

## Tracing and Transmission Cost Allocation of Active Power Flow Based On Extended Incidence Matrix

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**ABSTRACT:** In the present open access restructured power system market, it is necessary to develop an appropriate pricing scheme that can provide the useful economic information to the market participants such as generation, transmission companies and customers. Because of the introduction of competition in the electricity supply industry, it has become much more important to be able to determine which generators are supplying a particular load, how much use each generator is making of a transmission line and what is each generator's contribution to the system losses. However, accurately estimating and allocating the transmission cost in the transmission pricing scheme is a challenging task although many methods have been proposed. The objective is to introduce a simple transmission pricing scheme using a power flow tracing method, in which transmission service cost is considered. The proposed method is an analytical method for tracing of power flow based on the concept of extended incidence matrix (EIM). The proposed method can handle any power system with or without loop flows and is flexible to start with an AC or DC power flow solution. After obtaining the load flow solution, the extended incidence matrix, distribution factor matrix and load extraction matrix are developed. Using the extended incidence matrix, the contribution of generator to each load and line are calculated. Then the Transmission service charge is allocated among the generators and loads according to the 50/50 split savings method. The effectiveness of the proposed method is illustrated using a 4-bus, IEEE 14-bus system, IEEE 30-bus system.

**Keywords:** Extended incidence matrix, loop flows, Power flow tracing.

### I. INTRODUCTION

Tracing power flow (TPF) has been received more attention in recent years due to power industry restructuring. For power systems under the environment of deregulation and transmission open access, it is extremely important to calculate contributions of individual generators and loads to line flows, active power transfers between individual generators and loads, distributions of power losses and charges for the utilization of lines. Many methods have been presented to solve the TPF problem. The most popular approaches include topological generation distribution factors, nodal generation distribution factors and factors based on the generator domains. Presented an approach which is suitable for large-scale power systems. Based on the concept of extraction and contribution factor matrix, a method was suggested by using downstream and upstream tracing sequences. By using the concept of loop flow coefficient and series theory, proposed a TPF method based on graph theory which can be applied to networks with loop flows. Presented some models and algorithms for determining the contribution of individual generators and loads to power losses or energy losses. A main principle used to trace electricity flow in the published literatures so far is proportional sharing. The principle described in assumes that each outflow (a flow leaving a node) on a line is dependent only on the voltage gradient and impedance of the line, and that the contribution of each inflow (a line flow entering the node) to each outflow is in the same proportion as the inflow on each line divided by the total inflow of all lines at the node. The proportionality described in assumes that if the proportion of the inflow which can be traced to generator  $i$  is  $x_i$ , then the proportion of the outflow which can be traced to generator  $i$  is also  $x_i$ . The method proposed in is based on the assumption that the ratio of each bus load to the total system load is constant. All the principles can be easily utilized from an application point. This paper presents an analytical model for TPF based on the concept of extended incidence matrix (EIM). The proposed method does not need any assumption associated with the power sharing principle. It can handle any power system with or without loop flows and is flexible to start with an AC or DC power flow solution. The results of case studies indicate that the implicit distribution principle of the proposed method has the same effect as the explicit proportional principle in for cases without loop flow whereas it can also deal with cases with loop flow using the same formulation and procedure.

### II. TRACING OF POWER FLOW USING EXTENDED INCIDENCE MATRIX

The proposed method is an analytical method for tracing of power flow based on the concept of extended incidence matrix (EIM). The proposed method does not need any assumption associated with the power sharing principle. It can handle any power system with or without loop flows and is flexible to start with an AC or DC power flow solution. The elements of the EIM will represent the power flow relationship between the buses.

