OPEN ACCESS

International
Journal
Of Modern Engineering Research (IJMER)

Propabilistic Analysis of a Man-Machine System Operating Subject To Different Physical Conditions

G. S. Mokaddis[1], Y. M. Ayed[2], H. S. Al-Hajeri[3]

[1]*Ain shams University, Department of Mathematics, Faculty of Science, Cairo, Egypt.*
[2]*Suez University. Department of Mathematics, Faculty of Science, Suez, Egypt*
[3]*Kuwait University. Faculty of Science, Kuwait*

ABSTRACT: *This paper deals with the stochastic behavior of a single unit of man-machine system operating under different physical conditions. Assuming that the failure, repair and physical conditions (good - poor) times are stochastically independent random variables each having an arbitrary distribution. The system is analyzed by the semi-Markov process technique. Some reliability measures of interest to system designers as well as operations managers have been obtained. Explicit expressions for the Laplace-Stieltjes transforms of the distribution function of the first passage time, mean time to system failure, pointwise availability, and steady state availability of the system are obtained . Busy period by the server, expected number of visits by the server and the cost per unit time in steady state of the system are also obtained. Several important results have been derived as particular cases.*

Keywords: *Availability, Failure rate , Cost function, Busy period.*

I. Introduction and Description of The System

Many authors have studied the single-unit system under different conditions and obtain various reliability parameters by using the theory of regenerative process, Markov renewal process and semi-Markov process [2, 3].

This paper investigates the model of a single-unit operating by a person who may be in good or poor physical condition. The failure, physical conditions and repair times are stochastically independent random variables each having an arbitrary distribution. The unit may fail in one of three ways, the first is due to hardware failure, the second is due to human error when operator is in good physical condition and the third is due to human error when operator is in poor physical condition. The operator reports to work in good physical condition which may change to poor is generally distributed. He can revive to good physical condition with another arbitrary distribution. It is assumed that when the system is down and the operator is in good physical condition, it can’t determine as he is supposed to be at rest. Repair time distributions for the three types of failure are taken arbitrary. Repair facility is always available with the system to repair the failed unit and after repair of the unit becomes like new. Using the semi-Markov process technique, and the results of the regenerative process, several reliability measures of interest to system designers are obtained as the distribution time to the system failure. The mean time to system failure, pointwise availability and steady state availability, busy period by the server, expected number of visits by the server and the cost per unit time in a steady state of the system are also obtained. The results obtained by [5,6] are derived from the present paper as special cases. In this system the following assumptions and notations are used to analysis the system.

(1) The system consists of a single unit which can operate by a person in good or poor physical condition.
(2) The unit fails in one of three ways; the first is due to hardware failure, the second is due to human error when operator is in good physical condition and the third is due to human error when operator is in poor physical condition.
(3) Failure, physical conditions and repair times are stochastically independent random variables each having an arbitrary distribution.
(4) The operator reports to work in good physical condition which may change to poor and vice versa are stochastically independent random variables each having an arbitrary distribution.
(5) When the system is down and the operator is in good physical condition, it cannot deteriorate as he is supposed to be at rest.
(6) There is a single repair facility with the system to repair the failed unit.
(7) On repair of the failed unit, it acts like a new unit.
(8) All random variables are mutually independent.

II. Notations and States of the System

E_0 state of the system at epoch t = 0,

E set of regenerative states; { $S_0, S_1, S_2, S_3, S_4, S_5$ }, as in fig. 1,

$\bar{E}$ set of non-regenerative state;{ S_6, S_7 },as in fig. 1

$f(t), F(t)$ pdf and cdf of failure time of the unit due to hardware failure,

$f_1(t), F_1(t)$ pdf and cdf of failure time of the unit due to human error; where, the operator is in good physical condition,

$f_2(t), F_2(t)$ pdf and cdf of failure time of the unit due to human error; where, the operator is in poor physical condition,

$l(t), L(t)$ pdf and cdf of change of physical condition from good mode to poor mode,

$h(t), H(t)$ pdf and cdf of change of physical condition from poor mode to good mode,

$g(t), G(t)$ pdf and cdf of time to repair the unit from hardware failure,

$g_1(t), G_1(t)$ pdf and cdf of time to repair the unit from human error; where the operator is in good physical condition,

$g_2(t), G_2(t)$ pdf and cdf of time to repair the unit due to human error; where the operator is in poor physical condition,

$q_{ij}(t), Q_{ij}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state i or to a failed state j without visiting any other regenerative state in (0, t]; i, j $\in$ E,

$q_{ij}^{(k)}(t), Q_{ij}^{(K)}(t)$ pdf and cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]; $i, j \in E, K \in \bar{E}$,

p_{ij} one step transition probability from state I to state j; $i, j \in E,$

p_{ij}^{k} probability that the system in state i goes to state j passing through state k; $i, j \in E, K \in \bar{E}$

$\pi_i(t)$ cdf of first passage time from regenerative state i to a failed state,

