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A New Method Based On the Comparison of the Unique Chain Code to Detect Isomorphism among Kinematic Chains

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ABSTRACT: *This paper presents a new method for the detection of isomorphism in kinematic chains which plays an important role in structural synthesis of kinematic chains. The proposed method uses a new invariant i.e. a unique chain codec based on the degree of links and used as an identifier of a kinematic chain (KC). The proposed method is easy to compute, reliable and capable of detecting isomorphism in all types of KC, i.e. chains of single or multi degree of freedom with simple joints. This study will help the designer to select the best KC and mechanisms to perform the specified task at conceptual stage of design*

Keywords: *Degree of freedom, Isomorphism, Invariant, Mechanisms, Kinematic chain (KC)*

I. INTRODUCTION

One of the most difficult tasks in structural synthesis of kinematic chain is to check the isomorphism among two chains. The two kinematic chains KC1 and KC2 are said to be isomorphic, if there exists a one to one correspondence between the links of KC1 and KC2.

The detection of isomorphism among two kinematic chains with same link assortment is necessary to prevent duplication and omission of a potential useful chain. A lot of time and effort had been devoted in developing a reliable and computationally efficient technique; therefore a wealth of literature pertaining with this topic is available. However, there is a scope for an efficient, simple and reliable method and this paper is an attempt in this direction. Heuristic and visual methods [1] were only suitable for kinematic chains with a small number of links. Characteristic polynomial method [2] has the disadvantage of dealing with cumbersome calculations and later counter examples were also reported [3]. Quist and Soni presented a method of loops of a chain [4]. A unique index for isomorphism i.e. characteristic polynomial of structural matrix was proposed by Yan and Hwang [5], however this method is computationally uneconomical. Mruthyunjaya proposed the computerized method of structural synthesis which works on binary coding of chains [6]. Agarwal and Rao proposed Variable permanent function to identify multi loop kinematic chains [7]. Ambekar and Agarwal presents a method of Canonical coding of kinematic chains [8] but it becomes computationally uneconomical when applied to large kinematic chain. Hamming number technique [9] is very reliable and computationally efficient, however when the primary Hamming string fails, the time consuming computation of the secondary Hamming string is needed. Shin and Krishna Murthy presents some rules for relabeling its vertices canonically for a given kinematic chain [10]. However it tends to become computationally inefficient where a higher number of symmetry group elements in the kinematic chain are present. The degree code [11] of the contracted link adjacency matrix of a chain was also proposed to test the isomorphism. Yadav and Pratap present a method of link distance for the detection of isomorphism [12]. A method based on artificial neural network theory by Kong et al. was presented [13]. A method based on loop formations of a kinematic chain was proposed by Rao and Prasad [14]. A new method based on eigenvalues and eigenvectors of adjacent matrices of chains was also proposed [15]. The reliability of the existing spectral techniques for isomorphism detection was challenged by Sunkari, R.P., and Schmidt [16]. Huafeng Ding and Zhen Huang [17] shows that the characteristic polynomial and eigen value approach fails and proposed a method based on the perimeter topological graph and some rules for relabeling its vertices canonically and one-to-one descriptive method for the canonical adjacency matrix set of kinematic chains Hasan and Khan [18] presented a method based on degrees of freedom of kinematic pairs. All the above methods developed so far only uses the graphs of the KC or their adjacency matrices in one or the other way. The method presented in this paper uses ‘connection string’ to detect isomorphism in KCs.

II. PRELIMINARIES

The following definitions are to be understood clearly before applying this method.

(i) **Degree of link:** Degree of a particular link depends upon its type the degree of a binary link is 2, the degree of a ternary link is 3, and the degree of a quaternary link is 4 and so on.

(ii) **Adjacency Matrix:** A matrix which reveals the connectivity pattern of the kinematic chain is known as adjacency matrix and can be formed in the following manner

$A=[a_{ij}]$ (1)

aii = 0, since a link-i cannot connect to itself.

aij = 1, if link-i is directly connected to link-j.

aij = 0, if the vertices i and j are not directly connected.

