Performance Analysis of OFDM System Using PAPR Reduction Techniques

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Abstract: Orthogonal Frequency division Multiplexing (OFDM) is an efficient method of data transmission for high speed communication systems. However, the main drawback of OFDM system is the high Peak to Average Power Ratio (PAPR) of the transmitted signals. OFDM consist of large number of independent subcarriers, as a result of which the amplitude of such a signal can have high peak values. Coding, phase rotation and clipping are among many PAPR reduction schemes that have been proposed to overcome this problem. Here two different PAPR reduction methods e.g. partial transmit sequence (PTS) and selective mapping (SLM) are used to reduce PAPR. Significant reduction in PAPR has been achieved using these techniques. The performances of the three methods are then compared.

Index Terms: Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), and PAPR reduction techniques

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation (MCM) technique which seems to be an attractive candidate for fourth generation (4G) wireless communication systems. OFDM offer high spectral efficiency, immune to the multipath delay, low inter-symbol interference (ISI), immunity to frequency selective fading and high power efficiency. Due to these merits OFDM is chosen as high data rate communication systems such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access (mobile Wi-MAX). However OFDM system suffers from serious problem of high PAPR. In OFDM system output is superposition of multiple sub-carriers. In this case some instantaneous power output might increase greatly and become far higher than the mean power of system. To transmit signals with such high PAPR, it requires power amplifiers with very high power scope. These kinds of amplifiers are very expensive and have low efficiency-cost. If the peak power is too high, it could be out of the scope of the linear power amplifier. This gives rise to non-linear distortion which changes the superposition of the signal spectrum resulting in performance degradation. If no measure is taken to reduce the high PAPR, OFDM system could face serious restriction for practical applications [1]-[4]. PAPR can be described by its complementary cumulative distribution function (CCDF). In this probabilistic approach certain schemes have been proposed by researchers. These include clipping, coding and signal scrambling techniques. Under the heading of signal scrambling and distortion techniques there are three schemes included. Which is Partial transmit sequence (PTS) and Selected Mapping (SLM) and proposed companding. Although some techniques of PAPR reduction have been summarized in [5], it is still indeed needed to give a comprehensive review including some motivations of PAPR reductions, such as power saving, and to compare some typical methods of PAPR reduction through theoretical analysis and simulation results directly. An effective PAPR reduction technique should be given the best trade-off between the capacity of PAPR reduction and transmission power, data rate loss, implementation complexity and Bit-Error-Ratio (BER) performance etc.

In this paper, firstly the distribution of PAPR based on the characteristics of the OFDM signals are investigated then typical PAPR reduction techniques are analyzed.

II. OFDM Signal Characteristics

An OFDM symbol is made of sub-carriers modulated by constellations mapping. This mapping can be achieved from phase-shift keying (PSK) or quadrature amplitude modulation (QAM). For an OFDM system with N sub-carriers, the high-speed binary serial input stream is denoted as $\{a_i\}$.After serial to parallel (S/P) conversion and constellation mapping, a new parallel signal sequence $\{d_0,d_1,......d_i.....d_{N-1}\}$is obtained, d_i is a discrete complex valued signal[6]. Here, $d_i \in \{\pm1\}$ when BPSK mapping is adopted. When QPSK mapping is used, $d_i \in \{\pm1, \pm i\}$.Each element of parallel signal sequence is supplied to N orthogonal sub-carriers $\{e^{j2\pi f0t}, e^{j2\pi f1t} \ldots e^{j2\pi fN-1t}\}$ for modulation, respectively. Finally, modulated signals are added together to form an OFDM symbol. Use of discrete Fourier transform simplifies the OFDM system structure. The complex envelope of the transmitted OFDM signals can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_K \; e^{j2\pi f_k t}, \; 0 \le t \le NT \qquad (1)$$

Signals with large N become Gaussian distributed with Probability Density Function (PDF) is given by [5].

$$P_r\{x(t)\} = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{[x(t)]^2}{2\sigma^2}} \qquad (2)$$

Where σ is the variance of x(t).

