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# Inverted Pendulum Control: A Brief Overview

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***ABSTRACT:*** *Inverted Pendulum is a classic example of an inherently unstable system. This paper studies the Control of Inverted Pendulum. It focuses on the Mobile Inverted Pendulum. A few papers have been referred to get a brief overview of the work going on in this field. A short survey of these papers has been done for the purpose. Along with it, the basic idea of the mobile inverted pendulum and 2 control strategies have been discussed.*

***Keywords:*** *Ball-bot, Degree of Freedom, Fuzzy control, Inverted pendulum, PID control.*

## I. INTRODUCTION

Inverted pendulum is a system that is stable under specific conditions. It is inherently unstable but a few conditions can be matched to obtain stability. Much work has been done on inverted pendulum since last 50 years. In simple words, an inverted pendulum can be explained with the analogy of balancing a long stick on one finger, something that all of us have tried in childhood.

Inverted pendulum is a standard problem in Control system and is implemented in areas of precision control and robotics. The concept is applied for high precision robotic arms, launching of a rocket, control of a Vertical Take-Off and Land (VTOL) aircraft, etc. Due to the advent of humanoid robots, where this concept is applied for its balancing, recently a lot of work is being done to try and find new control methods.

Another very important place where this concept is used is the field of education. Here new students can get a very good idea of this classic problem.

The inverted pendulum has been implemented at various different levels or Degree of Freedom (DOF). The ones implementing 1st DOF are mounted on a fixed base either sliding along a straight direction (Linear Inverted Pendulum) or rotating around vertical axis (Rotary Inverted Pendulum). Other models with higher DOF (double or triple DOF) are mounted on a mobile platform or a Cart structure. Examples of which being JOE [1], Segway [2]. Segway is the commercial version of Two wheeled inverted pendulum used for personal transportation. Most papers consider a rigid pendulum, but for better precision now even the flexibility of the pendulum is being considered [3].

In recent years, people have tried to give more Degree of Freedom to this setup by making a Ball-bot [4]. Here the control and driving mechanism are placed on top of a ball, which behaves like an Omni-directional wheel for the setup. The driving mechanism consists of 3 or 4 Omni wheels that rotate the ball accordingly to keep the balance.

Most researches have mainly focused on the balance, while few have tried the driving control [5] and trajectory planning [6]. This means the controller has an additional burden of desired movement along with maintaining balance.

This paper attempts to overview the basic concepts involved in control of an inverted pendulum, along with the current work in the field. We shall focus on mobile inverted pendulum and not the fixed base ones.

## II. STRUCTURE OF MOBILE INVERTED PENDULUM

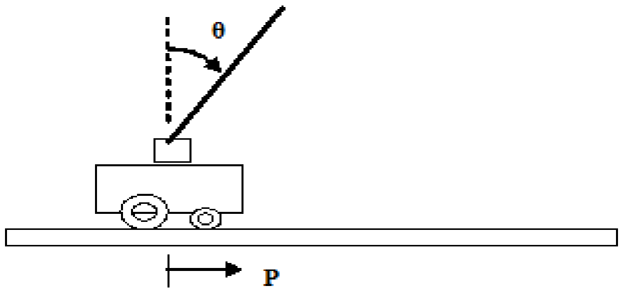


Fig. 1 Mobile Inverted Pendulum Setup

Fig. 1 shows a Four wheeled cart which has an inverted pendulum mounted on top of it. The cart is equipped with motors that provide horizontal motion. The cart position p, the tilt angle θ, the rate of tilting are measured and fed to the controller. The controller then generates and sends Pulse Width Modulated (PWM) signals to the motor drivers.

By applying the laws of dynamics on inverted pendulum system, the equations of motion obtained are [7]:

$$\ddot{p}\left(M - \frac{m_p l \cos^2(\theta)}{L}\right) = \frac{Km\,Kg}{Rr} V - \frac{Km^2\,Kg^2}{Rr^2}\dot{p} - \frac{m_p l g}{L}\cos(\theta)\sin\theta + m_p l \sin(\theta)(\dot{\theta})^2 \quad (1)$$

$$\ddot{\theta}\left(L - \frac{m_p l \cos^2(\theta)}{M}\right) - g\sin(\theta) - \frac{m_p l(\dot{\theta})^2}{L}\cos(\theta)\sin(\theta) - \frac{\cos(\theta)}{M}\left(\frac{Km\,Kg}{Rr}V - \frac{Km^2\,Kg^2}{Rr^2}p\right) \quad (2)$$