(iii) **Instantaneous Centre:** The combined motion of rotation and translation of the link may be assumed to be a motion of pure rotation about some centre, known instantaneous centre of rotation. The number of instantaneous centres of a mechanism is given by the relation given below

$$\text{Number of I Centers} = \frac{N(N-1)}{2} \quad (2)$$

Where N is the number of links in the chain

(iv) **Unique Chain Code:** The unique chain code is a number rounded off to the nearest whole number which can be calculated by the formula given in the equation below

$$\text{Unique Chain Code} = N\sum_{i=1}^{i=N} \frac{\text{Number of I centers of the chain}}{S_i} \quad (3)$$

S_i is the sum of the degrees of the links connected to the i^{th} link

III. ISOMORPHISM DEFINITION

Two kinematic chains are said to be isomorphic if there is a one-to-one correspondence between the links of one chain to those of the other chain. If there is no such correspondence between the links of the two chains they are considered to be non-isomorphic.

IV. IDENTIFICATION OF ISOMORPHISM

If the unique codes and the sum of the characteristics polynomial of the adjacency matrices of the two kinematic chains are same then it is said that the two chains has one to one correspondence then the two chains are isomorphic or otherwise. Although no mathematical proof of this method is being offered but no counter example has been found by the author among the known cases of planar kinematic chains. If in any case the unique codes are same but the sum of the characteristics polynomial of the adjacency matrices are not same then both the chains will be coded with the same numerical code followed by the '-1' and '-2'. For example if the code for two chains is 345 then they will be coded as 345-1 and 345-2

V. APPLICATION TO KINEMATIC CHAINS

The proposed method for isomorphism detection in this paper is tested on several kinematic chains available in the literature and found that it works well in all cases. Three examples of ten links kinematic chains are shown below to reveal the reliability of the method

Example 1:- The two Kinematic chains with 10 links, single-degree of freedom shown in Fig 1(a) and (b). The aim is to examine whether these two KC are isomorphic as reported in the literature [13].

The value of unique chain code for both the chains shown in Fig 1(a) and (b) comes out to be 638 as determined by the relation (3), and the sum of the absolute characteristic polynomials of the adjacency matrices A1 and B1 is 146 which reveals that the two chains are isomorphic which is already proved by other researchers and now our method also gives the same result.

Example 2:- The two Kinematic chains with 10 links, single degree of freedom having same characteristic polynomials for their adjacency matrices are shown in Fig 2(a) and 2(b) The aim is to examine whether these two KC are non-isomorphic as reported in the literature [3].

The value of unique chain code for both the chain shown in Fig 2(a) is 775 and Fig 2(b) is 770 as determined by the relation (3), the sum of the absolute characteristic polynomials of the adjacency matrices A2 and B2 is 238 which reveals that the two chains are nonisomorphic because one of two the necessary conditions for isomorphism proposed in this paper is not satisfied hence our method also gives the same result as already proved by other researchers.

$$A1=\begin{pmatrix}0&1&0&0&0&0&1&1&1&0\\1&0&1&0&0&0&0&0&0&0\\0&1&0&1&0&0&0&0&1&0\\0&0&1&0&1&0&0&0&0&1\\0&0&0&1&0&1&0&0&0&0\\0&0&0&0&1&0&1&1&0&0\\1&0&0&0&0&1&0&0&0&0\\1&0&0&0&0&1&0&0&0&0\\1&0&1&0&0&0&0&0&0&1\\0&0&0&1&0&0&0&0&1&0\end{pmatrix}\quad B1=\begin{pmatrix}0&1&0&0&0&0&1&1&1&0\\1&0&1&0&0&0&0&0&0&1\\0&1&0&1&0&0&0&0&0&0\\0&0&1&0&1&0&0&0&0&1\\0&0&0&1&0&1&0&0&0&0\\0&0&0&0&1&0&1&1&0&0\\1&0&0&0&0&1&0&0&0&0\\1&0&0&0&0&1&0&0&0&0\\1&0&0&0&0&0&0&0&0&1\\0&1&0&1&0&0&0&0&1&0\end{pmatrix}$$

