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Finite Element Analysis of Convective Micro Polar Fluid Flow through a Porous Medium in Cylindrical Annulus

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ABSTRACT: *We analyze a finite element solution for the mixed convection micro polar flow through a porous medium in a cylindrical annulus. The governing partial differential equations are solved numerically by using finite element technique. The effect of Darcy parameter and Heat source parameter and surface condition on velocity, micro rotation and temperature functions has been study.*

Keywords: *Porous medium, Micro polar Fluids, Convection, Cylindrical annulus*.

I. INTRODUCTION

In recent years there have been considerable advancement in the study of free convection in a porous annulus because of its natural occurrence and of its importance in many branches of science and engineering. This is of fundamental importance to a number of technological applications, such as underground disposal of radioactive waste materials, cooling of nuclear fuel in shipping flasks and water filled storage bays, regenerative heat exchangers containing porous materials and petroleum reservoirs, burying of drums containing heat generating chemicals in the earth, storage of agricultural products, ground water flow modeling, nuclear reactor assembly, thermal energy storage tanks, insulation of gas cooled reactor vessels, high performance insulation for building, and cold storage, free convective heat transfer inside a vertical annulus filled with isotropic porous medium has been studied by many researchers, notable among them are Havstad et. al., [7], Reda [15], Prasad et. al., [12,13], Prasad et. al., [14], Hickon et. al., [8], and Shiva kumara et. al., [16], they concluded that radius ratio and Rayleigh number influence the heat and fluid flow significantly.

Eringen [5] presented the theory of micro polar fluid which explains adequately the internal characteristics of the substructure particles subject to rotations and deformations, Ariman [1] Sylvester et. al., [17] and Ariman et. al., [2] confirmed that the micropolar fluid serves a better model for animal blood. The liquid crystals, suspension solutions and certain polymeric fluids consisting of randomly oriented barlike elements of dumbbell molecules, behave as micro polar fluid. Kazakai and Ariman [10] introduced the heat conducting micropolar fluid and investigated the flow between two parallel plates. Eringen [6] extended the theory further to include heat conduction and viscous dissipation. The application of this theory may be searched in biomechanics.

Berman [4] has studied the flow of a viscous liquid under a constant pressure gradient between two concentric cylinders at rest. Inger [9] solved the problem when the walls are permeable and the outer cylinder is sliding with a constant axial velocity relative to the stationery inner one. Mishra and Acharya [11] examined the elastico viscous effects in the flow considered by Inger [9]. Sastri and Maiti solved the problem of combined convective flow of micro polar fluid in an annulus.

The problem of massive blowing into aerodynamic body flow fields is a complex 1 for a realistic flow configuration. However the study the simplified flow models exhibits some of the essential physical features involving the interaction of blowing with a shear flow. The classical coutte – poiseylle shear flow is an idealized model for this purpose to investigate to micro polar structures at least theoretically on such a flow Agarwal and Dhanpal [3] have analyzed the connective micro polar fluid flow and heat transfer between two concentric porous circular cylinders when the outer cylinder moves parallel to itself with a constant velocity. A problem solved by Inger [9] in viscous fluids.

In this paper we make an investigation of connective heat transfer flow micro polar fluid in a cylindrical annulus between the porous concentric cylinders r = a and r = b. By employing Galerikin finite element analysis with line elements with three nodes the transport equations of linear momentum, angular momentum and energy are solved to obtain velocity, micro concentration and temperature distributions.

II. FORMULATION OF THE PROBLEM

Consider the steady motion of an incompressible micro polar fluid through an annulus of two infinitely long porous circular cylinders of radii a and b (a – b = h > 0) respectively. The fluid is injected through the inner cylinder with arbitrary radial velocity u_b and in view of continuity, also flows outward through the moving cylinder with a radial velocity u_a. The cylindrical polar co-ordinate system (γ, θ, z) with z co-ordinate along the axis of the cylinders is chosen to specify the problem.

