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Lossy Transmission Lines Terminated by Parallel Connected RC-Loads and in Series Connected L-Load (I)

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Abstract: *The present paper is the first part of investigations devoted to analysis of lossy transmission lines terminated by nonlinear parallel connected GC loads and in series connected L-load (cf. Fig. 1). First we formulate boundary conditions for lossy transmission line system on the base of Kirchhoff's law. Then we reduce the mixed problem for the hyperbolic system (Telegrapher equations) to an initial value problem for a neutral system on the boundary. We show that only oscillating solutions are characteristic for this case. Finally we analyze the arising nonlinearities.*

Keywords: *Fixed point theorem, Kirchhoff's law, Lossless transmission line, mixed problem for hyperbolic system, Neutral equation, Oscillatory solution*

I. Introduction

The transmission line theory is based on the Telegrapher equations, which from mathematical point of view presents a first order hyperbolic system of partial differential equations with unknown functions voltage and current. The subject of transmission lines has grown in importance because of the many applications (cf. [1]-[9]).

In the previous our papers we have considered lossless and lossy transmission lines terminated by various configuration of nonlinear (or linear) loads – in series connected, parallel connected and so on (cf. [10]-[16]). The main purpose of the present paper is to consider a lossless transmission line terminated by nonlinear *GCL*-loads placed in the following way: *GC*-loads are parallel connected and a *L*-load is in series connected (cf. Fig. 1).

The first difficulty is to derive the boundary conditions as a consequence of Kirchhoff's law (cf. Fig.1) and to formulate the mixed problem for the hyperbolic system. The second one is to reduce the mixed problem for the hyperbolic system to an initial value problem for neutral equations on the boundary. The third one is to introduce a suitable operator whose fixed point is an oscillatory solution of the problem stated. In the second part of the present paper by means of by fixed point method [17] we obtain an existence-uniqueness of an oscillatory solution.

The paper consists of four sections. In Section II on the base of Kirchhoff's law we derive boundary conditions and then formulate the mixed problem for the hyperbolic system or transmission line system. In Section III we reduce the mixed problem to an initial value problem on the boundary. In Section IV we analyse the arising nonlinearities and make some estimates which we use in the second part of the present paper.

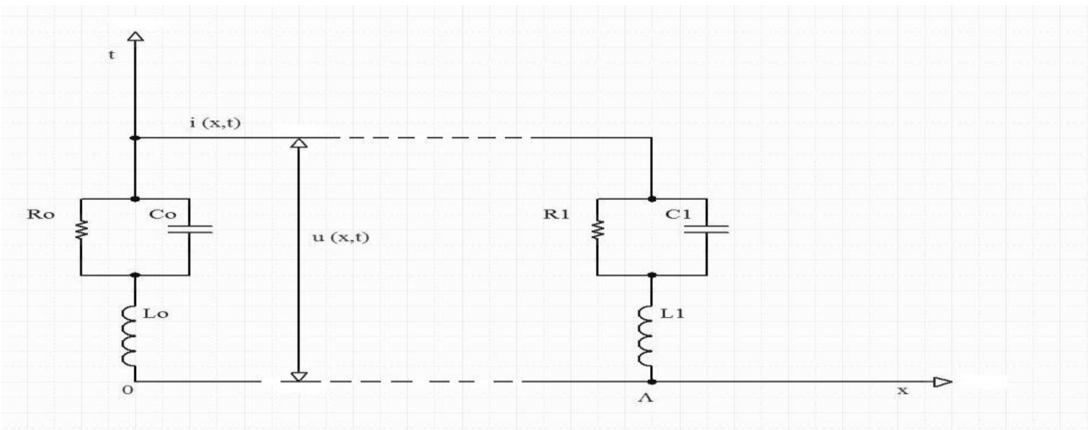

Fig. 1. Lossy transmission line terminated by circuits consisting of *RC*-elements in series connected to *L*-element

II. Derivation Of The Boundary Conditions And Formulation Of The Mixed Problem

In order to obtain the boundary conditions we have to take into account that if Λ is the length of the transmission line then, $T = \Lambda/(1/\sqrt{LC}) = \Lambda\sqrt{LC}$ where *L* is per unit length inductance and *C* – per unit-length capacitance

In accordance of Kirchhoff's *V*-law (cf. Fig. 1) we have to collect the currents of the elements G_p and C_p after that to collect the voltage of G_pC_p with the voltage of L_p $(p=0,1)$. But we deal with nonlinear elements, that is, $R_p(i)=\sum_{n=1}^{m} r_n^{(p)} i^n, (p=0,1)$ and $L_p(i)=\sum_{n=0}^{m} l_n^{(p)} i^n$, $\tilde{L}_p(i)=i\ .L_p(i)=i.\sum_{n=0}^{m} l_n^{(p)} i^n$, $\tilde{C}_p(u)=u\,C_p(u)$;

$$\frac{d\tilde{L}_p(i)}{dt}=\frac{d(i\,L_p(i))}{dt}=\frac{di}{dt}\left(L_p(i)+i\,\frac{dL_p(i)}{di}\right), (p=0,1);$$

$$\frac{d\tilde{C}_p(u)}{dt}=\frac{d(u\,C_p(u))}{dt}=\frac{du}{dt}\left(C_p(u)+u\,\frac{dC_p(u)}{du}\right),\ (p=0,1).$$