#### 2.1 Formation of EIM:

According to Kirchhoff's first law, the total inflow at a bus equals the total outflow from the same bus in any network. The inflow here is defined as the sum of powers injected by sources and powers imported to a bus from other buses. The outflow is defined as the sum of powers extracted from a bus by loads and powers exported to other buses. The elements of the EIM are formed based on the Eqn.(1) and it is denoted by matrix 'A'.

$$A_{ij} = \begin{cases} -P_{ij} & \text{for } i \neq j \text{ and } P_{ij} > 0 \\ 0 & \text{for } i \neq j \text{ and } P_{ji} > 0 \\ P_{Ti} & \text{for } i = j \end{cases} \dots (1)$$

where  $P_{Ti} = \sum_{k=1, k \neq i}^n P_{ki} + P_{Gi}$  and  $i, j = 1, 2, \dots, n$ ;  $n = \text{no. of buses}$  ....(2)  
 $P_{ki} > 0$

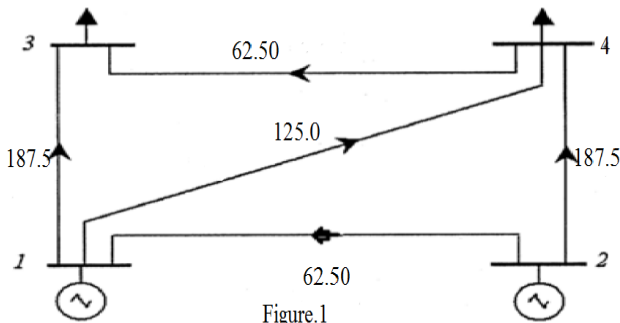


Figure.1

**2.2 Properties of Extended incidence matrix:**

**Property 1.** The sum of all elements in any row of an EIM equals the active load power at that bus and it is given in Eqn. 3

$$AE = P_L \dots (3)$$

For system in Fig 1 the sum of all elements in every column of the EIM is [250 250 0 0]. This is the row vector of generation outputs, i.e.,  $(P_G^T)$ .

**Property 2.** The sum of all elements in any column of an EIM equals the total active power of generators at that bus k. i.e.,

$$A^T E = P_G \dots (4)$$

**Property 3.** EIM is a diagonally dominant and full rank matrix. In other words, an EIM is an invertible matrix. The Inverse matrix B is

$$B = A^{-1} \dots (5)$$

From Eqns. (3) and (4)

$$E = A^{-1} P_L \dots (6)$$

and

$$E = (A^{-1})^T P_G \dots (7)$$

**2.3 Tracing of power flow:**

**2.3.1 Contribution of each generator to the each load:** The Generator capacity of the system can be represented in a matrix and the diagonal matrix is  $P_{GG} = \text{diag}(P_{G1}, P_{G2}, \dots, P_{Gn})$ . Then the individual generator capacity of the system is given in Eqn. (8)

$$P_G = P_{GG} E \dots (8)$$

By substituting the Eqn. (8) in the Eqn. (6), then

$$P_G = P_{GG} A^{-1} P_L \dots (9)$$

Eqn. (9) directly describes the relationship between  $P_G$  and  $P_L$ , which gives the contribution of each generator to each load. Rewriting the Eqn. (9)

$$P_G = K P_L \dots (10)$$

where,  $K = P_{GG} A^{-1}$

And K is known as Distribution-Factor-Matrix (DFM).

In general, the individual generator capacity is

$$P_{Gi} = \sum_{j=1}^n K_{ij} P_{Lj} \dots (11)$$

where  $K_{ij} P_{Lj}$  equals the active power contribution of generator output at bus i to the load at bus j. We denote:

$$P_{i \rightarrow j} = K_{ij} P_{Lj} \dots (12)$$

For the network shown in Fig 1,  $P_{GG} = \text{diag} [250 \ 250 \ 0 \ 0]$  and the DFM is obtained by

$$K = \begin{bmatrix} 0.8 & 0 & 0.68 & 0.32 \\ 0.2 & 1.0 & 0.32 & 0.68 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \dots(13)$$

It can be seen that the sum of all elements in every column of matrix K equals 1.