The velocity and micro rotation are taken in the form

$$v_r = u(r),\ \ v_\theta = v = 0,\ \ v_z = w(r)$$

$$v_r = 0,\ \ v_\theta = v = (r),\ v_z = 0 \qquad 2.1$$

the basic equations are:

Continuity: $$\frac{\partial u}{\partial r} + \frac{u}{r} = 0 \qquad 2.2$$

Momentum: $$\rho u \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} + (\mu + k)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) \qquad 2.3$$

$$\rho u \frac{\partial w}{\partial r} = -\frac{\partial p}{\partial z} + (\mu + k)\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{k}{r}\frac{\partial}{\partial r}(r N) \qquad 2.4$$

First stress Momentum:

$$\rho j u \frac{\partial N}{\partial r} = r\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} - \frac{N}{r^2}\right) - k\frac{\partial w}{\partial r} - 2kN \qquad 2.5$$

Energy: $$\rho C_p w \frac{\partial T}{\partial z} = k_f\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \qquad 2.6$$

Where T is the temperature, N is the micro rotation ρ the density, p the pressure, j the micro inertia, C_p the specific heat, k_f the thermal conductivity, μ the fluid viscosity, k is the material constants.

At r = b, u = ub, w = 0, v = 0, $T = T_0 + A_0 z$

At r = a, u = ua, w = 0 v = 0, $T = T_1 + A_0 z$ 2.7

Where the fluid is assumed to adhere to the solid boundaries.A_0 is the constant of proportionality and T_0, T_1 are the constant wall temperatures of inner and outer cylinders temperatures of inner and outer cylinders respectively at z = 0.

The integration of (2.2) yields

$$ur = C, \text{ a constant} \qquad 2.8$$

$$\Rightarrow ru = au_a = bu_b \Rightarrow u = \frac{au_a}{r}$$

Also inview of the boundary condition on temperature, we may write

$$T = T_0 + A_0 (z) + \theta(r) \qquad 2.9$$

Introducing the non-dimensional variables r′, w′, p′, θ and N as

$$r' = \frac{r}{a},\ \ w' = \frac{w}{a},\ \ p' = \frac{p}{\rho\, u^2 / a^2}$$

$$\theta = \frac{T - T_0}{T_i - T_0},\ \ N' = \frac{(u+k)\,w}{\rho\,\mu^2\,a^2} \qquad 2.10$$

The governing equations in the non-dimensional form are

$$\frac{d^2 w}{dr^2} + \left(1 - \frac{\lambda}{1+\Delta}\right)\frac{1}{r}\frac{dw}{dr} = -\pi_1 + D_1^{-1} - G\theta - \frac{\Delta}{r}\frac{d}{dr}(rN) \qquad 2.11$$

$$\frac{d^2 N}{dr^2} + (1 - \lambda A)\frac{1}{r}\frac{dN}{dr} - \left(\frac{1}{r^2} - \frac{2\Delta}{A}\right)N = \frac{\Delta}{A}\frac{1}{r}\frac{dw}{dr} \qquad 2.12$$

$$P_r\ N_T\ w = \frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} \qquad 2.13$$

where

$\Delta = \dfrac{k}{\mu}$ (Micropolar parameter) $\qquad A = \dfrac{r}{\mu J}$ (Micropolar parameter

$A_1 = \dfrac{\beta}{\mu\ a^2}$ $\qquad \lambda = \dfrac{a\ u_a}{\nu}$ (Suction parameter)

$D^{-1} = \dfrac{L^2}{k}$ (Darcy parameter) $\qquad G = \dfrac{\beta\, g\, \Delta T\, L^3}{\nu^2}$ (Grashof number)

$G_1 = \dfrac{G}{1+\Delta}$ $\qquad \Delta_1 = \dfrac{\Delta}{1+\Delta}$

$P_r = \dfrac{\mu\ C_p}{k_f}$ (Prandtl number)

The boundary conditions are

$w = 0, \quad \theta = 1, \quad N = 0 \quad$ on $\quad r = 1$

$w = 0, \quad \theta = 0, \quad N = 0 \quad$ on $\quad r = s$

In order to predict the heat and mass transfer behavior in the porous medium equations (2.11) – (2.13) are solved by using finite element method. A simple 3-noded triangular element is considered. ψ, θ and ϕ vary inside the element and can be expressed as

$$\psi = N_1\ \psi_1 + N_2\ \psi_2 + N_3\ \psi_3$$
$$\theta = N_1\ \theta_1 + N_2\ \theta_2 + N_3\ \theta_3$$
$$\phi = N_1\ \phi_1 + N_2\ \phi_2 + N_3\ \phi_3$$

