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Numerical Investigation of the Perfomance of Convergent Divergent Nozzle

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ABSTRACT: *The main objective of the work is to analyse the performance and flow characteristics of convergent divergent nozzle and also to compare the numerical values of the two methods i.e "HIT & TRIAL METHOD" AND "ANALYTICAL METHOD". In this paper we have determine the location and strength of normal shock wave in the divergent portion of the nozzle under varying operating conditions and with different nozzle geometry.*

KEYWORD: *Mach number, Sub-sonic, Super-sonic, Sonic, Compressible flow, Throat.*

I. INTRODUCTION

A nozzle is a relatively simple device, just a specially shaped tube through which hot gases flow. However, the mathematics, which describes the operation of the nozzle, takes some careful thought. Nozzles come in a variety of shapes and sizes. Simple turbojets, and turboprops, often have a fixed geometry convergent nozzle as shown on the left of the figure. Turbofan engines often employ a co-annular nozzle. The core flow exits the centre nozzle while the fan flow exits the annular nozzle. Mixing of the two flows provides some thrust enhancement and these nozzles also tend to be quieter than convergent nozzles. Afterburning turbojets and turbofans require a variable geometry convergent-divergent - CD nozzle. In this nozzle, the flow first converges down to the minimum area or throat, then is expanded through the divergent section to the exit at the right. The variable geometry causes these nozzles to be heavier than a fixed geometry nozzle, but variable geometry provides efficient engine operation over a wider airflow range than a simple fixed nozzle. Rocket engines also use nozzles to accelerate hot exhaust to produce thrust. Rocket engines usually have a fixed geometry CD nozzle with a much larger divergent section than is required for a gas turbine.

1.1 SHOCKS IN NOZZLE

A shock wave (also called shock front or simply "shock") is a type of propagating disturbance. Like an ordinary wave, it carries energy and can propagate through a medium (solid, liquid or gas) or in some cases in the absence of a material medium, through a field such as the electromagnetic field. Shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium. Across a shock there is always an extremely rapid rise in pressure, temperature and density of the flow. In supersonic flows, expansion is achieved through an expansion fan. A shock wave travels through most media at a higher speed than an ordinary wave. Unlike solutions (another kind of nonlinear wave), the energy of a shock wave dissipates relatively quickly with distance. Also, the accompanying expansion wave approaches and eventually merges with the shock wave, partially cancelling it out. Thus the sonic boom associated with the passage of a supersonic aircraft is the sound wave resulting from the degradation and merging of the shock wave and the expansion wave produced by the aircraft. When a shock wave passes through matter, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases. This, for example, creates additional drag force on aircraft with shocks. Shock waves can be

- Normal: at 90° (perpendicular) to the shock medium's flow directions.
- Oblique: at an angle to the direction of flow.
- Bow: Occurs upstream of the front (bow) of a blunt object when the upstream velocity exceeds Mach 1.

Shock waves form when the speed of a gas changes by more than the speed of sound. At the region where this occurs sound waves travelling against the flow reach a point where they cannot travel any further upstream and the pressure progressively builds in that region, and a high pressure shock wave rapidly forms. Shock waves are not conventional sound waves; a shock wave takes the form of a very sharp change in the gas properties on the order of a few mean free paths (roughly micro-meters at atmospheric conditions) in thickness. Shock waves in air are heard as a loud "crack" or "snap" noise. Over longer distances a shock wave can change from a nonlinear wave into a linear wave, degenerating into a conventional sound wave as it heats the air and loses energy. The sound wave is heard as the familiar "thud" or "thump" of a sonic boom, commonly created by the supersonic flight of aircraft.

II. METHODOLOGY

2.1 ANALYTICAL APPROACH

To determine shock location and shock strength in convergent divergent nozzle.

STEP 1: Find the pressure ratio that will produce a shock in the divergent portion of the nozzle.

STEP 2: Determine exit Mach number (M_e)

$$M_e^2 = \frac{-1}{\gamma-1} + \sqrt{\left(\frac{1}{\gamma-1}\right)^2 + \left(\frac{2}{\gamma-1}\right)\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{P_{01}}{P_e}\right)^2\left(\frac{A_t}{A_e}\right)^2}$$

STEP 3: Use M_e to determine P_e/P_{02}

$$\frac{P_{02}}{P_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}}$$

STEP 4: Since $M_e<1$, $P_e=P_b$

$$\frac{P_{02}}{P_{01}} = \frac{P_b}{P_{01}} * \frac{P_{02}}{P_e}$$

STEP 5: Determine M_1, by using the value of P_{02}/P_{01}

$$\frac{P_{02}}{P_{01}} = \left[\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1}{\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}}\right]^{\frac{1}{\gamma-1}}$$

