Profile modification of adhesively bonded cylindrical joint for maximum torque transmission capability

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ABSTRACT: *The increasing demand for light weight, high quality and more cost effective product has led adhesive bonding to emerge as one of the primary ways of fastening structural members. Adhesive joints have previously been designed empirically but now a day's data is available to design adhesive joints in an optimum way. The fixing of cylindrical components subjected to torque is a common requirement in industrial manufacture and there is a need to design and optimize the adhesively bonded cylindrical joint for maximum torque transmission capability facilitating keyless fastening. Some typical examples of adhesively bonded cylindrical joints are shaft to shaft, gear to shaft, rotor to shaft, fan to shaft, pulley to shaft etc. The present paper aims at the development of analytical model for adhesively bonded cylindrical joint subjected to torsion loading for determination of the joint profile geometry for maximum torque transmission capability. The analysis is based on classical torsion theory and constitutive, equilibrium and compatibility equations of theory of elasticity are used to obtain stress field in the adhesive layer and optimize joint profile. The analytical model developed is used to determine profile of adherends of bonded cylindrical joint for maximum torque transmission capability and with minimum weight.*

Keywords: *Bonded joint, Torsion, Analytical solution, Stress distribution, Optimization*

I. INTRODUCTION

As compared to former times, today's products are designed and optimized under completely different rules, with the main focus being on vital questions such as shortages of raw materials, total energy content and the environment-friendly disposal or reuse of a product. The increasing demand for light weight, high quality and more cost effective product has led adhesive bonding to emerge as one of the primary ways of fastening structural members.

The fixing of cylindrical components which are subjected to different kinds of loads like axial tensile, axial compressive, bending, torque, pressure etc. is a common requirement in industrial manufacture. In recent years, adhesives, especially anaerobic adhesives, have found many applications that range across many industries replacing traditional methods. Some typical cylindrical joints are Shaft to Shaft, Gear to Shaft, Rotor to Shaft, Bearing into Housing, Tube into Casting, Cylinder Liner into Engine Block, Pulley to Shaft, Fan to Shaft, Trunnions into Rollers and Bushings into Housings etc.

Adhesive joints have previously been designed empirically. For optimum design of adhesively bonded cylindrical joint, complete knowledge of stress distribution and the parameters affecting stress distribution in adhesive joint is necessary. Also for maximum torque transmission capability of the joint the stress distribution should be uniform.

The increased application of adhesive bonding was accompanied by development of mathematical and numerical methods to analyse and predict the behaviour of joints each with different assumptions and simplifications but at present also this is still an open problem.

II. LITERATURE SURVEY

The stress distribution in adhesive bonded tubular lap joints subjected to torsion was analysed by D. Chen and S. Cheng [1]. The analysis was based on the elasticity theory in conjunction with variational principle of complimentary energy, with two adherends may be having different materials and different thickness. The closed form solution so obtained was used to determine the stress intensities in adhesive layer and stress concentration factor.

A closed form solution for stress distribution of tubular lap joint in torsion whose adherends were of composite materials was obtained by Choon T. Chon [2]. The stress concentrations at and near the end was studied as function of various parameters such as wrap angles, overlap length and thickness of adhesive layer.

N. Pugno and G. Surace [3] analysed problem of torsion in adhesive bonded tubular joint and the stress field in the adhesive layer was obtained based on the elasticity theory and pure torsion theory. A special type of tubular joint was proposed by optimizing the tubular joint for torsional strength.

Zhenyu Ouyang , Guoqiang Li [4] obtained Cohesive Zone Model (CZM) based analytical solutions for the bonded pipe joints under torsion. The concept of the minimum interfacial cohesive shear slip is introduced and used in the fundamental expression of the external torsion load. The results obtained from Cohesive Zone Model were in good agreement with finite element analysis (FEA) results.

The experimental investigations for effect of different surface roughness values on bonding strength for both static and dynamic loading conditions was performed by Tezcan Sekercioglu, Alper Gulsoz, Hikmat Rende [5]. For the experimentation adherends of structural steel and anaerobic adhesive Loctite 638 were used.

Tezcan Sekercioglu [6] investigated experimentally the effect of parameters namely interference fit, bonding clearance, surface roughness, adherend materials and temperature on bonding strength of adhesive. For shear strength estimation of adhesively bonded tubular joints the nonlinear models (GASSEM) were developed using Genetic Algorithm (GA) approach.

Present paper aims at the development of analytical model for adhesively bonded cylindrical joint subjected to torsion loading for profile modification of the joint for maximum torque transmission capability and minimum weight.