One can formulate boundary conditions corresponding to Fig. 1: for $x=0$ $(i_{G_0}+i_{C_0}=i_{G_0C_0}=i(0,t))$

$$\begin{aligned}&\left[u_{G_0C_0}\frac{dC_0(u_{G_0C_0})}{du_{G_0C_0}}+C_0(u_{G_0C_0})\right]\frac{du_{G_0C_0}}{dt}=-i(0,t)-G_0(u_{G_0C_0}),\\&\left[i(0,t)\frac{dL_0(i(0,t))}{di_{G_0C_0}}+L_0(i(0,t))\right]\frac{di(0,t)}{dt}=u(0,t)-u_{G_0C_0}(t)+E_0(t)\end{aligned} \quad (1)$$

And for $x=\Lambda$ $(i_{G_1}+i_{C_1}=i_{G_1C_1}=i(\Lambda,t))$:

$$\begin{aligned}&\left[u_{G_1C_1}\frac{dC_1(u_{G_1C_1})}{du_{G_1C_1}}+C_1(u_{G_1C_1})\right]\frac{du_{G_1C_1}}{dt}=i(\Lambda,t)-G_1(u_{G_1C_1})\\&\left[i(\Lambda,t)\frac{dL_1(i(\Lambda,t))}{di_{G_1C_1}}+L_1(i(\Lambda,t))\right]\frac{di(\Lambda,t)}{dt}=-u(\Lambda,t)+u_{G_1C_1}(t)-E_1(t).\end{aligned} \quad (2)$$

Here we consider the following lossy transmission line system:

$$\begin{aligned}&C\frac{\partial u(x,t)}{\partial t}+\frac{\partial i(x,t)}{\partial x}+Gu(x,t)=0\\&L\frac{\partial i(x,t)}{\partial t}+\frac{\partial u(x,t)}{\partial x}+Ri(x,t)=0\end{aligned} \quad (3)$$

$$(x,t)\in\Pi=\{(x,t)\in\Pi^2:(x,t)\in[0,\Lambda]\times[0,\infty)\}$$

Where $u(x,t)$ and $i(x,t)$ are the unknown voltage and current, while *L*, *C*, *R* and *G* are prescribed specific parameters of the line and $\Lambda>0$ is its length.

For the above system (3) might be formulated the following mixed problem: to find $u(x,t)$ and $i(x,t)$ in Π such that the following initial conditions

$$u(x,0)=u_0(x),\ i(x,0)=i_0(x),\ x\in[0,\Lambda] \quad (4)$$

And boundary conditions (1) and (2) to be satisfied.

III. REDUCING THE MIXED PROBLEM TO AN INITIAL VALUE PROBLEM ON THE BOUNDARY

First we present (3) in the form:

$$\begin{aligned}&\frac{\partial u(x,t)}{\partial t}+\frac{1}{C}\frac{\partial i(x,t)}{\partial x}+\frac{G}{C}u(x,t)=0\\&\frac{\partial i(x,t)}{\partial t}+\frac{1}{L}\frac{\partial u(x,t)}{\partial x}+\frac{R}{L}i(x,t)=0\end{aligned} \quad (5)$$

And then write it in a matrix form

$$\frac{\partial U(x,t)}{\partial t} + A\frac{\partial U(x,t)}{\partial x} + BU(x,t) = 0 \tag{6}$$

Where

$$U(x,t) = \begin{bmatrix} u(x,t) \\ i(x,t) \end{bmatrix}, \quad \frac{\partial U(x,t)}{\partial t} = \begin{bmatrix} \frac{\partial u(x,t)}{\partial t} \\ \frac{\partial i(x,t)}{\partial t} \end{bmatrix}, \quad \frac{\partial U(x,t)}{\partial x} = \begin{bmatrix} \frac{\partial u(x,t)}{\partial x} \\ \frac{\partial i(x,t)}{\partial x} \end{bmatrix}, A = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{G}{C} & 0 \\ 0 & \frac{R}{L} \end{bmatrix}.$$

In order to transform the matrix A in a diagonal form we have to solve the characteristic equation: $\begin{vmatrix} -\lambda & 1/C \\ 1/L & -\lambda \end{vmatrix} = 0$ whose roots are $\lambda_1 = 1/\sqrt{LC}$, $\lambda_2 = -1/\sqrt{LC}$. For the eigen-vectors we obtain the following systems:

$$\left| \begin{aligned} -\frac{1}{\sqrt{LC}}\xi_1 + \frac{1}{L}\xi_2 = 0 \\ \frac{1}{C}\xi_1 - \frac{1}{\sqrt{LC}}\xi_2 = 0 \end{aligned} \right. \quad \text{And} \quad \left| \begin{aligned} \frac{1}{\sqrt{LC}}\xi_1 + \frac{1}{L}\xi_2 = 0 \\ \frac{1}{C}\xi_1 + \frac{1}{\sqrt{LC}}\xi_2 = 0 \end{aligned} \right. .$$

Hence $\left(\xi_1^{(1)}, \xi_2^{(1)}\right) = \left(\sqrt{C}, \sqrt{L}\right)$, $\left(\xi_1^{(2)}, \xi_2^{(2)}\right) = \left(-\sqrt{C}, \sqrt{L}\right)$

Denote by H the matrix formed by eigen-vectors $H = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix}$ and its inverse one $H^{-1} = \begin{bmatrix} \frac{1}{2\sqrt{C}} & -\frac{1}{2\sqrt{C}} \\ \frac{1}{2\sqrt{L}} & \frac{1}{2\sqrt{L}} \end{bmatrix}$. It is known that $A^{\text{can}} = HAH^{-1}$, where $A^{\text{can}} = \begin{bmatrix} 1/\sqrt{LC} & 0 \\ 0 & -1/\sqrt{LC} \end{bmatrix}$.

