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GA Based Multi-Objective Time-Cost Optimization in a Project with Resources Consideration

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ABSTRACT: *Project planner's main goal is to complete project with minimal time as well as minimal cost. Much effort (e.g. resources which demand cost) is required to complete the project in shorter time. There is a trade-off relationship between activity duration and its resources. This study employed a GA based multi-objective approach for time-cost trade-off using resources-time tradeoff as input. Combining with modified adaptive weight approach (MAWA), this proposed optimization model can find out best optimal solution and Pareto front which provides flexibility to planners and decision makers in making efficient time-cost decisions. This proposed model was developed in MATLAB and applied to test problems. However, it is illustrated that this model could be useful to solve bigger networks in practice.*

Key words: *Critical path method (CPM); Time-cost trade-off problem (TCTP); Resources-time trade-off; Multi-objective optimization; Genetic algorithm (GA).*

I. Introduction

A project is a group of activities. According to precedence relationship of the activities project planner has to plan the execution order to complete the project. So, all activities are arranged in a network using arrows and nodes, maintaining their logical sequence. Among the paths a critical path is the longest continuous path which defines the total project time. Hence, critical path method (CPM) is widely recognized project planning and scheduling tool which deals with deterministic time duration of activities to schedule the project.

In a project, there are three major elements, such as project duration, project cost and resources. These are depended to each other. The activity duration is a function of resources (i.e. crew size, equipments and materials) availability. On the other hand, resources demand direct costs. Therefore, the relationship between project time and direct cost of each activity is a monotonously decreasing curve. It means if activity duration is compressed then there is leading to an increase in resources and so that direct costs. But, project indirect costs increase with the project duration. Hence, relationship between project time and cost is trade-off (Geem, 2010). So, in a project it needs multi-objective approach to minimize both projects time and cost by varying options of the critical activities.

Traditionally, time-cost trade-off problem (TCTP) is a deterministic scheduling problems, it is assumed that all data are available. Several mathematical models such as linear programming (Kelly, 1961; Hendrickson and Au, 1989; Pagnoni, 1990), integer programming, or dynamic programming (Butcher, 1967; Robinson, 1975; Elmaghraby, 1993; De, Dunne and Wells, 1995) and LP/IP hybrid (Liu, Burns and Feng, 1995; Burns, Liu and Feng, 1996) models are used to solve TCTPs. Meyer and Shaffer (1963) and Patterson and Huber (1974) use mixed integer programming. However, for large number of activity in network and complex problem, integer programming needs a lot of computation effort (Feng, Liu and Burns, 1997). Since, these are suitable for small project.

Some heuristic models, such as Fondahl's (1961) method, effective cost slope model of Siemens (1971), structural stiffness model (Moselhi, 1993) and structural model (Prager, 1963) could only optimize one objective at a time but the solution is not guaranteed to be the global optimal (Xiong and Kuang, 2008). Li and Love (1997) and Hegazy (1999) introduced GA to solve deterministic TCT problems, but yet remained as a single-objective optimization tool.

Some researchers have tried to introduce evolutionary algorithms to find global optima such as genetic algorithm (GA) (Feng, Liu and Burns, 1997; Gen and Cheng, 2000; Zheng, Ng and Kumaraswamy, 2004) the particle swarm optimization algorithm (Yang, 2007), ant colony optimization (ACO) (Xiong and Kuang, 2008; Ng and Zhang 2008) and harmony search (HS) (Geem, 2010). The model of Feng, Liu and Burns (1997) might be inefficient to obtain optima for the large TCT problem. Gen and Cheng (2000) and Zheng, Ng and Kumaraswamy (2004) have drawback that they may be entrapped into local optima for TCT problems.

In this study another evolutionary algorithm, genetic algorithm is induced to solve TCT problem incorporating modified adaptive weight approach (MAWA) (Zheng, Ng and Kumaraswamy, 2004). Moreover, this study has used discrete relationship between activity time and resources except relationship between activity time and cost to find optima in TCT problem. Penalty for delay above the desired project completion time is also added for TCT in this study. When compared with models of Gen and Cheng (2000) and Zheng, Ng and Kumaraswamy (2004), the new model saved fittest solution of each generation in fittest table and evaluated performances of them again to obtain best optimal solution. Finally, this proposed algorithm solved two example test cases and found out best optimal solutions and Pareto set solutions.

