International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol.3, Issue.1, Jan-Feb. 2013 pp-342-345 ISSN: 2249-6645

Weak Triple Connected Domination Number of a Graph

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Abstract: *Recently the concept of triple connected graphs with real life application was introduced in [14] by considering the existence of a path containing any three vertices of a graph G. In [3], G. Mahadevan et. al., introduced triple connected domination number of a graph. Also in[10], the authors introduced the concept of strong triple connected domination number of a graph. A subset S of V of a nontrivial graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number and is denoted by* γ_{tc}. *A subset S of V of a nontrivial graph G is said to be strong triple connected dominating set, if S is a strong dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all strong triple connected dominating sets is called the strong triple connected domination number and is denoted by* γ_{stc}. *In this paper we introduce the concept of weak triple connected domination number of a graph.Let G = (V, E) a graph. A set* $D \subseteq V$ *is a weak dominating set of G if for every vertex* $y \in V - D$ *there is a vertex* $x \in D$ *with* $xy \in E$ *and* $d(y, G) \geq d(x, G)$. *The weak domination number* $\gamma_w(G)$ *is defined as the minimum cardinality of a weak dominating set. A subset S of V of a nontrivial graph G is said to be weak triple connected dominating set, if S is a weak dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all weak triple connected dominating sets is called the weak triple connected domination number and is denoted by* γ_{wtc}. *We determine this number for some standard graphs and obtain bounds for general graph. Its relationship with other graph theoretical parameters are also investigated.*

Keywords: *Domination Number, Triple connected graph, Weak Triple connected domination number AMS Subject Classification: 05C 69*

I. Introduction

By a graph we mean a finite, simple, connected and undirected graph G (V, E), where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has p vertices and q edges. Degree of a vertex v is denoted by d(v), the maximum degree of a graph G is denoted by $\Delta(G)$. We denote a cycle on p vertices by C_p, a path on p vertices by P_p, and a complete graph on p vertices by K_p. A graph G is connected if any two vertices of G are connected by a path. A maximal connected sub graph of a graph G is called a component of G. The number of components of G is denoted by $\omega(G)$. The complement $\overline{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A tree is a connected acyclic graph. A bipartite graph (or bigraph) is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge has one end in V_1 and another end in V_2. A complete bipartite graph is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1|=m$ and $|V_2|=n$, is denoted by $K_{m,n}$. A star, denoted by $K_{1,p-1}$ is a tree with one root vertex and p – 1 pendant vertices. A cut – vertex (cut edge) of a graph G is a vertex (edge) whose removal increases the number of components. A vertex cut, or separating set of a connected graph G is a set of vertices whose removal results in a disconnected. The connectivity or vertex connectivity of a graph G, denoted by $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut. The chromatic number of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colour. For any real numberx, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. A Nordhaus -Gaddum-type result is a (tight) lower or upper bound on the sum or product of a parameter of a graph and its complement. Terms not defined here are used in the sense of [2].

A subset S of V is called a dominating set of G if every vertex in V − S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced sub graph <S> is connected. The minimum cardinality taken over all connected dominating sets is the connected domination number and is denoted by γ_c. A subset S of V is called a weak dominating set of G, if for every vertex $y \in V(G) - D$ there is a vertex $x \in D$ with $xy \in E(G)$ and $d(y,G) \geq d(x,G)$. The weak domination number $\gamma_w(G)$ is defined as the minimum cardinality of a weak domination set. One can get a comprehensive survey of results on various types of domination number of a graph in [17, 18, 19].

Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [15, 16]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph J. et. al.,[14] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. In [3] Mahadevan G. et. al., introduced triple connected domination number of a graph and found many results on them.

A subset S of V of a nontrivial connected graph G is said to be triple connected dominating set, if S is a dominating set and the induced sub graph <S> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the triple connected domination number of G and is denoted by $\gamma_{tc}(G)$.

In [4, 5, 6, 7, 8, 9] Mahadevan G. et. al., introduced complementary triple connected domination number, paried triple connected domination number, complementary perfect triple connected domination number, triple connected two domination number, restrained triple connected domination number, dom strong triple connected domination number of a graph. In [10], the same author also introduced strong triple connected domination of a graph which is communicated to a journal for publication.

In this paper, we use this idea to develop the concept of weak triple connected dominating set and weak triple connected domination number of a graph

Notation 1.1 Let G be a connected graph with m vertices $v_1, v_2, \ldots, v_m$. The graph obtained from G by attaching n_1 times a pendant vertex of P_{l_1} on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on, is denoted by $G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, \ldots, n_mP_{l_m})$ where $n_i, l_i \geq 0$ and $1 \leq i \leq m$.