Introduce new variables $Z = HU$, (or $U = H^{-1}Z$)

$$Z = \begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix}, \quad H = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix}, \quad U = \begin{bmatrix} u(x,t) \\ i(x,t) \end{bmatrix}.$$

Then

$$\left| \begin{aligned} V(x,t) &= \sqrt{C}\, u(x,t) + \sqrt{L}\, i(x,t) \\ I(x,t) &= -\sqrt{C}\, u(x,t) + \sqrt{L}\, i(x,t) \end{aligned} \right. \tag{7}$$

or

$$\left| \begin{aligned} u(x,t) &= \frac{1}{2\sqrt{C}} V(x,t) - \frac{1}{2\sqrt{C}} I(x,t) \\ i(x,t) &= \frac{1}{2\sqrt{L}} V(x,t) + \frac{1}{2\sqrt{L}} I(x,t). \end{aligned} \right. \tag{8}$$

Replacing $U(x,t) = H^{-1}Z(x,t)$ in (6) we obtain

$$\frac{\partial\left(H^{-1}Z\right)}{\partial t} + A\frac{\partial\left(H^{-1}Z\right)}{\partial x} + B\left(H^{-1}Z\right) = 0 .$$

Since H^{-1} is a constant matrix we have

$$H^{-1}\frac{\partial Z(x,t)}{\partial t}+\left(AH^{-1}\right)\frac{\partial Z(x,t)}{\partial x}+\left(BH^{-1}\right)Z(x,t)=0\,.$$

After multiplication from the left by H we obtain $\frac{\partial Z}{\partial t}+H\left(AH^{-1}\right)\frac{\partial Z}{\partial x}+H\left(BH^{-1}\right)Z=0$, i.e.

$$\frac{\partial Z}{\partial t}+A^{can}\frac{\partial Z}{\partial x}+\left(HBH^{-1}\right)Z=0\,. \tag{9}$$

But

$$HBH^{-1}=\begin{bmatrix}\sqrt{C} & \sqrt{L}\\ -\sqrt{C} & \sqrt{L}\end{bmatrix}\begin{bmatrix}\frac{G}{C} & 0\\ 0 & \frac{R}{L}\end{bmatrix}\begin{bmatrix}\frac{1}{2\sqrt{C}} & -\frac{1}{2\sqrt{C}}\\ \frac{1}{2\sqrt{L}} & \frac{1}{2\sqrt{L}}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\left(\frac{G}{C}+\frac{R}{L}\right) & \frac{1}{2}\left(-\frac{G}{C}+\frac{R}{L}\right)\\ \frac{1}{2}\left(-\frac{G}{C}+\frac{R}{L}\right) & \frac{1}{2}\left(\frac{G}{C}+\frac{R}{L}\right)\end{bmatrix}.$$

Applying Heaviside condition $\frac{R}{L}=\frac{G}{C}$ we obtain

$$\begin{bmatrix}\frac{\partial V(x,t)}{\partial t}\\ \frac{\partial I(x,t)}{\partial t}\end{bmatrix}+\begin{bmatrix}\frac{1}{\sqrt{LC}} & 0\\ 0 & -\frac{1}{\sqrt{LC}}\end{bmatrix}\begin{bmatrix}\frac{\partial V(x,t)}{\partial x}\\ \frac{\partial I(x,t)}{\partial x}\end{bmatrix}+\begin{bmatrix}\frac{R}{L} & 0\\ 0 & \frac{R}{L}\end{bmatrix}\begin{bmatrix}V(x,t)\\ I(x,t)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}. \tag{10}$$

The new initial conditions we obtain from (4):

$$V(x,0)=\sqrt{C}\ u(x,0)+\sqrt{L}\ i(x,0)=\sqrt{C}\ u_0(x)+\sqrt{L}\ i_0(x)\equiv V_0(x),\ \ x\in[0,\Lambda] \tag{11}$$

$$I(x,0)=-\sqrt{C}\ u(x,0)+\sqrt{L}\ i(x,0)=-\sqrt{C}\ u_0(x)+\sqrt{L}\ i_0(x)\equiv I_0(x),\ \ x\in[0,\Lambda]. \tag{12}$$

One can simplify (10) by the substitution:

$$W(x,t)=e^{\frac{R}{L}t}V(x,t)$$
$$J(x,t)=e^{\frac{R}{L}t}I(x,t)$$

Or

$$\begin{aligned}V(x,t)&=e^{-\frac{R}{L}t}W(x,t)\\ I(x,t)&=e^{-\frac{R}{L}t}J(x,t)\end{aligned}\ . \tag{13}$$