$$A2=\begin{pmatrix}0&1&0&1&1&0&0&1&0&0\\1&0&1&0&0&0&0&0&1&0\\0&1&0&1&0&0&0&0&0&1\\1&0&1&0&0&0&0&0&0&0\\1&0&0&0&0&1&0&0&0&1\\0&0&0&0&1&0&1&0&1&0\\0&0&0&0&0&1&0&1&0&0\\1&0&0&0&0&0&1&0&0&0\\0&1&0&0&0&1&0&0&0&0\\0&0&1&0&1&0&0&0&0&0\end{pmatrix}\quad B2=\begin{pmatrix}0&1&0&1&1&0&0&0&0&0\\1&0&1&0&0&0&0&1&0&1\\0&1&0&0&0&1&1&0&0&0\\1&0&0&0&0&0&1&0&0&0\\1&0&0&0&0&1&0&0&0&0\\0&0&1&0&1&0&0&0&0&0\\0&0&1&1&0&0&0&0&1&0\\0&1&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&1&1&0&1\\0&1&0&0&0&0&0&0&1&0\end{pmatrix}$$

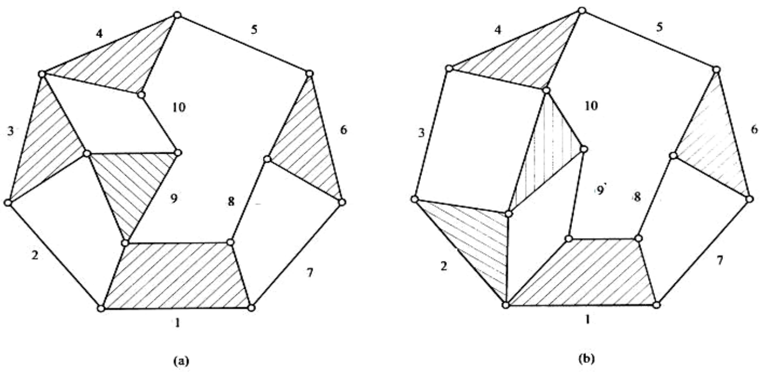


Fig. 1(a) and 1(b): The two ten link chains with single degree of freedom

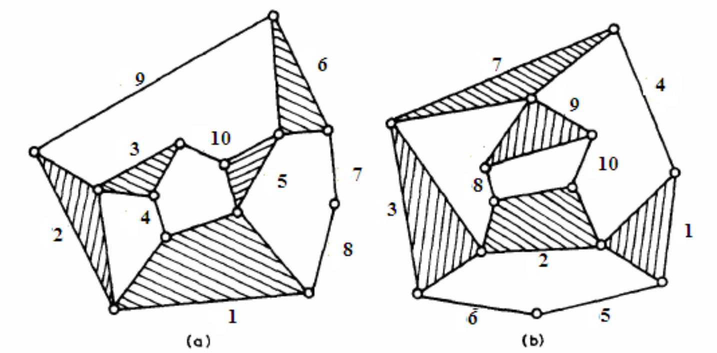


Fig. 2(a) and 2(b): The two ten link chains with single degree of freedom having same characteristic polynomial for their adjacency matrix.

Example 3:- The two Kinematic chains with two 10 link three degree of freedom, having same characteristic polynomials for their adjacency matrices are shown in Figs 3(a) and 3(b). The aim is to examine whether these two KC are non-isomorphic as reported in the literature [3].