$$E^T K = E^T \quad \dots(14)$$

Eqs.(9),(10),(14) indicates the fact that the load at every bus equals the power extracted from all generators and the DFM gives the distribution relationship between loads and generators. Based on Eqs. (12),(13), the power transfers from generator 1 to load 3 and generator 2 to load 4 in LPN shown in Fig 1 are

$$P_{1 \rightarrow 3} = 0.6800 \times 250 = 170$$

$$P_{2 \rightarrow 4} = 0.3200 \times 250 = 80$$

### 2.3.2 Contribution of each generator to the each line flow:

From the distribution-factor-matrix, the each generator contribution to the each load through the different transmission lines in the network is obtained. So therefore, the each generator ( $i$ ) contribution to power flow in each line flow( $s-t$ ) is given by

$$P_{i \rightarrow s-t} = K_{is} * P_{s-t} \quad \dots (15)$$

where  $P_{s-t}$  = Power flow in each line( $s-t$ )

$P_{i \rightarrow s-t}$  = Power flow in line( $s-t$ ) due to the generator ( $i$ )

The power transfer from generator 1 and 2 to line 4-3 in network is

$$P_{1 \rightarrow 4-3} = K_{14} P_{43} = 0.3200 \times 187.50 = 60$$

$$P_{2 \rightarrow 4-3} = K_{24} P_{43} = 0.200 \times 187.50 = 37.5$$

### 2.3.3 Power extracted by loads from generators:

The above description is based on the idea that each generator makes contribution to each load. We can also view the power flow tracing problem from another angle each load extracts power from each generator. Under this view, the EIM with loads extracting power from generators is the transformation matrix of the EIM with generators transferring power to loads.

The dual equation for Eqn. (9) is obtained as:

$$P_L = P_{LL} (A^{-1})^T * P_G \quad \dots (16)$$

Eqn. (16) directly describes the relationship between  $P_L$  and  $P_G$ , which gives the contribution of each generator to each load. Rewriting the Eqn.

$$P_L = L P_G \quad \dots (17)$$

where,

$$L = P_{LL} (A^{-1})^T$$

And  $L$  = Load extraction matrix.

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.68 & 0.32 & 1.0 & 0.2 \\ 0.32 & 0.68 & 0 & 0.80 \end{bmatrix} \quad \dots\dots\dots(18)$$

For instance, the power extracted by load 3 from generator 1 is.  $0.68 \times 250 = 170$ , which is the same as the power contributed from generator 1 to load 3, as calculated in section 2.3

### 2.3.4 Power extracted by loads from each line flow:

From the extraction factor matrix, the each load extracts the power from the different transmission lines in the network. So therefore, power extraction by each load ( $i$ ) from each line ( $s-t$ ) is given in Eqn. 2.16.

$$P_{i \rightarrow s-t} = L_{si} * P_{s-t} \quad \dots (19)$$

where  $P_{s-t}$  = Power flow in each line( $s-t$ )

$P_{i \rightarrow s-t}$  = Power extracted by load ( $i$ ) in flow line ( $s-t$ ).

**3.5 Allocation of Transmission service charge to generators and demands:**

Generally, the transmission service cost is obtained as the sum of the costs of each transmission line. The cost of each line is directly obtained from its reactance value and the cost is given by  $C = 1000 * x$  where reactance  $x$  is in p.u. and the total transmission service cost is considered as 50/50 split between generators and demands. Consider the usage cost of line  $s-t$  is denoted as  $C_{s-t}$  (in Rs/h), then the charge allocated to a generator for this usage cost of line  $s-t$  is given by

$$C_{i \rightarrow s-t} = \frac{C_{s-t}}{2} * \frac{P_{i \rightarrow s-t}}{P_{ij}} \quad \dots (20)$$

The total charge allocated to generators at any bus for the usage of all lines is

$$C_{i \rightarrow s-t} = \sum_{ij \in \emptyset} C_{i \rightarrow s-t} \quad \dots (21)$$

