

Performance of MMSE Denoise Signal Using LS-MMSE Technique

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Abstract: This paper presents performance of mmse denoises signal using consistent cycle spinning (ccs) and least square (LS) techniques. In the past decade, TV denoise technique is used to reduced the noisy signal. The main drawback is the low quality signal and high MMSE signal. Presently, we proposed the CCS-MMSE and LS-MMSE technique. The CCS-MMSE technique consists of two steps. They are wavelet based denoise and consistent cycle spinning. The wavelet denoise is powerful decorrelating effect on many signal domains. The consistent cycle spinning is used to estimation the MMSE in the signal domain. The LS-MMSE is better estimation of MMSE signal domain compare to CCS-MMSE. The experimental result shows the average MMSE reduction using various techniques.

Key words: CCS-MMSE, LS-MMSE, MMSE estimation, Total Variation Denoising, Wavelet Denoising.

I. Introduction

Wavelet regularization [1] has been shown to be particularly effective in reducing noise, while preserving important signal features. The performance of the method can be further improved at little computational cost, by using the technique known as cycle spinning [2]-[4]. Cycle spinning compensates for the lack of shift-invariance of the wavelet basis by considering different shifts of the signal. Total variation (TV) [5] regularization is another widely used Denoising method, which penalizes random oscillations in the signal, while allowing the discontinuities. Interestingly, for 1-D signals, Haar-wavelet shrinkage with cycle spinning has been shown to closely related to TV regularization [6]. In this paper, we exploit the link between the two estimation methods to derive a new wavelet- based method for efficiently solving TV type denoising problems in 1or2-D. The key observation is that TV-regularized least squares minimization can be reformulated as a constrained optimization problem in the wavelet domain. We use the augmented-Lagrangian method [7] to cast the problem as a sequence of unconstrained problems that can be solved by simple soft- thresholding. By replacing the soft-thresholding function by another scalar function or by a precompiled lookup table, we can efficiently extend our algorithm beyond traditional l_1 regularizes to general, possibly non-convex, potential functions. The rest of this paper is organized as follows. In section 2, we describe the CCS-MMSE. Section3 explain the LS-MMSE. Section4 GIVES some experimental results. Finally, a conclusion will be presented in section V.

II. CCS-MMSE

The CCS-MMSE technique follows two steps. They are wavelet denoise (Haar wavelet) and consistent cycle spinning explained in section-2a and 2-b. The algorithm of CCS-MMSE is given by

1. Initialize parameters
2. For snr = 1:20
3. For iteration 1:2000
4. X=Generate random signal
5. X₁=convert serial to parallel
6. B=2*X-1%convert binary signal
7. Pilot=[bit(1:m) pilot symbol bit (m+1:end)]%insert pilot signal
8. Wavelet = dwt2(pilot, 'haar wavelet')%add wave let
9. G=insert interval
10. W=HG+n % output signal
11. $\phi_{MMSE}(w) - \frac{1}{2}(\eta_{MMSE}^{-1}(w) - w)^2 - \log_{p_u}(\eta_{MMSE}^{-1}(w))\%(CCS-MMSE)$
12. Error=error+(|X- ϕ_{mmse})
13. End iteration
14. End snr

2.1 Haar Wavelet Denoise

The wavelet denoise is power is powerful decorrelating effect on many signal domains. The problem of estimation of wavelet denoise is given by

$$\hat{W} = f(W) (u) = \text{argmin} \left\{ \frac{1}{2} \|w - u\|_2^2 + \phi(w) \right\} \text{----- (1)}$$

ϕ_w is non-smooth convex function using wavelet soft threshold and is given by

$$\phi_w(w) = \lambda \sqrt{2} |w^d| \text{----- (2)}$$

μ is the vector of lag range multipliers.

The soft thresholding function η is applied to component-wise on the detail wavelet coefficients. The detail coefficients derived from haar-wavelet transform.

Improving the signal quality using haar wavelet denoise is used. It is removing noisy signal domain. These are the advantages of wavelet denoise technique. A major drawback of haar wavelet denoise is to estimate the MMSE estimation is not in general equivalent to MMSE estimation in the signal domain. It is happen only on wavelet transform is orthogonal.

