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Probabilistic Modeling and Performance Analysis of A 2(K)-Out-Of-3(N) System with Inspection Subject To Operational Restriction

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ABSTRACT: *In repairable redundant the failed units can either be repaired or replaced by identical standby to reduce the system down time. The failed units are inspected for repair/replacement. In this paper, one stochastic model for 2(k)-out-of-3(n) redundant system of identical units with repair and inspection are examined stochastically. The system is considered in up-state only if 2(k)-out-of-3(n) units are operative in this model. Normally, the server either attends the system promptly or may take some time, after failure. The system is studied under an operational restriction on the inspection i.e. in case when system has only one unit in operational mode the server has to attend the system for inspection. Semi- Markov processes and regenerative point technique is adopted to obtain the expressions for measures of system effectiveness such as transition probabilities, mean sojourn times, to system failure , steady state availability, busy period, expected number of visits etc. Cost-analysis is also carried out for the system model.*

Keywords: *Probabilistic modeling, performance analysis, regenerative point, semi-Markov process, mean sojourn times, availability, busy period.*

I. Introduction

Redundancy techniques are widely used to improve system performance in terms of reliability and availability. Among various redundancy techniques standby is the simplest and commonly accepted one. In general there are three types of standby; cold, worm and hot standby. Hot standby implies that the redundant (spare) unit or component has same failure rate as when it is in operation mode where as in case of cold standby the failure rate of the redundant unit or component is zero and it can't fail in standby mode. Between hot and cold there is an intermediate case known as worm standby. In this case the failure rate of redundant unit lies in between that of hot and cold standby.

In order to reduce the down time redundancy is necessary. In literature, many researchers have been discussed the reliability and availability of standby systems in detail by considering different cases and strategies such as by considering weather conditions [2] , replacement policy with spares [3], dissimilar unit system with perfect or imperfect switch [4].

In this paper a probabilistic model of a 2(k)-out-of -3 (n) worms standby systems are examined stochastically. Such system found applications in various fields including the process industry, network design and many more. For such a system when an operating unit fails the standby unit becomes operative after repaired and the system works if at least 2k-out-of-3(n) units are in operative mode. In this model , server attends the system promptly whenever needed and first inspects the failed unit to see the practicability of its repair. If repair of the unit is not practicable, it is replaced by new one so that unnecessary expanses on repair can be avoided. In real life, it is not always possible for the server to attend the system swiftly when required may because of his pre-occupation. In such a situation server may be allowed to take some time to reach the system. But it is urgently required that the server must arrive at the system promptly in case of urgent situation. In this model, the standby server do not takes time to arrive at the system when 2(k)-out-of 3(n) units are operative. While in case when the system has only one unit in operational mode the server has to attend the system swiftly for inspection due to operational restriction imposed on it, so that the down time of the system may be reduced. Failure time follows negative exponential distribution while repair and inspection times follow arbitrary distributions. All the random variable are mutually independent and un-correlated. The expressions for various measures of system performances such as transition probabilities, sojourn times, MTSF (mean time of system failure), availability, busy period of server and profit function are down for steady state.

II. Notations

N_0 Units in normal mode and operative.

$\overline{N_0}$ Units in normal mode but not working.

S_i i[th] transition state.

a/b Probability that repair is useful/not useful.

c/d Probability that repair is useful/not useful of the standby unit.

λ Constant failure rate of an operative unit.

α Constant failure rate of an warm standby unit.

$q_{ij}(t)/Q_{ij}(t)$ pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state without visiting any other regenerative state in (0,t].

$q_{ij.kr}(t)/Q_{ij.kr}(t)$ pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting states k, r once in (0,t].

h(t)/H(t) pdf/cdf of inspection time.

g(t)/G(t) pdf/cdf of repair time of the server.

$F_{wi}/F_{WI}/F_{ui}/F_{UI}$ Unit is completely failed and waiting for inspection/ waiting for inspection continuously from previous state/ under inspection/ under continuous inspection from previous state.

F_{ur}/F_{UR} Unit is completely failed and under repair/ under repair continuously from previous state.

$p_{ij}/p_{ij.kr}$ Probability of transition from regenerative state i to state j without visiting any other state in (0,t]/ visiting state k,r once in (0,t].

*/ & Laplace/ laplace-stiltje's transform.

III. Transition States

The following are the possible transition states of system. The state S_0, S_1, S_2, S_3, S_4 are regenerative states while states S_5, S_6, S_7, S_8, S_9, S_{10} are failed and non-regenerative stat.