$A_i(t)$ probability that the system is in upstate at instant t given that the system started from regenerative state i at time t = 0,

$M_i(t)$ probability that the system having started from state i is up at time t without making any transition into any other regenerative state,

$B_i(t)$ probability that the server is busy at time t given that the system entered regenerative state i at time t = 0,

$V_i(t)$ expected number of visits by the server given that the system started from regenerative state i at time t = 0,

μ_{ij} contribution mean sojourn time in state i when transition is to state j is $-\tilde{Q}_{ij}'(0) = q_{ij}^{*}(0)$,

μ_i Mean sojourn time in state i, $\mu_i = \sum_j [\mu_{ij} + \sum_k \mu_{ij}^{(k)}]$,

~ Symbol for Laplace-Stieltjes transform, e.g. $\tilde{F}(s) = \int e^{-st} d\,F(t)$,

* Symbol for Laplace transform, e.g. $f^*(s) = \int e^{-st} f(t)\, dt,$

Ⓢ Symbol for Stieltjes convolution, e.g. A(t) Ⓢ B(t) = $\int_0^t B(t-u)\, dA(u)$,

© Symbol for ordinary convolution, e.g. $a(t) © b(t) = \int_0^t a(u)\, b(t-u)\, du$

For simplicity, whenever integration limits are (0, ∞), they are not written.

Symbols used for the state

o Operative unit,
d The physical condition is good,
p The physical condition is poor,
r The failed unit is under repair when failed due to hardware failure,

r_1 The failed unit is under repair when failed due to human error; where the operator is in good physical condition,

r_2 The failed unit is under repair when failed due to human error; where the operator is in poor physical condition,

R The unit is in continued repair; where the failure is due to hardware failure,
R_2 The unit is in continued repair when failed due to human error; where the operator is in poor physical condition.

Considering these symbols, the system may be in one of the following states at any instant where the first letter denotes the mode of unit and the second corresponds to physical condition

$S_0 \equiv (o\,,\,d)$, $S_1 \equiv (o\,,\,p)$, $S_2 \equiv (r\,,\,d)$, $S_3 \equiv (r_1\,,\,d)$,
$S_4 \equiv (r\,,\,p)$, $S_5 \equiv (r_2\,,\,p)$, $S_6 \equiv (R\,,\,d)$, $S_7 \equiv (R_2\,,\,d)$.

Stated and possible transitions between them are shown in Fig. 1.

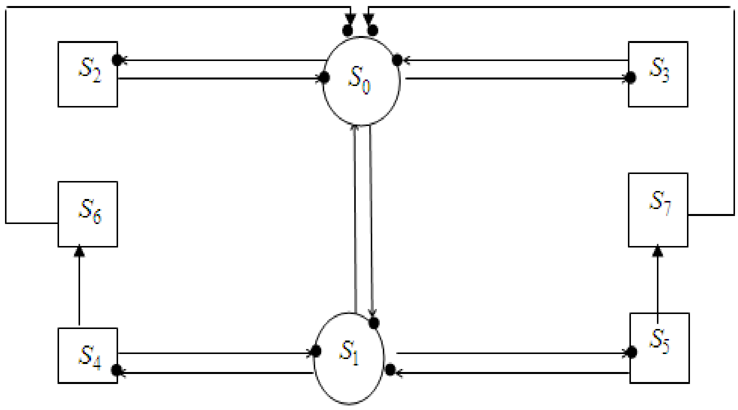

Fig.1 state transition diagram

III. Transition Probabilities And Mean Sojourn Times

It can be observed that the time points of entry into $S_i \in E$,i=0,1,2,3,4,5 are regenerative points so these states are regenerative. Let T_0 (≡ 0) , T_1 , T_2 , . . . denote the time points at which the system enters any state $S_i \in E$ and X_n denotes the state visited at the time point T_{n+1} , i.e. just after the transition at T_{n+1} , then $\{X_n , T_n\}$ is a Markov-renewal process with state space E and

$Q_{ij} = P\,[\, X_{n+1} = j\,,\ T_{n+1} = T_n < t \mid X_n = i\,]$

is a semi-Markov kernel over E. The stochastic matrix of the embedded Markov chain is

$P = (p_{ij}) = (Q_{ij}(\infty)) = Q(\infty)$ and the nonzero elements p_{ij} are

$p_{01} = \int \ell(t)\, \overline{F}(t)\, \overline{F}_1(t)\, dt$, $p_{02} = \int f(t)\, \overline{L}(t)\, \overline{F}_1(t)\, dt$,

$p_{03} = \int f_1(t)\,\bar{L}(t)\,\bar{F}(t)\,dt$, $p_{10} = \int h(t)\,\bar{F}(t)\,\bar{F}_2(t)\,dt$,

$p_{14} = \int f(t)\,\bar{F}_2(t)\,\bar{H}(t)\,dt$, $p_{15} = \int f_2(t)\,\bar{F}(t)\,\bar{H}(t)\,dt$,