II. Problem Formulation

In TCT problems, there are twin objectives-minimize both project time and cost. The durations of each activity and their corresponding resources (i.e. crew sizes, equipments and materials) are included to measure direct costs in present project cost function.

$$C = \sum_{i=1}^{l} \sum_{j=1}^{r} C_{d_{ij}} + C_{id} \quad (1)$$

Where, C is the total project cost, $C_{d_{ij}}$ the direct cost of resources type j at activity i, C_{id} the indirect cost of project, *r* total number of resource types and *l* the total number of activity.

For any type of resource at any activity, the direct cost depends on whether the number required exceeds that available limit (Supply limit). When demanded resource is available, the normal cost rate of that resource is applied. However, if the activity requires more than available limit of resource the extra premium cost rate is added for each extra unit of resource. Thus,

$$C_{d_{ij}} = R^r_{ij} \times M_{ij} \times t_i + P_{ij} \times t_i \times (R^r_{ij} - R^n_{ij}) \quad (2)$$

Where, R^r_{ij} is number of resource type j required by activity i on time t, R^n_{ij} the number of resource type j of activity i on time t available at normal condition, M_{ij} the cost rate of resource type j of activity i, P_{ij} the premium rate of resource type j of activity i above normal condition and t_i the time of activity i.

The premium is added when number of resources j of activity i exceeds of availability, so that the P_{ij} is given by

$$P_{ij} = \begin{cases} V_{ij} - M_{ij}, & R^r_{ij} > R^n_{ij} \\ 0, & R^r_{ij} \le R^n_{ij} \end{cases} \quad (3)$$

Where, V_{ij} is the cost rates of resources type j at activity i above normal availability limit.
The indirect cost of project C_{id} depends on total project time and desired project time, thus

$$C_{id} = T \times C_L + D \times (T - T_a) \quad (4)$$

$$D = \begin{cases} C_p, & T > T_a \\ 0, & T \le T_a \end{cases} \quad (5)$$

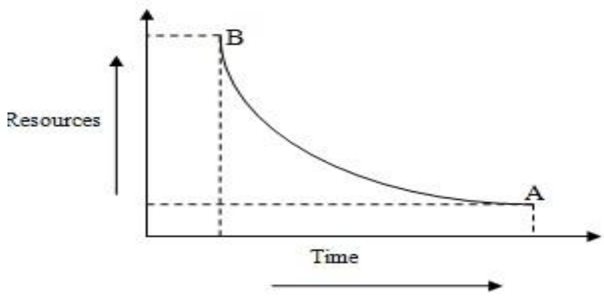
Where, C_L the indirect cost rate, C_p the delay fine rate for unit time delay and desired project time T_a. If the project duration is more than the desired project duration, then delay fine will be added.
The total project time T is given by

$$T = \max \{EST_i + t_i \mid i = 1,2,\dots,l\} \quad (6)$$

Where, EST_i the earliest starting time of activity i.
The objective functions for TCTP are Eq. (1) and (6), to minimize project cost, C and time. Eq. (6) is also used to find the critical path and total duration of the project.

For TCTP, resources-time trade-off curve acts as input in this study shown in Figure 1 which was originally developed by (Chua, Chan and Govinda, 1997). The curve was developed from a consideration of varying of resources such as crew sizes, equipments and materials, their productivity and cost rates. It is shown in Figure 1 that, resources of activity decreases with as activity duration increases. That means much resources (correlated direct cost) are required to complete the activity in shorter time. Hence, the purpose of this study is to minimize resources so that project cost of shorted schedule for TCT.

Figure 1 level of resources-time trade-off input.