2.2 Consistent Cycle Spinning

The CCS method is used to estimate the MMSE in the different signal domain. In cycle spinning technique is used to expanding the signal in a wavelet frame. It has fast convergence. It is fast and minimizing the signal noise, the cost function decreases monotonically until the algorithm reaches a fixed point. Cycle spinning implies a redundant representation. Thus, not every set of wavelet-domain coefficients can be perfectly inverted back to the signal domain. When the estimated coefficients violate the invertibility condition a problem will arise. It can be resolved through consistency. The algorithm of cycle spinning as shown below

Input: $y, s \in \mathbb{R}^N, \tau, \epsilon \in \mathbb{R}^0$

Set: $k=0, \lambda_0=0, u=Ay$;

Repeat

$$z^{k+1} = \text{prox}_{\phi} \left(\frac{1}{1+\mu} (u + \mu A s^k + \lambda^k); \frac{\tau}{1+\mu} \right)$$

$$s^{k+1} = A^\dagger \left(z^{k+1} - \frac{1}{\mu} \lambda^k \right)$$

$$\lambda^{k+1} = \lambda^k - \mu (z^{k+1} - A s^{k+1})$$

k=k+1

Until stopping criterion

Return $S=S^k$

The characterize of iterative MMSE shrinkage in CCS-MMSE IS given by

$$\phi_{MMSE}(w) = -\frac{1}{2} (\eta_{MMSE}^{-1}(w) - w)^2 - \log_{p_u} (\eta_{MMSE}^{-1}(w)) \text{----- (3)}$$

It is minimizing the cost function and better performance technique compare to TV denoise technique.

The objective function with the new penalty function is given by

$$\mathcal{L}(W, X) = \mathcal{J}(u, w) + \frac{\tau}{2} \|w - W_X\|_2^2 - \mu^T (w - W_X) \text{----- (4)}$$

Where $\tau > 0$ is the penalty parameter and μ is the vector of lag range multipliers.

2.3 Discrete Fourier Transform

The DFT is used to compute the Fourier transform of discrete data. The Wavelet performance is better than DFT because wavelet can localized in both frequency and time. The DFT maps a discrete signal into the frequency domain. In the DFT we use sine and cosine waves, it commonly called as DFT basis functions.

The DFT basis generating functions are

$$C_k[i] = \cos(2 \pi k i / N),$$

$$S_k[i] = \sin(2 \pi k i / N)$$

Where

$c_k[i]$ is the cosine wave for the amplitude held in real part

$s_k[i]$ is the cosine wave for the amplitude held in imaginary part.

The DFT can be calculated in three different waves. Such as

- 1) Set of simultaneous equations
- 2) Correlation
- 3) FFT [Fast Fourier Transform]

In this paper we are using the Fast Fourier Transform. Fast Fourier Transform is ingenious algorithm it decomposes a DFT with N points. The Fast Fourier Transform is 100 times faster than the Set of simultaneous equations, Correlation. Here we notice that, DFT has less than 32 points than we used correlation method otherwise we go for the Fast Fourier Transform.

III. LS-MMSE

The method of least squares-mmse is estimating the mmse by minimizing the squared discrepancies between observed data and their expected signal values. We will study the method in the context of regression problem, where the variation in one variable, called the response variable Y, multiple linear regression co-variables x. The prediction of Y is given by

$$Y=f(X) + \text{noise} \dots \dots (5)$$

Where

f= egression function. It is used to estimate the co-variables and their responses.

The block diagram of LS-MMSE is given by

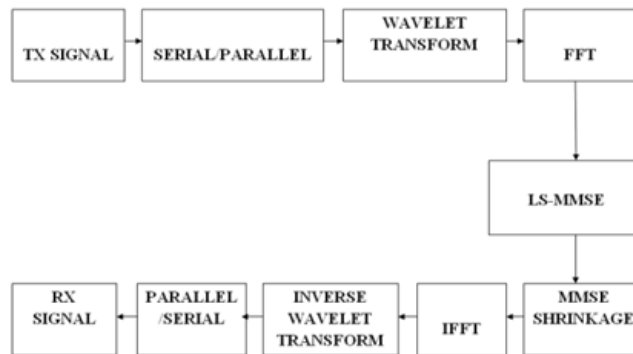


Fig.3.1 block diagram of LS-MMSE technique

The least squares criterion is a computationally better compare to CCS-MMSE. It corresponds to maximum likelihood estimation when the noise is normally distributed with equal variances. Other measures of fit are sometimes used, for example, least absolute derivations, which is more robust against outliers.

The least squares estimator, denoted by β , is that value of b that minimizes

$$\sum_{i=1}^n (y_i - f_b(x_i))^2 \dots \dots \dots (6)$$

The covariance matrix of estimator β is equal to

$$(X'X)\sigma^2$$

Where σ^2 is the variance of the noise. As an estimator of σ^2 , we consider

$$\hat{\sigma}^2 = \frac{1}{n-p} \|y - X\hat{\beta}\|^2 = \frac{1}{n-p} \sum_{i=1}^n \hat{e}_i^2 \dots \dots \dots (7)$$

The algorithm of LS-MMSE is given by

1. Initialize parameters
2. For snr = 1:20
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10. W=HG+n %output signal
11. $\hat{\sigma}^2 = \frac{1}{n-p} \|y - X\hat{\beta}\|^2 = \frac{1}{n-p} \sum_{i=1}^n \hat{e}_i^2$
12. Error=error+(|X-Ømmse)
13. End iteration
14. End snr

Improving the signal quality, removing noisy signal domain and minimum MMSE denoise signal are the advantages of LS-MMSE technique.

