

## Meshfree Simulation of the Penetration of Thick Metal Structure by Rigid Penetrator

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**Abstract:** The Reproducing Kernel Particle Method (RKPM) has been used for the simulation of the penetration of thick metal blocks by rigid penetrator. The particle to segment contact algorithm has been used for the contact detection as well the contact constraints implementation. In the study it is found that the finite element method without remeshing fails in earlier stage of the penetration process while the RKPM goes beyond very large deformation. The effectiveness of the RKPM for the simulation of the large deformation of mechanical structure is confirmed.

**Keywords:** large deformation, mechanical contact, penetration, RKPM.

### I. Introduction

Nowadays, the meshless or meshfree methods are widely used in mechanical science and engineering applications because of their advantages over the finite element methods (FEM); especially in the simulation of problems involving large deformations and distortions of the structure. Among the meshfree methods are: Smoothed Particle Hydrodynamics (SPH) [1, 2]; Diffuse Element Method (DEM) [3]; Element-Free Galerkin (EFG) [2, 4, 5, 6]; Reproducing Kernel Particle Method (RKPM) [7-17]; Partition of Unity Method (PUM) [18] and so forth.

The simulation of the penetration requires the development of contact algorithms which can be classified into two categories, contact searching algorithm and contact constraints algorithm. The contact searching consists of finding the contacting boundaries (contacting particles/nodes). The contact constraints algorithm is concerned with the implementation of the so-called impenetrability condition which does not allow overlapping between bodies, in other words two bodies cannot occupy the same space at the same time.

In the framework of FEM, several contact searching algorithm have been developed. These contact searching algorithms include the master-slave contact algorithm [19, 20]; the single surface contact algorithm [19, 20], the hierarchy territory contact algorithm [21], and the pinball contact algorithm [22, 23]. In the framework of meshless method, the particle to particle contact algorithm [24, 25], the meshfree contact-detection algorithm [11, 12], the particle to segment contact algorithm [13, 15-17],... were developed. In the particle to segment contact algorithm the boundaries of the bodies are represented by particles located on the boundary, and these particles are interconnected to form polygons fitting the boundary without overlapping. The algorithm is developed for the simulation of the contact between a flexible body and several rigid bodies as encountered in metal forming where the work piece is deformable and the tools are usually assumed to be rigid. This algorithm was found to very effective as it has the advantage to allow the correct evaluation of the interpenetration used for the determination of the contact constraints [13, 15, 16]. Therefore in this work, it will be used for the simulation of the penetration process.

### II. RKPM discretization

The reproducing kernel particle method (RKPM) is systematically formulated in [7-11]. The RKPM uses the finite integral representation of a function  $u(\mathbf{x})$  in a domain  $\Omega_x$ .

$$u^a(\mathbf{x}) = \int_{\Omega_x} \Phi_a(\mathbf{x} - \mathbf{y})u(\mathbf{y})d\Omega_x \quad (1)$$

Where  $u^a(\mathbf{x})$  is the approximation of function  $u(\mathbf{x})$  and  $\Phi_a(\mathbf{x} - \mathbf{y})$  is the kernel function with compact support  $a$ .

Discrediting the domain  $\Omega_x$  by a set of particles  $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{NP}\}$ , where  $\mathbf{x}_I$  is the position vector of particle  $I$ , and  $NP$  is the total number of particles; the integral is approximated by the following summation:

$$u^h(\mathbf{x}) = \sum_{I=1}^{NP} \mathbf{N}_I(\mathbf{x})u(\mathbf{x}_I) \quad (2)$$

Where  $\mathbf{N}_I(\mathbf{x})$  is the RKPM shape function defined to be?

$$\mathbf{N}_I(\mathbf{x}) = \mathbf{C}(\mathbf{x}; \mathbf{x} - \mathbf{x}_I)\Phi_a(\mathbf{x} - \mathbf{x}_I)\Delta\mathbf{V}_I \quad (3)$$

$\mathbf{C}(\mathbf{x}; \mathbf{x} - \mathbf{x}_I)$  is the correction function introduced to improve the accuracy of the approximation near the boundaries and  $\Delta V_I$  is the volume of particle  $I$  and the subscript  $h$  is associated with a discretized domain.