III. Modified Adaptive Weight Approach

This study applies the modified adaptive weight approach (MAWA) proposed by Zheng, Ng and Kumaraswamy (2004) to deal with the multi-objective TCTPs. This utilizes some useful information from the current population to generate an adaptive weight for each objective, and thereby exerts a search pressure towards the ideal point. Under the MAWA (Zheng, Ng and Kumaraswamy, 2004), the adaptive weights are formulated through the following four conditions:

1. For $Z_c^{max} \neq Z_c^{min}$ and $Z_t^{max} \neq Z_t^{min}$;

$$\upsilon_c = \frac{Z_c^{min}}{Z_c^{max} - Z_c^{min}} \quad 7$$

$$\upsilon_t = \frac{Z_t^{min}}{Z_t^{max} - Z_t^{min}} \quad 8$$

$$\upsilon = \upsilon_c + \upsilon_t \quad 9$$

$$w_c = \upsilon_c / \upsilon \quad 10$$

$$w_t = \upsilon_t / \upsilon \quad 11$$

$$w_c + w_t = 1 \quad 12$$

2. For $Z_c^{max} = Z_c^{min}$ and $Z_t^{max} = Z_t^{min}$;

$$w_c = w_t = 0.5 \quad 13$$

3. For $Z_t^{max} = Z_t^{min}$ and $Z_c^{max} \neq Z_c^{min}$

$$w_c = 0.1,\ w_t = 0.9 \quad 14$$

4. For $Z_t^{max} \neq Z_t^{min}$ and $Z_c^{max} = Z_c^{min}$

$$w_c = 0.9,\ w_t = 0.1 \quad 15$$

Zheng, Ng and Kumaraswamy (2004) propose a fitness formula in accordance with the proposed adaptive weight:

$$f(x) = w_t \frac{Z_t^{max} - Z_t + \gamma}{Z_t^{max} - Z_t^{min} + \gamma} + w_c \frac{Z_c^{max} - Z_c + \gamma}{Z_t^{max} - Z_t^{min} + \gamma} \quad 16$$

Here, γ = random number (between 0 and 1).

Where, Z_c^{max}, Z_t^{max} = maximal value for the objective of total cost and time, respectively, in the current population; Z_c^{min}, Z_t^{min} = minimal value for the objective of total cost and time, respectively, in the current population; w_c, w_t = adaptive weights, respectively, on cost and time derived from the last generation, υ_c, υ_t = value for the criterion of cost and time respectively; υ = value for the project; w_c = adaptive weight for the criterion of cost; and w_t = adaptive weight for the criterion of time, Z_C represents the total cost of the *x* th solution in the current population; Z_t represents the time of the *x* th solution in the current population.

IV. Genetic Algorithm

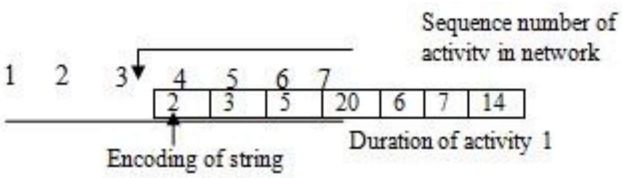
Genetic algorithms (GAs) are global search and optimization techniques modeled from natural genetics, and they explore the search space by incorporating a set of candidate solutions in parallel (Holland, 1975). GA for TCTP is used for selecting an optimal option to perform each activity of a project. This study applied simple GA for TCTP.

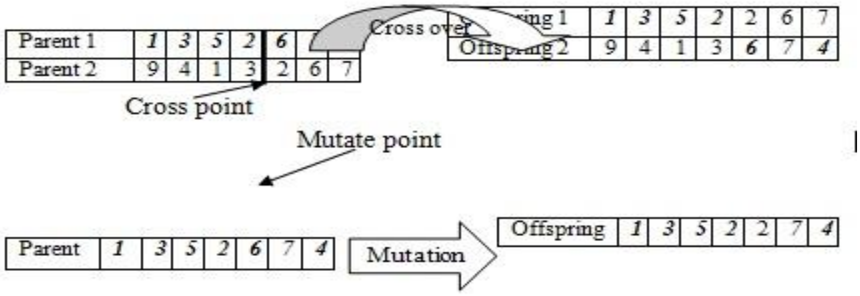
GAs procedure begins by generating an initial collection of random solutions that are encoded in the form of strings (or chromosome). A string is collection of genes which represent decision variables of problem. The number of string in population is referred to as population size. Strings are evaluated on their performances with respect to the fitness function in the population and fitter strings are selected by selection operator. Best strings exchange information and mutate to produce offspring chromosomes that are evaluated and can replace less fit member in the population to create new population for next generation. Thus, the best solutions are evolved through successive generation.