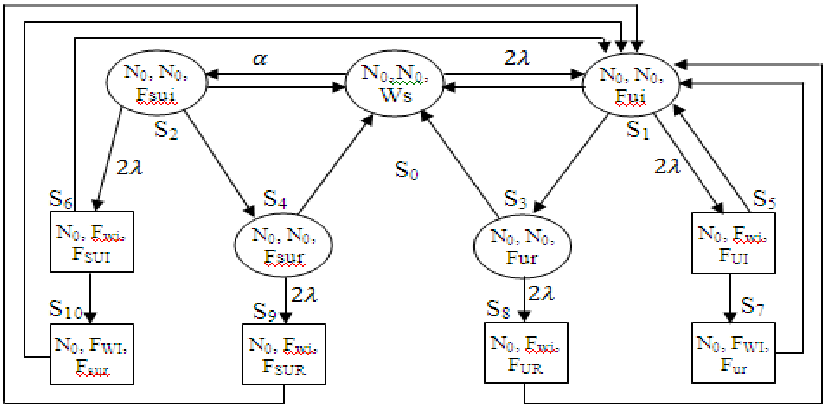
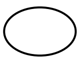
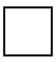


Figure. 1 : State Transition diagram


Up state


Down state

Transition states:-

$S_0(N_0, N_0, W_s)$: two units are operative and the other unit is kept as worm standby

$S_1(N_0, N_0, F_{ui})$: two units are operative and the other unit is under inspection

$S_2(N_0, N_0, F_{sui})$: two unit are operative and the standby unit is under inspection

$S_3(N_0, N_0, F_{ur})$: two units are operative and the other unit is under repair

$S_4(N_0, N_0, F_{sur})$: two units are operative and the standby unit is under repair

$S_5(\bar{N}_0, F_{wi}, F_{UI})$: one unit is normal but not working , the second unit is wait for inspection and the third unit is still under inspection

$S_6(\bar{N}_0, F_{wi}, F_{SUI})$: one unit is normal but not working , the second unit is wait for inspection and the standby unit is still under inspection

$S_7(\bar{N}_0, F_{WI}, F_{ur})$: one unit is normal but not working , the second unit is still wait for inspection and the third unit is under repair

$S_8(\bar{N}_0, F_{wi}, F_{UR})$: one unit is normal but not working , the second unit is wait for inspection and the third unit is still under repair

$S_9(\bar{N}_0, F_{wi}, F_{SUR})$: one unit is normal but not working , the second unit is wait for inspection and the standby unit is still under repair

$S_{10}(\bar{N}_0, F_{WI}, F_{sur})$: one unit is normal but not working , the second unit is still wait for inspection and the standby unit is under repair

IV. Transition Probabilities

1. We shall consider the state transition from state S_0, there are two transitions one to state S_1 and one to S_2. Therefore,

$$P_{01} = \int_0^\infty 2\lambda e^{2\lambda t} e^{-\alpha t}\, dt = \frac{2\lambda}{2\lambda+\alpha},\ P_{02} = \int_0^\infty \alpha e^{2\lambda t} e^{-\alpha t}\, dt = \frac{\alpha}{2\lambda+\alpha}$$

Thus, it can easily regarded that

$$P_{01} + P_{02} = 1.$$

2. In state S_1, there are three transitions one to state S_0, one to state S_3, and one to state S_5. Therefore,

$$P_{10} = b\int_0^\infty h(t)e^{-2\lambda t}\, dt, \quad P_{13} = a\int_0^\infty h(t)e^{-2\lambda t}\, dt, \quad P_{15} = 2\lambda\int_0^\infty e^{-2\lambda t}\overline{H}(t)dt,$$

Thus, it can easily regarded that

$$P_{10} + P_{13} + P_{15} = 1$$

3. In state S_2, there are three transitions one to state S_0, one to state S_4, and one to state S_6. Therefore,

$$P_{20} = c\int_0^\infty h(t)e^{-2\lambda t}\, dt, \quad P_{24} = d\int_0^\infty h(t)e^{-2\lambda t}\, dt, \quad P_{26} = 2\lambda\int_0^\infty e^{-2\lambda t}\overline{H}(t)dt,$$

Thus, it can easily regarded that

$$P_{20} + P_{24} + P_{26} = 1.$$

4. In state S_3, there are two transitions one to state S_0, one to state S_8. Therefore,

$$P_{30} = \int_0^\infty g(t)e^{-2\lambda t}\, dt, \qquad P_{38} = 2\lambda\int_0^\infty e^{-2\lambda t}\overline{G}(t)dt,$$

Thus, it can easily regarded that

$$P_{30} + P_{38} = 1.$$

5. In state S_4, there are two transitions one to state S_0, one to state S_9. Therefore,

$$P_{40} = \int_0^\infty g_s(t) e^{-2\lambda t}\, dt, \qquad P_{49} = 2\lambda \int_0^\infty e^{-2\lambda t} \overline{G}_s(t) dt,$$

Thus, it can easily regarded that

$P_{40} + P_{49} = 1$.