Example 1.2 Let $v_1, v_2, v_3, v_4, v_5, v_6$ be the vertices of K_6. The graph $K_6(P_2, 3P_2, P_3, P_2, P_4, P_2)$ is obtained from K_6 by attaching 1 time a pendant vertex of P_2 on v_1, 3 time a pendant vertex of P_2 on v_2, 1 time a pendant vertex of P_3 on v_3 and 1 time a pendant vertex of P_2 on v_4, 1 time a pendant vertex of P_4 on v_5, 1 time a pendant vertex of P_2 on v_6 and is shown in Figure 1.1.

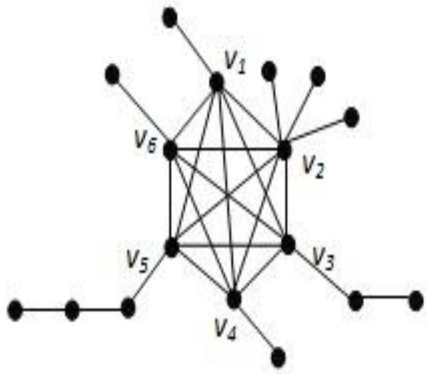

Figure 1.1: K_6 (P2, $3P_2$, P_3, $2P_4$)

II. Weak Triple connected domination number

Definition 2.1 A subset S of V of a nontrivial graph G is said to be a weak triple connected dominating set, if S is a weak dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all weak triple connected dominating sets is called the weak triple connected domination number of G and is denoted by $\gamma_{wtc}(G)$. Any weak triple connected dominating set with γ_{wtc} vertices is called a γ_{wtc} -set of G.

Example 2.2 for the graph G_1 in Figure 2.1, S = {v_1, v_2, v_3} forms a γ_{wtc} -set of G. Hence $\gamma_{wtc}(G_1) = 3$.

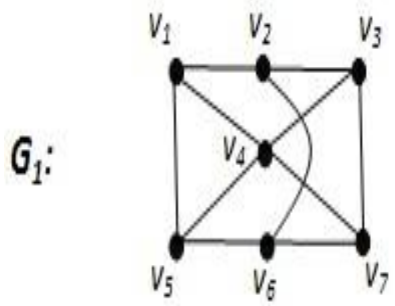

Figure 2.1

Observation 2.3 Weak triple connected dominating set (wtcd set) does not exists for all graphs and if exists, then $\gamma_{wtc}(G) \geq 3$.

Example 2.4 for the graph G_2 in Figure 2.2, any minimum triple connected dominating set must contain the v_5 and any triple connected dominating set containing v_5 is not a weak triple connected and hence γ_{wtc} does not exists.

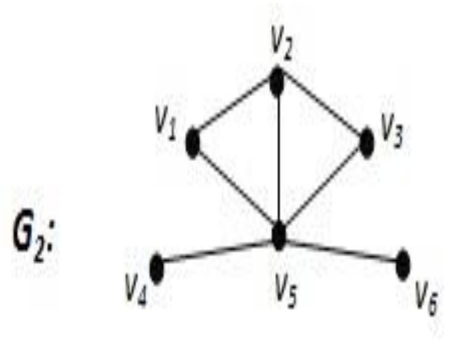

Figure 2.2: Graph with no wtcd set

Throughout this paper we consider only connected graphs for which weak triple connected dominating set exists.

Observation 2.5 The complement of the weak triple connected dominating set need not be a weak triple connected dominating set.

Observation 2.6 every weak triple connected dominating set is a triple dominating set but not conversely.

Observation 2.7 every weak triple connected dominating set is a dominating set but not conversely.

Observation 2.8 For any connected graph G, $\gamma_{tc}(G) \leq \gamma_{wtc}(G)$ and for K_p, $(p > 3)$ the bound is sharp.

Exact value for some standard graphs:

1) For any cycle of order $p > 3$, $\gamma_{wtc}(C_p) = \begin{cases} 3 & \text{if } p < 5 \\ p-2 & \text{if } p \geq 5. \end{cases}$
2) For any complete graph of order $p > 3$, $\gamma_{wtc}(K_p) = 3$.

Theorem 2.9 For any connected graph G with $p > 3$, we have $3 \leq \gamma_{wtc}(G) \leq p - 1$ and the bounds are sharp.

Proof the lower and upper bounds follows from Definition 2.1. For the cycle C_4, the lower bound is attained and for K_4, the upper bound is attained.