III. ANALYTICAL SOLUTION

The following assumptions are made in the current study-

- Two shafts and adhesive layer forming the cylindrical joint are governed by an isotropic linear elastic law.
- Deformations considered are very small and Classical Torsion theory is used.
- The variation of stress across the thickness of adhesive layer is neglected as the thickness of adhesive layer is very very small and torsion load carried by thin adhesive layer is ignored [10].

Consider two shafts bonded adhesively by a thin adhesive layer with $2c$ as bond length, as shown in fig.(1) subjected to torsion moment T.

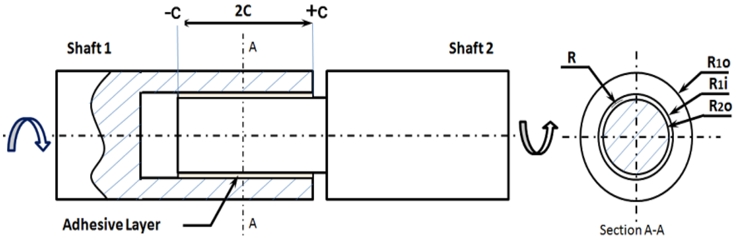

Figure 1: Two shafts bonded adhesively by thin adhesive layer

Torque transfer takes place gradually from shaft 1 to shaft 2 and at any cross section torque $T1$ and torque $T2$ are produced in shaft 1 and shaft 2 respectively. The external torsion load is assumed to be resisted by shafts only and hence the sum of the moments absorbed by the two shafts must be equivalent to the applied torsional moment T for every cross section. Here torsion load carried by thin adhesive layer is ignored.

$$T = T_1(z) + T_2(z) \qquad [1]$$

The torsional moment $T_i(z)$ at any section z of shaft i can be expressed as function of z.

$$T_1(z) = T * f(z) \qquad [2]$$

$$T_2(z) = T * (1 - f(z)) \qquad [3]$$

With Boundary conditions

$$T_1(-c) = T \quad , \quad T_1(+c) = 0 \qquad [4]$$

$$T_2(-c) = 0 \quad , \quad T_2(+c) = T$$

Consider an differential element of length dz $(-c \le z \le +c)$ belonging to the shaft 1 as shown in fig.(2) and imposing rotational equilibrium the stress field in the adhesive layer equivalent to applied torsional moment can be written as –

$$\tau(z) = \frac{-1}{2\pi R^2} \frac{dT_1(z)}{dz} \qquad [5]$$

Where R is the mean radius of adhesive surface and $T_1(z)$ is torsional moment in shaft 1 at section 'z'.

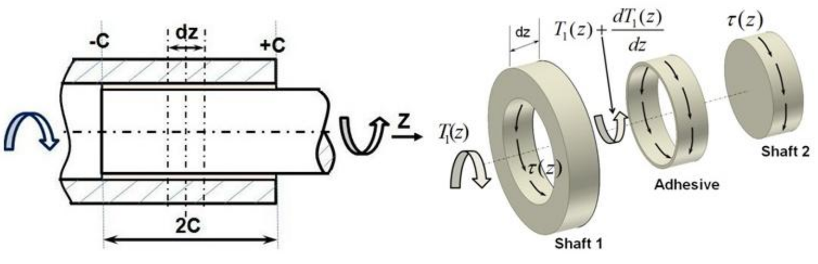

Figure 2: Bonded cylindrical joint under torsion

From equation (5) it can be seen that the shear stress in adhesive layer depends on rate of change of torque in bonded shafts. Now if the joint profile is modified in such a way that the rate of change of torque in the shafts along bond length remains constant then the shear stress in adhesive layer will be constant along bond length as shown in fig.(3).