STEP 6: Determine Shock Location A_s/A_t

$$\frac{A_s}{A_t} = \frac{1}{M_1}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

STEP 7: Determine Shock Strength

$$\frac{P_y - P_x}{P_x} = \left[\frac{2\gamma}{\gamma+1}\left(M_1^2-1\right)\right]$$

STEP 8: Determine Temperature Ratio across the shock

$$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)\left(\frac{2\gamma}{\gamma-1}M_1^2-1\right)}{\left(\frac{(\gamma+1)^2}{2(\gamma-1)}\right)M_1^2}$$

STEP 9: Determine Pressure Ratio across the shock

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}$$

STEP 10: Determine density ratio across the shock

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

2.2 HIT AND TRIAL APPROACH

To determine the shock location and shock strength in convergent divergent nozzle

Let us consider a convergent divergent nozzle with inlet and outlet section specified in the diagram as 1 and 5 respectively. In the diagram section 2 represent the throat while the section 3 and 4 represent the flow condition before and after the shock.

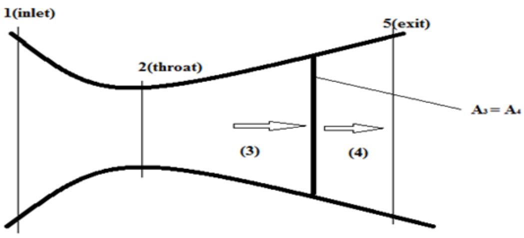

Fig.2.2.1 Convergent divergent nozzle

Let,
A_e/A_t=exit area/throat area=1.53
Inlet condition P_0=1atm.
Now let us determine the various flow characteristics for this case.

STEP 1:
If we assume chocked flow at the throat then from the IFT for A_e/A_t we have two solutions

1) For FIRST critical point(fully isentropic subsonic flow)
   From IFT at (A_e/A_t)=1.53,we have
   (P_e/P_o)=.886
   M_e=.42

For THIRD critical point(fully isentropic supersonic flow)
   From IFT at (A_e/A_t)=1.53,we have
   (P_e/P_o)=.154
   M_e=1.88

Now as we know that normal shock takes place in the divergent portion of the nozzle i.e in the supersonic flow. Therefore for supersonic flow conditions we have
M_e=1.88 and
$P_e/P_o(P_3/P_{03})$=.154

STEP 2:
Now from the normal shock table for M_e=1.88, we have
M_e(aftershock)=.599
P_y/P_x(after shock)=3.957
And the operating pressure ratio will be
$P_{rec}/P_0=P_4/P_{01}=P_4/P_3*P_3/P_{03}*P_{03}/P_{01}$
P_4/P_{01}=3.957*.154*1
P_4/P_{01}=.609378
Thus for our C-D nozzle with A_e/A_t=1.53,any operating pressure ratio between .886 and .609378 will cause a normal shock to be located some where in the divergent portion of the nozzle.

STEP 3:
Now let us find shock location at the operating pressure ratio of 0.75
Let us assume shock wave is located at A_s/A_t=1.204
Note-this value is selected because it is one of the numbers in IFT. by selecting this value we don't need to interpolate.
Key equation:
$P_e=P_e/P_{02}*P_{02}/P_{01}*P_{01}$
From IFT, corresponding to A_s/A_t=1.204, we have
M_1=1.54
From NST corresponding to M_1=1.54, we have
M_2=.687 and P_{02}/P_{01}=.917
From IFT corresponding to M_2=.687,we have
A_s/A^*=1.1018
$A_e/A^*=A_e/A_t*A_t/A_s*A_s/A^*$
=1.53*1/1.204*1.1018
=1.4
From IFT corresponding to A_e/A^*=1.4, we have
M_e=0.47
P_{02}/P_e=1/.859=1.163