Where  $C_k$  is the total allocated charge for the generators at bus  $k$ , and  $\emptyset$  is the branch set. Similarly, the charge allocated to a load for the use of all lines is calculated by Eqn. (20) and the total charge allocated to loads at any bus for the usage of all lines is calculated by Eqn. (21)

**III. ALGORITHM STEPS FOR THE TRACING OF ACTIVE POWER FLOW USING EIM:**

- Step 1: Read the bus and line data (If phase shifting transformer is present Read the Phase shift Angle)  
 Step 2: Form the Y-bus using the Singular Transformation method in AC power flow  
 (Or) Form the susceptance matrix in the case of DC power flow.  
 Step 3: Obtain the line flows and line losses from the NR method in case of AC power flow and DC power flow  
 Step 4: Form the EIM using the Eqn.(1)  
 Step 5: Obtain the B matrix using the Eqn. (5)  
 Step 6: Obtain the Distribution Factor matrix (K) using the Eqn. (10)  
 Step 7: Obtain the contribution of each generator to each load using the Eqn.(11)  
 Step 8: Obtain the contribution of each generator to each line flow using the Eqn.(15)  
 Step 9: Obtain the Load extraction matrix (L) using the Eqn. (17)  
 Step10: Obtain the contribution of each line flow to the load using the Eqn. (19)  
 Step 11: Allocate the transmission service charge to the each generator and the each load using Eqn.(20)  
 Step 12: Print the results  
 Step 13: STOP

**IV. SIMULATION RESULTS AND DISCUSSION****IEEE 30-BUS SYSTEM(AC power flow)****Table 1 Generator contribution to each load (AC power flow)**

Bus	Load(MW)	Gen1(MW)	Gen 2(MW)	Gen5 (MW)	Gen8 (MW)	Gen 11(MW)	Gen 13 (MW)
1	0.0	0	0	0	0	0	0
2	21.7	13.1369	8.5631	0	0	0	0
3	2.4	2.400	11.7050	0	0	0	0
4	7.6	6.4815	0	0	0	0	0
5	94.2	43.9766	8.6694	24.1808	0.1591	0	0
6	0.0	0.0000	0	0	0	0	0
7	22.8	16.5430	5.9824	0	0.2746	0	0
8	30.0	0	0	0	30	0	0
9	0.0	0	0	0	0	0	0
10	5.8	2.5212	0.9117	0	0	2.3252	0
11	0.0	0	0	0	0.0418	0	0
12	11.2	5.9166	1.0210	0	0	0	4.2623
13	0.0	0	0	0	0	0	0
14	6.2	3.2753	0.5652	0	0	0	2.3595
15	8.2	4.3318	0.7475	0	0	0	3.1206
16	3.5	1.8490	0.3191	0	0	0	1.3320
17	9.0	4.0254	1.3349	0	0.0562	3.1233	0.4603
18	3.2	1.6905	0.2917	0	0	0	1.2178
19	9.5	4.4573	1.2621	0	0.0433	2.404	1.3328
20	2.2	0.9563	0.3458	0	0.0159	0.8820	0
21	17.5	7.6070	2.7509	0	0.1263	7.0158	0
22	0.0	0	0	0	0	0	0
23	3.2	1.6905	0.2917	0	0	0	1.2178
24	8.7	4.1360	1.1549	0	0.1080	2.0025	1.2985
25	0.0	0	0	0	0	0	0
26	3.5	1.9389	0.7011	0	0.8600	0	0
27	0.0	0	0	0	0	0	0
28	0.0	0	0	0	0	0	0
29	2.4	1.3295	0.4808	0	0.5897	0	0
30	10.6	5.8720	2.1235	0	2.6046	0	0

From the Table 1 for the load 2, the contribution of the generator 1 is 13.1369MW and the contribution of the generator 2 is 8.5631MW the remaining generators 8, 11, 13 do not contribute to the load 2. From the proposed method the cost allocation to the generator is calculated using Eqn. 2.17 and the results are tabulated in Table 2