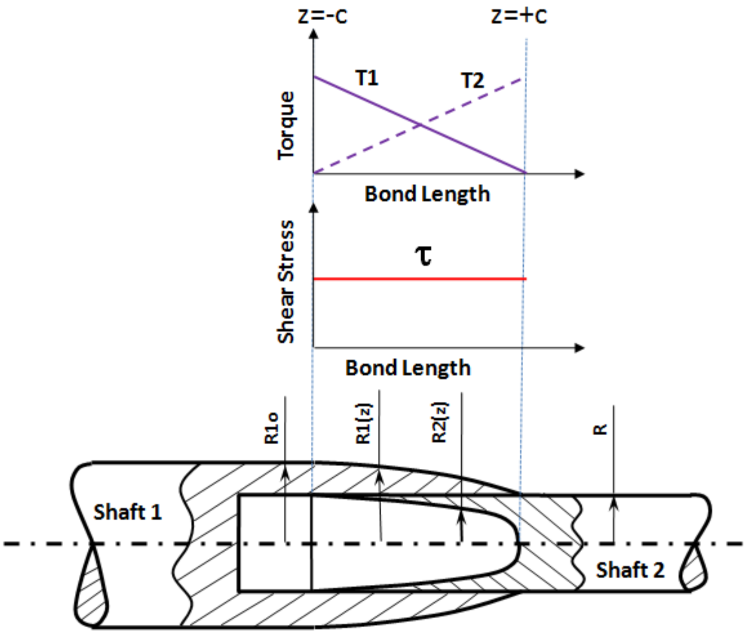

Figure 3 : Profile modification of Cylindrical Joint for uniform strength

The boundary conditions mentioned in equation (4) and linear rate of change of torque is satisfied by the following function.

$$f(z) = \left(\frac{c-z}{2c}\right) \qquad [6]$$

At any cross section if the rotations of shaft 1 and shaft 2 are identical then there is no relative displacement between them at that cross section. But if at any cross section if the rotations of shaft 1 and shaft 2 are different from each other, a relative rotation occurs resulting in circumferential relative displacement θ(z) at the bond layer which can be written as

$$\theta(z) = \theta_2(z) - \theta_1(z) \qquad [7]$$

From the Torsional moments absorbed by the two shafts at the joint, the rotations $\theta_1(z)$ and $\theta_2(z)$ of the cross sections of shaft 1 and shaft 2 respectively can be obtained from the compatibility and the circumferential relative displacements at the bond layer and θ(z) can be written as –

$$\theta(z) = \int_{-c}^{z} \frac{T_2(z)}{G_2 J_2}\, dz - \int_{-c}^{z} \frac{T_1(z)}{G_1 J_1}\, dz + \Delta\theta_i \qquad [8]$$

Where G_i = Shear modulus of material of shaft i , J_i = polar moment of inertia of shaft i and $\Delta\theta_i$ is the difference in the absolute rotations of shaft 1 and shaft 2 at the initial section $(z = -c)$.

The strain field γ (z) in the adhesive layer can be obtained from the stress field equation (5) and deformed geometry of adhesive layer as shown in fig.(4).

$$\gamma(z) = \frac{-1}{2\pi R^2 G_a} \frac{dT_1(z)}{dz} = \frac{R\ \theta(z)}{t_a} \qquad [9]$$

Where t_a is adhesive layer thickness and G_a Shear modulus of adhesive.

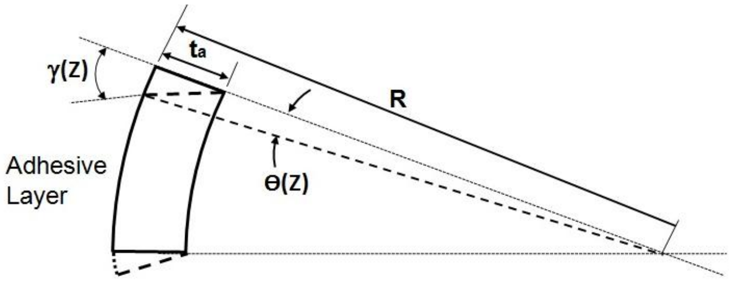

Figure 4: Shearing strain in adhesive layer

The rate of change of torque in shaft 1 along bond length can be obtained by substituting values of $\theta(z)$, τ(z) from equations (8) and (5) in equation (9).

$$\frac{dT_1(z)}{dz} = \frac{-2\pi R^3 G_a}{t_a}\left(\int_{-c}^{z}\frac{T_2(z)}{G_2 J_2}\,dz - \int_{-c}^{z}\frac{T_1(z)}{G_1 J_1}\,dz + \Delta\theta_i\right) \quad [10]$$

The following condition is obtained after differentiating equation (10) w.r.t. z and substituting values of $T_1(z)$, $T_2(z)$, f(z) from equations (2), (3), (6).

$$\frac{G_2 J_2(z)}{G_1 J_1(z)} = \frac{(c+z)}{(c-z)} \quad [11]$$

For unique solution of optimized geometry profile an additional condition must be considered of symmetric torsional stiffness of shafts section by section which can be written as-

$$G_1 J_1(z) = G_2 J_2(-z) \quad [12]$$

From the geometry of shaft 1 the following equations can be written for the torsional stiffness of shaft at different locations along the bond length.