Gela kin's method is used to convert the partial differential equation (2.11) – (2.13) into matrix form of equations. Details of FEM formulations and good understanding of the subject is given in the books [3, 4]. The matrix equations are, assembled to get global matrix equations for he whole domain, which is then solved iteratively, to obtainθ, ψ and ϕ in porous medium. In order to get accurate results, tolerance level of solution for θ, ψ and ϕ are set at 10^{-5} and 10^{-9} respectively. Element size in domain varies. Large number of elements are located near the walls where large variations in θ, ψ and ϕ are expected. The mesh is symmetrical about central horizontal and vertical lines of the cavity. Sufficiently dense mesh is chosen to make the solution mesh invariant. The mesh size of 3200 elements has good accuracy in predicting the heat transfer behavior of the porous medium. The computations are carried out on high-end computer.

III. RESULTS AND DISCUSSION

On solving the equations, given by the finite element technique the velocity, micro rotation and temperature distributions are obtained. The Prandtl number p_r, material constants A & A are taken to be constant, at 0.733, 1 and 1 respectively whereas the effect of other important parameters, namely micro polar parameter Δ, the section Reynolds number λ, Grashof number G and Darcy parameter D^{-1} has been studied for these functions and the corresponding profiles are shown in figs. 1.

Fig. 1 depicts the variation of velocity function W with Grashof number G. The actual axial velocity W is in the vertically downwards direction and $w < 0$ represents the actual flow. Therefore $w > 0$ represents the reversal flow. We notice from fig. 1 that $w>0$ for $G > 0$ and $w < 0$ for $G < 0$ except in the vicinity of outer cylinder r = 2. the reversal flow exists everywhere in the region ($1.1 \le r \le 1.8$) for $G > 0$ and in the neighborhood of r = 2 for $G < 0$. The region of reversed flow enlarges with $G \le 2x10^3$ and shrinks with higher $G \ge 3x10^3$. Also it grows in size with |G| (<0). |w| enhances with $G \le 2x10^3$ and reduces with higher $G \ge 3x10^3$. |w| experiences a depreciation in the case of cooling of the boundaries with maximum at r = 1.5.The variation of w with Darcy parameter D^{-1}. It is found that lesser the permeability of the porous medium larger |w| in the flow region (fig. 2). The influence of micro rotation parameter Δ on w is shown in fig. 3. As the micro polar parameter $\Delta \le 3$ increases, the velocity continuously increases and decreases with higher $\Delta \ge 5$, with maximum attained in the vicinity of r = 1. Fig. 4 represents w with suction parameterλ. IT is found that the axial velocity experiences an enhancement with increase in $\lambda \le 0.03$ and depreciates with higher $\lambda \ge 0.05$. Fig. 5 depicts w with the width of the annular region. We notice that the axial velocity continuously decreases with increase in the width S of the annular region. Thus the velocity enhances in the narrow gap region and depreciates in the wide gap case. The effect of radiation parameter N_1 on w is exhibited in Fig. 6. IT is observed that the velocity w enhances with increase in $N_1 \le 1.0$ except in a narrow region adjacent to r = 2 and for higher $N_1 \ge 1.5$, it experiences an enhancement in the entire flow region.

The micro rotation (N) in shown in figs 7-12 for different values of G, Δ, λ, S and N_1. It is found that the values of micro rotation for G > 0 are negative and positive for G < 0. An increase in $|G| \le 2x10^3$ editor an enhancement in N and for higher $|G| \ge 3x10^3$, it reduces in the region adjacent to r = 1 and enhances in the reform adjacent to r = 2 with maximum at r = 1.5 (fig. 7). From fig. 8 we find that lesser the permeability of the porous medium larger the micro oration everywhere in the flow region.The effect of N on micro polar parameter Δ in shown in Fig. 9. We notice that an increase in Δ ≤ 3 leads to an enhancement in |N| and for higher Δ ≥ 5, it enhances the first half (1.1 ≤ r ≤ 1.5) and reduces in the second half (1.6 ≤ r ≤ 1.9). Fig. 10 illustrates that the micro oration enhances in the first half and reduces in the second half with increase in λ ≤ 0.02 and for higher values of λ ≥ 0.03 we notice an increment in |N| everywhere in the flow region. Form fig. 11 we find that the micro rotation depreciates in the narrow gap case and enhances in the wide gap case. Fig. 12 illustrates that the micro rotation |N| enhances with increase in the radiation parameter $N_1 \le 1.5$ and for higher $N_1 \ge 2.5$, the micro rotation enhances in the let half and depreciates in the second half of the flow region.