Substituting in (8) we obtain

$$\left|\begin{aligned}u(x,t)&=\frac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}W(x,t)-\frac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}J(x,t)\\ i(x,t)&=\frac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}W(x,t)+\frac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}J(x,t).\end{aligned}\right. \tag{14}$$

Then with respect to the variables $W(x,t)$ and $J(x,t)$ (10) looks like:

$$\begin{aligned}\frac{\partial W(x,t)}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial W(x,t)}{\partial x}&=0,\\ \frac{\partial J(x,t)}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial J(x,t)}{\partial x}&=0.\end{aligned} \tag{15}$$

The mixed problem for (1) - (4) can be reduced to an initial value problem for a neutral system. The neutral system is a nonlinear one in view of the nonlinear characteristics of the *RGLC*-elements.
From now on we propose two manners to obtain a neutral system for unknown voltage and current functions.

First manner: The solution of (15) is a pair of functions $W(x,t)=\Phi_W(x-vt)$ and $J(x,t)=\Phi_J(x+vt)$, where Φ_W and Φ_J are arbitrary smooth functions. From (14) we obtain

$$u(x,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{C}}\left[\Phi_W(x-vt)-\Phi_J(x+vt)\right]$$
$$i(x,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{L}}\left[\Phi_W(x-vt)+\Phi_J(x+vt)\right]. \qquad (16)$$

Hence

$$\Phi_W(x-vt)=e^{\frac{R}{L}t}\left(\sqrt{C}\,u(x,t)+\sqrt{L}\,i(x,t)\right)$$
$$\Phi_J(x+vt)=e^{\frac{R}{L}t}\left(\sqrt{L}\,i(x,t)-\sqrt{C}\,u(x,t)\right). \qquad (17)$$

For $x=\Lambda$ we obtain

$$\Phi_W(\Lambda-vt)=e^{\frac{R}{L}t}\left[\sqrt{C}\,u(\Lambda,t)+\sqrt{L}\,i(\Lambda,t)\right]$$
$$\Phi_J(\Lambda+vt)=e^{\frac{R}{L}t}\left[\sqrt{L}\,i(\Lambda,t)-\sqrt{C}\,u(\Lambda,t)\right] \qquad (18)$$

Let us put $\Lambda-vt=-vt'\Rightarrow t=t'+\Lambda/v\equiv t'+T \quad (T=\Lambda/v)$ and then replacing t by $t'+T$ in the first equation of (18) we get

$$\Phi_W(-vt')=e^{\frac{R}{L}(t'+T)}\left[\sqrt{C}\,u(\Lambda,t'+T)+\sqrt{L}\,i(\Lambda,t'+T)\right].$$

For the second equation of (18) we put
$\Lambda+vt=vt''\Rightarrow t=t''-\Lambda/v\equiv t''-T \quad (T=\Lambda/v)$ And then we have

$$\Phi_J(vt'')=e^{\frac{R}{L}(t''-T)}[\sqrt{L}i(\Lambda,t''\text{-}T)\text{-}\sqrt{C}u(\Lambda,t''\text{-}T)].$$

So we obtain

$$\Phi_W(-vt)=e^{\frac{R}{L}(t+T)}\left[\sqrt{C}\,u(\Lambda,t+T)+\sqrt{L}\,i(\Lambda,t+T)\right] \qquad (19)$$
$$\Phi_J(vt)=e^{\frac{R}{L}(t-T)}\left[\sqrt{L}\,i(\Lambda,t-T)-\sqrt{C}\,u(\Lambda,t-T)\right]. \qquad (20)$$

From (16) by $x=0$ we have

$$u(0,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{C}}\left[\Phi_W(-vt)-\Phi_J(vt)\right]$$
$$i(0,t)=\frac{e^{-\frac{R}{L}t}}{2\sqrt{L}}\left[\Phi_W(-vt)+\Phi_J(vt).\right] \qquad (21)$$

Substituting $\Phi_W(-vt)$ and $\Phi_J(vt)$ from (19) and (20) into (21) we obtain:

$$u(0,t)=\frac{1}{2\sqrt{C}}\left[e^{\frac{R(t+T)}{L}}\left(\sqrt{C}\,u(\Lambda,t+T)+\sqrt{L}\,i(\Lambda,t+T)\right)-e^{\frac{R(t-T)}{L}}\left(\sqrt{L}\,i(\Lambda,t-T)-\sqrt{C}\,u(\Lambda,t-T)\right)\right]$$

$$i(0,t)=\frac{1}{2\sqrt{C}}\left[e^{\frac{R(t+T)}{L}}\left(\sqrt{C}\,u(\Lambda,t+T)+\sqrt{L}\,i(\Lambda,t+T)\right)+e^{\frac{R(t-T)}{L}}\left(\sqrt{L}\,i(\Lambda,t-T)-\sqrt{C}\,u(\Lambda,t-T)\right)\right].$$