$$At \quad z=-c \qquad G_1 J_1 = G_1\left(\frac{\pi}{2}\right)\left(R^4{}_{1o} - R^4\right) \quad [13]$$

$$z=+c \qquad G_1 J_1 = 0 \quad [14]$$

$$at \quad any \quad z \qquad G_1 J_1 = G_1\left(\frac{\pi}{2}\right)\left(R^4{}_1(z) - R^4\right) = G_1\left(\frac{c-z}{2c}\right)\left(\frac{\pi}{2}\right)\left(R^4{}_{1o} - R^4\right) \quad [15]$$

Rearranging equation (15) for $R_1(z)$ the following equation can be obtained

$$R_1(z) = \sqrt[4]{R^4 + \frac{c-z}{2c}\left(R_{1o}^4 - R^4\right)} \quad [16]$$

Now applying symmetric torsional stiffness condition (12) for shafts, the inner radius of shaft 2, $R_2(+z)$ can be obtained as-

$$R_2(+z) = \sqrt[4]{\left(R^4 - \frac{G_1}{G_2}\left(\frac{c+z}{2c}\right)\left(R_{1o}^4 - R^4\right)\right)} \quad [17]$$

The equations (16) and (17) obtained are for the geometry of shaft 1 and shaft 2 as shown in fig.(3) in the bond region and which satisfies both the conditions as mentioned in equations (11) and (12).

The mean radius of adhesive surface (R) can be obtained from equation (12) that the torsional stiffness of shaft 1 at z=-c is equal to torsional stiffness of shaft 2 at z=+c.

$$G_1\left(\frac{\pi}{2}\right)\left(R^4{}_{1o} - R^4\right) = G_2\left(\frac{\pi}{2}\right)\left(R^4 - R_{2i}^4\right) \qquad [18]$$

Limits of inner radius (R_{2i}) of shaft 2 will be $0 \le R_{2i} < R$ and which can be written as –

$$R_{2i} = \eta R \text{ where } 0 \le \eta < 1 \qquad [19]$$

Inserting value of R_{2i} in terms of R from equation (19) in to equation (18) and after rearrangement the outer radius (R_{1o}) of shaft 1 can be written as-

$$R_{10} = \xi R \qquad [20]$$

$$where \quad \xi = \sqrt[4]{\frac{G_1 + G_2 - G_2\eta^4}{G_1}}$$

The maximum torsional shear stress in the shafts due to applied torque occurs at the external radius (R_{1o}) of shaft 1. This maximum shear stress should be less than the allowable shear stress (τ_a) for the material of shaft 1.

$$\tau_{\max} = \frac{T}{\left(\frac{\pi}{2}\right)\left(\frac{R_{1o}^4 - R^4}{R_{10}}\right)} \le \tau_a \qquad [21]$$

Inserting value R_{1o} in terms of R from equation (20) in to the equation (21) , the mean radius of adhesive surface can be obtained as –

$$R \ge \sqrt[3]{\left(\frac{2T}{\pi\tau_a}\right)\left(\frac{\xi}{\psi}\right)} \qquad [22]$$

$$where \quad \psi = \left(\frac{G_2(1-\eta^4)}{G_1}\right)$$

The external radius (R_{1o}) of shaft 1 can be obtained from equations (19) and (20) as –

$$R_{1o} \ge \sqrt[3]{\left(\frac{2T}{\pi\tau_a}\right)\left(\frac{\xi^4}{\psi}\right)} \qquad [23]$$

From the joint profile obtained the weight of joint is proportional to –

$$\text{Joint Weight } \alpha \quad \xi^2 - \eta^2 \qquad [24]$$

The torsional shear stress in adhesive layer can be obtained by inserting equations (2) and (6) into (5)

$$\tau(z) = \frac{T}{4\pi R^2 c} \qquad [25]$$

From the above equation it can be seen that the shear stress in adhesive layer is constant and do not vary along the bond length and hence the joint is of uniform strength.

IV. Conclusion

i. Varying torsional stiffness profile for shaft 1 and shaft 2 as determined by the equations (16), (17) gives joint of uniform torsional strength and shear stress in adhesive layer remains constant (i.e. independent of z) equal to the average shear stress in adhesive layer.

ii. For cylindrical joint with uniform torsional strength the torque transmission capability increases as compared to the constant diameter shaft profile. The torque transmission capability increases by the ratio of maximum value of shear stress with constant diameter profile to the mean or average value of shear stress.

iii. When shaft 2 is solid (i.e. R_{2i}=0 or η=0) the outer radius of shaft 1 is minimum but the joint weight is maximum. For minimum weight hollow shafts are preferred and hence when shaft 2 is made hollow (i.e. η→1) the outer radius of shaft 1 increases but joint weight decreases.

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