Analytic Model of Wind Disturbance Torque on Servo Tracking Antenna

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ABSTRACT: *The paper develops an analytic wind disturbance torque model that can be used to simulate antenna servo errors. The forces due to wind acting on antenna induce torque that creates servo errors which become significant in slow tracking antennas. The model developed is based on the vector sum of torque acting on the antenna and is analytically computed by disintegrating the forces due to wind in mutually orthogonal directions. The demarcation between the existing field models of wind disturbances and the model presented in this article is brought out by the generalization of torque induced with the only variable antenna specific parameter being the porosity.*

Keywords: *Antenna servo errors, antenna torque, field model, mutually orthogonal, wind disturbance*

I. INTRODUCTION

Large antennas are required to capture feeble signals, especially those received from space. Radio telescopes that track astronomical phenomenon and inter planetary satellite tracking antennas fall under these categories. These tracking antennas require very high pointing accuracies and thereby a robust servo control system. The large structures associated with these antennas in ideal conditions require a second-order time invariant feedback controller[1]. However, these large structures are affected by torques generated by wind disturbances. The torque results in large servo errors, which reduce the pointing accuracies of antennas. The requirement of high pointing accuracy antennas call for wind torque disturbance models that can accommodate the induced servo errors. The existing models[2] are field models specific to antennas and thus, require extensive fieldwork before they can be used for designing the servo control loop. The model presented here is generalized for large antenna structures and depends only on one antenna specific parameter. The parameter is the resistance offered by the antenna to wind and is characterized by the porosity of the antenna[2].

The article presented develops an analytic method which employs vector mechanics to calculate the torques generated about a point. The model was conceptualized at Tata Institute of Fundamental Research during servo control design for 45 meter Giant Meterwave Radio Telescope at Pune.

II. THE ANALYTIC MODEL FOR WIND DISTURBANCE TORQUE

The wind disturbance generates torque that is the main cause of servo errors for tracking antenna. The wind torque generated can be analytically expressed as vector sum of torques generated in two mutually perpendicular axes. The determination of wind force from wind velocity is the preliminary modeling step. The force due to wind is calculated using the pressure exerted by wind by virtue of its velocity.

2.1 Obtaining Wind Force from Wind Velocity

An antenna exposed to uniformly blowing wind is acted upon by pressure given by $1/2\ \rho v_w^2$ where
$\rho$is the density of air;
$v_w$is the uniform velocity with which wind is blowing.
The force acting on an area A due to pressure P in quantized as the product of the P and A. Generalized force on antenna of cross sectional area A due to wind is given by Eqn (1).

$$F = \eta_1 \eta_2 PA \quad (1)$$

Where
η_1 is a measure of porosity of the antenna
η_2 is a factor less than 1 that takes into account the relative orientation of the wind, w.r.t. the antenna.
Fig 1 illustrates the interaction of wind with the cross sectional area of an antenna.

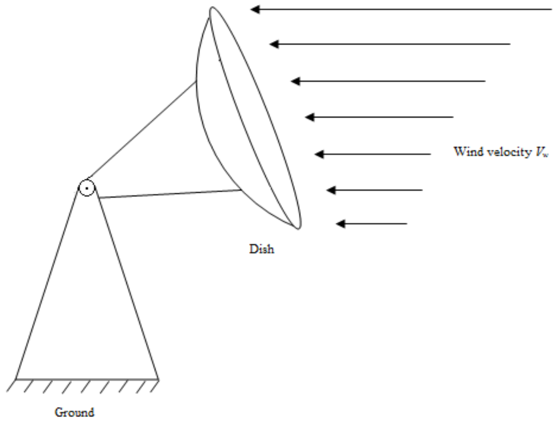

Fig 1: The force generated is a function of wind velocity, area offered by the antenna against wind, porosity of the antenna and the relative orientation of the antenna w.r.t wind.

The wind velocity so far is assumed to be constant. However, it is a function of the height above ground level. The forces experienced by the antenna therefore, vary along the height. The unbalanced forces at the top of the antenna and bottom of the antenna generate torque. The unbalanced force vectors are illustrated in Fig 2.