Substitute the above expressions into the boundary conditions (1), (2) we have

$$\frac{du_{G_0C_0}(t)}{dt}=\frac{e^{\frac{R(t+T)}{L}}\left(u(\Lambda,t+T)+Z_0\ i(\Lambda,t+T)\right)+e^{\frac{R(t-T)}{L}}\left(Z_0\ i(\Lambda,t-T)-u(\Lambda,t-T)\right)}{2Z_0\dfrac{d\widetilde{C}_0(u_{G_0C_0})}{du_{G_0C_0}}}-\frac{G_0(u_{G_0C_0})}{\dfrac{d\widetilde{C}_0(u_{G_0C_0})}{du_{G_0C_0}}},$$

$$\frac{1}{2Z_0}\frac{d}{dt}\left[e^{\frac{R(t+T)}{L}}\left(u(\Lambda,t+T)+Z_0\ i(\Lambda,t+T)\right)+e^{\frac{R(t-T)}{L}}\left(Z_0\ i(\Lambda,t-T)-u(\Lambda,t-T)\right)\right]=$$

$$=\frac{\left[e^{\frac{R(t+T)}{L}}\left(u(\Lambda,t+T)+Z_0\ i(\Lambda,t+T)\right)-e^{\frac{R(t-T)}{L}}\left(Z_0\ i(\Lambda,t-T)-u(\Lambda,t-T)\right)\right]-2u_{G_0C_0}(t)+2E_0(t)}{2d\widetilde{L}_0(i(0,t))/di_{G_0C_0}},$$

$$\frac{du_{G_1C_1}(t)}{dt}=\frac{i(\Lambda,t)-G_1(u_{G_1C_1})}{d\widetilde{C}_1(u_{G_1C_1})/du_{G_1C_1}},$$

$$\frac{di(\Lambda,t)}{dt}=\frac{-u(\Lambda,t)+u_{G_1C_1}(t)-E_1(t)}{d\widetilde{L}_1(i(\Lambda,t))/di_{G_1C_1}}.$$

Let us put $\tau\equiv t+T$ .Then we arrive at a system that we cannot formulate an initial value problem.

Second manner

We proceed from (14) and obtain

$$\left|\begin{array}{l}u(0,t)=\dfrac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}W(0,t)-\dfrac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}J(0,t)\\ i(0,t)=\dfrac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}W(0,t)+\dfrac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}J(0,t)\end{array}\right.\quad\text{And}\quad\left|\begin{array}{l}u(\Lambda,t)=\dfrac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}W(\Lambda,t)-\dfrac{1}{2\sqrt{C}}e^{-\frac{R}{L}t}J(\Lambda,t)\\ i(\Lambda,t)=\dfrac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}W(\Lambda,t)+\dfrac{1}{2\sqrt{L}}e^{-\frac{R}{L}t}J(\Lambda,t).\end{array}\right.$$

Substituting in (2.1) and (2.2) we obtain

$$\frac{du_{G_0C_0}(t)}{dt}=\frac{e^{-\frac{R}{L}t}W(0,t)+e^{-\frac{R}{L}t}J(0,t)-2\sqrt{L}G_0(u_{G_0C_0}(t))}{2\sqrt{L}\ d\widetilde{C}_0(u_{G_0C_0})/du_{G_0C_0}};$$

$$\frac{d}{dt}\left(e^{-\frac{R}{L}t}W(0,t)+e^{-\frac{R}{L}t}J(0,t)\right)=\frac{Z_0\ e^{-\frac{R}{L}t}W(0,t)-Z_0e^{-\frac{R}{L}t}J(0,t)-2\sqrt{L}u_{G_0C_0}(t)+2\sqrt{L}E_0(t)}{d\widetilde{L}_0(i(0,t))/di_{G_0C_0}};$$

$$\frac{du_{G_1C_1}(t)}{dt}=\frac{e^{-\frac{R}{L}t}W(\Lambda,t)+e^{-\frac{R}{L}t}J(\Lambda,t)-2\sqrt{L}G_1(u_{G_1C_1}(t))}{2\sqrt{L}\ dC_1(u_{G_1C_1})/du_{G_1C_1}};$$

$$\frac{d}{dt}\left(e^{-\frac{R}{L}t}W(\Lambda,t)+e^{-\frac{R}{L}t}J(\Lambda,t)\right)=\frac{-Z_0\ e^{-\frac{R}{L}t}W(\Lambda,t)+Z_0e^{-\frac{R}{L}t}J(\Lambda,t)+2\sqrt{L}u_{G_1C_1}(t)-2\sqrt{L}E_1(t)}{d\widetilde{L}_1(i(\Lambda,t))/di_{G_1C_1}}.$$

But

$$W(0,t)=W(\Lambda,t+T),\ J(0,t+T)=J(\Lambda,t)\ \Rightarrow\ W(0,t-T)=W(\Lambda,t),\ J(0,t)=J(\Lambda,t-T).$$

