, optionally, a fraction of the true objective. This optimization terminates either with a message that the problem is infeasible or with a feasible solution. Beware if an infeasibility message is produced, because the program may have become stuck at a local minimum of the Phase I objective (or too large a part of the true objective is incorporated), and the problem may actually have feasible solutions. The suggested remedy, in this case, is to choose different starting values for the variables (or reduce the proportion of the true objective) and try again.

Phase II begins with a feasible solution, either found by Phase I or with the user provided starting point if it is feasible, and attempts to optimize the objective function. At the conclusion of Phase II, a full optimization cycle has been completed and summary output is provided.

2.1.2 Interior Point Algorithm

Interior point methods (also referred to as barrier methods) are a certain class of algorithms to solve linear and nonlinear optimization problems. The interior point algorithm is used for general nonlinear optimization. It is especially useful for large-scale problems that have sparsity or structure, and tolerates user-defined objective and constraint function evaluation failures. It is based on a barrier function, and optionally keeps all iterates strictly feasible with respect to bounds during the optimization run.

In interior point method, the slack variables are introduced in to the simple non linear program, to make all inequality constraint in to non negativity, these non negativity constraints are replaced with logarithmic barrier terms in the objective. Incorporate the equality constraints into the objective using Lagrange multipliers. Newton's method is applied to compute search directions. Iterations are carried out and results are obtained using MATLAB.

2.2 Heuristic optimization techniques

In the last three decades, heuristic methods have been rapidly developed to solve optimization problems. These methods are principally intuitive and do not have theoretical support. Heuristic methods such as genetic algorithms (GAs), simulated annealing (SA) and tabu search (TS) provide general ways to search for a good but not necessarily the best solution.

2.2.1 Genetic Algorithm

Genetic algorithms (GA) are numerical optimization techniques inspired by the natural evolution laws. A GA starts searching design space with a population of designs, which are initially created over the design space at random. In the basic GA, every individual of population (design) is described by a binary string (encoded form). GA uses four main operators, namely, selection, creation of the mating pool, crossover and mutation to direct the population of designs towards the optimum design. In the selection process, some designs of a population are selected by randomized methods for GA operations, for example in creation of the mating pool, some good designs in the population is selected and copied to form a mating pool. The better (fitter) designs have a greater chance to be selected. Crossover allows the characteristics of the designs to be altered. In this process different digits of binary strings of each parent are transferred to their children (new designs produced by the crossover operation). Mutation is an occasional random change of the value of some randomly selected design variables. The mutation operation changes each bit of string from 0 to 1 or vice versa in a design's binary code depending on the mutation probability. Mutation can be considered as a factor preventing from premature convergence.

III. Problem Formulation

The general form of an optimization problem is as follows

1. Given - Constant parameters
2. Find - Design variables
3. Minimize - Objective function
4. Satisfy - Design constraint

3.1 Constant Parameters

In this work, optimal design of doubly reinforced beam has been done for different material combinations of M20, M25 grades of concrete and Fe415, Fe500 grades of steel. The cost of materials for different grades and form work are given in Table 1.

Concrete Grade	C_c (Rs/m^3)	Steel Grade	C_s (Rs/kg)	C_f Rs/m^2
M20	4366	Fe415	58	320
M25	5610	Fe500	60	

3.2 Design variables

Width of beam = b = x_1
Tension reinforcement = $A_{st} = x_2$
Compression reinforcement = $A_{sc} = x_3$
Nominal cover = d' = x_4

3.3 Objective function
The objective function to be minimized:

$$F(x) = C_c [b (d + d') – (A_{st} + A_{sc})] + C_s [A_{st} + A_{sc}] + C_f [b + 2 (d+ d')]$$

$$F(x) = C_c [X_1 (d + X_4) – (X_2+ X_3)] + C_s [X_2+ X_3] + C_f [X_1+ 2 (d+ X_4)]$$

Where, C_c is cost of concrete, C_s is cost of steel and C_f is cost of formwork.