$p_{20} = p_{30} = 1$, $p_{41} = \int \mathbf{g(t)\,\bar{H}(t)\,dt}$,

$p_{46} = \int h(t)\,\bar{G}(t)\,dt$, $p_{40}^{(6)} = \iint h(u)\,g(t)\,\bar{L}(t-u)\,dt\,du$,

$p_{51} = \int g_2(t)\,\bar{H}(t)\,dt$, $p_{57} = \int h(t)\,\bar{G}_2(t)\,dt$,

$$p_{50}^{(7)} = \iint h(u)\,g_2(t)\,\bar{L}(t-u)\,du\,dt. \tag{3.1}$$

The mean sojourn times μ_i in state S_i are

$\mu_0 = \int \bar{F}(t)\,\bar{F}_1(t)\,\bar{L}(t)\,dt$, $\mu_1 = \int \bar{F}(t)\,\bar{F}_2(t)\,\bar{H}(t)\,dt$,

$\mu_2 = \int \bar{G}(t)\,\bar{L}(t)\,dt$, $\mu_3 = \int \bar{G}_1(t)\,\bar{L}(t)\,dt$,

$\mu_4 = \int \bar{G}(t)\,\bar{H}(t)\,dt$, $\mu_5 = \int \bar{G}_2\,\bar{H}(t)\,dt$, (.3.2)

IV. Mean Time To System Failure

Time to system failure can be regarded as the first passage to failed states S_6 , S_7 which are considered as absorbing. By probabilistic arguments, the following recursive relations for $\pi_i(t)$ are obtained

$\pi_0(t) = Q_{02}(t) + Q_{03}(t) + Q_{01}(t)$ Ⓢ $\pi_1(t)$,

$\pi_1(t) = Q_{14}(t) + Q_{15}(t) + Q_{10}(t)$ Ⓢ $\pi_0(t)$ (4.1)

Taking Laplace-Stieltjes transforms of equations (4.1) and solving for $\tilde{\pi}_0(s)$, dropping the argument “s” for brevity, it follows

$\tilde{\pi}_0(s) = N_0(s) / D_0(s)$, (4.2)

where

$N_0(s) = \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{01}(\tilde{Q}_{14} + \tilde{Q}_{15})$

and

$D_0(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{10}$. (4.3)

The mean time to system failure with starting state S_0 is given by

$MTSF = N_0 / D_0$, (4.4)

where

$N_0 = \mu_0 + p_{01}\,\mu_1$

and

$D_0 = 1 - p_{01}\,p_{10}$. (4.5)

V. Availability Analysis

Elementary probability arguments yield the following relations for $A_i(t)$

$A_0(t) = M_0(t) + q_{01}(t)$ © $A_1(t) + q_{02}(t)$ © $A_2(t) + q_{03}(t)$ © $A_3(t)$,

$A_1(t) = M_1(t) + q_{10}(t)$ © $A_0(t) + q_{14}(t)$ © $A_4(t) + q_{15}(t)$ © $A_5(t)$,

$A_2(t) = q_{20}(t)$ © $A_0(t)$,

$A_3(t) = q_{30}(t)$ © $A_0(t)$,

$A_4(t) = q_{41}(t)$ © $A_1(t) + q_{40}^{(6)}(t)$ © $A_0(t)$,

$A_5(t) = q_{51}(t) © A_1(t) + q_{50}^{(7)}(t) © A_0(t)$. (5.1)

where

$M_0(t) = \overline{F}(t)\,\overline{F}_1(t)\,\overline{L}(t)$, $M_1(t) = \overline{F}(t)\,\overline{F}_2(t)\,\overline{H}(t)$. (5.2)

Taking Laplace transforms of equations (5.1) and solving for $A_0^*(s)$, it gives

$A_0^*(s) = A_1(s) / D_1(s)$. (5.3)

where

$N_1(s) = M_0^*(1-q_{14}^*q_{41}^* - q_{15}^*q_{51}^*) + M_1^*q_{01}^*$

and

$D_1(s) = (1-q_{02}^*q_{20}^* - q_{03}^*q_{30}^*)(1-q_{14}^*q_{41}^* - q_{15}^*q_{51}^*) - q_{01}^*(q_{10}^* + q_{14}^*q_{40}^{(6)*} + q_{15}^*q_{50}^{(7)*})$,

(5.4)

The steady state availability of the system is

$A_0(\infty) = N_1 / D_1$, (5.5)

where

$N_1 = \mu_0\,(1 - p_{14}p_{41} - p_{15}p_{51}) + \mu_1 p_{01}$

and

$D_1 = (1 - p_{01})\,(\mu_{14}p_{41} + p_{14}\mu_{41} + p_{15}\mu_{51} + \mu_{15}p_{51})$

$+ (1 - p_{14}p_{41} - p_{15}p_{51})\,(\mu_{02} + p_{02}\mu_{20} + p_{03}\mu_{30} + \mu_{03}) + \mu_{01}\,(p_{10} + p_{14}\,p_{40}^{(6)} + p_{15}\,p_{50}^{(7)}) + p_{01}\,(\mu_{10} + \mu_{14}\,p_{40}^{(6)} + p_{14}\,\mu_{40}^{(6)}$