IV. Experiment Results

We have used MATLAB to perform simulations of the CCS-MMSE and LS-MMSE are discussed in section II, III. As can be seen in Fig 2,3 and 4, both CCS-MMSE and LS-MMSE result in the best MSE reduction for all signals and noise levels. The performance of LMMSE and TV methods heavily depends on the type of signal.

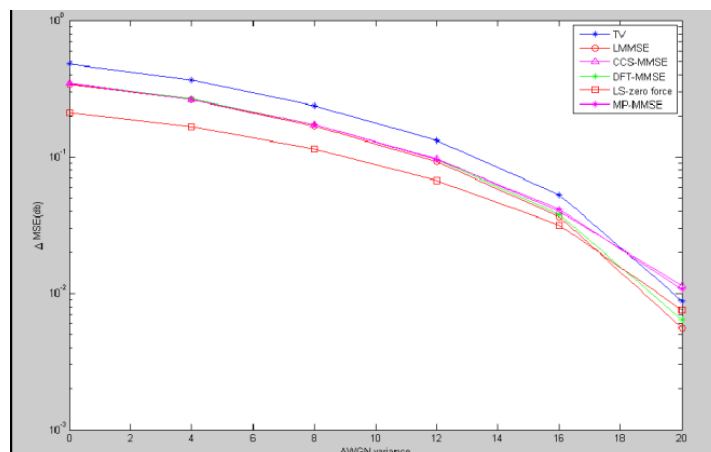


Fig4.1 the average MSE reduction is plotted against AWGN variance with Gaussian noise

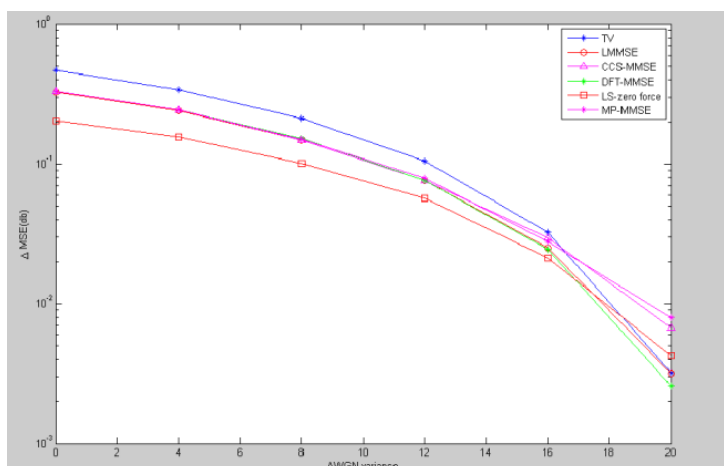


Fig4.2.the average MSE reduction is plotted against AWGN variance with Laplace noise

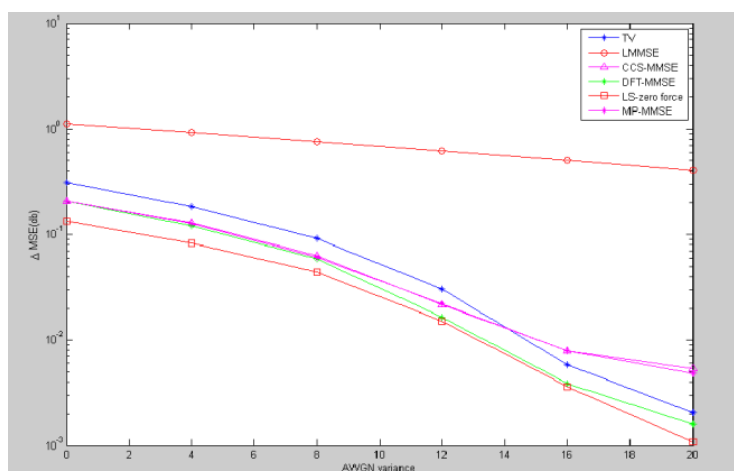


Fig4.3.the average MSE reduction is plotted against AWGN variance with Cauchy noise

V. Conclusion & Future Scope

We present the performance of average MSE reduction using various Bayesian methods. The LS-MSE technique is better performance to existing methods in terms of mmse reduction. Although, the results presented here focus on 1D signal, the extensions to 2D OR 3D data can be achieved by using higher-dimensional Haar-transforms in CCS-MMSE technique.

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