3.4 Design constraint

Geometrical constraints:
1. Ductility constraint: $xu \leq xa$
2. Constraint for minimum area of tension reinforcement:

$$A_{st} > \frac{0.85\, b\, d}{fy}$$

3. Constraint for maximum area of tension reinforcement:

$$A_{st} \leq 0.04\, b\, D$$

4. Constraint for maximum area of compression reinforcement:

$$A_{sc} \leq 0.04\, b\, D$$

5. Depth to width ratio constraint: $r = d/b$; $1.5 \leq r \leq 4$

Behavioral constraints:
1. Durability constraint: nominal cover ≥ 40 mm
2. Moment-equilibrium constraint: $Mu \leq Mc$
3. Deflection constraint (serviceability constraint): $\delta tot \leq \delta all$

Where, xu is balance position of neutral axis in mm and xa is actual position of neutral axis. Mu is bending moment due to given loading and self weight in km.m, Mc is moment capacity of beam in kn.m.δtot and δall are sum of short term and long term deflection and allowable deflection in mm respectively.

IV. System Performance

In present study the attempt is made to optimize Doubly Reinforced Beam using Genetic Algorithm (MATLAB Toolbox) and performance analysis is performed of this element. Variation in parameters such as cost function, design variables are examined for several trial giving initial values obtained by Limit State Method. For justification of variations, Interior point Algorithm using Matlab and Generalized reduced gradient method (Microsoft Office Excel Solver Tool) are used.

The input of simply supported doubly reinforced beam (DRB) consists of 6 inputs, viz. load, span, d to b ratio, fck and fy, and depth of beam. The output are cover to reinforcement (d') and optimum tensile steel reinforcement (pt). and optimum compression steel reinforcement (pc). In this paper five problems are solved by using three optimization techniques i.e. GRGM, IP and G.A., results are shown in Table 2.

4.1 Design example: S.S doubly reinforced beam with UDL

Design a simply supported doubly reinforced beam of span 9 m, depth 673.62 mm, subjected to following load and specification: Superimposed load = 10 kn/m, Live load = 20 kn/m, using M20 grade concrete and Fe415 grade steel.

4.1.1 Conventional limit state solution:
M_u= 577.29 kn/m, P_t = 0.9789, P_c = 0.015,
M_c = 573.14 kn/m, Cost = 2020.4 Rs/m.
4.1.2 Solution by proposed technique:
M_u= 577.29 kn/m, P_t = 1.002, P_c = 0.04450,
M_c = 577.30 kn/m, Cost = 1968.26 Rs/m.
Design given by proposed technique is safe while that given by conventional limit state method fails in moment capacity.

V. Conclusions

The main conclusions drawn from the current research are summarized as follows:

1. The results obtained from the Genetic Algorithm optimization technique showed a cost that is less than the cost obtained from the Generalized Reduced Gradient technique and Interior Point optimization technique. This comparison showed the superiority of the Genetic Algorithm technique over the classical Generalized Reduced Gradient technique and Interior Point optimization technique

2. It was shown that the Genetic Algorithm optimizer does a remarkable effort on minimizing the expensive material in the objective function of the numerical examples. This effort is devoted to the total cost. Therefore, one can conclude that the Genetic Algorithm search and optimization technique is powerful and intelligent

3. The performance of the Genetic Algorithm using different methods of crossover and selection can vary from one problem to another. Therefore, several values of each of the operators of the Genetic Algorithm should be examined in order to reach the best value for that operator for the problem under consideration. This is called tuning of the Genetic Algorithm operators.

4. It can be said that researches carried out for finding optimum design of concrete structures are of great value to practicing engineers. The optimum solution satisfies the provisions of the code and minimizes the cost of the structure.

Additional research studies should be carried out on geometry and layout optimization of structural elements within the RC structure. Extra research should be carried out on other types of structures such as frames and trusses. The researcher recommends that there is a need for research on cost optimization of realistic RC three-dimensional large-scale structures.

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Table 2: Results for optimal design for SS - Doubly Reinforced Beam

Sr	Input	Method	Pt	Pc	Mu	Mc	F(x)
1	W=350 kN L=4 m, r=1.5 M20, Fe415 d=660.07mm	GRGM I.P G.A.	0.98655 0.9904 1.03281	0.02967 0.03628 0.07377	548 548 548	548.09 548.176 555.23	1950.93 1942.8 1888.475
2	W=40 Kn/m L=10m, r=1.5 M20, Fe500 d=781.38mm	GRGM I.P G.A	0.8367 0.8404 0.8484	0.08094 0.08385 0.08741	950.0 950.0 950.0	950.49 951.44 950.51	2588.98 2577.35 2549.12
3	W=30 Kn/m L = 9 m, r=1.5 M20, Fe415 d=673.62mm	GRGM I.P G.A	0.9789 0.9967 1.002	0.0218 0.0367 0.04450	577.29 577.29 577.29	577.28 577.90 577.30	2017.11 1989.522 1968.26