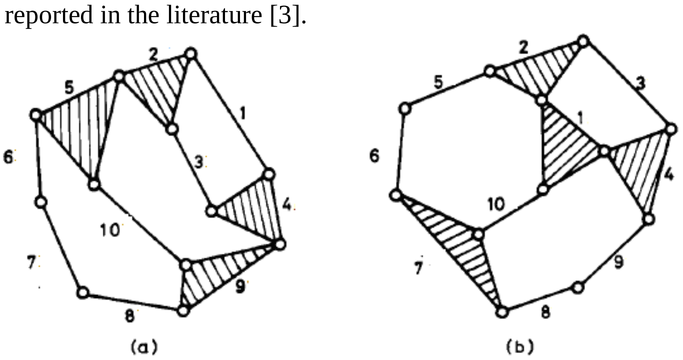


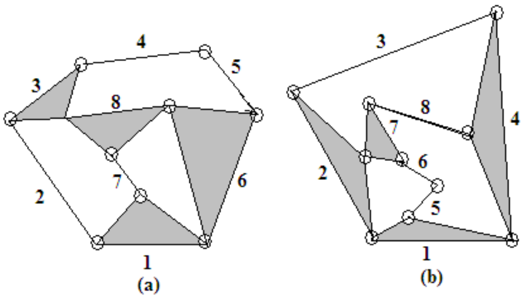
Fig. 3(a) and (b): The two ten link chains with three degree of freedom having same characteristic polynomial for their adjacency matrix.

$$A3 = \begin{pmatrix} 0&1&0&1&1&0&0&1&0&0\\ 1&0&1&0&0&0&0&0&1&0\\ 0&1&0&1&0&0&0&0&0&1\\ 1&0&1&0&0&0&0&0&0&0\\ 1&0&0&0&0&1&0&0&0&1\\ 0&0&0&0&1&0&1&0&1&0\\ 0&0&0&0&0&1&0&1&0&0\\ 1&0&0&0&0&0&1&0&0&0\\ 0&1&0&0&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0 \end{pmatrix} \quad B3 = \begin{pmatrix} 0&1&0&1&1&0&0&0&0&0\\ 1&0&1&0&0&0&0&1&0&1\\ 0&1&0&0&0&1&1&0&0&0\\ 1&0&0&0&0&0&1&0&0&0\\ 1&0&0&0&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1&1&0&1\\ 0&1&0&0&0&0&0&0&1&0 \end{pmatrix}$$

The value of unique chain code for the chain shown in Fig 3(a) is 653 and Fig 3(b) is 655 as determined by the relation (3), and the sum of the absolute characteristic polynomials of the adjacency matrices A3 and B3 is 238 reveals that the two chains are nonisomorphic because one of two the necessary conditions for isomorphism proposed in this paper is not satisfied hence our method also gives the same result as already proved by other researchers.

Example 4:- The two Kinematic chains having 8 links single degree of freedom, were proven isomorphic by cubllio[19],are shown in Figs 4(a) and 4(b). The aim is to examine whether these two KC are non-isomorphic as reported in the literature.

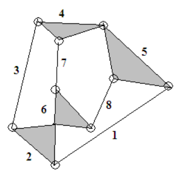
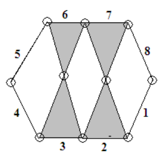
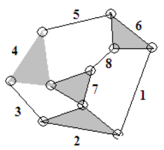
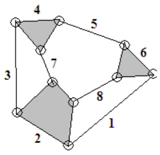
The value of unique chain code for the chain shown in Fig 3(a) & 3(b) is 284 as determined by the relation (3) and the sum of the absolute characteristic polynomials of the adjacency matrices A4 and B4 is 74 hence the necessary conditions for isomorphism proposed in this paper is are satisfied so the two chains are isomorphic which shows the reliability our method as the proposed gives the same result that is available in the literature.