The application of the principle of virtual work and the particles approximation to the equation of the conservation of the linear momentum lead to the equation of motion for contact problems [13]:

$$\mathbf{f}_I^{ext} - \mathbf{f}_I^{int} + \mathbf{f}_I^{cont} = \mathbf{M}_{IJ} \ddot{\mathbf{d}}_J \quad (4)$$

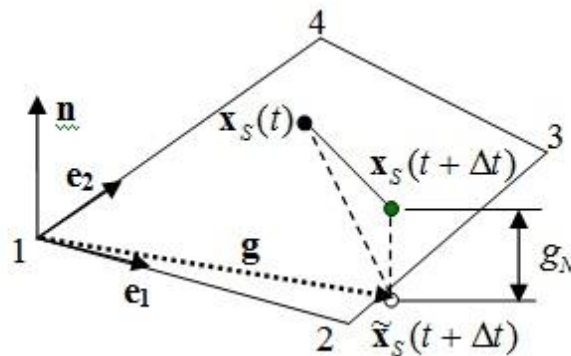
Where:

- $\mathbf{f}_I^{ext}$  : the external force of particle  $I$
- $\mathbf{f}_I^{int}$  : the internal force of particle  $I$
- $\ddot{\mathbf{d}}_J$  : the generalized acceleration of particle  $J$
- $\mathbf{f}_I^{cont}$  : the contact force of particle  $I$  (see **part III**)
- $M_{IJ} = \int_{\Omega_0} \rho_0 N_I N_J d\Omega$  is the consistent mass matrix which can be approximated by row sum technique.

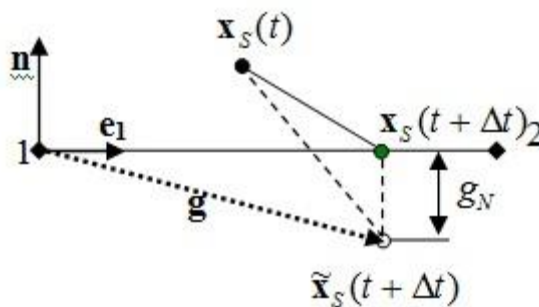
### III. Particle To Segment Contact Algorithm

In this work, the particle to segment contact algorithm is used for the contact simulation. The particle to segment contact algorithm was developed by G. Li et al [13] for 3D problems and by K. Sidibe et al. [15] for 2D problems. Applications of the contact algorithm to sheet metal forming are done by K. Sidibe et al. [16, 17]. An updated version of this algorithm for flexible bodies called ‘reversible particle to segment contact algorithm’ was developed by K. Sidibe et al. [16] to handle the mechanical contact between deformable bodies. Designated for the simulation of the metal forming analysis, the particle to segment contact algorithm modelled the tools as rigid bodies and the work piece as flexible (deformable) body. In 2D, the boundaries of the rigid tools are discretized by piecewise linear segments while in 3D the boundaries of tools assumed to be rigid are modelled by flat segments.

Every time step, prior to the calling to the contact-subroutine, the trial accelerations, velocities and displacements are computed from the explicit time routine. The trial positions of the particles, obtained from the trial displacements, are then used to check whether there is overlapping between the bodies. Whenever any overlapping is found the contact forces are evaluated and applied to cancel the interpenetration of the bodies as shown in Figure 1.



(a) 3D



(b) 2D

**Figure 1.** Correction of trial position of the penetrating slave particle S

In **Figure 1**:  $\mathbf{x}_S(t)$  is the position of S at time  $t$  (before penetration),  $\tilde{\mathbf{x}}_S(t + \Delta t)$  trial position of S at time  $t + \Delta t$  and  $\mathbf{x}_S(t + \Delta t)$  corrected position of S at time  $t + \Delta t$  (after application of contact forces);  $\mathbf{n}$  is the normal unit vector of the contact segment pointing out from the segment toward the flexible body,  $\mathbf{e}_1$  and  $\mathbf{e}_2$  are the tangential unit vectors on the edges of the segment and  $g_N$  is the normal gap.

As shown in **Figure 1**, the normal gap is given by:

$$g_N = \mathbf{g} \cdot \mathbf{n} = (\tilde{\mathbf{x}}_S(t + \Delta t) - \mathbf{x}_1(t + \Delta t)) \cdot \mathbf{n} \quad (5)$$

For each penetrating slave particle, the penalty force necessary to cancel the penetration is evaluated by:

$$\mathbf{f}_N(s) = -\frac{m_s g_N}{\Delta t^2} \mathbf{n} = f_n \cdot \mathbf{n} \quad (6)$$

The Coulomb friction model is adopted to evaluate the friction between the contacting bodies. Frictional force applied to oppose the relative tangential displacement at the contact interface is given by:

$$\mathbf{f}_T(s) = -\min\left(\mu f_n, \left\| \frac{m_s}{\Delta t} \mathbf{v}_r \right\| \right) \frac{\mathbf{v}_r}{\|\mathbf{v}_r\|} \quad (7)$$

Where  $\mu$  the friction coefficient on the contact is interface and  $\mathbf{v}_r$  is the tangential component of the relative velocity of the slave particle S with respect to the contact segment.