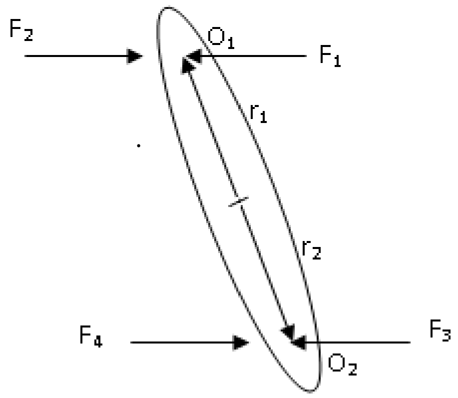

Fig 2: Unbalanced forces due to varying wind velocity generate torque.

The force due to wind is distributed over the entire area of the antenna. However, single vector sum of forces acting on O_1 and O_2 can be calculated which shall produce the same torque as distributed forces would have produced. The vectors are given by Eqns (2-3).

$$\overrightarrow{F_1} - \overrightarrow{F_2} = \overrightarrow{F_{up}} \quad (2)$$

$$\overrightarrow{F_3} - \overrightarrow{F_4} = \overrightarrow{F_{down}} \quad (3)$$

O_1 and O_2 are points on the upper and lower half of discs, where a resultant single vector force shall produce the same effects as distributed force.

2.2 Variation of Wind Velocity: Hellman's Equation

Hellman's equation relates the velocity of wind w.r.t altitude above the earth's surface. (The equation is assumed valid as long as density of air remains constant, within the altitude range or in other words, thinning of air as one goes up is not taken into consideration).

$$v_w(h) = v_o \left(\frac{h}{h_o}\right)^{\alpha} \quad (4)$$

Where

α is the Hellman coefficient

v_o is velocity of wind at reference altitude

$v_w(h)$ is velocity of wind at height h

α can be measured on a day when winds are steady. To measure α , velocity at two points of antenna can be measured and the equation can be solved for α. The measurement can be repeated for precision. To further nullify the effect of wind-gusts, velocity of wind can be taken over finite time duration and wind-gusts being random the velocity of wind will deviate about the steady state wind velocity. Since wind gusts follow Davenport spectrum, the measured α deviates as dictated by Davenport spectrum.

2.3 The Case of Face on Wind: Perpendicular Attack

The perpendicular attack case is the worst case as the whole cross-section of the antenna is exposed to the wind. Fig 3 illustrates the case of perpendicular attack. To calculate force we need an elemental area over which the force is constant. The area is basically a horizontal strip that is at a distance x from the center and is of width dx. The strip subtends an angle θ at the center of parabolic disc. The strip's center subtends an angle ϕ at O as illustrated in Fig 4. The strip is the differential element for torque calculation.

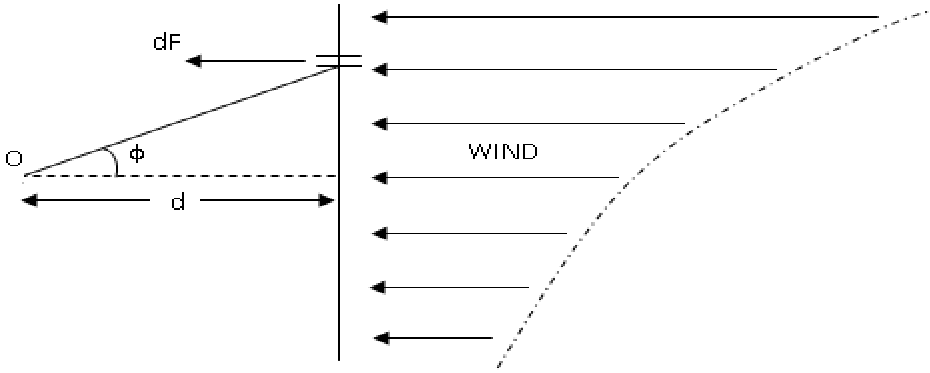

Fig 3: The varying wind velocities are illustrated with different vector lengths. The shaded strip is the part of antenna that encounters equal velocity wind. The differential element for torque calculation is the strip that faces perpendicular force of wind.