The non-dimensional temperature (θ) is shown in figs. 13 – 17 for different values of G, λ, Δ, S and N_1. Fig. 13 illustrates that non-dimensional temperature is positive for all variations. The actual temperature enhances with increase in $G \le 2x10^3$ and depreciates with higher $G \ge 3x10^3$. Also it enhances with G < 0. The variation of θ with D^{-1} shows that lesser the permeability of the porous medium larger the actual temperature in the flow region (Fig. 14), Fig. 15 illustrates that an increase in the micro rotation parameter Δ increases the actual temperature continuously with maximum attained at r=1.5. The variation of θ with suction parameter λ shows that the temperature enhances with increase in λ ≥ 0.03. For further increase in λ ≥ 0.05, the temperature depreciates in the first half and enhances in the second half. The influence of the suction of the boundary on θ is shown in fig. 16. IT is found that increase in S ≤ 0.6 reduces θ in the left half and enhances it in the right half and for S = 0. 7, θ reduces in the flow region except in a region adjacent to r =1. For further increase in S we notice an enhancement in the central flow region and depreciation in θ in the regions abutting the cylinders r = 1 & 2. Fig. 17 represents the temperature with radiation parameter N_1. It is found that an increase in N_1 enhances θ in the left half and depreciates in the right half and for higher values $N_1 \ge 1.5$ we observe depreciation in the first half and enhancement in the second half.