**Table 2 Allocation of transmission cost to the Generators (AC power flow)**

Line (s-t)	Gen-1 cost(Rs/hr)	Gen-2 cost(Rs/hr)	Gen-5 cost(Rs/hr)	Gen-8 cost(Rs/hr)	Gen-11 cost(Rs/hr)	Gen-13cost(Rs/hr)	Transmission cost(C=x*1000)
1-2	28.7500	0	0	0	0	0	57.5000
1-3	82.6000	0	0	0	0	0	165.2000
2-4	52.5780	34.2720	0	0	0	0	173.7000
3-4	18.9500	0	0	0	0	0	37.9000
2-5	60.0243	39.1257	0	0	0	0	198.3000
6-2	53.3650	34.7850	0	0	0	0	176.3000
6-4	17.6536	3.0464	0	0	0	0	41.4000
5-7	42.0832	15.2184	0	0.6985	0	0	116.0000
6-7	29.7484	10.7578	0	0.4938	0	0	82.0000
6-8	0	0	0	21.0000	0	0	42.0000
6-9	108.1102	39.0954	0	1.7944	0	0	298.0000
6-10	201.7090	72.9431	0	3.3479	0	0	556.0000
9-11	0	0	0	0	104.0000	0	208.0000
9-10	17.8729	6.4633	0	0.2966	30.3672	0	110.0000
4-12	109.1621	18.8379	0	0	0	0	256.0000
12-13	0	0	0	0	0	70.0000	140.0000
12-14	67.5924	11.6643	0	0	0	48.6933	255.9000
12-15	34.4433	5.9438	0	0	0	24.8128	130.4000
12-16	52.4838	9.0570	0	0	0	37.8091	198.7000
14-15	52.7480	9.1026	0	0	0	37.9994	199.7000
16-17	50.7934	8.7653	0	0	0	36.5913	192.3000
15-18	57.7137	9.9595	0	0	0	41.5767	218.5000
18-19	34.1264	5.8891	0	0	0	24.5845	129.2000
19-20	14.7794	5.3446	0	0.2453	13.6307	0	68.0000
10-20	45.4248	16.4268	0	0.7539	41.8945	0	209.0000
10-17	18.3655	6.6414	0	0.3048	16.9382	0	84.5000
10-21	16.2790	5.8869	0	0.2702	15.0138	0	74.9000
22-10	32.5798	11.7817	0	0.5407	30.0477	0	149.9000
22-21	5.1293	1.8549	0	0.0851	4.7307	0	23.6000
15-23	53.3555	9.2075	0	0	0	38.4371	202.0000
22-24	38.9045	14.0689	0	0.6457	35.8809	0	179.0000
23-24	71.3167	12.3070	0	0	0	51.3763	270.0000
25-24	91.1818	32.9737	0	40.4445	0	0	329.2000
25-26	105.2524	38.0620	0	46.6856	0	0	380.0000
25-27	57.8057	20.9040	0	25.6402	0	0	208.7000
28-27	109.6841	39.6646	0	48.6513	0	0	396.0000
27-29	115.0298	41.5977	0	51.0224	0	0	415.3000
27-30	166.9359	60.3683	0	74.0458	0	0	602.7000
29-30	125.5551	45.4039	0	55.6910	0	0	453.3000
28-8	0	0	0	100.0000	0	0	200.0000
6-28	21.7309	7.8584	0	0.3607	0	0	59.9000
Total cost	2261.8	705.3	0	473	292.5	411.9	8289 Rs/hr

From the Table 2 It is observed that total cost allocated for generators is 4144.5 Rs/hr (2261.8+705.3+473+292.5+411.9) which is half of the total cost of the transmission service cost which is 8289 Rs/hr.