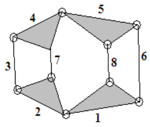
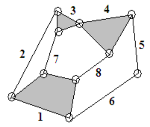
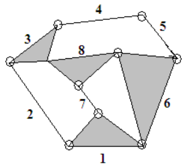
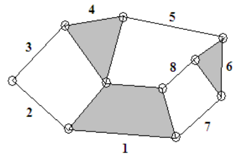
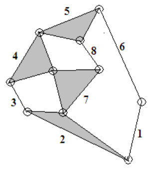
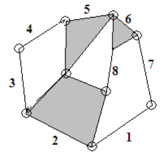
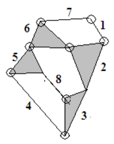
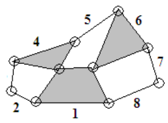
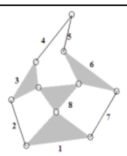
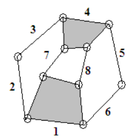
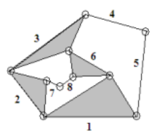
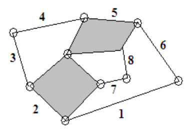


$$A4 = \begin{pmatrix} 0&0&1&1&0&0&1&0\\ 0&0&1&1&0&0&0&0\\ 1&1&0&0&1&0&0&0\\ 1&1&0&0&0&1&0&0\\ 0&0&1&0&0&1&0&1\\ 0&0&0&1&1&0&0&0\\ 1&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&1&0 \end{pmatrix} \quad B4 = \begin{pmatrix} 0&1&0&0&1&0&0&0\\ 1&0&1&0&0&1&0&0\\ 0&1&0&1&0&0&0&0\\ 0&0&1&0&1&0&0&1\\ 1&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&1\\ 0&0&0&1&0&0&1&0 \end{pmatrix}$$

Fig-4(a) & (b): eight link isomorphic chains

Table-I: 16 Different Possible Combinations for 8 Link Single Degree of Freedom Chain.

S.No	KINEMATIC GRAPH OF THE CHAIN	SUM OF ABS CHAR POLY	CHAIN CODE	S.No	KINEMATIC GRAPH OF THE CHAIN	SUM OF ABS CHAR POLY	CHAIN CODE
1		12	379	9		45	291-2
2		72	279	10		51	268

3		52	277	11		70	270
4		74	284	12		38	283
5		12	1045	13		59	279
6		43	293	14		45	280
7		56	285	15		68	261
8		87	291-1	16		52	269

VI. RESULTS

The proposed method can detect isomorphism among all kinematic chains of single or multi degree of freedom having simple joints, up to 10-links. All 16 eight-bar single-DOF kinematic chains which are shown in 'Table-I' with their unique codes, 40 nine-bar two-DOF kinematic chains, 98 ten-bar three DOF kinematic chains and 230 ten-bar single-DOF kinematic chains have been tested with this method and no counterexamples have been found.

VII. CONCLUSIONS

The reliability of this method is based on the fact it has been found to be successful in not only distinguishing all known 16 kinematic chains of 8-links, 230 kinematic chains of 10-links having 1-degree of freedom and 40 kinematic chains of 9-links 2-degree of freedom but also on all the counter examples reported earlier literature. The method is so simple that one can detect isomorphism without computer program, whereas most of the methods developed so far requires sophisticated algorithms.

REFERENCES

[1] T.H. Davies and F.E. Crossley; "Structural analysis of plane linkages by Franke's condensed notation", J. Mech., Vol. 1(2), pp. 171-183, 1966.

[2] J.N. Yadav and C.R. Pratap; "Computer aided detection of isomorphism among kinematic chains and mechanisms using the link-link multiplicity distance concept", Mech. Mach. Theory, Vol. 31(4), pp. 873-877, 1996. DOI:0094-114X(95)0078X

[3] T.S. Mruthyunjaya and H.R. Balasubramanian; "In quest of reliable and efficient computational test for detection of isomorphism in kinematic chains", Mech. Mach. Theory, Vol. 22(2), pp. 131-139, 1987. DOI:0094-114X/87

[4] F.F. Quist and A.H. Soni; "Structural synthesis and analysis of kinematic chains using path matrices", In. Proceedings of the 3rd World Congress for Theory of Machines and Mechanisms, pp. D161–D176, 1971.