$+ p_{15}\,\mu_{50}^{(7)} + \mu_{15}\,p_{50}^{(7)})$. (5.6)

VI. Busy Period Analysis

Elementary probability arguments yield the following relations for $B_i(t)$

$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t) + q_{03}(t) © B_3(t)$,

$B_1(t) = q_{10}(t) © B_0(t) + q_{14}(t) © B_4(t) + q_{15}(t) © B_5(t)$,

$B_2(t) = V_2(t) + q_{20}(t) © B_0(t)$,

$B_3(t) = V_3(t) + q_{30}(t) © B_0(t)$,

$B_4(t) = V_4(t) + q_{41}(t) © B_1(t) + q_{40}^{(6)}(t) © B_0(t)$,

$B_5(t) = V_5(t) + q_{51}(t) © B_1(t) + q_{50}^{(7)}(t) © B_0(t)$,

(6.1)

where

$V_2(t) = \overline{G}(t)\,\overline{L}(t)$, $V_3(t) = \overline{G}_1(t)\,\overline{L}(t)$,

$V_4(t) = \overline{G}(t)\,\overline{H}(t)$, $V_5(t) = \overline{G}_2(t)\,\overline{H}(t)$.

Taking Laplace transforms of equations (6.1) and solving for $B_0^*(s)$, it gives

$B_0^*(s) = N_2(s) / D_1(s)$, (6.2)

where

$N_2(s) = (q_{02}^*V_2^* + q_{03}^*V_3^*)(1-q_{14}^*q_{41}^* - q_{15}^*q_{51}^*) + q_{01}^*(q_{14}^*V_4^* + q_{15}^*V_5^*)$, (6.3)

and

$D_1(s)$ is given by (5.4).

In long run the fraction of time for which the server is busy is given by

$B_0(\infty) = N_2 / D_1$, (6.4)

where

$N_2 = (p_{02}\mu_2 + p_{03}\mu_3)\,(1 - p_{14}p_{41} - p_{15}p_{51}) + p_{01}\,(p_{14}\mu_4 + p_{15}\mu_5)$ (6.5)

and

D_1 is given by (5.6) .

The expected busy period of server facility in (0, t] is

$\mu_b(t)$ = expected busy time of the repairman in (0, t] .

The repairman may be busy during (0, t] starting from initial state S_0 .

Hence

$$\mu_b(t) = \int_0^t B_0(u)\,du ,$$

so that

$$\mu_b^*(s) = B_0^*(s)/s .$$

Thus one can evaluate $\mu_b(t)$ by taking inverse Laplace transform of $\mu_b^*(s)$.
Expected idle time of the repairman in (0, t] is
$\mu_1(t) = 1 - \mu_b(t)$.

VII. Expected Number of Visits by The Repairman

Elementary probability arguments yield the following relations for $B_i(t)$

$V_0(t) = Q_{01}(t) \circledS [1 + V_1(t)] + Q_{02}(t) \circledS [1 + V_2(t)] + Q_{03}(t) \circledS [1 + V_3(t)]$,
$V_1(t) = Q_{10}(t) \circledS [1 + V_0(t)] + Q_{14}(t) \circledS [1 + V_4(t)] + Q_{15}(t) \circledS [1 + V_5(t)]$,
$V_2(t) = Q_{20}(t) \circledS V_0(t)$,
$V_3(t) = Q_{30}(t) \circledS V_0(t)$,
$V_4(t) = Q_{41}(t) \circledS V_1(t) + Q_{40}^{(6)}(t) \circledS V_0(t)$,
$V_5(t) = Q_{51}(t) \circledS V_1(t) + Q_{50}^{(7)}(t) \circledS V_0(t)$, (7.1)

Taking Laplace-Stieltjes transforms of equations (7.1) and solving for $V_0^*(s)$, dropping the argument "s" for brevity, it follows

$$V_0^*(s) = N_3(s) / D_2(s) , \quad (7.2)$$

where

$$N_3(s) = (1-\tilde{Q}_{01}+\tilde{Q}_{02}+\tilde{Q}_{03})(1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51})+\tilde{Q}_{01}(\tilde{Q}_{10}+\tilde{Q}_{14}+\tilde{Q}_{15})$$

and

$$D_2(s) = (1-\tilde{Q}_{02}\tilde{Q}_{20}-\tilde{Q}_{03}\tilde{Q}_{30})(1-\tilde{Q}_{14}\tilde{Q}_{41}-\tilde{Q}_{15}\tilde{Q}_{51}) - \tilde{Q}_{01}(\tilde{Q}_{10}+\tilde{Q}_{14}\tilde{Q}_{40}^{(6)}+\tilde{Q}_{15}\tilde{Q}_{50}^{(7)}) \quad (7.3)$$

In steady state, number of visits per unit is given by

$$V_0(\infty) = N_3 / D_2 , \quad (7.4)$$

where

$N_3 = 1 + p_{01} - p_{14}p_{41} - p_{15}p_{51}$

and

$D_2 = p_{01}\,[1 - p_{10} - p_{14}(p_{41} + p_{40}^{(6)}) - p_{15}(p_{51} + p_{50}^{(7)})]$.