### IEEE 30 bus system (DC POWER FLOW)

**Table 3 Generator contribution to each load (DC power flow)**

Bus	Load(MW)	Gen1(MW)	Gen2(MW)	Gen5(MW)	Gen8(MW)	Gen11(MW)	Gen13(MW)
1	0.0	0	0	0	0	0	0
2	21.7	12.8588	8.8412	0	0	0	0
3	2.4	2.4	0	0	0	0	0
4	7.6	6.429	1.1710	0	0	0	0
5	94.2	42.9855	26.4960	24.56	0.1584	0	0
6	0.0	0	0	0	0	0	0
7	22.8	16.3170	6.2243	0	0.2587	0	0
8	30.0	0	0	0	30.00	0	0
9	0.0	0	0	0	0	0	0
10	5.8	2.4693	0.9420	0	0.0391	2.3496	0
11	0.0	0	0	0	0	0	0
12	11.2	5.7578	1.0487	0	0	0	4.3935
13	0.0	0	0	0	0	0	0
14	6.2	3.1873	0.5805	0	0	0	2.4321
15	8.2	4.2155	0.7678	0	0	0	3.2167
16	3.5	1.7993	0.3277	0	0	0	1.3730
17	9.0	4.1212	1.2363	0	0.0386	2.3183	1.2856

18	3.2	1.6451	0.2996	0	0	0	1.2553
19	9.5	4.2727	1.3653	0	0.0467	2.8024	1.0129
20	2.2	0.9366	0.3573	0	0.0148	0.8912	0
21	17.5	7.4506	2.8421	0	0.1181	7.0892	0
22	0.0	0	0	0	0	0	0
23	3.2	1.6451	0.2996	0	0	0	1.2553
24	8.7	3.9575	1.3306	0	0.2469	2.4794	0.6856
25	0.0	0	0	0	0	0	0
26	3.5	1.9073	0.7276	0	0.8651	0	0
27	0.0	0	0	0	0	0	0
28	0.0	0	0	0	0	0	0
29	2.4	1.3079	0.4989	0	0.5932	0	0
30	10.6	5.7764	2.2035	0	2.6201	0	0

Table 4 Allocation of transmission cost to the Generators

Line (s-t)	Gen-1 cost(Rs/hr)	Gen-2 cost(Rs/hr)	Gen-5 cost(Rs/hr)	Gen-8 cost(Rs/hr)	Gen-11 cost(Rs/hr)	Gen-13 cost(Rs/hr)	Transmission cost(C=x*1000)
1-2	28.7500	0	0	0	0	0	57.5000
1-3	82.6000	0	0	0	0	0	165.2000
2-4	51.4650	35.3850	0	0	0	0	173.7000
3-4	18.9500	0	0	0	0	0	37.9000
2-5	58.7537	40.3963	0	0	0	0	198.3000
6-2	52.2353	35.9147	0	0	0	0	176.3000
6-4	17.5106	3.1894	0	0	0	0	41.4000
5-7	41.5082	15.8338	0	0.6581	0	0	116.0000
6-7	29.3420	11.1928	0	0.4652	0	0	82.0000
6-8	0	0	0	21.0000	0	0	42.0000
6-9	106.6331	40.6764	0	1.6906	0	0	298.0000
6-10	198.9530	75.8929	0	3.1542	0	0	556.0000
9-11	0	0	0	0	104.0000	0	208.0000
9-10	17.5950	6.7118	0	0.2790	30.4142	0	110.0000
4-12	108.2780	19.7220	0	0	0	0	256.0000
12-13	0	0	0	0	0	70.0000	140.0000
12-14	65.7772	11.9808	0	0	0	50.1920	255.9000
12-15	33.5184	6.1051	0	0	0	25.5765	130.4000
12-16	51.0744	9.3028	0	0	0	38.9728	198.7000
14-15	51.3314	9.3496	0	0	0	39.1690	199.7000
16-17	49.4293	9.0032	0	0	0	37.7175	192.3000
15-18	56.1638	10.2298	0	0	0	42.8564	218.5000
18-19	33.2099	6.0489	0	0	0	25.3412	129.2000
19-20	14.4754	5.5218	0	0.2295	13.7733	0	68.0000
10-20	44.4906	16.9715	0	0.7054	42.3326	0	209.0000
10-17	17.9878	6.8617	0	0.2852	17.1153	0	84.5000
10-21	15.9442	6.0821	0	0.2528	15.1709	0	74.9000
22-10	31.9098	12.1723	0	0.5059	30.3620	0	149.9000
22-21	5.0238	1.9164	0	0.0796	4.7801	0	23.6000
15-23	51.9226	9.4573	0	0	0	39.6201	202.0000
22-24	38.1044	14.5354	0	0.6041	36.2561	0	179.0000
23-24	69.4015	12.6409	0	0	0	52.9575	270.0000
25-24	89.6975	34.2161	0	40.6864	0	0	329.2000
25-26	103.5390	39.4961	0	46.9649	0	0	380.0000
25-27	56.8647	21.6917	0	25.7936	0	0	208.7000
28-27	107.8985	41.1591	0	48.9424	0	0	396.0000
27-29	113.1572	43.1651	0	51.3277	0	0	415.3000
27-30	164.2183	62.6429	0	74.4888	0	0	602.7000
29-30	123.5111	47.1147	0	56.0242	0	0	453.3000
28-8	0	0	0	100.0000	0	0	200.0000
6-28	21.4340	8.1762	0	0.3398	0	0	59.9000
Total cost	2222.7	730.7	0	474.5	294.2	422.4	8289 Rs/hr