III. PAPR (Peak-To-Average Power Ratio)

In general, the PAPR [3] of OFDM signals x (t) is defined as the ratio between the maximum instantaneous power and its average power

$$PAPR[X(t)] = \frac{P_{PEAK}}{P_{AVERAGE}} = 10\ \log_{10} \frac{\max[|X(n)|^2]}{E[|X_n|^2]} \tag{3}$$

Where P $_{PEAK}$ represents peak output power, P $_{AVERAGE}$ means average output power. E[·] denotes the expected value, x_n represents the transmitted OFDM signals which are obtained by taking IFFT operation on modulated input symbols Xk [7]. x_n is expressed as:]

$$x_n = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_k\, W_N^{nk} \tag{4}$$

The instantaneous output of an OFDM system often has large fluctuations compared to traditional single-carrier systems. This requires that system devices, such as power amplifiers, A/D converters and D/A converters, must have large linear dynamic ranges. If this is not satisfied, a series of undesirable interference is encountered when the peak signal goes into the non-linear region of devices at the transmitter, such as high out of band radiation and inter-modulation distortion. PAPR reduction techniques are therefore of great importance for OFDM systems. Also due to the large fluctuations in power output the HPA (high power amplifier) should have large dynamic range. This results in poor power efficiency.

IV. PAPR(Peak-To-Average Power Ratio) Reduction Techniques

Several PAPR reduction techniques have been proposed in the literature [6]. These techniques are divided into two groups - signal scrambling techniques and signal distortion techniques which are given below:

a) Signal Scrambling Techniques

- Block Coding Techniques
- Block Coding Scheme with Error Correction
- Selected Mapping (SLM)
- Partial Transmit Sequence (PTS)
- Interleaving Technique
- Tone Reservation (TR)
- Tone Injection (TI)

b) Signal Distortion Techniques

- Peak Windowing
- Envelope Scaling
- Peak Reduction Carrier
- Clipping and Filtering

One of the most pragmatic and easiest approaches is clipping and filtering which can snip the signal at the transmitter is to eliminate the appearance of high peaks above a certain level. But due to non-linear distortion introduced by this process, orthogonality [8] is destroyed to some extent which results in In-band noise and Out-band noise. In-band noise cannot be removed by filtering, it decreases the bit error rate (BER). Out-band noise reduces the bandwidth efficiency but frequency domain filtering [7] can be employed to minimize the out-band power. Although filtering has a good effect on noise suppression, it may cause peak re-growth. To overcome this drawback, the whole process is repeated several times until a desired situation is achieved. Here, two signal scrambling techniques are used to overcome these problems.

1. Selection Mapping Technique (SLM)

The CCDF of the original signal sequence PAPR above threshold PAPR0 is written as *Pr {PAPR> PAPR₀}* Thus for K statistical independent signal waveforms, CCDF can be written as $[Pr\ \{PAPR > PAPR_0\}]^R$ so the probability of PAPR exceed the same threshold. The probability of PAPR larger than a threshold Z can be written as

$$P(PAPR < Z) = F(Z)^N = (1 - \exp(-Z))^N \tag{5}$$

Assuming that M-OFDM symbols carry the same information and that they are statistically independent of each other. In this case, the probability of PAPR greater than Z is equals to the product of each independent probability. This process can be written as

$$P(\ PAPR_{LOW} > Z) = (P\ \{PAPR > Z\})^M = \left((1 - \exp(-Z))^N\right)^M \tag{6}$$

In selection mapping method, firstly M statistically independent sequences which represent the same information are generated, and next, the resulting M statistically independent data blocks

$S_m=[S_{m,0}, S_{m,1}, S_{m,N-1}]^T$, for m=1,2,...,M are then forwarded into IFFT operation simultaneously. $x_m=[x_1,x_2,x_3]^T$ in discrete time-domain are acquired and then the PAPR of these M vectors are calculated separately. Eventually, the sequences x with the smallest PAPR is selected for final serial transmission. Figure 1 shows the basic block diagram of selection mapping technique for suppressing the high PAPR.