6. In state S_5, there are two transitions one to state S_1, one to state S_7. Therefore,

$$P_{51} = b \int_0^\infty h(t) dt, \qquad P_{57} = a \int_0^\infty h(t) dt,$$

Thus, it can easily regarded that

$P_{51} + P_{57} = 1$.

7. In state S_6, there are two transitions one to state S_1, one to state S_{10}. Therefore,

$$P_{61} = c \int_0^\infty h(t) dt, \qquad P_{610} = d \int_0^\infty h(t) dt,$$

Thus, it can easily regarded that

$P_{61} + P_{610} = 1$.

8. In state S_7, there is one transition to state S_1 . Therefore,

$$P_{71} = \int_0^\infty g(t) dt$$

9. In state S_8, there is one transition to state S_1 . Therefore,

$$P_{81} = \int_0^\infty g(t) dt$$

10. In state S_9, there is one transition to state S_1 . Therefore,

$$P_{91} = \int_0^\infty g_s(t) dt$$

11. In state S_{10}, there is one transition to state S_1 . Therefore,

$$P_{101} = \int_0^\infty g_s(t) dt$$

The mean sojourn times μ_i, in state S_i is given by: -

$$\mu_0 = \int_0^\infty e^{-2\lambda t} e^{-\alpha t}\, dt, \quad \mu_1 = \int_0^\infty \overline{H}(t) e^{-2\lambda t} dt, \quad \mu_2 = \int_0^\infty \overline{H}(t) e^{-2\lambda t}\, dt,$$

$$\mu_3 = \int_0^\infty \overline{G}(t) e^{-2\lambda t}\, dt, \quad \mu_4 = \int_0^\infty \overline{G}_s(t) e^{-2\lambda t}\, dt. \tag{4.1}$$

V. MTSF Analysis

On the basis of arguments used for regenerative processes, we obtain the expressions for cdf ($\phi_i(t)$)of first passage times from regenerative state i to a failed states

$$\phi_i(t) = \sum_{ij} \phi_{ij}(t) \& \phi_{ij}(t)$$

Their for

$$\pi_0(t) = Q_{01}(t) \& \pi_1(t) + Q_{02}(t) \& \pi_2(t),$$
$$\pi_1(t) = Q_{10}(t) \& \pi_0(t) + Q_{13}(t) \& \pi_3(t) + Q_{15},$$
$$\pi_2(t) = Q_{20}(t) \& \pi_0(t) + Q_{24}(t) \& \pi_3(t) + Q_{26},$$
$$\pi_3(t) = Q_{30}(t) \& \pi_0(t) + Q_{38}(t),$$
$$\pi_4(t) = Q_{40}(t) \& \pi_0(t) + Q_{49}(t). \tag{5.1}$$

Taking Laplace-Stieltjes transform for this equations and solving for $\tilde{\pi}_0(0)$ we get the Mean Time to System Failure (MTSF) which is given by

$$\frac{-d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{(D_1'(0) - N_1'(0))}{D_1(0)}, \tag{5.2}$$

where,

$$\text{MTSF}=\frac{\mu_0 + P_{01}\mu_1 + P_{02}\mu_2}{1 - P_{01}P_{10} + P_{02}P_{20}}. \tag{5.3}$$

VI. Availability Analysis

The following probabilistic arguments

$$M_0(t) = e^{-2\lambda t}e^{-\alpha t}, \qquad M_1(t) = e^{-2\lambda t}\overline{H}(t),$$
$$M_2(t) = e^{-2\lambda t}\overline{H}(t), \qquad M_3(t) = e^{-2\lambda t}\overline{G}(t),$$
$$M_4(t) = e^{-2\lambda t}\overline{G}_s(t). \tag{6.1}$$