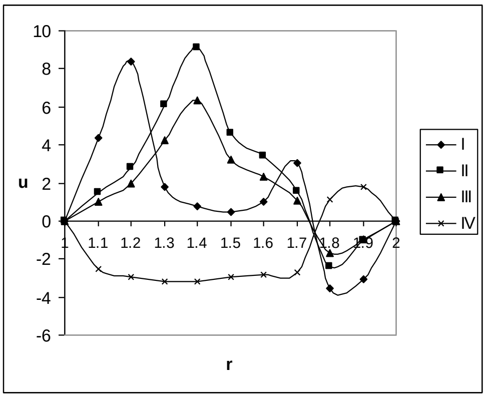

Fig. 1 : Variation of velocity (u) with G

	I	II	III	IV
G	10^3	$3x10^3$	$5x10^3$	-10^3

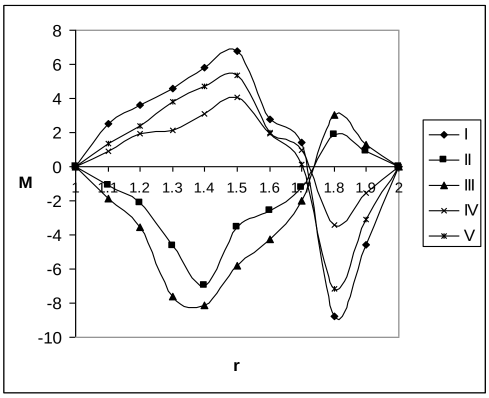

Fig. 2 : Variation of velocity (u) with G

	I	II	III	IV	V
G	$-3x10^3$	$-5x10^3$	10^3	$2x10^3$	$3x10^3$

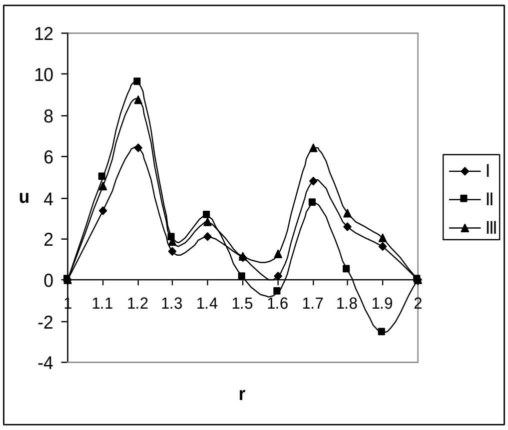

Fig. 3 : Variation of velocity (u) with Δ

	I	II	III
Δ	1	3	5

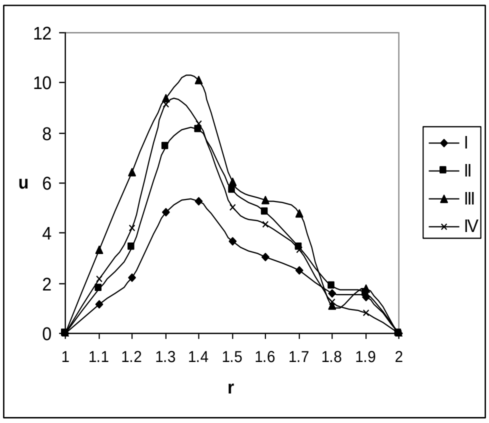

Fig. 4 : Variation of velocity (u) with λ

	I	II	III	IV
λ	0.01	0.02	0.03	0.05

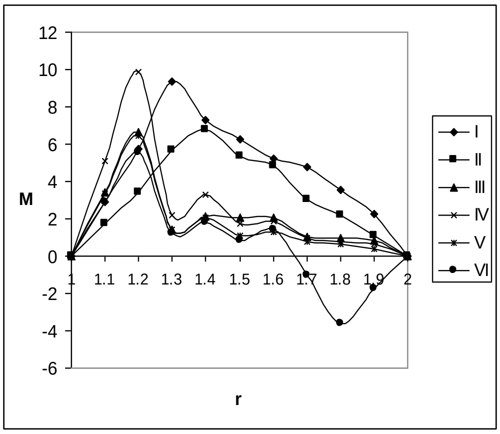

Fig. 5 : Variation of velocity (u) with S

	I	II	III	IV	V	VI
S	0.4	0.5	0.6	0.7	0.8	0.9

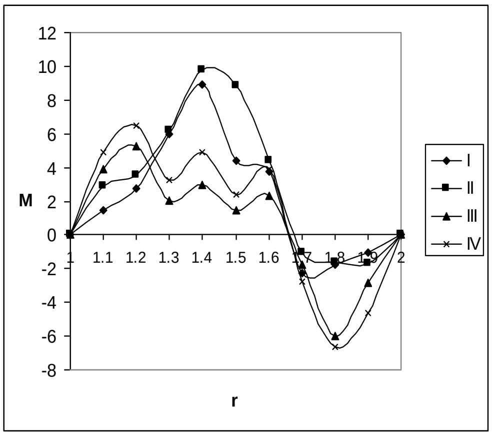

Fig. 6 : Variation of velocity (u) with N_t

	I	II	III	IV
N_t	0.5	1.0	1.5	2.5

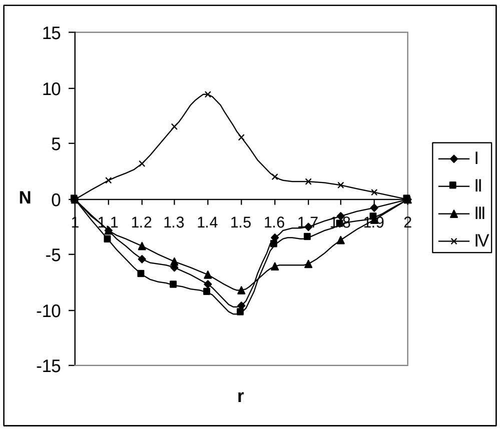

