

\widehat{g}^* S-closed sets in topological spaces

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Abstract: In this paper, we define and study about a new type of generalized closed set called, \widehat{g}^* s-closed set. Its relationship with already defined generalized closed sets are also studied.

Keywords: \widehat{g}^* s-closed sets, $S\widehat{g}$ space

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I. Introduction

Norman Levine introduced the notion of semi open sets[8] and generalized closed(briefly,g-closed) sets[7] in a topological space (X,τ) in 1963 and 1970 respectively. The initiation of the study of generalized closed sets was done by Aull[3] in 1968 as he considered sets whose closure is contained in every open super set. Since then extensive research on generalization of closed sets has been going on. The notion of ‘generalized semi closed sets’ was introduced by Arya and Nour[2] in 1990. In 1987, Bhattacharya and Lahiri[14] defined and studied the concept of ‘semi generalised closed sets’ via the notion of semi closed sets. In 2009, A.I.El.Maghrabi and A.A.Nasef introduced and studied a new class of sets, namely g^* s-closed sets[6], which is properly placed between the class of all semi closed sets and the class of all gs-closed sets. The authors introduce the class of \widehat{g}^* s-closed sets which happen to lie between the class of all g^* s-closed sets and the class of all gs-closed sets.

Throughout this paper, (X,τ) denotes a topological space in which no separation axiom is assumed unless explicitly stated.

II. Prelimineries

Definition 2.1: A subset A of a topological space (X,τ) is called

- (i) semi-open[8] if $A \subseteq \text{cl}(\text{int}(A))$ and semi-closed if $\text{int}(\text{cl}(A)) \subseteq A$,
- (ii) pre-open[11] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed if $\text{cl}(\text{int}(A)) \subseteq A$,
- (iii) α -open[12] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and α -closed if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
- (iv) Semi pre open or β -open[1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ and semi pre-closed β -closed if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

The semi closure (respectively, pre-closure, α -closure and semi pre closure) of a subset A of a space (X,τ) is the intersection of all semi-closed sets(respectively, pre-closed, α -closed and semi pre closed) sets containing A and is denoted by $\text{scl}(A)$ (respectively, $p\text{-cl}(A)$, $\alpha\text{-cl}(A)$ and $\text{sp cl}(A)$).

The semi interior (respectively, pre-interior) of a subset A of a space (X,τ) is the union of all semi open(respectively, pre-open) sets contained in A and is denoted by $\text{sint}(A)$ (respectively, $\text{pint}(A)$).

Definition 2.2 [20]: A subset A of a topological space (X,τ) is called \widehat{g} -closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ) .

Remark 2.3: Note that \widehat{g} -closed sets are called ω -closed sets by P.Sundaram and M.Sheik John[15] in 1995 and s^*g -closed sets[4] by K.Chandrasekara Rao and K.Joseph in 2000.

Definition 2.4 [7]: A subset A of a topological space (X,τ) is called a g -closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ) .

Definition 2.5 [14]: A subset A of a topological space (X,τ) is called a sg -closed if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X,τ) .

Definition 2.6 [18]: A subset A of a topological space (X, τ) is called a g^* -closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .

Definition 2.7 [6]: A subset A of a topological space (X, τ) is called a g^*s -closed if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .

Definition 2.8 [2]: A subset A of a topological space (X, τ) is called a gs -closed if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.9 [19]: A subset A of a topological space (X, τ) is called a \hat{g}^* -closed if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in (X, τ) .

Definition 2.10 [16]: A subset A of a topological space (X, τ) is called a $\hat{g}^*\alpha$ -closed if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in (X, τ) .

Definition 2.11 [9]: A subset A of a topological space (X, τ) is called a $g\alpha$ -closed if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .

Definition 2.12 [10]: A subset A of a topological space (X, τ) is called a αg -closed if $\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.13 [5]: A subset A of a topological space (X, τ) is called a generalized semi pre closed set (briefly, gsp -closed), if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.14 [13]: A subset A of a topological space (X, τ) is called a $\hat{\eta}^*$ -closed set if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is ω -open in (X, τ) .

Definition 2.15 [21]: A subset A of a topological space (X, τ) is called a pre semi closed set if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .

Definition 2.16 [17]: A subset A of a topological space (X, τ) is called a ψ -closed set if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is sg -open in (X, τ) .