From the Tables 1,2,3,4, it is observed that the power contributed by the generator to the loads are different in both AC and DC power flows, but the total cost allocated for generators is 4144.5 Rs/hr which is half of the total cost of the transmission service cost which is 8289 Rs/hr is same in case of both AC and DC power flows.

## V. Conclusions

In the analysis of the IEEE 30 bus system, it was concluded that the each generator contribution to the load is different in AC and DC power flows, but the total cost allocated to generators are same in both AC and DC power flows. From the total analysis, it is concluded that the proposed method is used for any system like AC, DC, without loop and with loop power flow systems.

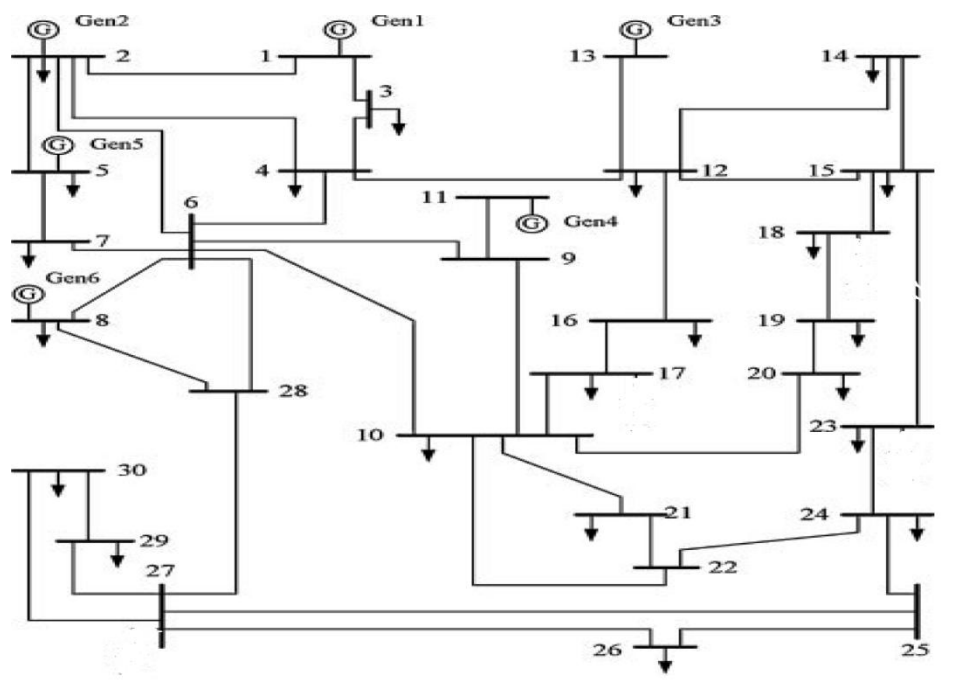


**FUTURE SCOPE OF THE WORK**

- In this project, the proposed method is developed without considering the type of the conductor. It can be further developed by considering the type of the conductor.
- The proposed method is applied for the single area interconnected system. It can be further developed for the multiple area interconnected systems.
- The proposed method is developed with considering the real power flow. It can be further developed by considering the reactive power flow.

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**Fig. 1 Single line Diagram of IEEE 30 bus system****Table 5 IEEE 30 bus system Line data**

Sending bus	Receiving bus	R(p.u)	X(p.u)	B(p.u)	Taps	Cost(Rs/hr)
1	2	0.0192	0.0575	0.0528	1.0000	57.5000