Theorem 2.10 For a connected graph G with 4 vertices, $\gamma_{wtc}(G) = p - 1$ if and only if G is isomorphic to $G \cong C_4, K_4, K_4 - \{e\}$.

Proof Suppose $G \cong C_4, K_4 - \{e\}, K_4$, then $\gamma_{wtc}(G) = 3 = p - 1$. Conversely, let G be a connected graph with p vertices such that $\gamma_{wtc}(G) = p - 1$. Let $S = \{v_1, v_2, v_3\}$ be a γ_{wtc} -set of G and $V - S = \{v_4\}$. Since S is a γ_{wtc} -set of G, $\langle S\rangle = P_3$ or C_3.

Case(i) $\langle S\rangle = P_3 = v_1v_2v_3$.

Since G is connected, v_4 is adjacent to v_1 (or v_3) or v_4 is adjacent to v_2. Hence $G \cong P_4$ or $K_{1,3}$, but for both the graphs a γ_{wtc} – set does not exists. By increasing the degrees, no graph exists.

Case(i) $\langle S\rangle = C_3 = v_1v_2v_3v_1$.

Since G is connected, v_4 is adjacent to v_1 (or v_2 or v_3). Hence $G \cong C_3(P_2)$, but for $C_3(P_2)$ γ_{wtc} –set does not exists. On increasing the degress, $G \cong C_4, K_4, K_4 - \{e\}$.

The Nordhaus – Gaddum type result is given below:

Theorem 2.24 Let G be a graph such that G and $\overline{G}$ have no isolates of order $p > 3$. Then

(i) $\gamma_{wtc}(G) + \gamma_{wtc}(\overline{G}) \leq 2(p - 1)$

(ii) $\gamma_{wtc}(G).\ \gamma_{wtc}(\overline{G}) \leq (p - 1)^2$ and the bound is sharp.

Proof The bound directly follows from Theorem 2.9. For the cycle C_4, both the bounds are attained.

III. Relation with Other Graph Theoretical Parameters

Theorem 3.1 For any connected graph G with $p > 3$ vertices, $\gamma_{wtc}(G) + \kappa(G) \leq 2p - 2$ and the bound is sharp if and only if $G \cong K_4$.

Proof Let G be a connected graph with $p > 3$ vertices. We know that $\kappa(G) \leq p - 1$ and by Theorem 2.9, $\gamma_{wtc}(G) \leq p - 1$. Hence $\gamma_{wtc}(G) + \kappa(G) \leq 2p - 2$. Suppose G is isomorphic to K_4. Then clearly $\gamma_{wtc}(G) + \kappa(G) = 2p - 2$. Conversely, Let $\gamma_{wtc}(G) + \kappa(G) = 2p - 2$. This is possible only if $\gamma_{wtc}(G) = p - 1$ and $\kappa(G) = p - 1$. But $\kappa(G) = p - 1$, and so $G \cong K_p$ for which $\gamma_{wtc}(G) = 3 = p - 1$ so that $p = 4$. Hence $G \cong K_4$.

Theorem 3.2 for any connected graph G with $p > 3$ vertices, $\gamma_{wtc}(G) + \chi(G) \leq 2p - 1$ and the bound is sharp if and only if $G \cong K_4$.

Proof Let G be a connected graph with $p > 3$ vertices. We know that $\chi(G) \leq p$ and by Theorem 2.9, $\gamma_{wtc}(G) \leq p - 1$. Hence $\gamma_{wtc}(G) + \chi(G) \leq 2p - 1$. Suppose G is isomorphic to K_4. Then clearly $\gamma_{wtc}(G) + \chi(G) = 2p - 1$. Conversely, let $\gamma_{wtc}(G) + \chi(G) = 2p - 1$. This is possible only if $\gamma_{tc}(G) = p - 1$ and $\chi(G) = p$. Since $\chi(G) = p$, G is isomorphic to K_p for which $\gamma_{wtc}(G) = 3 = p - 1$ so that $p = 4$. Hence $G \cong K_4$.

Theorem 3.3 for any connected graph G with $p > 3$ vertices, $\gamma_{wtc}(G) + \Delta(G) \leq 2p - 2$ and the bound is sharp.

Proof Let G be a connected graph with $p > 3$ vertices. We know that $\Delta(G) \leq p - 1$ and by Theorem 2.9, $\gamma_{wtc}(G) \leq p - 1$. Hence $\gamma_{wtc}(G) + \Delta(G) \leq 2p - 2$. For K_4, the bound is sharp.

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