VIII. Cost Analysis

The cost function of the system obtained by considering the mean-up time of the system, expected busy period of the server and the expected number of visits by the server, therefore, the expected profit incurred in (0, t] is

C(t) = expected total revenue in (0, t]
 – expected total service cost in (0, t]
 – expected cost of visits by server in (0, t]
 $= K_1\,\mu_{up}(t) - K_2\,\mu_b(t) - K_3\,V_0(t)$. (8.1)

The expected profit per unit time in steady-state is

$$C = K_1 A_0 - K_2 B_0 - K_3 V_0 \quad (8.2)$$

where K_1 is the revenue per unit up time, K_2 is the cost per unit time for which system is under repair and K_3 is the cost per visit by repair facility.

IX. Special Cases

9.1. The single unit with failure and repair exponentially distributed :

Let

α failure rate of the unit due to hardware failure ,

β	failure rate of the unit due to human error; where the operator is in good physical condition ,
γ	failure rate of the unit due to human error; where the operator is in poor physical condition ,
δ	change of physical condition rate from good mode to poor mode ,
θ	change of physical condition rate from poor mode to good mode ,
ω	repair rate of the unit from hardware failure ,
λ	repair rate of the unit from human error; where the operator is in good physical condition ,
ε	repair rate of the unit from human error; where the operator is in poor physical condition .

Transition probabilities are

$p_{01} = \delta / (\delta + \alpha + \beta)$, $p_{02} = \alpha / (\delta + \alpha + \beta)$, $p_{03} = \beta / (\delta + \alpha + \beta)$, $p_{10} = \theta / (\theta + \alpha + \gamma)$,
$p_{14} = \alpha / (\theta + \alpha + \gamma)$, $p_{15} = \gamma / (\theta + \alpha + \gamma)$, $p_{41} = \omega / (\omega + \theta)$, $p_{46} = \theta / (\omega + \theta)$,

$p_{51} = \varepsilon / (\varepsilon + \theta)$, $p_{57} = \theta / (\varepsilon + \theta)$, $p_{40}^{(6)} = \theta\, \omega / (\theta + \omega)\, (\omega + \delta)$,

$p_{50}^{(7)} = \theta\, \varepsilon\, \omega / (\theta + \varepsilon)\, (\varepsilon + \delta)$.

The mean sojourn times are

$\mu_0 = 1 / (\alpha + \beta + \delta)$, $\mu_1 = 1 / (\alpha + \gamma + \theta)$, $\mu_2 = 1 / (\omega + \delta)$, $\mu_3 = 1 / (\lambda + \delta)$,
$\mu_4 = 1 / (\omega + \theta)$, $\mu_5 = 1 / (\varepsilon + \theta)$.

MTSF= $\hat{N}_2 / \hat{D}_1$ where $\hat{N}_0 = \frac{1}{(\delta+\alpha+\beta)}\left[1+\frac{\delta}{(\alpha+\gamma)}\right]$, $\hat{D}_0 = 1-\frac{\delta\theta}{(\alpha+\beta+\delta)(\alpha+\gamma+\theta)}$

in this case, $\hat{M}_i(t)$ are

$$\hat{M}_0(t) = e^{-(\alpha+\beta+\delta)t} , \qquad \hat{M}_1(t) = e^{-(\alpha+\gamma+\theta)t} .$$

The steady state availability of the system is

$\hat{A}_0(\infty) = \hat{N}_1 / \hat{D}_1$ where, $\hat{N}_1 = \frac{1}{(\delta+\alpha+\beta)}\left\{1-\frac{1}{(\alpha+\gamma+\theta)}\left[\frac{\alpha\omega}{(\omega+\theta)}+\frac{\gamma\varepsilon}{(\varepsilon+\theta)}\right]+\frac{\delta}{(\alpha+\gamma+\theta)}\right\}$,

$$\hat{D}_1 = \frac{1}{(\alpha+\gamma+\theta)},\left[1-\frac{\delta}{(\alpha+\beta+\delta)}\right]$$
$$\left\{\frac{\alpha\omega}{(\omega+\theta)}\left[\frac{1}{(\alpha+\gamma+\theta)}+\frac{1}{(\omega+\theta)}\right]+\frac{\gamma\theta}{(\varepsilon+\theta)}\left[\frac{1}{(\alpha+\gamma+\theta)}+\frac{1}{(\omega+\theta)}\right]\right\}$$
$$+\frac{1}{(\alpha+\beta+\delta)}\left\{1-\frac{1}{(\theta+\alpha+\delta)}\left[\frac{\alpha\omega}{(\omega+\theta)}+\frac{\gamma\varepsilon}{(\varepsilon+\theta)}\right]\right\}$$
$$\left[\frac{(\alpha+\beta)}{(\alpha+\beta+\delta)}+\frac{\alpha}{\omega}+\frac{\beta}{\lambda}\right]+\frac{\delta\theta}{(\alpha+\beta+\delta)^2(\alpha+\gamma+\theta)}$$
$$\left\{\left[1+\frac{\alpha\omega}{(\theta+\omega)(\omega+\delta)}+\frac{\gamma\varepsilon}{(\theta+\varepsilon)(\varepsilon+\delta)}\right]+2(\delta+\theta)\left[\frac{\alpha\omega^2}{(\omega+\theta)^2+(\omega+\delta)^2}+\frac{\gamma\varepsilon^2}{(\theta+\varepsilon)^2(\delta+\varepsilon)^2}\right]\right\},$$