Where:-
$m_c$ is the cart mass
$m_p$ is the pendulum mass
I is the rotational inertia
l is the half-length of the pendulum
R is the motor armature resistance
r is the motor pinion radius
Km is the motor torque constant
Kg is the gearbox ratio

For simplicity :

$$M = m_c + m_p \tag{3}$$

$$L = \frac{I + m_p l^2}{m_p l} \tag{4}$$

Note that the relationship between force, F and voltage V for motor is:

$$F = \frac{Km\,Kg}{Rr} V - \frac{Km^2\,Kg^2}{Rr^2} \dot{p} \tag{5}$$

A modification to this structure is a two wheeled inverted pendulum system, where the controller and rider (if any), form the pendulum bob. Examples of same given before: Joe [1], Segway [2].

## III. Control Mechanism

Controller design is a key content of the Inverted Pendulum system. Controllers are used to stabilize the unstable system and make it robust to disturbances. Several techniques have been used for achieving the same, e.g. Sliding Mode Technique [8], Fuzzy Logic Controller [9], Partial Feedback Linearization [10], Fuzzy Servo Control Method [11], Real-Time Control [12]. We take a brief view of conventional PID type controller and rule based Fuzzy Logic controller for inverted pendulum-cart system [9]. The framework of this Inverted Pendulum-cart system Controller is presented in Fig.2.

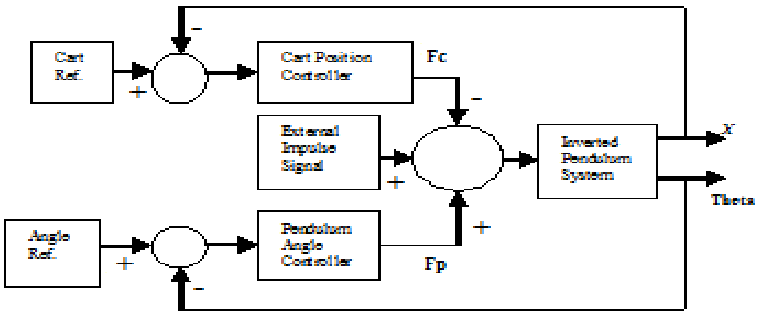


Fig. 2 Block diagram of Inverted Pendulum-cart controller system

### 3.1 Conventional PID Controller

An inverted pendulum-cart system, as mentioned before, is an unstable model. It can be stabilized by using the controllers. We see two PID controllers been designed, first one for Pendulum angle control and other or the Cart position control. These controllers try to correct the error between measured values and the desired values. This is done by calculating and then outputting a corrective action that can adjust the motion of the cart accordingly. For both PID controllers, the structure is taken as:

$$u = K_P\, e + K_I \int e\, dt + K_D \frac{de}{dt} \tag{6}$$

Where:-
u is PID output control action,
e is the error i.e. difference between set point input and actual output
e = yref - yactual,
KP, KI, KD are the proportional, integral and derivative gains respectively.
We can see that the output of pendulum angle controller and the cart position controller are of opposite signs.
The selection of PID controller parameters (KP, KI, KD) is important as incorrect selection of these parameters can make controlled process input unstable. The control parameters are adjusted to optimum values for the desired response. This is called Tuning of the control loop.

### 3.2 Fuzzy Logic Controller (FLC)

FLC is one of the most successful applications of fuzzy set theory. It uses linguistic variables instead of numerical variables. Linguistic variables, defined as variables whose values are sentences in natural language (such as small and large) are represented by fuzzy sets. This makes the instruction set more Human.

A crisp set is one where an element can only belong to a set (full membership) or not belong to at all (no membership). A Fuzzy set is an extension of crisp set as it allows partial membership, which means that an element may partially belong to more than one set.

A fuzzy set A is characterized by a membership function µA that assigns membership to each object and it can range from 0 (no membership) to 1 (full membership), we therefore write:

$\mu A : X \rightarrow [0, 1]$

Which means that the fuzzy set A belongs to a universal set X (usually called universe of discourse) defined in a specific problem. A fuzzy set A is called a fuzzy singleton when there is only one element xo with µA(xo) = 1 , while all other elements have a membership grade which equal to zero.

This approach allows characterization of the system behaviour through simple relations (fuzzy rules) between linguistic variables. These fuzzy rules are expressed in the form of fuzzy conditional statements Ri of the type,

Ri : if x is small THEN y is large.

Where x and y are fuzzy variables, and small and large are labels of fuzzy set.

If there are i =1 to n rules, the rule set is represented by union of these rules,

R = R1 else R2 else…….Rn

A fuzzy logic controller is based on a collection of R control rules. The execution of these rules is governed by the compositional rule of inference [9].