Can be used in the theory of regenerative process in order to find the point wise availabilities $A_i(t)$ as shown in following recessive relations:-

$$A_0(t) = M_0(t) + Q_{01}(t) © A_1(t) + Q_{02}(t) © A_2(t),$$
$$A_1(t) = M_1(t) + Q_{10}(t) © A_0(t) + Q_{13}(t) © A_3(t) + Q_{11}^{5}(t) © A_1(t)$$
$$+ Q_{11}^{57}(t) © A_1(t),$$
$$A_2(t) = M_2(t) + Q_{20}(t) © A_0(t) + Q_{24}(t) © A_4(t) + Q_{21}^{6}(t) © A_1(t)$$
$$+ Q_{21}^{610}(t) © A_1(t),$$
$$A_3(t) = M_3(t) + Q_{30}(t) © A_0(t) + Q_{31}^{8}(t) © A_1(t),$$
$$A_4(t) = M_4(t) + Q_{40}(t) © A_0(t) + Q_{41}^{9}(t) © A_1(t). \tag{6.2}$$

Using Laplace transform for equations and solving for $A_0^*(s)$ we can calculate the steady state availability such that

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} \tag{6.3}$$

$$D_2'(0) = \mu_0\left(p_{10} + p_{13}p_{30}\right) + \mu_1\left(1 - p_{02}\left(p_{20} + p_{24}p_{40}\right)\right) + \mu_2\left(p_{02}\left(p_{10} + p_{13}p_{30}\right)\right) +$$
$$\mu_3 p_{13}\left(1 - p_{02}\left(p_{20} + p_{24}p_{40}\right)\right) + \mu_4 p_{02}p_{24}\left(p_{10} + p_{13}p_{30}\right).$$

$$N_2 = \mu_0\left(1 - p_{11}^5 - p_{13}p_{31}^8 - p_{11}^{57}\right) + \mu_1\left(p_{01} + p_{02}\left(p_{21}^6 + p_{24}p_{41}^9 + p_{21}^{610}\right)\right) +$$
$$\mu_2\left(p_{02}\left(1 - p_{11}^5 - p_{13}p_{31}^8 - p_{11}^{57}\right)\right) + \mu_3 p_{13}\left(p_{01} + p_{02}\left(p_{21}^6 + p_{24}p_{41}^9 + p_{21}^{610}\right)\right) +$$
$$\mu_4 p_{02}p_{24}\left(1 - p_{11}^5 - p_{13}p_{31}^8 - p_{11}^{57}\right).$$

VII. Busy Period Analysis

The following probabilistic arguments

$$W_1(t) = e^{-2\lambda t}\overline{H}(t), \qquad W_2(t) = e^{-2\lambda t}\overline{H}(t),$$
$$W_3(t) = e^{-2\lambda t}\overline{G}(t), \qquad W_4(t) = e^{-2\lambda t}\overline{G}_s(t). \tag{7.1}$$

Can be used in the theory of regenerative process in order to find the busy period analysis for the expert repairman $B_i(t)$ (the probability that the operative unit is under repair at time t) as shown in following recessive relations:-

$$B_0(t) = Q_{01}(t)\& B_1(t) + Q_{02}(t)\& B_2(t),$$
$$B_1(t) = W_1(t) + Q_{10}(t)\&\mathrm{B}_0(t) + Q_{13}(t)\&\mathrm{B}_3(t) + Q_{11}^5(t)\&\mathrm{B}_1(t)$$
$$+ Q_{11}^{57}(t)\&\mathrm{B}_1(t),$$
$$B_2(t) = W_2(t) + Q_{20}(t)\&\mathrm{B}_0(t) + Q_{24}(t)\&\mathrm{B}_4(t) + Q_{21}^6(t)\&\mathrm{B}_1(t)$$
$$+ Q_{21}^{610}(t)\&\mathrm{B}_1(t),$$
$$B_3(t) = W_3(t) + Q_{30}(t)\&\mathrm{B}_0(t) + Q_{31}^8(t)\&\mathrm{B}_1(t),$$
$$B_4(t) = W_4(t) + Q_{40}(t)\&\mathrm{B}_0(t) + Q_{41}^9(t)\&\mathrm{B}_1(t). \tag{7.2}$$

After using Laplace transform for equations and solving for $B_0^*(s)$ we can calculate the busy period analysis steady such that

$$B_0 = \lim_{s\to o} s\widetilde{B}_0(s) = \frac{N_3(0)}{D_2'(0)} \tag{7.3}$$

where,

$$N_3 = \mu_1\left(p_{01} + p_{02}\left(p_{21}^6 + p_{24}p_{41}^9 + p_{21}^{610}\right)\right) + \mu_2\left(p_{02}\left(1 - p_{11}^5 - p_{13}p_{31}^8 - p_{11}^{57}\right)\right) +$$
$$\mu_3 p_{13}\left(p_{01} + p_{02}\left(p_{21}^6 + p_{24}p_{41}^9 + p_{21}^{610}\right)\right) + \tag{7.4}$$
$$\mu_4 p_{02}p_{24}\left(1 - p_{11}^5 - p_{13}p_{31}^8 - p_{11}^{57}\right).$$

VIII. The Expected Profit Gained In (0, T]

Profit = total revenue in (0,t]- total expenditure incurred in (0,t] i.e.
At steady state the net expected profit per unit of time is:

$$\text{Profit} = \lim_{t\to\infty} G(t)/t = C_1A_0 - C_2B_0, \tag{8.1}$$

where,

C_1 : is revenue per unit uptime by the system.