in this case $\hat{V}_i(t)$ are

$$\hat{V}_2(t) = e^{-(\omega+\delta)t}, \hat{V}_3(t) = e^{-(\lambda+\delta)t}$$
$$\hat{V}_4(t) = e^{-(\omega+\theta)t}, \hat{V}_5(t) = e^{-(\varepsilon+\theta)t} .$$

In long run, the function of time for which the server is busy is given by

$\hat{B}_0(\infty) = \hat{N}_2 / \hat{D}_1$,

where

$$\hat{N}_2 = \frac{1}{(\delta+\alpha+\beta)}\left[\frac{1}{(\omega+\delta)}+\frac{1}{(\lambda+\delta)}\right]\left\{1-\frac{1}{(\theta+\alpha+\gamma)}\left[\frac{\alpha\omega}{(\omega+\theta)}+\frac{\gamma\varepsilon}{(\varepsilon+\theta)}\right]\right\}$$

$$+\frac{\delta}{(\delta+\alpha+\beta)(\theta+\alpha+\gamma)}\left[\frac{\alpha}{(\omega+\theta)}+\frac{\gamma}{(\varepsilon+\theta)}\right].$$

In steady state, number of visits per unit is given by

$\hat{V}_0(\infty)=\hat{N}_3/\hat{D}_2$, where , $\hat{N}_3=1+\frac{\delta}{(\delta+\alpha+\beta)}-\frac{1}{(\theta+\alpha+\gamma)}\left[\frac{\alpha\omega}{(\omega+\theta)}+\frac{\gamma\varepsilon}{(\varepsilon+\theta)}\right]$,

$$\hat{D}_2=\frac{\delta}{(\delta+\alpha+\beta)}\left\{1-\frac{\theta}{(\theta+\alpha+\gamma)}-\frac{\alpha\omega}{(\theta+\alpha+\gamma)(\omega+\theta)}\left[1+\frac{\theta}{(\omega+\theta)}\right]\quad\frac{\gamma\varepsilon}{(\theta+\alpha+\gamma)(\varepsilon+\theta)}\left[1+\frac{\theta}{(\varepsilon+\delta)}\right]\right\}$$

The expected profit per unit time in steady state is

$$\hat{C}=K_1\hat{A}_0-K_2\hat{B}_0-K_3\hat{V}_0\ .$$

9.2 Numerical Example :

Let $K_1 = 2000$, $K_2 = 100$, $K_3 = 50$, $\beta = 0.3$, $\gamma = 0.7$, $\theta = 0.5$, $\omega = 0.6$, $\lambda = 0.4$, $\varepsilon = 0.1$

Table 1

α	C		
	δ = 0.3	δ = 0.5	δ = 0.8
0.1	1024.2690	1298.3750	1564.9260
0.2	826.5411	1075.1180	1322.3090
0.3	639.8087	890.6446	1123.4460
0.4	476.2774	734.9694	957.9896
0.5	330.6919	601.2720	818.3685
0.6	199.2988	484.7160	698.9977
0.7	100.3174	381.7681	595.7105
0.8	51.9091	189.7883	505.3575
0.9	23.8333	206.7642	425.5283

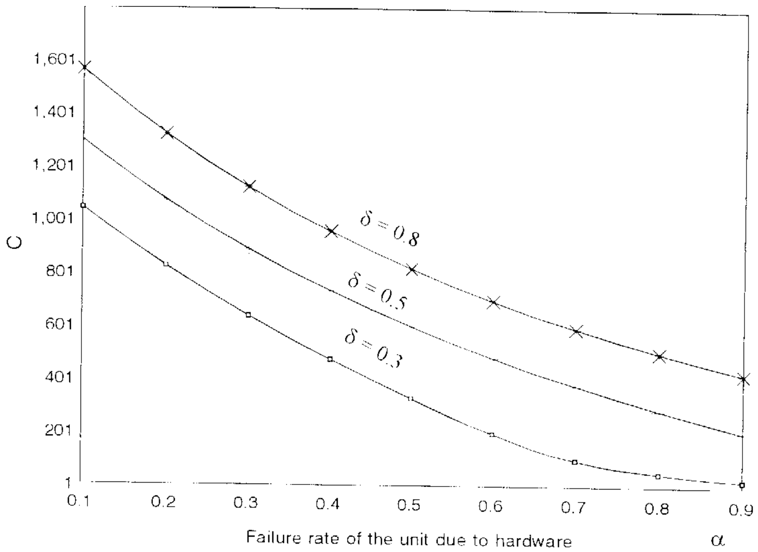

Fig. 2

Relation between the failure rate of the unit due to hardware failure and the cost per unit time.