The resultant of the contact forces, on a given slave particle J is calculated by

$$\mathbf{f}_J = \mathbf{f}_N(J) + \mathbf{f}_T(J) \quad (8)$$

The force vectors calculated above are the exact nodal force vectors for each penetrating particle, to satisfy the impenetrability and friction conditions at the interface. Therefore the exact nodal force is re-distributed to a non-local 'fictitious force'. The fictitious force vector for a particle  $I$  is calculated as follows.

$$\mathbf{f}_I^{cont} = \sum_J \mathbf{f}_J N_I(\mathbf{X}_J) \quad (9)$$

#### IV. Numerical Applications

Our current RKPM computer codes were tested and validated through standard test by G. Li et al. [13] for 3D formulation and K. Sidibe et al. [14] for the 2D formulation. Also the 3D implementation of the particle to segment contact algorithm was validated by G. Li et al. [13] and the 2D implemented by K. Sidibe et al. [15] for bulk metal forming. This work deals with the extension of these codes to the penetration simulation, the basic formulation staying the same. Details about the implementation in the computer code can be found in K. Sidibe et al. [16].

To further investigate on the effectiveness of the RKPM for large deformation analysis, the problem of penetration is considered in this section. Both two and three-dimensional penetration simulations are treated. The material of the work pieces is assumed to be an elastoplastic with isotropic non-linear strain hardening law defined as

$$\sigma = \mathbf{K}(\varepsilon_0 + \varepsilon_p)^n \quad (10)$$

Where  $\sigma$  is the stress and  $\varepsilon_p$  the plastic strain, the parameters defining the material used for the simulation are given in Table 1.

##### 4.1 Two-dimensional penetration simulation

A rigid penetrator of width 4mm with round nose is moved downward with a constant velocity of 10m/s, and penetrates a work piece of  $100 \times 25 \text{mm}^2$ . The statement of the problem is shown in **Figure 2**. A plane strain formulation is used. The contact between the penetrator and the target is assumed to be frictionless. The work piece is discretized by a set of 4992 particles distributed with various particles densities as shown in **Figure 3-a**. The damaged shape of the workpiece is shown in **Figure 3-b**. Using the finite element method (FEM) commercial code LS-DYNA3D without remeshing, mesh overlapping is observed in earlier stage of the deformation as shown in **Figure 4**.

##### 4.2 Three-dimensional penetration simulation

A cylindrical rigid penetrator (radius 5.11mm) with round nose penetrates a thick steel plate ( $71.12 \times 15.24 \times 25.4 \text{mm}^3$ ). The penetrator is initially inclined at  $45^\circ$  and attacks the target with constant velocity of (50m/s, 0,

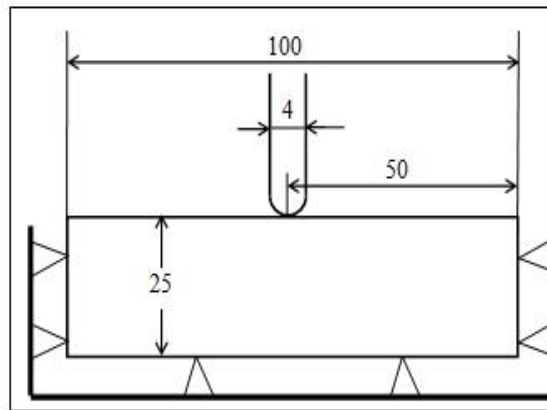
-50m/s). The target is discretized by a set of 5330 particles distributed with various particles densities as shown in **Figure 5-a**. The damage shape of the plate is shown in **Figure 5-b**.