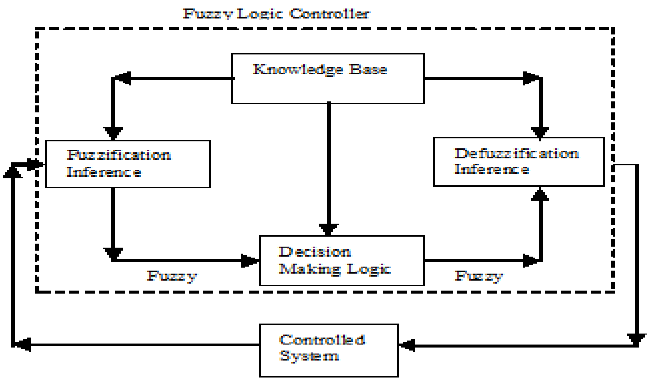


Fig. 3 Basic configuration of FLC

The general structure of an FLC is represented in Fig.3 and comprises four principle components:

1) A fuzzyfication inference converts input data into suitable linguistic values;
2) A knowledge base consists of a data base with the necessary linguistic definitions and control rule set;
3) A decision making logic infers the fuzzy control action from the knowledge of the control rules and the linguistic variable definitions;
4) A defuzzyfication inference yields a non-fuzzy control action from an inferred fuzzy controlled action.

### 3.3 Performance

Following figures shown are the graphical results of PID and Fuzzy Logic controller respectively. These figures have been taken from [9] for the sole purpose of comparing the two controllers. The results are from MATLAB Simulink simulations.

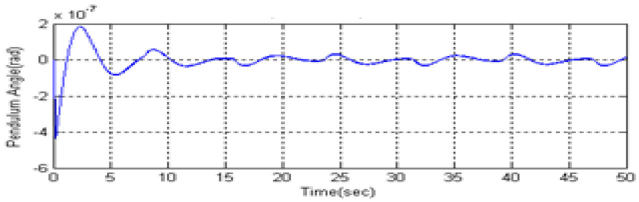


Fig. 4.a PID Response for disturbance 0.000001 unit

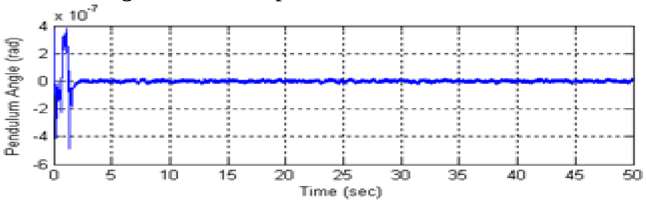


Fig. 4.b Fuzzy Response for disturbance 0.000001 unit

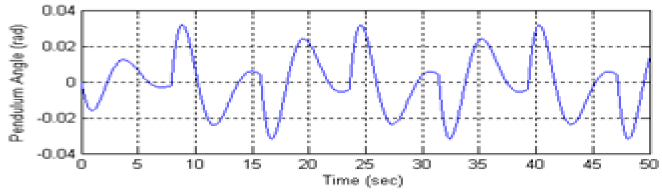


Fig. 5.a PID Response for disturbance 1 unit

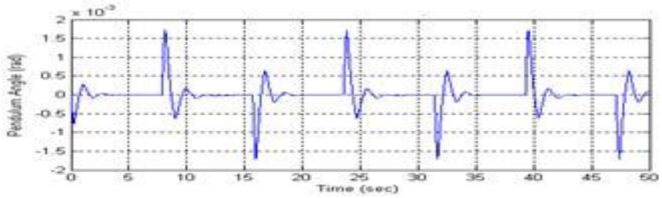


Fig. 5.b Fuzzy Response for disturbance 1 unit

The two controllers were tested for different magnitudes of disturbances for comparing their performances, two of which have been shown above. The performance comparison clearly shows that Fuzzy Controller offers a much better control as compared to the PID controller owing to its more human like approach. The PID controller does its job of maintaining the pendulum angle, but its response is sluggish as compared to the Fuzzy Controller.

## IV. CONCLUSION

A brief overview of The Inverted Pendulum is taken in this paper. Two control strategies have been discussed and compared. It has been observed that the PID controller is a basic control technic with a sluggish response whereas the Fuzzy controller response is more crisp.

The paper reinstates that inverted pendulum system is a fundamental benchmark in education and research in control theory. Application of Mobile Inverted Pendulum in robotics and personal transportation has boosted the amount of work done in this field in recent times. This area of research has a lot of scope and a bright future.

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