P_e=1/1.163*.917*1
P_e=.788 atm.
But it is given that P_e=.75 atma . , hence we to move the shock a bit downstream.
STEP 4:
Let us assume shock wave is located at A_s/A_t=1.301
We have M_1=1.66,from NST corresponding to M_1=1.66, we have
M_2=.6512 and P_{02}/P_{01}=.872
From IFT corresponding to M_2=.6512, we have A_s/A^*=1.1356
$A_e/A^*=A_e/A_t*A_t/A_s*A_s/A^*$
=1.53*1/1.301*1.1356
=1.335
From IFT corresponding to A_e/A^*=1.335,we have
M_e=0.5
P_{02}/P_e=1/.843=1.1862
P_e=1/1.1862*.872*1
P_e=.735atm
Again the pressure ratio does not exactly match our given exit pressure. Now we can interpolate between the two assumed values. So
A_s/A_t=1.301-(1.301-1.204)*(.75-.735)/(.788-.735)
A_s/A_t=1.274
STEP 5:
Let us assume that shock wave is located at A_s/A_t=1.274
From IFT corresponding to A_s/A_t=1.274, we have M_1=1.63
From NST corresponding to M_1=1.63,we have M_2=.6595 and P_{02}/P_{01}=.8838
From IFT corresponding to M_2=.6595,we have A_s/A^*=1.1265
$A_e/A^*=A_e/A_t*A_t/A_s*A_s/A^*$
=1.53*1/1.274*1.1265
=1.353
From IFT corresponding to A_e/A_*=1.353, we have
M_e=0.49 and P_{02}/P_e=1/.848=1.178
P_e=1/1.178*.8838*1
=.750atm
Thus the properties obtained for a C-D nozzle of area ratio A_e/A_t=1.53 at operating pressure ratio of 0.75 are
M_1=1.63
P_{02}/P_{01}=.8838
M_e=0.49
P_{02}/P_e=1.178
P_e=0.75
A_s/A_t=1.274

III. RESULTS AND DISCUSSIONS

TABLE.1 FOR AREA RATIO 2.035

	Pe/Po						
Properties across shock		0.90	0.84	0.78	0.69	0.6	0.5
	M_e	.312923	.334810	.359950	.405523	.464070	.552165
	P_{o2}/P_e	1.070239	1.080692	1.093671	1.119925	1.159045	1.230187
	P_{02}/P_{01}	.963215	.907781	.853063	.772748	.695427	.615094
	M_1	1.380000	1.566000	1.707000	1.889000	2.055000	2.229000
	$(P_y-P_x)/P_x$	1.055133	1.694415	2.232824	2.996374	3.760196	4.629848
	T_2/T_1	1.241814	1.364657	1.463361	1.599362	1.732700	1.882576
	P_2/P_1	2.055133	2.694415	3.232824	3.996374	4.760196	5.629848
	ρ_2/ρ_1	1.654945	1.974427	2.209177	2.498731	2.747271	2.990502
	A_s/A_t	1.104193	1.223588	1.344237	1.541706	1.767480	2.057396

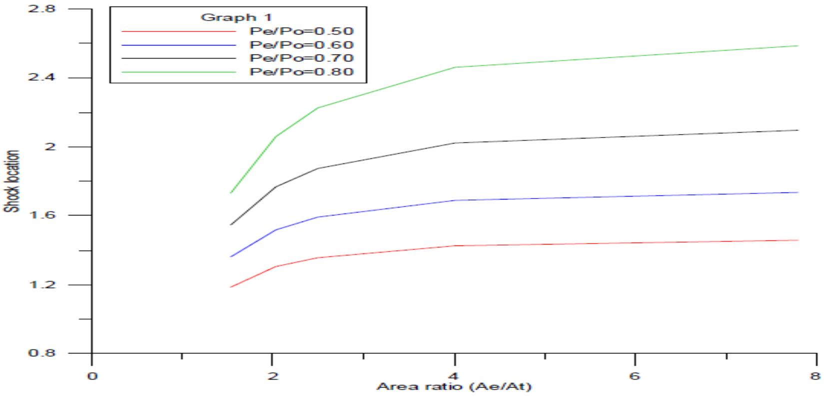

Fig.3.1variation of shock location with area ratio

TABLE 2.Comparison between analytical and hit & trial method

Properties	Analytical method	Hit and trial method
Exit mach number(M_e)	.3812	.38
Stagnation pressure ratio(P_{02}/P_{01})	.6632	.6628
Inlet mach number(M_1)	2.1240	2.12
Temperature ratio(T_2/T_1)	1.7908	1.787
Pressure ratio(P_2/P_1)	5.0966	5.077
Shock location(A_s/A_t)	1.8755	1.869
Shock strength$(P_y-P_x)/P_x$	4.096	4.077

The above values have been calculated for properties across Normal shock waves, where working fluid consider here is air with specific heat ratio γ=1.4 for a nozzle geometry of area ratio of 2.494 and the operating pressure ratio of 0.60 which has been chosen arbitrarily in between the critical pressure ratios to compare the result obtained by both the methods i.e. "Analytical method" and "Hit& trial method".

IV. CONCLUSIONS

1) Shock strength increases significantly by increasing (M_1),whereas shock location do not vary much with (M_1).
2) For constant operating pressure ratio, shock strength shows significant variation for smaller area ratio(A_e/A_t).
3) For a constant area ratio (A_e/A_t), shock location varies significantly for higher pressure ratios as compared to lower pressure ratios.
4) Shock location moves towards exit, by decreasing (P_e/P_o).

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