Notations Used:

- (i) $\hat{g}^*sC(X, \tau)$ denotes the class of all \hat{g}^* s-closed sets in (X, τ) .
- (ii) $\hat{g}^*sO(X, \tau)$ denotes the class of all \hat{g}^* s-open sets in (X, τ) .
- (iii) $SC(X, \tau)$ denotes the class of all semi-closed sets in (X, τ) .
- (iv) $SO(X, \tau)$ denotes the class of all semi-open sets in (X, τ) .
- (v) $gC(X, \tau)$ denotes the class of all g -closed sets in (X, τ) .
- (vi) $gO(X, \tau)$ denotes the class of all g -open sets in (X, τ) .
- (vii) $\hat{g}^*C(X, \tau)$ denotes the class of all \hat{g}^* -closed sets in (X, τ) .
- (viii) $\hat{g}^*\alpha C(X, \tau)$ denotes the class of all $\hat{g}^*\alpha$ -closed sets in (X, τ) .
- (ix) $g^*sC(X, \tau)$ denotes the class of all g^*s -closed sets in (X, τ) .
- (x) $\psi C(X, \tau)$ denotes the class of all ψ -closed sets in (X, τ) .
- (xi) $sgC(X, \tau)$ denotes the class of all sg -closed sets in (X, τ) .
- (xii) $gsC(X, \tau)$ denotes the class of all gs -closed sets in (X, τ) .
- (xiii) $SPC(X, \tau)$ denotes the class of all semi pre-closed sets in (X, τ) .
- (xiv) $\hat{\eta}^*C(X, \tau)$ denotes the class of all $\hat{\eta}^*$ -closed sets in (X, τ) .
- (xv) $\hat{g}C(X, \tau)$ denotes the class of all \hat{g} -closed sets in (X, τ) .
- (xvi) $\hat{g}\text{cl}(A)$ denotes the \hat{g} -closure of A in (X, τ) .
- (xvii) A^c or $X-A$ denotes the complement of A in (X, τ) .
- (xviii) $P(X)$ denotes the power set of X .

III. \hat{g}^* s-closed sets

Definition 3.1: A subset A of a topological space (X, τ) is called a \hat{g}^* s-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open.

Example 3.2: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$.

Example 3.3: Let (X, τ) be a topological space where $X = \{1, 2, 3, 4, \dots\}$ with $\tau = \{\emptyset, X, \{1\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 3, 4\}, \dots\}$.

We can easily find out that $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{2, 3, 4, \dots\}, \{1, 3, 4, \dots\}, \{1, 2, 4, \dots\}, \{1, 2, 3, 5, 6, \dots\}, \dots, \{3, 4, 5, \dots\}, \{2, 4, 5, \dots\}, \{2, 3, 5, 6, \dots\}, \{2, 3, 4, 6, 7, \dots\}, \dots, \{1, 4, 5, \dots\}, \{1, 3, 5, \dots\}, \{1, 3, 4, 6, \dots\}, \dots, \{1, 5, 6, 7, \dots\}, \{1, 4, 6, 7, \dots\}, \{1, 4, 5, 7, \dots\}, \dots\}$.

Remark 3.4: The following example shows that the intersection of two \hat{g}^* s-closed sets need not be a \hat{g}^* s-closed set.

Example 3.5: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Here $A = \{a, b, d\}$ and $B = \{a, b, c\}$ are \hat{g}^* s-closed sets. But $A \cap B = \{a, b\}$ is not a \hat{g}^* s-closed set.

Remark 3.6: The following example shows that the union of two \hat{g}^* s-closed sets need not be a \hat{g}^* s-closed set.

Example 3.7: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Here $A = \{a\}$ and $B = \{b\}$ are \hat{g}^* s-closed sets. But $A \cup B = \{a, b\}$ is not a \hat{g}^* s-closed set.

Proposition 3.8: Every semi closed set in a topological space X is \hat{g}^* s-closed in X.

Remark 3.9: The following example shows that the converse of the above proposition is not true.

Example 3.10: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Here $\{a, b, d\}$ is \hat{g}^* s-closed but not semi closed.

Proposition 3.11: Let A be a \hat{g}^* s-closed subset of X. Then $scl(A) - A$ contains no nonempty \hat{g} -closed subset.