We choose $W(0,t)=W(t),\ J(t)=J(\Lambda,t)$ to be unknown functions and then the above system becomes

$$\frac{du_{G_0C_0}(t)}{dt}=\frac{e^{-\frac{R}{L}t}W(t)+e^{-\frac{R}{L}(t-T)}J(t-T)-2\sqrt{L}G_0(u_{G_0C_0}(t))}{2\sqrt{L}\ d\widetilde{C}_0(u_{G_0C_0})/du_{G_0C_0}};$$
$$\frac{d}{dt}\left(e^{-\frac{R}{L}t}W(t)+e^{-\frac{R}{L}(t-T)}J(t-T)\right)=\frac{Z_0\,e^{-\frac{R}{L}t}W(t)-Z_0e^{-\frac{R}{L}(t-T)}J(t-T)-2\sqrt{L}\,u_{G_0C_0}(t)+2\sqrt{L}E_0(t)}{d\widetilde{L}_0(i(0,t))/di_{G_0C_0}};$$
$$\frac{du_{G_1C_1}(t)}{dt}=\frac{e^{-\frac{R}{L}(t-T)}W(t-T)+e^{-\frac{R}{L}t}J(t)-2\sqrt{L}G_1(u_{G_1C_1}(t))}{2\sqrt{L}\,dC_1(u_{G_1C_1})/du_{G_1C_1}};\tag{22}$$
$$\frac{d}{dt}\left(e^{-\frac{R}{L}(t-T)}W(t-T)+e^{-\frac{R}{L}t}J(t)\right)=\frac{-Z_0\,e^{-\frac{R}{L}(t-T)}W(t-T)+Z_0e^{-\frac{R}{L}t}J(t)+2\sqrt{L}u_{G_1C_1}(t)-2\sqrt{L}E_1(t)}{d\widetilde{L}_1(i(\Lambda,t))/di_{G_1C_1}}.$$

We notice that if (22) has a periodic solution

$$\left(u_{G_0C_0}(t),W(t),u_{G_1C_1}(t),J(t)\right)$$

Then functions $e^{-\frac{R}{L}t}W(t),\ e^{-\frac{R}{L}t}J(t)$ are oscillatory ones and vanishing exponentially at infinity. Therefore we put $\widehat{W}(t)=e^{-\frac{R}{L}t}W(t),\ \widehat{J}(t)=e^{-\frac{R}{L}t}J(t)$ and then we can state the problem for existence-uniqueness of an oscillatory solution vanishing at infinity of the following system (we denote by $\widehat{W}(t),\widehat{J}(t)$ again by $W(t),J(t)$) and obtain:

$$\frac{du_{G_0C_0}(t)}{dt}=\frac{W(t)+J(t-T)-2\sqrt{L}G_0(u_{G_0C_0}(t))}{2\sqrt{L}\ d\widetilde{C}_0(u_{G_0C_0})/du_{G_0C_0}},t\in[T;\infty);$$
$$\frac{dW(t)}{dt}=-\frac{dJ(t-T)}{dt}+\frac{Z_0\,W(t)-Z_0J(t-T)-2\sqrt{L}\,u_{G_0C_0}(t)+2\sqrt{L}E_0(t)}{d\widetilde{L}_0(i(0,t))/di_{G_0C_0}},\ t\in[T;\infty);$$
$$\frac{du_{G_1C_1}(t)}{dt}=\frac{W(t-T)+J(t)-2\sqrt{L}G_1(u_{G_1C_1}(t))}{2\sqrt{L}\,dC_1(u_{G_1C_1})/du_{G_1C_1}},\ t\in[T;\infty);\tag{23}$$
$$\frac{dJ(t)}{dt}=-\frac{dW(t-T)}{dt}+\frac{-Z_0\,W(t-T)+Z_0J(t)+2\sqrt{L}u_{G_1C_1}(t)-2\sqrt{L}E_1(t)}{d\widetilde{L}_1(i(\Lambda,t))/di_{G_1C_1}},\ t\in[T;\infty);$$

$$u_{G_0C_0}(T)=u^T_{G_0C_0},\ u_{G_1C_1}(T)=u^T_{G_1C_1},\ W(t)=W_0(t),J(t)=J_0(t),\ \frac{dW(t)}{dt}=\frac{dW_0(t)}{dt},\ \frac{dJ(t)}{dt}=\frac{dJ_0(t)}{dt},t\in[0,T].$$

IV. ANALYSIS OF THE ARISING NONLINEARITIES

First we precise the definition domains of the functions:

$$\frac{d\widetilde{C}_p(u)}{dt}=\frac{d(C_p(u).u)}{dt}\ ,(p=0,1)$$

Where $C_p(u)=\dfrac{c_p}{\sqrt[h]{(1-u/\Phi_p)}}=\dfrac{c_p\sqrt[h]{\Phi_p}}{\sqrt[h]{\Phi_p-u}}$, $c_p>0,\Phi_p>0,h\in[2,3]$ are constants and $|u|\le\phi_0<\min\{\Phi_0,\Phi_1\}$. We have to show an interval for u where

$$\widetilde{C}_p(u)=u.C_p(u)=c_p\sqrt[h]{\Phi_p}\,\frac{u}{\sqrt[h]{\Phi_p-u}}$$