In TCTP perspective, each gene in the chromosome represents duration of an activity in the network. Crossover and mutation operations are performed to exploit and explore potential solution. In this study, single point crossover and uniform mutation are used. To evaluate the performance of strings Eq. (16) is used as a fitness equation (or objective function) in GA search.

GA is a robust and random search algorithm for TCTP. The GAs differ from conventional searching tools in that they operate on a set of solutions rather than a single solution, and hence multiple frontiers are searched simultaneously within a single run. This advantage makes GAs the most suitable searching engine for complicated multi-criteria optimization problems (Zheng and Ng, 2005).

Figure 2 Mechanism of GA.

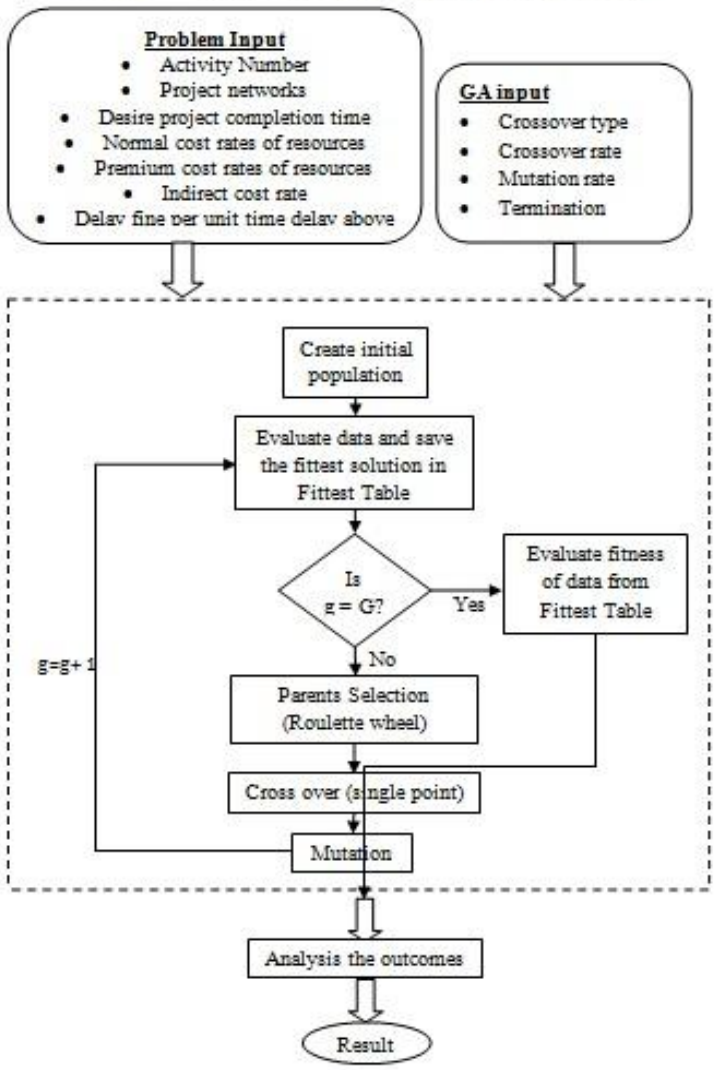




V. GA Based TCTP Solver

The operation of GA based model for TCTP is shown in Fig. 3. The proposed multi-objective time-cost optimization problem has been solved with genetic algorithm technique which is coded in MATLAB 7.7.0.471 (R2008b) and run on personal computer having Intel (R) Pentium (R) Dual CPU 2.80 GHz and 512 MB of RAM. One folder is created where inputs are given into text files. For each activity, one text file is used for time schedules and their corresponding resources and another text file is used for resource cost rates at available and above available limit and resources available limit. All GA parameters are inputted through dialogs.