[5] H.S. Yan and W.M. Hwang; "A method for identification of planar linkage Chains", J. Mech. Tran. Auto. Des. ASME Trans., Vol. 105(4), pp. 658-662, 1983.

[6] T.S. Mruthyunjaya; “A computerized methodology for structural synthesis of kinematic chains: Part 1- formulation”, Mech. Mach. Theory, Vol. 19(6), pp. 487–495, 1984.

[7] V.P. Agarwal and J.S. Rao; “Identification of multi-loop kinematic chains and their paths”, J. Int. Eng. (I) ME, Vol. 66, pp. 6-11, 1985.

[8] A.G. Ambekar and V.P. Agarwal; “Canonical numbering of kinematic chains, mechanisms, path generators and function generators using min codes”, Mech. Mach. Theory, Vol. 22(5), pp. 453-461, 1987.

[9] A.C. Rao and D. Raju Varda; “Application of the hamming number technique to detect isomorphism among kinematic chains and inversions”, Mech. Mach. Theory, Vol. 26(1), pp. 55-75, 1991.

[10] J.K. Shin and S. Krishna Murthy; “On identification and conical numberings of pin jointed kinematic chains”, J. Mech. Des ASME, Vol. 116(1), pp. 182-188, 1994. http://dx.doi.org/10.1115/1.2919344

[11] W.M. Hwang and Y.W. Hwang; “Computer aided structural synthesis of planar kinematic chains with simple joints”, Mech. Mach. Theory, Vol. 27(2), pp. 189–199, 1992.

[12] J.N. Yadav, C.R. Pratap and V.P. Agrawal; “Computer aided detection of isomorphism among kinematic chains and mechanisms using the link-link multiplicity distance concept”, Mech. Mach. Theory, Vol. 31(7), pp. 873–877, 1996.

[13] F.G. Kong, Q. Li and W.J. Zhang; “An artificial neural network approach to mechanism kinematic chain isomorphism identification”, Mech. Mach. Theory, Vol. 34(2), pp. 271–283, 1999. DOI:S0094-114X(01)00084-2

[14] A.C. Rao and V.V.N. Raju Prasad; “Loop based detection of isomorphism among chains, inversions and type of freedom in multi degree of freedom chain”, J. Mech. Des., Vol. 122(1), pp. 31-41, 2000. DOI:S1050-0472~00!70801-4

[15] Z. Chang, C. Zhang, Y. Yang and Y. Wang; “A new method to mechanism kinematic chain isomorphism Identification”, Mech. Mach. Theory, Vol. 37(4), pp. 411–417, 2002. DOI:S0094-114X(01)00084-2

[16] R.P. Sunkari and L.C. Schmidt; “Reliability and efficiency of the existing spectral methods for isomorphism detection”, J. Mech. Des., Vol. 128(6), pp. 1246–1252, 2006. DOI: 10.1115/1.2336253

[17] H. Ding and Z. Huang; “The establishment of the canonical perimeter topological graph of kinematic chains and isomorphism identification”, J. Mech. Des., Vol. 129(9), pp. 915–923, 2007. DOI: 10.1115/1.2748451

[18] A. Hasan and R.A. Khan; “Isomorphism and inversions of kinematic chains up to ten links using degrees of freedom of kinematic pairs”, Int. J. Comp. Methods, Vol. 5(2), pp. 329–339, 2008. DOI: 10.1142/S0219876208001492

[19] J.P. Cubillo and Jinbao Wan; “Comments on mechanism kinematic chain isomorphism identification using adjacent matrices” Mech. Mach. Theory, Vol. 40(2), pp. 131–139, 2005.