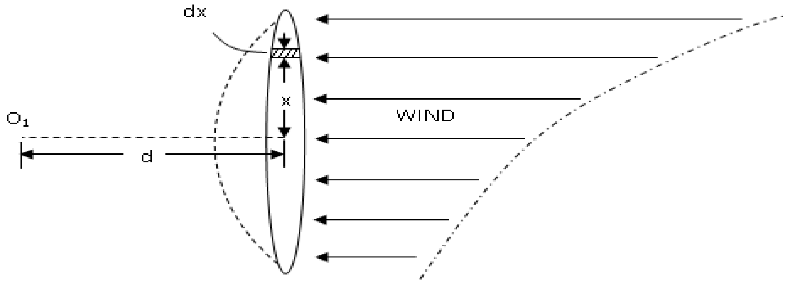

Fig 4: The differential element subtends an angle of ϕat O, which is the point about which the moment is calculated. O being *d* distance away from the center of disc.

Torque calculation requires calculating the force acting on the elemental strip as a function of wind velocity and distance from the center of the parabolic disc A. Fig 5 serves as reference for further analysis.

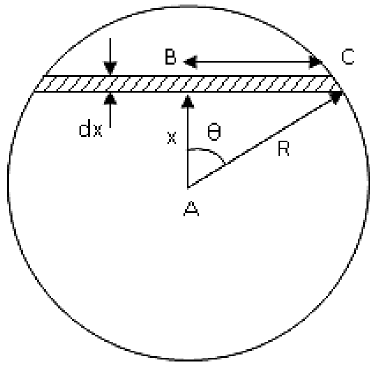

Fig 5: The elemental strip faces constant velocity wind. The center of the circular cross-section is A. The strip subtends θ at A, is *x* distance away from A and is of width *dx*.

The area of the elemental strip in polar coordinates is given by Eqn (5).

$$dA = -2R^2 sin^2\theta d\theta \qquad (5)$$

The inclination of antenna is 90^o w.r.t wind. Therefore, $\eta_2$is 1. Referring to Eqn (1) the force on the elemental strip can be given as in Eqn (6).

$$dF = 1/2\, \eta_1 \rho v_w^2 \times dA \qquad (6)$$

The velocity of wind as a function of height above ground level can be substituted as per Eqn (4), while the elemental area of the strip is substituted from Eqn (5). Eqn (6) modifies to Eqn (7).

$$dF = 1/2\, \eta_1 \rho \left(v_o \left(\frac{h}{h_o}\right)^{\alpha} \right)^2 \times (-2R^2 sin^2\theta d\theta) \quad (7)$$

If center of the circle shown in Fig 5 is taken as the reference, the height parameter in Eqn (7) equals *x*. Substituting *x* in polar coordinates in Eqn (7) in place of *h* yields Eqn (8).

$$dF = 1/2\, \eta_1 \rho \left(v_o \left(\frac{Rcos\theta}{h_o}\right)^{\alpha} \right)^2 \times (-2R^2 sin^2\theta d\theta) \quad (8)$$

The force on the differential element produces a turning moment about the hinge of antenna. The magnitude of the generated torque is analytically found by multiplying the rotational arm length for the element or the distance of the differential element from O with the component of *dF* perpendicular to the rotational arm. Fig 6 illustrates the force vectors and the rotational arm lengths about O.