Proof: Let F be a \hat{g} -closed subset of $scl(A) - A$. Now $F \subseteq scl(A) - A$ and $A \subseteq X - F$ where A is \hat{g}^* s-closed and $X - F$ is \hat{g} -open. Thus $scl(A) \subseteq X - F$ or equivalently $F \subseteq X - scl(A)$. But $F \subseteq scl(A)$. Therefore $F \subseteq [X - scl(A)] \cap [scl(A)]$, i.e., $F = \emptyset$. Hence the proof.

Corollary 3.12: If A is \hat{g}^* s-closed in (X, τ) , then A is semi closed iff $scl(A) - A$ is \hat{g} -closed in X.

Proof:

NECESSITY

If A is semi closed, then $scl(A) - A = \emptyset$ and hence $scl(A) - A$ is \hat{g} -closed in X.

SUFFICIENCY

Let $scl(A) - A$ be \hat{g} -closed in X. By Proposition 3.11, $scl(A) - A$ contains no nonempty \hat{g} -closed subset in X. Then $scl(A) - A = \emptyset$ and hence A is semi closed.

Remark 3.13: The following example shows that the converse of the Proposition 3.11 is not true.

Example 3.14: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, b, c\}\}$. Then $SC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$; $\hat{g}c(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{d\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$. Consider $A = \{a, c\}$. It is clear that $scl(A) - A = \{b, d\}$ contains no nonempty \hat{g} -closed subset in X. But $A = \{a, c\}$ is not \hat{g}^* s-closed.

Proposition 3.15: In a topological space (X, τ) , for each $x \in X$, $\{x\}$ is \hat{g} -closed in X or $\{x\}^c$ is \hat{g}^* s-closed in X.

Proof: If $\{x\}$ is not \hat{g} -closed in X , then the only \hat{g} -open set containing $X - \{x\}$ is X . Then $\text{scl}(X - \{x\}) \subseteq X$. Therefore, $X - \{x\}$ is \hat{g}^* s-closed.

Proposition 3.16: In a topological space (X, τ) , every \hat{g} -open set is semi closed iff every subset of X is \hat{g}^* s-closed.

Proof:

NECESSITY

Suppose that every \hat{g} -open set is semi closed. Let A be a subset of X such that $A \subseteq U$ whenever U is \hat{g} -open. But $\text{scl}(A) \subseteq \text{scl}(U) = U$. Therefore A is \hat{g}^* s-closed.

SUFFICIENCY

Suppose that every subset of X is \hat{g}^* s-closed. Let U be \hat{g} -open. Since U is \hat{g}^* s-closed, we have $\text{scl}(U) \subseteq U$. Therefore, $\text{scl}(U) = U$. Hence the proof.

Proposition 3.17: If A is both \hat{g} -open and \hat{g}^* s-closed, then A is semi closed.

Proof:

Let $A \subseteq A$ and A is both \hat{g} -open. Since A is \hat{g}^* -closed, $\text{scl}(A) \subseteq A$. Therefore, $\text{scl}(A) = A$ and hence, A is semi closed.

Proposition 3.18: Every \hat{g}^* -closed set is \hat{g}^* s-closed.

Remark 3.19: The following example shows that the converse of the Proposition 3.18 is not true.

Example 3.20: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then $\hat{g}^*C(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$. Here $\{b\}$ is \hat{g}^* s-closed but not \hat{g}^* -closed.

Proposition 3.21: Every g^* s-closed set is \hat{g}^* s-closed.

Remark 3.22: The following example shows that the converse of the Proposition 3.21 is not true.

Example 3.23: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}, \{a, d\}\}$. Then $g^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$. Here $\{a, b, d\}$ is \hat{g}^* s-closed but not g^* s-closed.

Proposition 3.24: Every ψ -closed set is \hat{g}^* s-closed.

Remark 3.25: The following example shows that the converse of the Proposition 3.24 is not true.

Example 3.26: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then $\psi C(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$. Here $\{a, b, d\}$ is \hat{g}^* s-closed but not ψ -closed.

Proposition 3.27: Every \hat{g}^* s-closed set is $\hat{\eta}^*$ -closed.

Remark 3.28: The following example shows that the converse of the Proposition 3.27 is not true.

Example 3.29: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $\hat{\eta}^*C(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$. Here $\{a\}$ is $\hat{\eta}^*$ -closed but not \hat{g}^* s-closed.

Proposition 3.30: Every \hat{g}^* s-closed set is gs -closed.

