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Stability of Simply Supported Square Plate with Concentric Cutout

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ABSTRACT: *The finite element method is used to obtain the elastic buckling loads for simply supported isotropic square plate containing circular, square and rectangular cutouts. ANSYS finite element software had been used in the study. The applied inplane loads considered are uniaxial and biaxial compressions. In all the cases the load is distributed uniformly along the plate outer edges. The effects of the size and shape of concentric cutouts with different plate thickness ratios and having all-round simply supported boundary condition on the plate buckling strength have been considered in the analysis. It is found that cutouts have considerable influence on the buckling load factor k and the effect is larger when cutout ratios greater than 0.3 and for thickness ratio greater than 0.15.*

Keywords: *Elastic buckling load, Finite element method, inplane compression loads, Plate thickness ratio, Plate with cutout*

I. Introduction

Steel plates are often used as the main components of steel structures such as webs of plate girders, box girders, ship decks and hulls and platforms on oil rigs. Perforations are often included in the stressed skin cover of air plane wings. In plates cutouts are provided to decrease the self-weight, to provide access, services and even aesthetics. When these structures are loaded, the presence of cutouts will cause changes in the member mechanical properties, consequently there will be change in the buckling characteristics of the plate as well as on the ultimate load carrying capacity of the structure. The cutout shape and size, different plate thickness ratios and the type of load applied, influence the performance of such plates. However, though the cutouts are provided to achieve certain structural advantages, it is worth to mention here that they may inadvertently affect the stability of the plate component in the form of buckling. This always can be accomplished by thicker plate but the design solution will not be economical in terms of the weight of material used. It is possible to design an adequately strong and rigid structural plate element by keeping its thickness as small as possible. Hence the study of plate stability behavior is of paramount importance. Although much information is available regarding the buckling strength of perforated plate under simply supported boundary conditions, very little published information is available in the literature concerning the influence of different shape and size of cutout at centre and plate thickness on the elastic buckling strength of plates, this is because of the difficulties involved in determining the buckling strength of such plates by using classical methods of analysis. Owing to the complexity of the problem caused by plate thicknesses and the cutout, it appears that a numerical method such as the finite element method would be the most suitable for solving this problem.

The stability of plates under various compressive loading and boundary conditions has been the subject and studied by Herrmann and Armenkas [1], Timoshenko and Gere [2] and many others. Thin plate theory is based on several approximations, the most important of which is the neglect of transverse shear deformations. The errors of such a theory naturally increase as the thickness of plate increases, Srinivas and Rao [3]. Chiang–Nan Chang and Feung–Kung Chiang [4] observed the change of mechanical behaviors due to the interior holes cut from plate structures and proved the importance to study the buckling behaviors to avoid the structure instability. They used FEM and considering the incremental deformation concept to study the buckling behavior of Mindlin thick plate with interior cutout for different plate boundaries and different opening ratios. Christopher J. Brown, Alan L.Yettram and Mark Burnett [5] have used the conjugate load/displacement method to predict the elastic buckling load of square plate with centrally located rectangular holes under different types of loads. Shanmugam, Thevendran and Tan [6] have used FEM to develop a design formula to determine the ultimate load carrying capacity of axially compressed square plates with centrally located perforations, circular or square.

They concluded that the ultimate load capacity of the square perforated plate is affected significantly by the hole size and the plate slenderness ratios. Ultimate strength of square plate with rectangular opening under axial compression using non-linear finite element analysis was studied by Suneel Kumar, Alagusundaramoorthy and Sundaravadivelu [7]. El-Sawy and Nazmy [8] have used the FEM to investigate the effect of plate aspect ratio and hole location on elastic buckling of uniaxially loaded rectangular plates with eccentric holes with simply supported edges in the out-of-plane direction. The study concluded that the use of a rectangular hole, with curved corners, with its short dimension positioned along the longitudinal direction of the plate is better option than using a circular hole, from the plate stability point of view. A general purpose finite element software ANSYS was used for carrying out the study. Jeom Kee Paik [9] studied the ultimate strength of perforated steel plate under combined biaxial compression and edge shear loads for the circular cutout located at the centre of the plate. A series of ANSYS elastic-plastic large deflection finite element analysis has been carried out on perforated steel plates with varying plate dimension.

In the present paper it has been attempted to investigate the effect of the size and shape of concentric circular, square and rectangular cutouts and the impact of plate thickness on the buckling load of all-round simply supported isotropic square plate subjected to uniform inplane uniaxial and biaxial compression loadings. To carry out the study, ANSYS software has been used with 8SHELL93 element [10]. The finite element mesh used to model the plate has been decided upon carrying out a series of convergence tests and considered 10 x 10 mesh shows nearly the accurate results and hence considered in the analysis.