Has a strictly positive lower bound.
First we calculate the derivatives

$$\frac{dC_p(u)}{du}=\frac{c_p\sqrt[h]{\Phi_p}}{h}(\Phi_p-u)^{-\frac{1+h}{h}};\quad \frac{d^2C_p(u)}{du^2}=\frac{c_p\sqrt[h]{\Phi_p}}{h}\frac{1+h}{h}(\Phi_p-u)^{-\frac{1+2h}{h}};$$

$$\frac{d\tilde{C}_p(u)}{du}=C_p(u)+u\frac{dC_p(u)}{du}=c_p\sqrt[h]{\Phi_p}(\Phi_p-u)^{-\frac{1}{h}}+c_p\sqrt[h]{\Phi_p}\frac{u}{h}(\Phi_p-u)^{-\frac{1}{h}-1}=c_p\sqrt[h]{\Phi_p}\left[\Phi_p-\left(\frac{h-1}{h}\right)u\right](\Phi_p-u)^{-1-\frac{1}{h}};$$

$$\frac{d^2\tilde{C}_p(u)}{du^2}=c_p\sqrt[h]{\Phi_p}\left[-\left(\frac{h-1}{h}\right)(\Phi_p-u)^{-1-\frac{1}{h}}-\left(\Phi_p-\left(\frac{h-1}{h}\right)u\right)\left(1+\frac{1}{h}\right)(\Phi_p-u)^{-2-\frac{1}{h}}\right]=\frac{c_p\sqrt[h]{\Phi_p}}{h^2}\;\frac{-2h^2\Phi_p+(2h^2+h+1)u}{(\Phi_p-u)^{2+\frac{1}{h}}}.$$

Since $|u|\le\phi_0<\min\{\Phi_0,\Phi_1\}<\min\left\{\frac{h}{h-1}\Phi_0,\frac{h}{h-1}\Phi_1\right\}$ the derivative $\frac{d\tilde{C}_p(u)}{du}>0$.

Then $\min\{\tilde{C}_p(u):u\in[-\phi_0.\phi_0]\}=\min\left\{c_p\sqrt[h]{\Phi_p}\frac{\phi_0}{\sqrt[h]{\Phi_p-\phi_0}},c_p\sqrt[h]{\Phi_p}\left|\frac{-\phi_0}{\sqrt[h]{\Phi_p+\phi_0}}\right|\right\}=\frac{c_p\sqrt[h]{\Phi_p}\,\phi_0}{\sqrt[h]{\Phi_p+\phi_0}}=\hat{C}_p>0.$

Further on we have

$$\left|\frac{d\tilde{C}_p(u)}{du}\right|\le\frac{2c_p\sqrt[h]{\Phi_p}}{h(\Phi_p-\phi_0)^{\frac{1}{h}+1}}+|u|\frac{(1+h)2c_p\sqrt[h]{\Phi_p}}{h^2(\Phi_p-\phi_0)^{\frac{1}{h}+2}}=\frac{2c_p\sqrt[h]{\Phi_p}\left[h(\Phi_p-\phi_0)+|u|(1+h)\right]}{h^2\;\sqrt[h]{(\Phi_p-\phi_0)^{1+2h}}}\le.$$

$$\le\frac{2c_p\sqrt[h]{\Phi_p}\left[h(\Phi_p-\phi_0)+\phi_0(1+h)\right]}{h^2\;\sqrt[h]{(\Phi_p-\phi_0)^{1+2h}}}=\frac{2c_p\sqrt[h]{\Phi_p}\;(h\Phi_p+\phi_0)}{h^2\;\sqrt[h]{(\Phi_p-\phi_0)^{1+2h}}}\equiv\breve{C}_p^{(1)}.$$

For $-\phi_0\le u\le\phi_0<\min\left\{\frac{h}{h+1},\Phi_0,\Phi_1\right\}$ it follows

$$\frac{d\tilde{C}_p(u)}{du}=\frac{c_p\sqrt[h]{\Phi_p}}{(\Phi_p-u)^{\frac{1+h}{h}}}\left(\Phi_p-\frac{h-1}{h}u\right)\ge\frac{c_p\sqrt[h]{\Phi_p}}{(\Phi_p+\phi_0)^{\frac{1+h}{h}}}\left(\Phi_p-\frac{h-1}{h}\phi_0\right)\equiv\hat{C}_p^{(1)}.$$

Therefore

$$\min\{\tilde{C}_p(u):\,u\in[-\phi_0,\phi_0]\}=\tilde{C}_p(-\phi_0)=\frac{c_p\sqrt[h]{\Phi_p}}{h}\;\frac{h\Phi_p+(h-1)\phi_0}{(\Phi_p+\phi_0)^{1+\frac{1}{h}}}=\hat{C}_p,$$

$$\left|\frac{d^2\tilde{C}_p(u)}{du^2}\right|\le\frac{c_p\sqrt[h]{\Phi_p}}{h^2}\left|\frac{-2h^2\Phi_p+(2h^2+h+1)u}{(\Phi_p-u)^{2+\frac{1}{h}}}\right|\le\frac{c_p\sqrt[h]{\Phi_p}}{h^2}\;\frac{2h^2\Phi_p+(2h^2+h+1)\phi_0}{(\Phi_p-\phi_0)^{2+\frac{1}{h}}}\equiv\breve{C}_p^{(2)}.$$

For the *I-V* characteristics we assume $R_p(i)=\sum_{n=1}^{m}r_n^{(p)}i^n,(p=0,1)$ and $L_p(i)=\sum_{n=0}^{m}l_n^{(p)}i^n$ then

$$\tilde{L}_p(i)=i\;.L_p(i)=i.\sum_{n=0}^{m}l_n^{(p)}i^n.$$

For $\tilde{L}_p(i)$ we get $\frac{d\tilde{L}_p(i)}{di}=i\frac{dL_p(i)}{di}+L_p(i)=i\sum_{n=1}^{m}nl_n^{(p)}i^{n-1}+\sum_{n=1}^{m}l_n^{(p)}i^n=\sum_{n=1}^{m}(n+1)l_n^{(p)}i^n$.