Fig. 7 : Variation of micro rotation (N) with G

	I	II	III	IV
G	10^3	$3x10^3$	$5x10^3$	-10^3

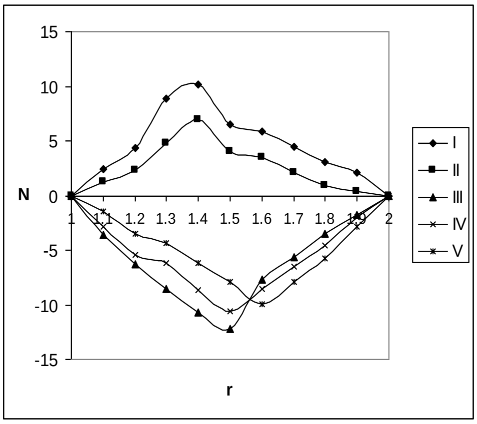

Fig. 8 : Variation of micro rotation (N) with G

	I	II	III	IV	V
G	$-3x10^3$	$-5x10^3$	10	20	30

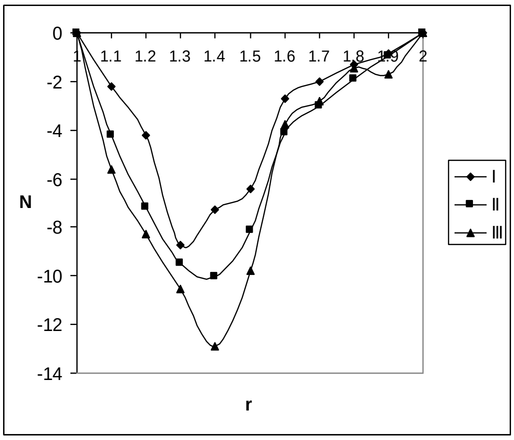

Fig. 9 : Variation of micro rotation (N) with Δ

	I	II	III
Δ	1	3	5

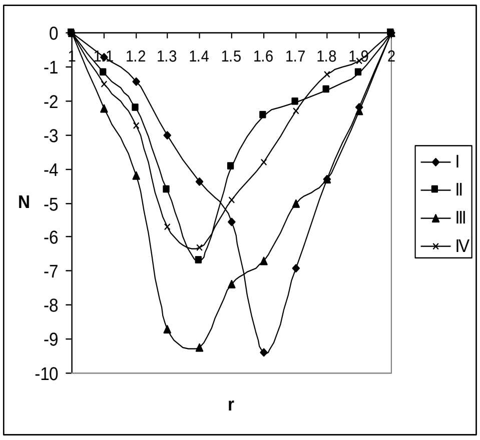

Fig. 10 : Variation of micro rotation (N) with λ

	I	II	III	IV
λ	0.01	0.02	0.03	0.05

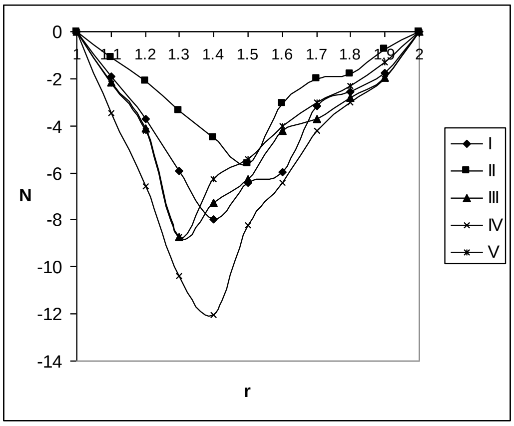

Fig. 11 : Variation of micro rotation (N) with S

	I	II	III	IV	V	VI
S	0.4	0.5	0.6	0.7	0.8	0.9

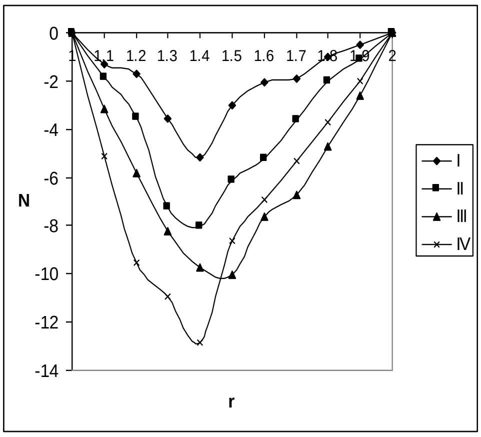

Fig. 12 : Variation of micro rotation (N) with N_t

	I	II	III	IV
N_t	0.5	1.0	1.5	2.5

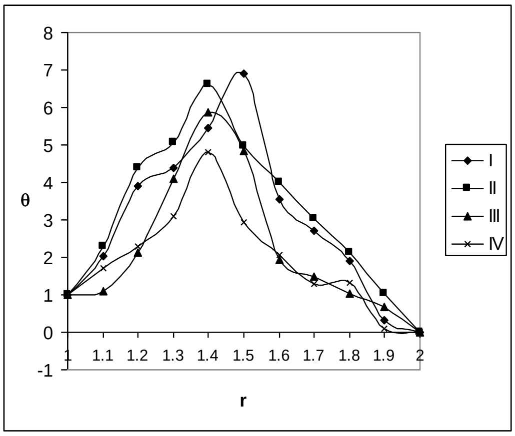