Let $K_1 = 2000$, $K_2 = 100$, $K_3 = 50$, $\alpha = 0.5$, $\gamma = 0.4$,$\theta = 0.5$, $\omega = 0.6$, $\lambda = 0.5$, $\varepsilon = 0.1$

Table 2

β	C		
	δ = 0.4	δ = 0.6	δ = 0.8
0.1	998.1833	1226.7330	1401.331
0.2	785.9746	1014.0950	1183.079
0.3	612.1279	842.9411	1008.1970
0.4	464.8877	700.7023	863.8236
0.5	336.9454	579.4864	741.7718
0.6	223.4917	474.0720	636.5713
0.7	121.2211	380.8579	544.4236
0.8	27.7779	297.2745	462.6036
0.9	10.5644	221.4357	389.1040

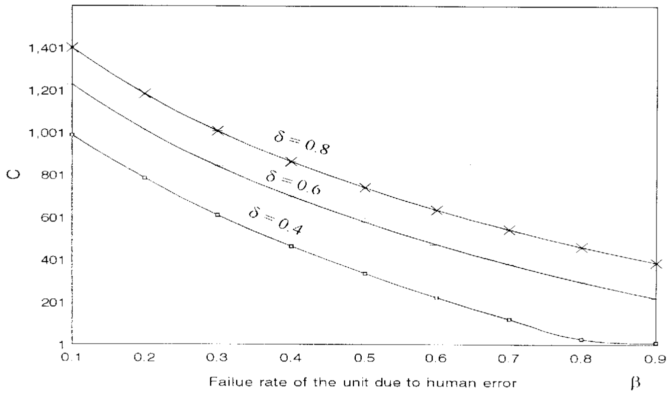

Fig. 3

Relation between the failure rate of the unit due to human error ; where the operator is in good physical condition and the cost per unit time

Let $K_1 = 5000$, $K_2 = 150$, $K_3 = 20$, $\alpha = 0.3$, $\beta = 0.1$,$\delta = 0.7$, $\omega = 0.1$, $\lambda = 0.1$, $\varepsilon = 0.1$

Table 3

γ	C		
	θ = 0.2	θ = 0.4	θ = 0.6
0.1	1884.574	1657.749	1479.912
0.2	1808.792	1628.560	1474.749
0.3	1754.425	1603.204	1466.649
0.4	1717.041	1582.434	1458.143
0.5	1693.305	1566.145	1450.294
0.6	1680.808	1553.974	1443.551
0.7	1677.841	1545.511	1438.081
0.8	1673.205	1540.367	1433.920
0.9	1669.083	1538.083	1431.036

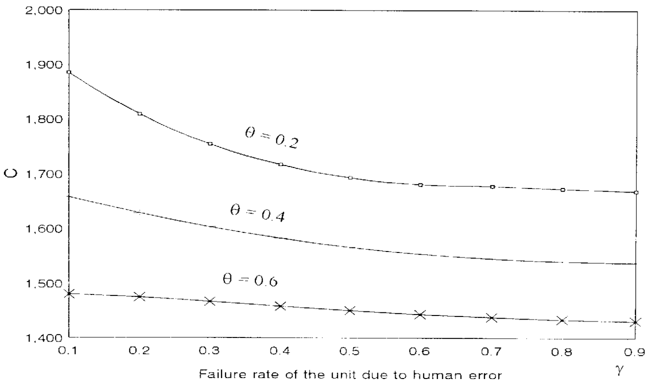

Fig. 4

Relation between the failure rate of the unit due to human error ; where the operator is in poor physical condition and the cost per unit time.

Let β = 0.3 , γ = 0.7 , θ = 0.5 .

Table 4

α	MTSF		
	δ = 0.2	δ = 0.5	δ = 0.8
0.1	2.2059	1.9565	1.8103
0.2	1.8182	1.6522	1.5493
0.3	1.5455	1.4286	1.3529
0.4	1.3433	1.2575	1.2000
0.5	1.1875	1.1224	1.0776
0.6	1.0638	1.0132	0.9774
0.7	0.9633	0.9231	0.8940
0.8	0.8800	0.8475	0.8235
0.9	0.8099	0.7831	0.7632

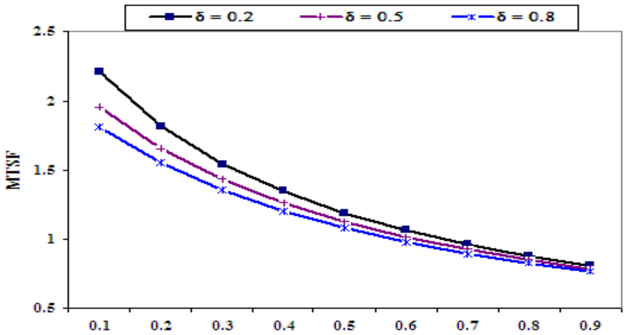

Fig. 5. α

Relation between the failure rate of the unit due to hardware failure and the mean time to system failure.