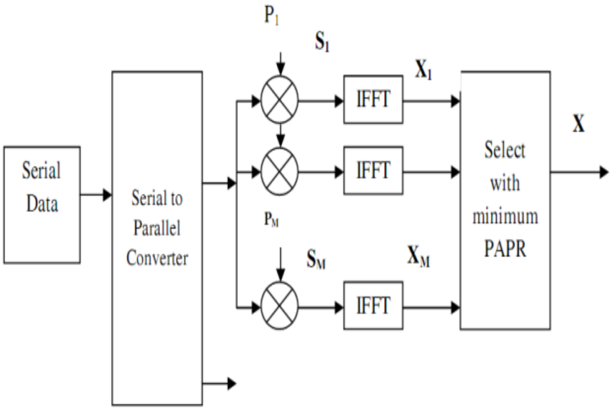

Figure 1. The Block Diagram of Selected Mapping Technique

2. Partial Transmit Sequence (PTS)

Partial Transmit Sequence (PTS) algorithm is a technique for improving the statistics of a multi-carrier signal. The basic idea of partial transmit sequences algorithm is to divide the original OFDM sequence [9] into several sub-sequences and for each sub-sequences multiplied by different weights until an optimum value is chosen.

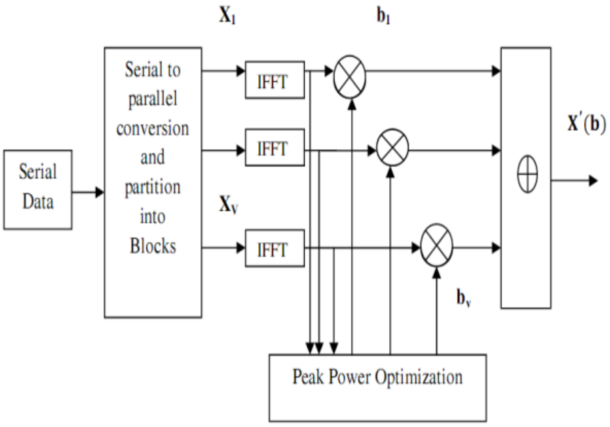

Figure 2. The Block diagram of PTS Technique

Figure 2 [10] is the block diagram of PTS technique. From the left side of diagram, the data information in frequency domain X is separated into V non-overlapping sub-blocks and each sub block vectors has the same size N. So for each and every sub-block it contains N/V nonzero elements and set the rest part to zero. Assume that these sub-blocks have the same size and no gap between each other. The sub-block vector is given by

$$X = \sum_{v=1}^{V} b_v X_v \qquad (7)$$

where $b_v=e^{j\varphi v}(\varphi_v \in[0,2\pi])\{v=1,2,X_v \}$ is a weighting factor been used for phase rotation. The signal in time domain is obtained by applying IFFT operation [11] on, that is

$$\hat{x} = \mathrm{IFFT}(X) = \sum_{v=1}^{V} b_v\ \mathrm{IFFT}(X_v) = \sum_{v=1}^{V} b_v\ X_v \qquad (8)$$

For the optimum result one of the suitable factor from combination b = [b1,b2,….b_v] is selected and the combination is given by

$$b = [b_1, b_2, \ldots b_v] = argmin_{(b_1,b_2,\ldots b_v)}(max_{1 \leq n \leq N|\sum_{v=1}^{V} b_v X_v|^2}) \quad (9)$$

Where arg min [(·)] is the condition that minimize the output value of function.