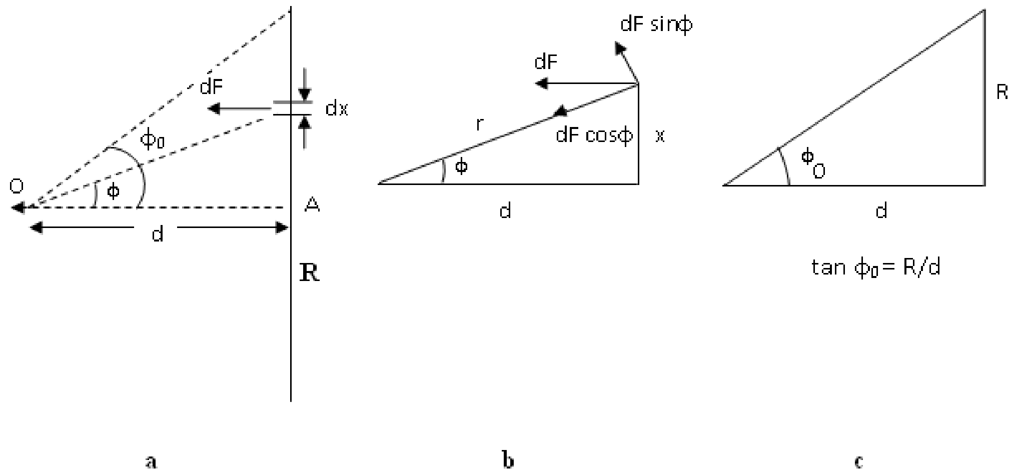

Fig 6: (a) Figure illustrating the angular span of the parabolic disc about O. The radius of the dish being R. (b) Figure showing the resolution of forces perpendicular to the rotational arm *r*. (c) Figure illustrating the relation between radius R of the antenna dish, angular span ϕ_o and distance *d*.

The elemental torque ($d\tau_{up}$) on the elemental strip in the upper half of the parabolic strip is given by Eqn (9).

$$d\tau_{up} = dFsin\phi \times r \quad (9)$$

The force on the elemental strip is calculated in Eqn (8) and substituting *r* as $d/_{cos\emptyset}$ from Fig 6, we have the torque on the elemental strip as

$$d\tau_{up} = -1/2\, \eta_1 \rho \left(v_o \left(\frac{Rcos\theta}{h_o}\right)^{\alpha} \right)^2 \times 2R^2 sin^2\theta d\theta \times d \times tan\emptyset \times d\emptyset \quad (10)$$

Integrating Eqn (10) with limits for θ and $\emptyset$ the torque on the upper half disc can be calculated as shown in Eqn (11).

$$\int_0^{\tau} d\tau_{up} = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\phi_o} \eta_1 \rho \left(v_o \left(\frac{Rcos\theta}{h_o}\right)^{\alpha} \right)^2 \times R^2 sin^2\theta d\theta \times d \times tan\emptyset \times d\emptyset\, d\theta \quad (11)$$

The limits for θ are reversed to absorb the negative sign in *dF* expression. The torque for the lower half of the dish can be found by placing proper limits and appropriate expression for the velocity of wind. The antenna here faces the wind perpendicularly; however, tracking antenna may be inclined at some angle to the wind velocity. The next section takes into account the parameter η_2.

2.4 Antenna Inclined w.r.t the Wind: Angular Attack

The most generalized torque generating case is that of an inclined antenna against arbitrary direction of wind. The assumption here is that the wind follows Hellman's velocity relation, only the direction is arbitrary. Consider the antenna to be inclined at angle ξas illustrated in Fig 7. Wind force is resolved into three mutually perpendicular components and torque due to each component is measured and generalized for parameter ξ.

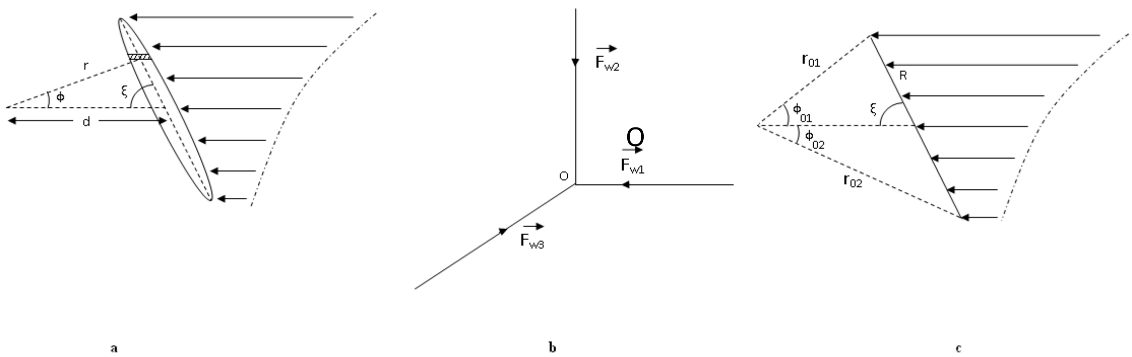

Fig 7: (a) Figure illustrating the inclination of the antenna w.r.t wind. (b) The resolved components of force vector. (c) Figure illustrating the angular span of the antenna about reference O.