Figure 3 Operation of the GA based model for TCTP



VI. Problem solving and results

6.1 Example 1

A project of seven activities is taken as an example which was derived by Zheng, Ng and Kumaraswamy (2004). Table 1 shows available activity options and corresponding durations and costs. This study was developed including the resources-time trade-off as input, so that the cost of each option was taken as resources to fit the case into the proposed model. Indirect cost rate was $1500/day. The robustness of the new proposed model in the deterministic situation was compared with two other previous models: (1) MAWA model (Zheng, Ng and Kumaraswamy, 2004) and (2) Gen and Cheng (2000) model.

Table 1 Options for seven activity example 1.

Activity Number	Precedent Activity	Option	Duration (days)	Direct cost ($)
1		1	14	23,000
		2	20	18,000
		3	24	12,000
2	1	1	15	3,000
		2	18	2,400
		3	20	1,800
		4	23	1,500
		5	25	1,000
3	1	1	15	4,500
		2	22	4,000
		3	33	3,200
4	1	1	12	45,000
		2	16	35,000
		3	20	30,000
5	2,3	1	22	20,000
		2	24	17,500
		3	28	15,000
		4	30	10,000
6	4	1	14	40,000
		2	18	32,000
		3	24	18,000
7	5,6	1	9	30,000
		2	15	24,000
		3	18	22,000

Table 2 Results of example 1 of the three models

Models	Optimal solutions								
	Time (day)	Cost ($)	Durations of activity (Day)						
Gen and Cheng (2000)	79	256,400	24	18	15	16	22	14	15
MAWA (Zheng, Ng and Kumaraswamy, 2004)	66	236,500	14	15	15	20	28	18	9
The new model	60	233500	14	15	15	12	22	24	9
	62	233000	14	15	15	20	24	18	9
	63	225500	14	15	15	16	24	24	9
	67	224000	14	15	15	20	28	24	9
	68	220500	14	15	15	20	30	24	9

The new model was able to find Pareto optimal front. Since, Gen and Cheng (2000) and Zheng, Ng and Kumaraswamy (2004) did not propose any Pareto front. Moreover, the finally obtained solutions by these two models as shown in table 2 respectively were not global optimal solutions. The new proposed model showed Pareto optimal solutions and best optimal solution (project time = 60 days and cost = $233500) in Table 2.

6.2 Example 2

A hypothetical case of 8 activities was developed to fit into the proposed model. Here, activities had several options of time and resources. A detail of example 2 is shown in table 3. Delay fine ($2500/day) was also added if project completion time is more than 50 days. Indirect cost rate of project was taken $1000/day. The parameters of the GA model were selected to solve the example 2, such as the generation number = 1000, population size = 200, crossover rate = 0.2 and mutation rate = 0.5.

The proposed model found the Pareto front as shown in Table 4 and Figure 5. The result of project example 2 is shown in Figure 4. The best optimal solution was the solution of time= 47 days, cost= \$ 2.552×10^5 and activity duration 7-6-7-15-5-9-10-3. The critical path of the best optimal solution is 1→3→4→5→7→8.

Table 3 Activity data of the project example 2.

Activity	Precedent activity	Time (days)	Resources (units/day)	Resources cost rates at available and above available limit (\$/unit/day) and available limit (units/day) respectively for each resources
1		6	10 7 19	400,500,7; 200,350,5;
		7	7 5 15	75,120,15
2	1	4	9 16	200,350,7; 125, 200,12
		6	7 10	
		8	5 7	
		9	4 6	
3	1	7	13 9	200,350,9; 50,150,7
		9	9 6	
		10	7 5	
4	2,3	12	5 16 9 5	400,500,3; 200,350,10;
		15	3 10 6 2	70,200,12;
		18	2 8 4 2	120,160,3
5	4	5	15	300,500,12
6	3	5	20 12	200,350,15; 30,70,9
		7	15 8	
		9	8 6	
		12	6 5	
7	2,5,6	9	17	200,350,14
		10	14	
8	7	3	4	200,350,2
		5	2	

Table 4 Pareto front for example 2.