Remark 3.31: The following example shows that the converse of the Proposition 3.30 is not true.

Example 3.32: Let (X, τ) be a topological space where $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then $gsC(X, \tau) = \{\emptyset, X, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c\}, \{a\}\}$. Here $\{a, c\}$ is gs -closed but not \hat{g}^* s-closed.

Proposition 3.33: Every \hat{g}^* α -closed set is \hat{g}^* s-closed.

Remark 3.34: The following example shows that the converse of the Proposition 3.33 is not true.

Example 3.35: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then $\hat{g}^*\alpha C(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{d\}, \{c\}, \{b\}, \{a\}\}$. Here $\{a\}$ is \hat{g}^* s-closed but not ψ -closed.

Remark 3.36: The following two examples show that g -closedness and \hat{g}^* s-closedness are independent of each other.

Example 3.37: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then $gC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$. Here $\{c\}$ is \hat{g}^* s-closed but not g -closed.

Example 3.38: Let (X, τ) be a topological space where $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c\}, \{a\}\}$; $gC(X, \tau) = P(X)$. Here $\{b\}$ is g -closed but not \hat{g}^* s-closed.

Remark 3.39: The following two examples show that sg -closedness and \hat{g}^* s-closedness are independent of each other.

Example 3.40: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Then $sgC(X, \tau) = P(X)$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{a, b\}, \{c, d\}\}$. Here $\{a\}$ is sg -closed but not \hat{g}^* s-closed.

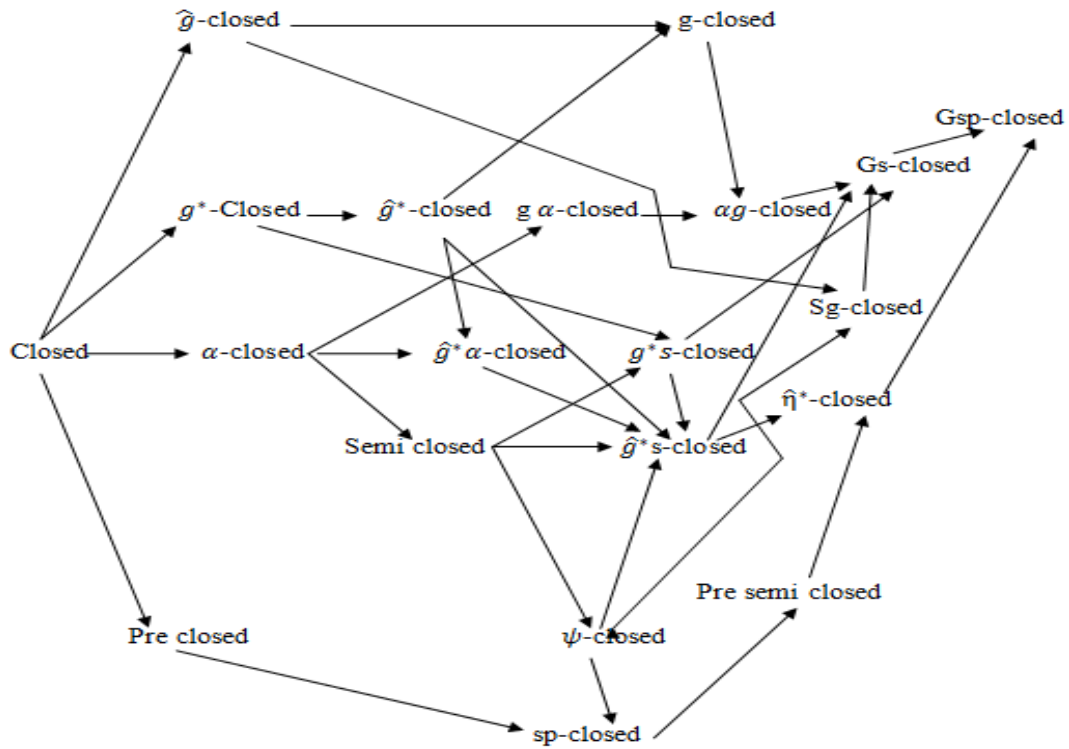
Example 3.41: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}\}$. Then $sgC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{a, c\}, \{a, b\}, \{d\}, \{c\}, \{b\}\}$. Here $\{a, b\}$ is \hat{g}^* s-closed but not sg -closed.