II. Problem Definition

The problem of elastic buckling of a square plate subjected to inplane compression loadings along its ends, Fig.1, having different cutouts such as circular, square and rectangular shapes with all-round simply supported boundary condition are considered. The plate has thickness h and dimensions A and B in x and y-directions, respectively and circular cutouts with diameter d, square and rectangular cutout of size axb. Here concentric cutout ratio is defined as the ratio of size of cutout to side of plate. The buckling load factor k is assessed with respect to the concentric cutout, vary between 0.1 to 0.6 for circular and square where as for rectangular cutout it is 0.1 to 0.5 and also with respect to the plate thickness ratios η, 0.01 to 0.3.

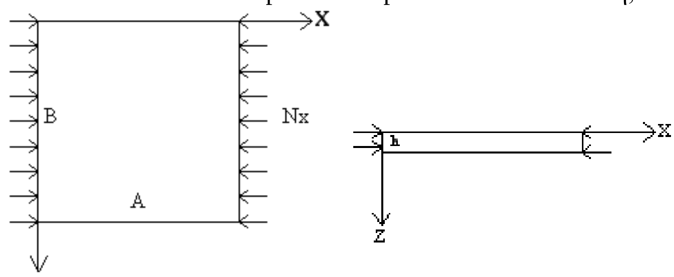


Fig.1: Plate coordinate system

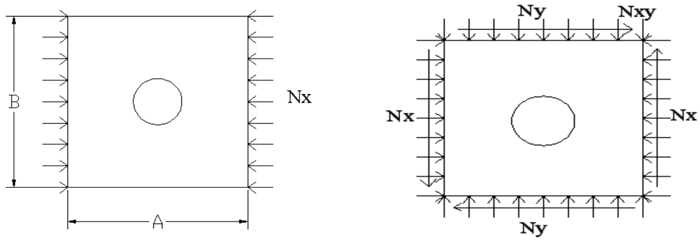
III. Finite Element Analysis Procedure

The finite element software program ANSYS was employed in this study. 8SHELL93 element is used to model the perforated plate because it shows satisfactory performance in verification work previously described by El-Sawy [8]. The elastic SHELL93 element has eight nodes possessing six degrees of freedom per node. The material of the plate was assumed to be homogeneous, isotropic and elastic. The material properties for Young's modulus E=210924N/mm^2 and Poision's ratio μ=0.3, were used.

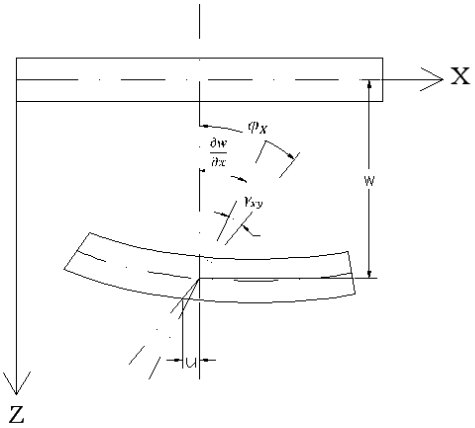
3.1 Buckling of Simply Supported Thick Plate under uniaxial compression

R.D. Mindlin in 1951, published the famous thick plate theory. This two dimensional theory of flexural motions of isotropic elastic plates is deduced from the three dimensional equations of elasticity. The theory includes the effects of rotary inertia and shear in the same manner as Timoshenko's one dimensional theory of bars [2]. The following assumptions are also applied.

1. The straight line that is vertical to the neutral surface before deformation remains straight but not necessarily vertical to the neutral surface after deformation.
2. Displacement is small so that small deformation theory can be applied.
3. Normal stress in the z-direction is neglected.
4. Body force is neglected.



(a) Geometry **(b) Inplane stress resultants**



(c) Thick plate deformation

Fig.2: Plate with opening and inplane stress resultants.

A plate of size A and B subjected to a system of general external inplane loadings and the internal stress resultants at the edges of an element due to the external inplane loads be N_X, N_Y and N_{xy} as depicted in Fig 2. The total potential energy of the plate due to flexure and the work done by the membrane stress resultants, taking the shear deformation into account may be written as:

$$U=\frac{D}{2}\int_0^{A_p-A_c}\left\{\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]^2-2(1-\mu)\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]\right\}dA+\frac{D}{2}\int_0^{A_p-A_c}\frac{6\chi(1-\mu)}{h^2}\left[\phi_x^2+\phi_y^2\right]dA$$

$$+\frac{1}{2}\int_0^{A_p-A_c}\left[N_x\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right\}+N_y\left\{\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right\}+2N_{xy}\left\{\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}+\frac{\partial v}{\partial z}\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right\}\right]dA \quad (1)$$