3. Proposed Companded Technique

OBI is the spectral leakage into alien channels. Quantification of the OBI caused by companding requires the knowledge of the power spectral density (PSD) of the companded signal. Unfortunately analytical expression of the PSD is in general mathematically intractable, because of the nonlinear companding transform involved. Here we take an alternative approach to estimate the OBI. Let (x) be a nonlinear companding function, and (t) = sin(ωt) be the input to the compander. The companded signal (t) is:

$$y(t) = f[x(t)] = f[\sin(\omega t)] \quad (10)$$

Since (t) is a periodic function with the same period as (t), (t) can then be expanded into the following Fourier series:

$$y(t) = \sum_{k=-\infty}^{+\infty} c(k)e^{j\omega t}, \quad (11)$$

Where the coefficients (k) is calculated as:

$$c(k) = c(-k)\frac{1}{T}\int_0^T y(t)e^{-j\omega t}\ dt, \quad (12)$$

Notice that the input x in this case is a pure sinusoidal signal, any $(k) \neq 0$ for $|k| > 1$ is the OBI produced by the nonlinear companding process. Therefore, to minimize the OBI, (k) must approach to zero fast enough as k increases. It has been shown that $(k) \cdot k-(m+1)$ tends to zero if $y(t)$ and its derivative up to the m-th order are continuous [8], or in other words, $c(k)$ converges at the rate of $\mathbf{k}^{-(m+1)}$. Given an arbitrary number n, the n-th order derivative of (t), $d^n y/dt^n$ is a function of $d^i f(x)/dx^i$, (i = 1, 2, · · ·, n), as well as sin(ωt) and cos(ωt), i.e.:

$$\frac{d^n y}{dt^n} = \ g\left(\frac{d^n f(x)}{dt^n}, \frac{d^{n-1} f(x)}{dt^{n-1}}, \ldots, \frac{df(x)}{dx}, \sin(wt), \cos(wt)\right) \quad (13)$$

sin(ωt) and and cos(ωt) are continuous functions, $d^n y/dt^n$ is continuous if and only if $d^i f(x)/dx^i$ are continuous. Based on this observation we can conclude:

Companding introduces minimum amount of OBI if the companding function (x) *is infinitely differentiable*.

The functions that meet the above condition are the smooth functions.

We now propose a new companding algorithm using a smooth function, namely the airy special function. The companding function is as follows

$$f(x) = \beta.sign(x).[airy(0) - airy(\alpha.|x|)] \quad (14)$$

Where airy (·) is the airy function of the first kind. α is the parameter that controls the degree of companding (and ultimately PAPR). β is the factor adjusting the average output power of the compander to the same level as the average input power:

$$\beta = \sqrt{\frac{E[|x|^2]}{E[|airy\ (0) - airy\ (\alpha.|x|)|^2]}} \quad (15)$$

Where E[·] denotes the expectation.

The decompanding function is the inverse of (x):

$$f^{-1}(x) = \frac{1}{\alpha}.sign(x).airy^{-1}[airy(0) - \frac{|x|}{\beta}] \quad (16)$$

Where the superscript -1 represents the inverse operation. Notice that the input to the decompander is a quantized signal with finite set of values.

We can therefore numerically pre- compute $f^{-1(}x)$ and use table look-up to perform the decompanding in practice.

V. Simulation Results

Figure 4 shows the CCDF as a function of PAPR distribution when SLM method is used with 64 numbers of subcarrier. M takes the value of 1 (without adopting SLM method), 2, 4, 8 and 16. It is seen in Figure 4 that with increase of branch number M, PAPR's CCDF gets smaller.