### 4.3 Discussion

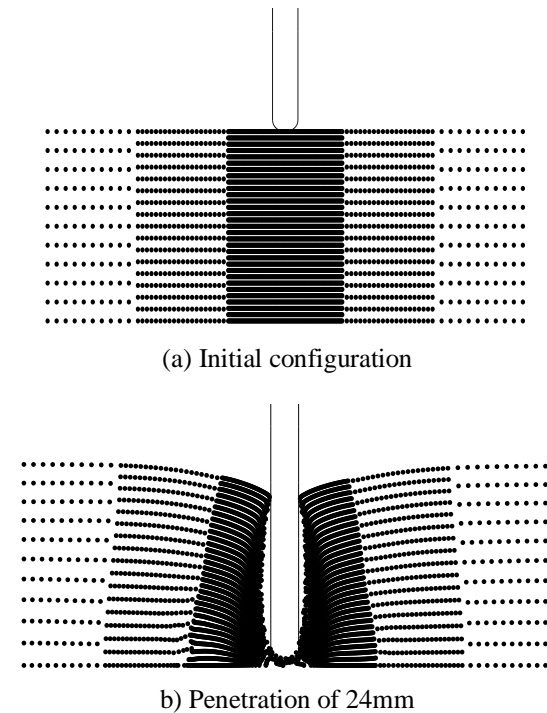
Analysing the results obtained, we can see that successful simulation of the penetration has been done. For a block of metal such large deformations might lead to mechanical fracture or rupture. Our current RKPM code does not take into account the mechanical fracture. The implementation of the mechanical fracture in our RKPM code is under investigation and might be a subject of future papers.

**Table 1** Material parameters of the work pieces for the simulation

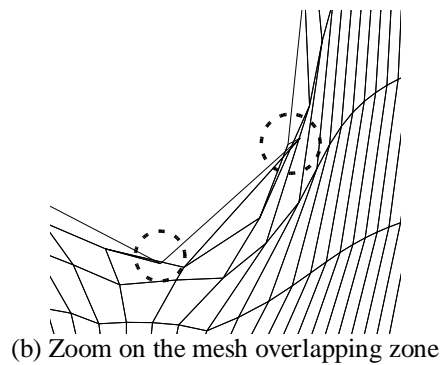
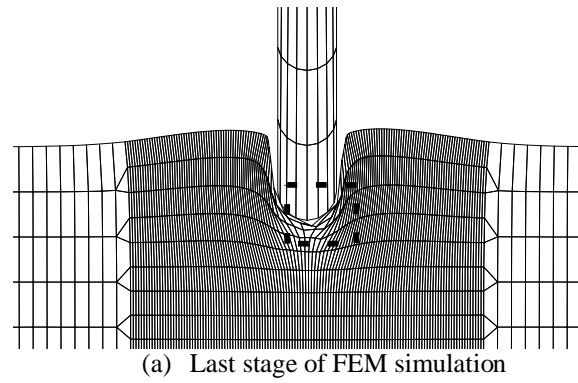
Parameter	value
Young's modulus ( $E$ )	71Gpa
Poisson's ration ( $\nu$ )	0.33
$\epsilon_0$	0.0166
K	576.79Mpa
n	0.3593
Mass density	1700kg/m3



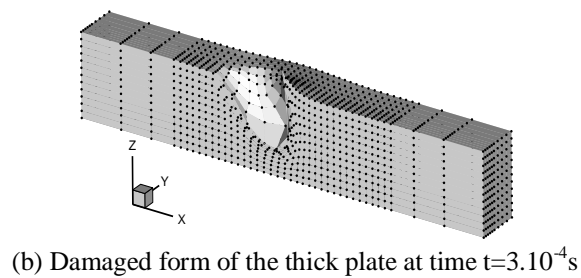
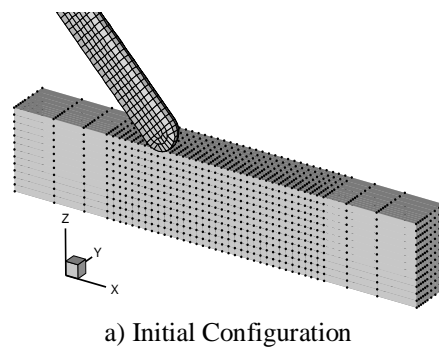
**Figure 2** Statement of the two-dimensional penetration problem



**Figure 3** Simulation of two-dimensional penetration



**Figure 4** Mesh overlapping when using FEM without adaptivity for the simulation of two-dimensional penetration



**Figure 5** Simulation of the 3D-penetration

## V. Conclusion

The particle to segment contact algorithm, correctly implemented in the Reproducing kernel Particle Method, has been successfully used for the simulation of the damage-penetration of thick metal. Both 2D and 3D problems are treated. As seen through this work, earlier failure of the computation is observed when using the FEM. The effectiveness of RKPM for large deformation is confirmed and to make more realistic simulation mechanical rupture model must be implemented in our current RKPM code.

## VI. Acknowledgement

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