Optimal solutions									
Time (days)	Cost (\$) (10^5)	Durations of activities							
45	2.709	6	6	7	15	5	9	9	3
46	2.607	7	6	7	15	5	9	9	3
47	2.552	7	6	7	15	5	9	10	3
48	2.527	7	9	9	15	5	9	9	3
49	2.47	7	9	9	15	5	12	10	3
50	2.458	7	9	10	15	5	12	10	3

Figure 4 Results of example 2.

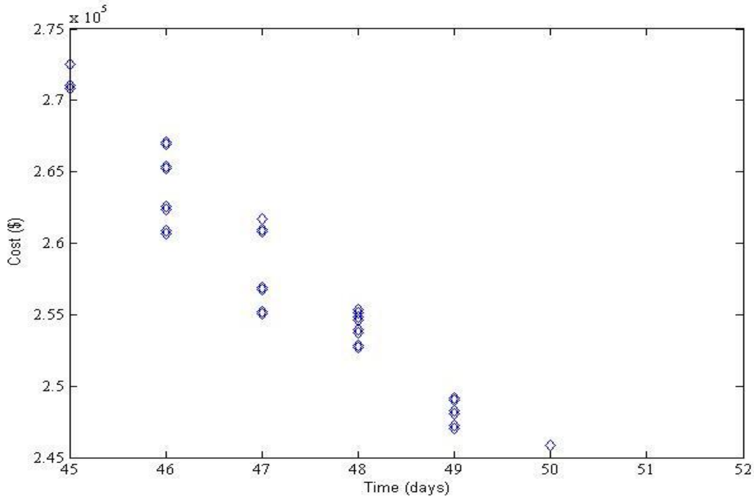
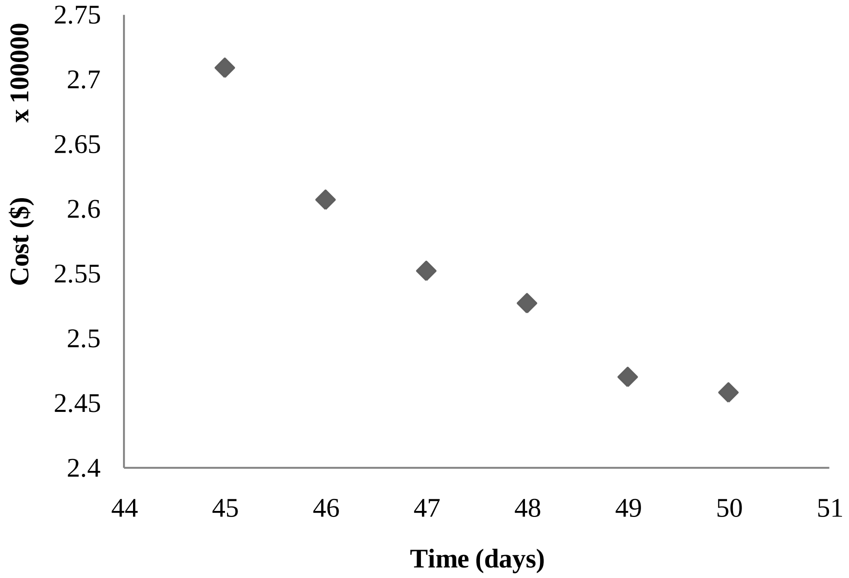


Figure 5 Pareto front of example 2.



VII. Conclusions

The GA based multi-objective time-cost trade-off problem solver was originally developed to minimize both project time and cost by assigning one optimal option of time and resources to each activity in the project network. Here, activity resources and time were directly used to calculate project cost. Incorporating MAWA, the TCTP solver integrated the project time and cost in single objective. Moreover, it was illustrated that this algorithm can define the Pareto front of the problem. So that, the decision maker gets options to solve project according to his desire. This GA based TCTP solver took much iterations but it was a robust and random searching algorithm so that it resisted the trials to entrap in local optima. Thereby, the development of the GA based multi-objective approach provides an efficient model to solving project time-cost optimization with resources consideration.

The performance of the presented model can be further analyzed in terms of number of iteration by comparing it with other best known algorithms for multi-objective TCT project scheduling. The algorithm can also be extended by considering limited resources time-cost trade-off with other different types of fuzzy numbers.

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