The three mutually perpendicular force vectors are depicted in Fig 7 (b). F_{w1} is the direction considered earlier for perpendicular attack. F_{w3} sees the parabolic lateral face analyzed in the next section and F_{w2} is in the vertical direction. The case of wind blowing vertically is seldom encountered and especially when the region experiences tornados and hurricanes.

The differential force on the element is illustrated in Fig 8. The corresponding parameters are depicted in the triangle. The component of the force perpendicular to the moment arm is $dF_{w1}sin\phi$. The element is at a distance r from O. As compared to the case of perpendicular case the element is at an inclination of ξ w.r.t the wind. InΔABO, the relation between sides and angles can be determined by applying Sine Law as in Eqn (12).

$$\frac{r}{sin\xi} = \frac{d}{\sin(\pi-(\phi+\xi))} \quad (12)$$

Applying Sine Law for the triangles in Fig 7 (c) we arrive at Eqns (13-14)

$$\frac{R}{sin\,\phi_{01}} = \frac{d}{\sin((\phi_{01}+\xi))} \quad (13)$$

$$\frac{R}{sin\,\phi_{02}} = \frac{d}{\sin((\xi-\phi_{02}))} \quad (14)$$

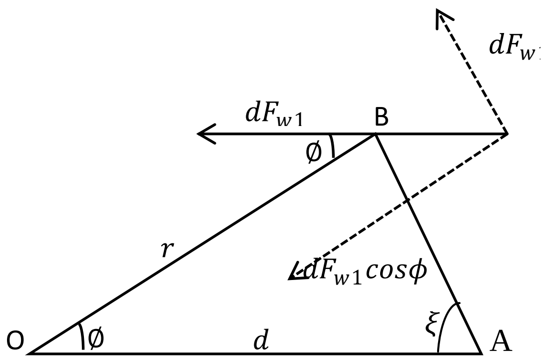

Fig 8: Figure illustrating the force vectors for the differential element. *r* is the distance of the element from O, *d* is the distance of center A from O. and being the angle subtended by the elemental strip at O and A respectively.

The differential torque is given by Eqn (9). Substituting *r* from Eqn (12) and *dF* from Eqn (8) in Eqn (9), the differential torque on the element can be expressed by Eqn (12).

$$d\tau_{up} = -\eta_1\rho\left(v_o\left(\frac{Rcos\theta}{h_o}\right)^{\alpha}\right)^2 \times R^2 sin^2\theta d\theta \times \frac{dsin\xi}{\sin(\xi+\phi)} \times sin\emptyset \times d\emptyset \quad (15)$$

The torque on the upper half of the antenna can be obtained by integrating Eqn (15) with limits for θ and ϕ. The limits for ϕ is from *0* toϕ_{01}, where $\phi_{01}$is obtained from Eqn (13).

$$\int_0^{\tau} d\tau_{up} = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\phi_{01}} \eta_1\rho\left(v_o\left(\frac{Rcos\theta}{h_o}\right)^{\alpha}\right)^2 \times R^2 sin^2\theta d\theta \times \frac{dsin\xi}{\sin(\xi+\phi)} \times sin\emptyset \times d\emptyset\, d\theta \quad (16)$$

2.5. Torque on the Lateral Surface

The antenna employed for tracking is mostly a parabolic antenna. The lateral cross- section thus is a parabola. Fig 9 shows the parameters for a parabolic cross-section. The parabola can be mathematically represented by Eqn (17).