Fig. 13 : Variation of temperature (θ) with G

	I	II	III	IV
G	10^3	$3x10^3$	$5x10^3$	-10^3

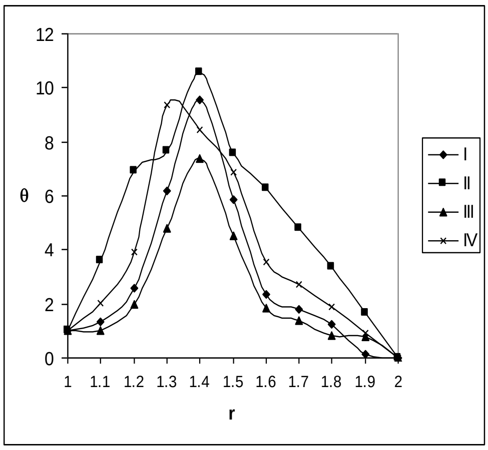

Fig. 14 : Variation of temperature (θ) with G

	I	II	III	IV	V
G	$-3x10^3$	$-5x10^3$	10	20	30

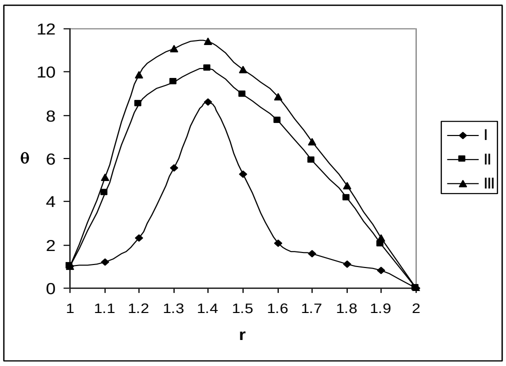

Fig. 15: Variation of temperature (θ) with Δ

	I	II	III
Δ	1	3	5

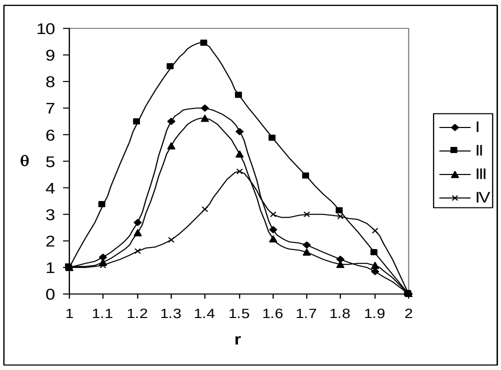

Fig. 16: Variation of temperature (θ) with λ

	I	II	III	IV
λ	0.01	0.02	0.03	0.05

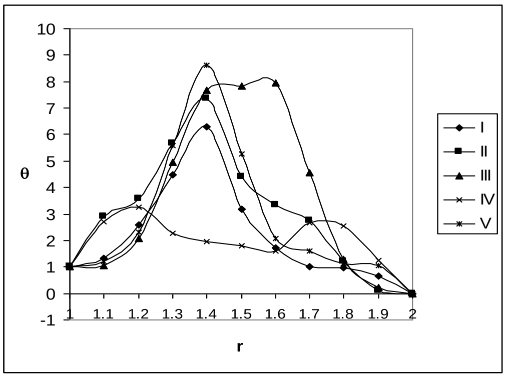

Fig. 17: Variation of temperature (θ) with S

	I	II	III	IV	V	VI
S	0.4	0.5	0.6	0.7	0.8	0.9

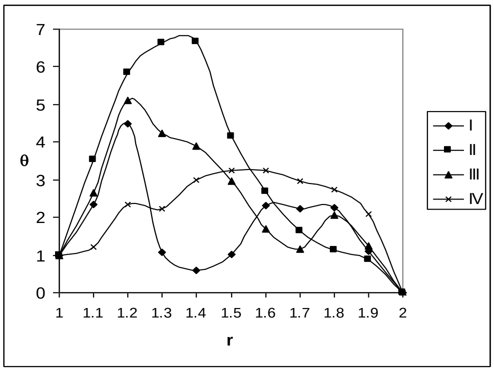

Fig. 18: Variation of temperature (θ) with N_t

	I	II	III	IV
N_t	0.5	1.0	1.5	2.5

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