Remark 3.42: The following two examples show that \hat{g} -closedness and \hat{g}^* s-closedness are independent of each other.

Example 3.43: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a\}, \{a, b, c\}\}$. Then $\hat{g}C(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{d\}\}$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{a, d\}, \{d\}, \{c\}, \{b\}\}$. Here $\{c\}$ is \hat{g}^* s-closed but not \hat{g} -closed.

Example 3.44: Let (X, τ) be a topological space where $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then $\hat{g}C(X, \tau) = P(X)$; $\hat{g}^*sC(X, \tau) = \{\emptyset, X, \{b, c\}, \{a\}\}$. Here $\{b\}$ is \hat{g} -closed but not \hat{g}^* s-closed.

Remark 3.45: From the above Propositions and Remarks, we obtain the following diagram.



Definition 3.46: A topological space (X, τ) is called a $S\hat{g}$ space if the intersection of a semi closed set with a \hat{g} -closed set in it is \hat{g} -closed.

Example 3.47: Let (X, τ) be a topological space where $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $SC(X, \tau) = \{\emptyset, X, \{c, d\}, \{d\}, \{c\}\}$; $\hat{g}c(X, \tau) = \{\emptyset, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}, \{c\}\}$. As the intersection of a semi closed set with a \hat{g} -closed set is \hat{g} -closed, the above topological space (X, τ) is a $S\hat{g}$ space.

Proposition 3.48: For a subset A of a $S\hat{g}$ space (X, τ) , the following are equivalent.

- (i) A is \hat{g}^* s-closed.
- (ii) $\hat{g}cl(\{x\}) \cap A \neq \emptyset$ for each $x \in scl(A)$.
- (iii) $scl(A) - A$ contains no nonempty \hat{g} -closed set.

Proof: (i) \Rightarrow (ii)

Let $x \in scl(A)$. Suppose $\hat{g}cl(\{x\}) \cap A = \emptyset$. Then $A \subseteq (X - \hat{g}cl(\{x\}))$. Since A is \hat{g}^* s-closed, $scl(A) \subseteq (X - \hat{g}cl(\{x\}))$ which is a contradiction to $x \in scl(A)$. Therefore $\hat{g}cl(\{x\}) \cap A \neq \emptyset$.

(ii) \Rightarrow (iii)

Let F be a \hat{g} -closed set such that $F \subseteq scl(A) - A$. Suppose $F \neq \emptyset$. Let $x \in F$. Then $\hat{g}cl(\{x\}) \subseteq F$. Therefore, by (ii), $\emptyset \neq \hat{g}cl(\{x\}) \cap A \subseteq F \cap A \subseteq (scl(A) - A) \cap A = \emptyset$ which is a contradiction. Therefore, $F = \emptyset$.

(iii) \Rightarrow (i)

Let $A \subseteq G$ and G be \hat{g} -open in X . Suppose $scl(A)$ is not a subset of G . Then $scl(A) \cap (X - G)$ is nonempty \hat{g} -closed subset of $scl(A) - A$ which is a contradiction. Therefore $scl(A) \subseteq G$ and hence A is \hat{g}^* s-closed.

Proposition 3.49: In a $S\hat{g}$ space (X, τ) , the converse of Proposition 3.11 holds.

Proof: (iii) \Rightarrow (i) of Proposition 3.48.

Definition 3.50: Let A be subset of a topological space (X, τ) . We define $\Lambda_{\hat{g}}(A)$ as follows. $\Lambda_{\hat{g}}(A) = \bigcap \{ U : U \supseteq A, U \in \mathcal{G}\mathcal{O}(X, \tau) \}$, where $\mathcal{G}\mathcal{O}(X, \tau)$ denotes the collection all \mathcal{G} -open sets in (X, τ) .

Proposition 3.51: A subset A of a topological space (X, τ) is \hat{g}^* s-closed iff $\text{scl}(A) \subseteq \Lambda_{\hat{g}}(A)$.

Proof:

NECESSISITY

Since A is \hat{g}^* s-closed, $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open. Therefore $\text{scl}(A) \subseteq \Lambda_{\hat{g}}(A)$.

SUFFICIENCY

Let U be \hat{g} -open such that $A \subseteq U$. By assumption, $\text{scl}(A) \subseteq \Lambda_{\hat{g}}(A) \subseteq U$. Therefore A is \hat{g}^* s-closed.

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