Let α = 0.3 , γ = 0.9 , θ = 0.5 .

Table 5

β	MTSF		
	δ = 0.2	δ = 0.5	δ = 0.8
0.1	2.0652	1.7188	1.5244
0.2	1.7431	1.5172	1.3812
0.3	1.5079	1.3580	1.2626
0.4	1.3287	1.2291	1.1628
0.5	1.1875	1.1224	1.0776
0.6	1.0734	1.0329	1.0040
0.7	0.9794	0.9565	0.9398
0.8	0.9005	0.8907	0.8834
0.9	0.8333	0.8333	0.8333

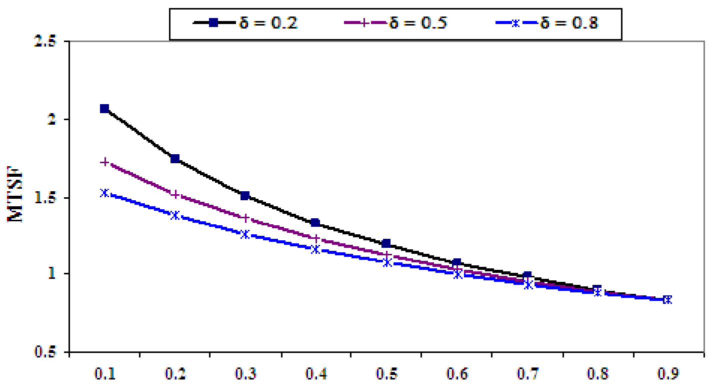

Fig. 6. β

Relation between the failure rate of the unit due to human error ; where the operator is in good physical condition and the mean time to system failure.

Let $\alpha = 0.2$, $\beta = 0.9$, $\delta = 0.1$.

Table 6

	MTSF		
γ	θ = 0.1	θ = 0.4	θ = 0.7
0.1	1.0638	1.000	0.9735
0.2	1.0169	0.9783	0.9600
0.3	0.9859	0.9615	0.9489
0.4	0.9639	0.9483	0.9396
0.5	0.9474	0.9375	0.9317
0.6	0.9346	0.9286	0.9249
0.7	0.9244	0.9215	0.9189
0.8	0.9160	0.9146	0.9137
0.9	0.9091	0.9091	0.9091

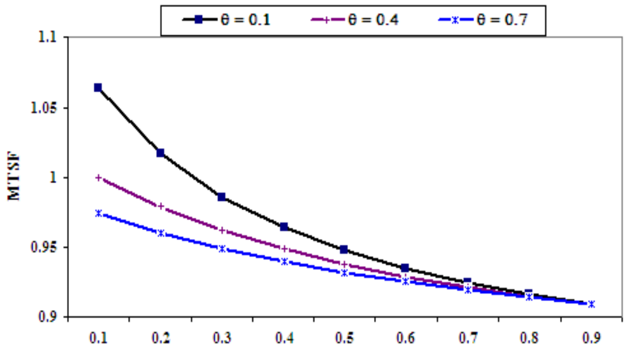

Fig. 7. γ

Relation between the failure rate of the unit due to human error ; where the operator is in poor physical condition and the mean time to system failure.

X. Summary

Expressions for various system performance characteristics are drawn by using semi-Markov processes and regenerative point technique. By using these expressions, the analytical as well numerical solutions of measures of performance can be obtained for the system in transient and steady states.

In each figure we vary the parameter in question and fix the reset for consistency. It is evident from figures 2-7 that the increase in failure rates (hardware failure and human error where the operating is in good /bad physical condition) induces decrease in MTSF, and cost profit.

REFERENCES

[1]. Barlow, R.E And Proschan, F., "Mathematical Theory Of Reliability", New York, John Wiley, 1965.
[2]. Dhillon, B.S., "Stochastic Models For Producing Human Reliability", Microelectron. Reliab., 22, 491, 1982.
[3]. Dhillon, B.S., "On Human Reliability Bibliography", Microelectron. Reliab., 20, 371, 1980.
[4]. Feller, W., "An Introduction To Probability Theory And Its Applications", Col. 2, New York, John Wiley, 1957.
[5]. Goel, L.R., Kumar, A., Rastogi, A.K., "Stochastic Behaviour Of Man-Machine Systems Operating Under Different Weather Conditions", Microelectron. Reliab., 25, No. 1, 87-91, 1985.
[6]. Mokaddis, G.S., Tawfek, M.L., El-Hssia, S.A.M., "Reliability Analysis Of Man-Machine System Operating Subject To Physical Conditions", Has Been Accepted For Publication In "Microelectronics And Reliability", England, 1996.