$$y = \frac{K}{R^2}m^2 \quad (17)$$

The elemental area in this case is the product of length of the strip with its width and can be expressed by Eqn (18)

$$dA = l \times dx \quad (18)$$

The length of the strip is mathematically the difference between the depth of the parabola *K* and *z*.

$$l = K - z = K\left(1 - \left(\frac{x^2}{R^2cos^2\xi}\right)\right) \quad (19)$$

The differential area thus obtained can be substituted in the expression for torque dictated by Eqn (10). The porosity of the antenna may differ as the lateral side now offers resistance to wind. The moment arm in this case is *x* as the torque is generated about *mm'*

$$d\tau_{up} = 1/2\,\eta_{1'}\rho\left(v_o\left(\frac{Rcos\theta}{h_o}\right)^{\alpha}\right)^2 \times K\left(1 - \left(\frac{x^2}{R^2cos^2\xi}\right)\right) \times dx \times x \quad (20)$$

Eqn (20) can be integrated over the upper half antenna to obtain the total torque experienced by the upper half part of the dish. Since, this expression does not use polar coordinates the torque expression is a single integral as shown in Eqn (21).

$$\int_0^{\tau_{up}} d\tau_{up} = \int_{x=0}^{Rsin\xi} 1/2\,\eta_{1'}\rho\left(v_o\left(\frac{Rcos\theta}{h_o}\right)^{\alpha}\right)^2 \times K\left(1 - \left(\frac{x^2}{R^2cos^2\xi}\right)\right) \times dx \times x \quad (21)$$

The torque for the lower half of the antenna can be expressed with the same equation but with different limits.

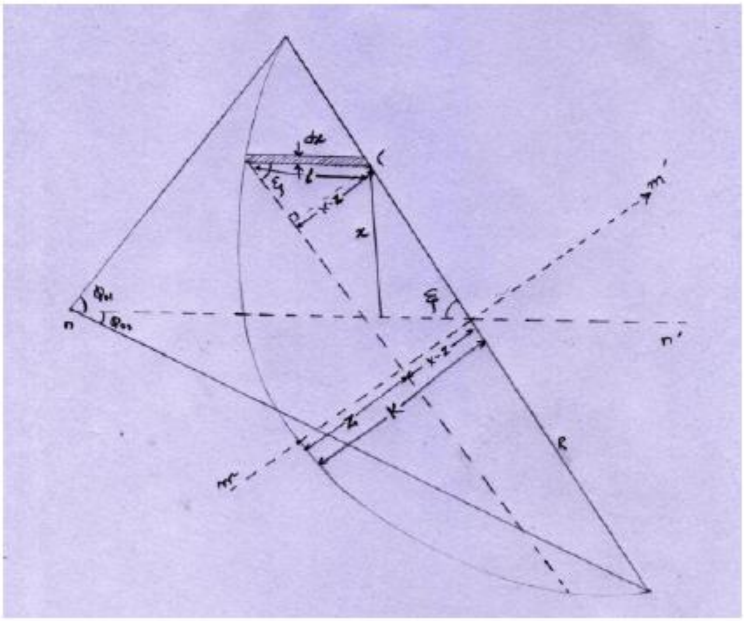

Fig 9: Figure illustrating the lateral cross-section of the parabola with its parameters

III. CONCLUSION AND FUTURE SCOPE

The paper develops a mathematical model for wind disturbance torque on tracking antenna. The vector approach to torque calculation in case of antenna requires only porosity of the antenna as the variable parameter to be as compared to the numerous antenna specific coefficients that need to be calculated while opting for field models of wind disturbance torques. The model here has considered antenna as a single entity, however, a finite element analysis of antenna structure still needs to be explored.

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Tushar Golani received his B.Tech from Indian Institute of Space Science and Technology, Thiruvananthapuram. He joined ISRO Satellite Centre in 2011 and is currently working in Spacecraft Checkout Group. His areas of interest include control systems, analog and digital circuit design and embedded systems. He is elected as a life member of the Astronautical Society of India (ASI) and Associate Member of the Aeronautical Society of India (AeSI).

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