C_2 : is per unit repair cost.

IX. Special Cases

The failure and repair times are exponential distributions

$$G(t)=1-e^{-\beta t} \qquad ,G_s(t)=1-e^{-\theta t}, \qquad H(t)=1-e^{-\gamma t}. \tag{9.1}$$

3.10- GRAPHICAL REPRESENTATION

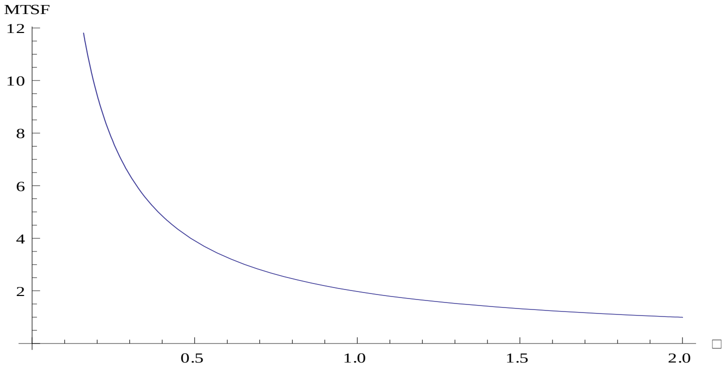


Fig. 2 MTSF vs. failure rate λ

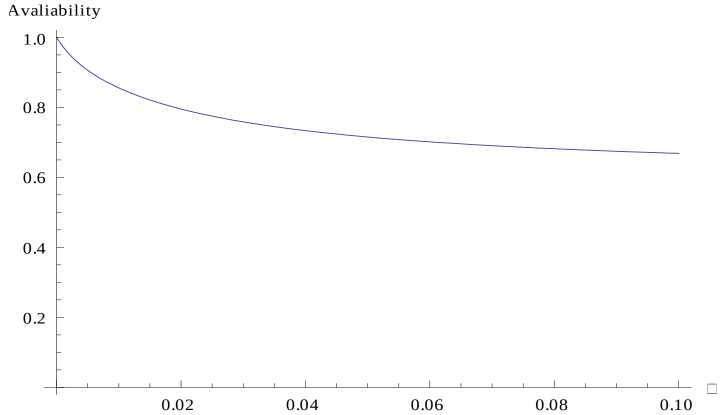


Fig. 3 Availability vs. failure rate λ

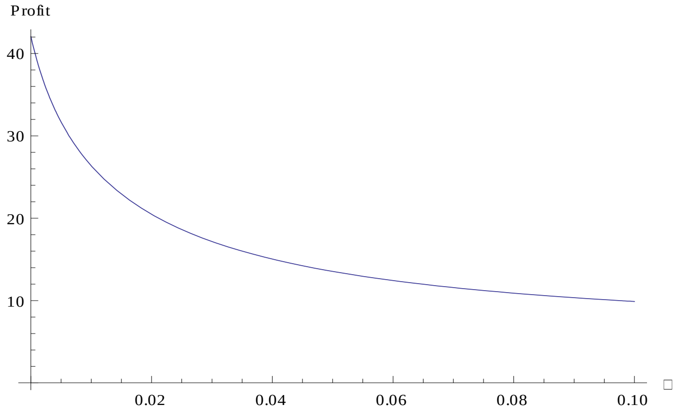


Fig. 4 Profit vs. failure rate λ

X. Summary

A 2(k)-out-of-3(n) worm standby system of identical units with arbitrary distribution of repair and inspection under operational restrictions is studied. Expressions for various system performance characteristics are drawn by using semi-Markov processes and re-generative point technique. By using these expressions, the analytical as well numerical solutions of measures of performance can be obtained for the system in transient and steady states.

In each figure we vary the parameter in question and fix the rest for consistency. It is evident from figures 2 – 4 that the increase in deterioration or failure rates induces decrease in MTSF, availability and profit.

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