1	3	0.0452	0.1652	0.0408	1.0000	165.2000
2	4	0.057	0.1737	0.0368	1.0000	173.7000
3	4	0.0132	0.0379	0.0084	1.0000	37.9000
2	5	0.0472	0.1983	0.0418	1.0000	198.3000

6	2	0.0581	0.1763	0.0374	1.0000	176.3000
6	4	0.0119	0.0414	0.0090	1.0000	41.4000
5	7	0.0460	0.1160	0.0204	1.0000	116.0000
6	7	0.0267	0.082	0.0170	1.0000	82.0000
6	8	0.0120	0.0420	0.0090	1.0155	42.0000
6	9	0	0.2080	0	0.9629	298.0000
6	10	0	0.5560	0	1.0000	556.0000
9	11	0	0.2080	0	1.0000	208.0000
9	10	0	0.1100	0	1.0130	110.0000
4	12	0	0.2560	0	1.0000	256.0000
12	13	0	0.1400	0	1.0000	140.0000
12	14	0.1231	0.2559	0	1.0000	255.9000
12	15	0.0662	0.1304	0	1.0000	130.4000
12	16	0.9450	0.1987	0	1.0000	198.7000
14	15	0.2210	0.1997	0	1.0000	199.7000
16	17	0.0824	0.1923	0	1.0000	192.3000
15	18	0.1073	0.2185	0	1.0000	218.5000
18	19	0.0639	0.1292	0	1.0000	129.2000
19	20	0.034	0.0680	0	1.0000	68.0000
10	20	0.0936	0.2090	0	1.0000	209.0000
10	17	0.0324	0.0845	0	1.0000	84.5000
10	21	0.0348	0.0749	0	1.0000	74.9000
22	10	0.7270	0.1499	0	1.0000	149.9000
22	21	0.0116	0.0236	0	1.0000	23.6000
15	23	0.1000	0.2020	0	1.0000	202.0000
22	24	0.1150	0.1790	0	1.0000	179.0000
23	24	0.1320	0.2700	0	1.0000	270.0000
25	24	0.1885	0.3292	0	1.0000	329.2000
25	26	0.2544	0.3800	0	1.0000	380.0000
25	27	0.1093	0.2087	0	1.0000	208.7000
28	27	0	0.396	0	1.0000	396.0000
27	29	0.2198	0.4153	0	1.0000	415.3000
27	30	0.32020	0.6027	0	1.0000	602.7000
29	30	0.2399	0.4533	0	1.0000	453.3000
28	8	0.0636	0.2000	0.0428	1.0000	200.0000
6	28	0.0169	0.0590	0.1300	1.0000	59.9000