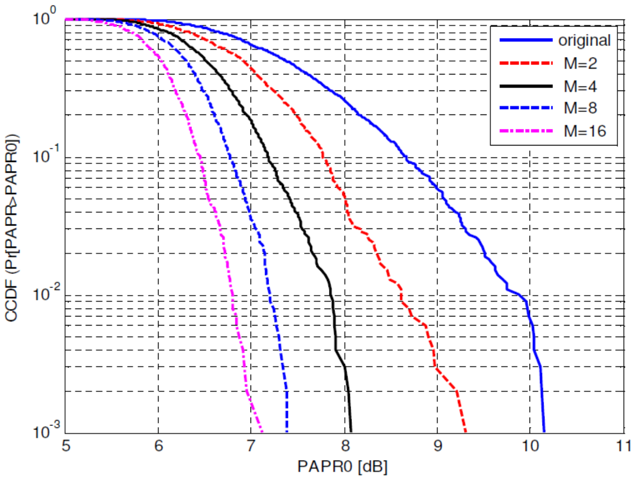

Figure. 4. PAPR’s CCDF using SLM method with N=64

Now discussed the simulation result for PTS technique, there are varying parameters which impact the PAPR reduction performance these are: 1) The number of sub-blocks V, which influences the complexity strongly; 2) The number of possible phase value W, which impacts the complexity; and 3) The sub-block partition schemes. Here, only one parameter is considered that is sub-block size V=4.

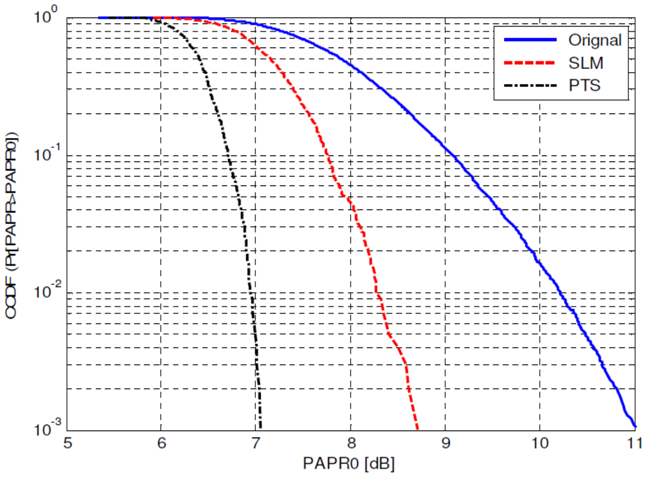

Figure 5. PAPR’s CCDF using SLM and PTS method with N=64

Figure.6 depicts the CCDF of the three companding schemes. The new algorithm is roughly 1.5dB inferior to the exponential, but surpasses the μ-law by 2dB

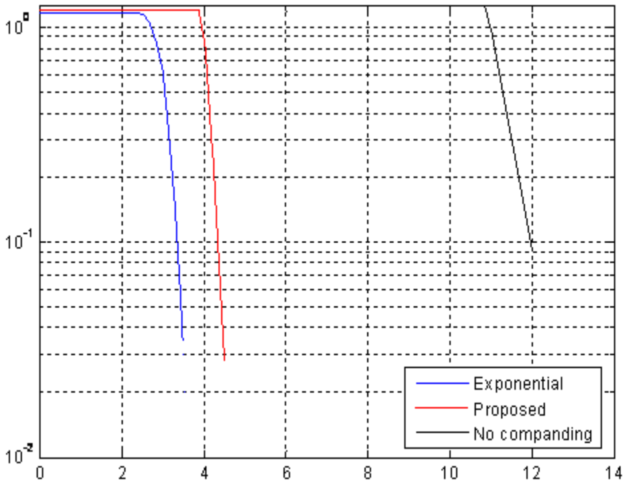

Figure.6.Complementary cumulative distribution function of original and companded signals

From the above figures it is clear that new proposed companded technique provides a better PAPR reduction performance than compared to PTS method and SLM method

VI. Conclusion

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high PAPR when the input sequences are highly correlated. In this paper, several important aspects are described as well as mathematical analysis is provided, including the distribution of the PAPR used in OFDM systems. Three typical signal scrambling and distortion techniques, SLM, PTS and Companding are investigated to reduce PAPR, all of which have the potential to provide substantial reduction in PAPR. Proposed Companding method performs better than PTS method and SLM method in reducing PAPR.

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