Assumptions (C):

$|u(0,t)|\le\frac{e^{\mu T_0}(W_0+J_0)}{2}\le\phi_0$; $|u(\Lambda,t)|\le\frac{e^{\mu T_0}(W_0+J_0)}{2}\le\phi_0$; $|i(0,t)|\le\frac{e^{\mu T_0}(W_0+J_0)}{2Z_0}\le I_0$, $|i(\Lambda,t)|\le\frac{e^{\mu T_0}(W_0+J_0)}{2Z_0}\le I_0$.

Assumptions (L): $|i|\le I_0<\infty\quad\Rightarrow\quad\frac{d\tilde{L}_p(i)}{di}=\sum_{n=1}^{m}(n+1)l_n^{(p)}i^n\ge\hat{L}_p^{(1)}>0,$

$$\left|\frac{d\widetilde{L}_p(i)}{di}\right| \le \sum_{n=1}^{m}(n+1)l_n^{(p)}\mathrm{I}_0{}^n = \breve{L}_p^{(1)}, \quad \left|\frac{d\widetilde{L}_p^2(i)}{di^2}\right| \le \sum_{n=1}^{m-1} n(n+1)l_n^{(p)}\mathrm{I}_0{}^{n-1} = \breve{L}_p^{(2)} \ .$$

Assumptions (G): $G_0(i_{R_0L_0}) = \sum_{n=1}^{m} g_n^{(0)}(i_{R_0L_0})^n$, $G_1(i_{R_1L_1}) = \sum_{n=1}^{m} g_n^{(1)}(i_{R_1L_1})^n$.

V. Conclusion

Here we have investigated lossy transmission lines terminated by circuits different from parallel or in series connected *RGLC*-elements. It turned out that in this case one obtains more number of equations which leads to more complicated boundary conditions at both ends of the line. First difficulty is to find independent unknown functions − voltages and currents and to obtain a system of neutral differential equations. We show that just oscillatory solutions are specific for the lossy transmission lines and in the second part of the paper we formulate conditions for existence-uniqueness of an oscillatory solution. They can be easily applied to concrete problem because they are explicit type conditions − just inequalities between specific parameters of the line and characteristics of the circuit.

References

[1] G. Miano, and A. Maffucci, Transmission lines and lumped circuits (Academic Press, NY, 2-nd ed., 2010)
[2] D. Pozar, Microwave engineering (J. Wiley & Sons, NY, 1998).
[3] C.R. Paul, Analysis of multi-conductor transmission lines (A Wiley-Inter science Publication, J. Wiley &Sons, NY, 1994).
[4] S. Ramom, J.R. Whinnery, and T.van Duzer, Fields and waves in communication electronics (J. Wiley & Sons, Inc. NY, 1994).
[5] S. Rosenstark, Transmission lines in computer engineering (Mc Grow-Hill, NY, 1994).
[6] P. Vizmuller, RF design guide systems, circuits and equations (Artech House, Inc., Boston - London, 1995).
[7] P.C. Magnusson, G.C. Alexander, and V.K. Tripathi, Transmission lines and wave propagation (CRC Press. Boka Raton, 3rd ed., 1992).
[8] J. Dunlop, and D.G. Smith, Telecommunications engineering (Chapman & Hall, London, 1994).
[9] V. Damgov, Nonlinear and parametric phenomena: Theory and applications in radio physical and mechanical systems (World Scientific: New Jersey, London, Singapore, 2004).
[10] V.G. Angelov, and M. Hristov, Distortion less lossy transmission lines terminated by in series connected RCL-loads. Circuits and Systems, 2(4), 2011, 297-310.
[11] V.G. Angelov, and D.Tz. Angelova, Oscillatory solutions of neutral equations with polynomial nonlinearities, in Recent Advances in Oscillation Theory − special issue published in International Journal of Differential Equations, Volume 2011, Article ID 949547, 2011, 12 pages.
[12] V.G. Angelov, Periodic regimes for distortion less lossy transmission lines terminated by parallel connected RCL-loads, in Transmission Lines: Theory, Types and Applications, (Nova Science Publishers, Inc., 2011) 259-293.
[13] V.G. Angelov, Oscillations in lossy transmission lines terminated by in series connected nonlinear RCL-loads, International Journal of Theoretical and Mathematical Physics, 2(5), (2012).
[14] V.G. Angelov, Various applications of fixed point theorems in uniform spaces. 10Th International Conference on Fixed Point Theory and its Applications. Cluj-Napoca, 2012.
[15] V.G. Angelov, Lossless transmission lines terminated by in series connected RL-loads parallel to C-load. International Journal of Theoretical and Mathematical Physics, 3(1), 2013.
[16] V.G. Angelov, A method for analysis of transmission lines terminated by nonlinear loads (submitted for publication).
[17] V.G. Angelov, Fixed points in uniform spaces and applications (Cluj University Press “Babes-Bolyia”, Romania, 2009).