Where χ is the Reissner's shear correction factor, A_p is the area of the plate including the cutout, A_c is the area of the cutout, Φ_x, Φ_Y are average shear strains, N_X, N_Y, N_{XY} are inplane stress resultants and D is the flexural rigidity of the plate. The stiffness matrix [K] is the combination of $[K_b]$ and $[K_s]$ such that $[K] = [K_b] - [K_s]$, in which the former is due to flexure [4] and the later is due to the work associated with the inplane stress resultants. Thus static buckling equilibrium equation becomes,

$$[[K_b] - [K_s]]\ \{q\} = 0 \quad (2)$$

The stress resultants N_X, N_Y and N_{XY} are functions of geometrical ordinates (X,Y) for the plate and depend upon the magnitude of the external inplane loads,{q} is nodal displacement vector. Choosing a factor λ, by which the inplane stress resultants can be gradually increased, equation (2) may be written as.

$$[[K_b] - \lambda\,[K_\sigma]]\ \{q\} = 0, \text{ in which } [K_S] = \lambda\,[K_\sigma] \quad (3)$$

The Eigen values of the equation (3) give the critical loads λ_i of the plate under investigation. The lowest Eigen value correspond to the fundamental critical load 'λ_{cr}'

Let P_{cr} be the critical bucking load and by replacing λ by P_{cr} the governing equation for the static stability problem modified to

$$[\,[K_b] - P_{cr}\,[K_\sigma]\,]\ \{q\} = 0 \quad (4)$$

The general equation of stability given in equation (4) contain the structural properties in matrix form, viz., $[K_b]$ and $[K_s]$. The very basic assumption in the derivation of these equations is that the displacement model of the entire structure which satisfies the equilibrium equations and compatibility conditions. Developing such a true displacements model is a tedious exercise. In the present study an alternate method, using the finite element technique through ANSYS software, has been used.

IV. Results and Discussions

Results on the effect of shape and size of circular, square and rectangular cutout and plate thickness ratio on the buckling load factor k of the square plate having simply supported plate boundary conditions, subjected to inplane uniaxial and biaxial compression loading cases are presented and discussed in this section. Here, the concentric cutout ratio, defined as the ratio of side of the cutout to plate side, for circular β and square δ lies between 0.1 – 0.6 and for rectangular cutout it is 0.1–0.5, along x-direction γ and along y-direction γ'. The thickness ratio η, defined as the ratio of plate thickness to the plate side and varies from 0.01 - 0.3. The critical buckling load factor is non-dimensionalised and represented as follows:

$$k = \frac{(N_{cr})B^2}{\pi^2 D} \qquad (5)$$

Where, k = Buckling load factor.

N_{cr} = Critical buckling load.

D = Plate flexural rigidity = $\frac{Eh^3}{12(1-\mu^2)}$

μ = Poisson's ratio of isotropic plate.

h = Thickness of the plate.

4.1 Comparitive Study

In order to verify the present analysis, a comparison with existing results in the literature on buckling of square plate with and without cutout has been performed. The results of present study with the available values are tabulated in Table 1 and 2. It can be observed that the results from the present work are in good agreement with the established work.

Table 1

Comparison of buckling load factor k for isotropic simply supported plate with concentric circular cutout subjected to inplane uniaxial compression load.

Sl. no	Cutout ratio, □	Buckling load factor, k	
		Present study	Reference value
1	0.1	3.8434	3.80[11]
2	0.2	3.5310	3.50[11]
3	0.3	3.2642	3.20[11]
4	0.4	3.1124	3.10[11]
5	0.5	3.0460	2.99[11]
6	0.6	2.8547	Present study

Table 2

Comparison of buckling load factor k for isotropic simply supported plate with concentric square cutout subjected to inplane biaxial compression load.

Sl. no	Ratio of size of hole to plate side □=b/A	Buckling load factor, k	
		Present study	Reference value
1	0	1.9980	2.0[2]
2	0.1	1.9051	Present study
3	0.2	1.7496	Present study
4	0.25	1.6986	1.65[12]
5	0.3	1.6684	Present study
6	0.4	1.7028	Present study
7	0.5	1.8891	1.55[12]
8	0.6	2.5182	Present study

4.2 Case of plate with simply supported boundary conditions

The results obtained are plotted as shown in Figs. 3 - 10 for plate with different cutout ratios and plate thickness ratios having simply supported plate boundary condition subjected to inplane uniaxial and biaxial compression loading cases. In all these Figs the variation of the buckling load factor k are plotted against thickness ratio η and cutout ratios. It can be noticed in these figures that, stability of the square plate with cutout is greatly affected by increase in thickness ratios.

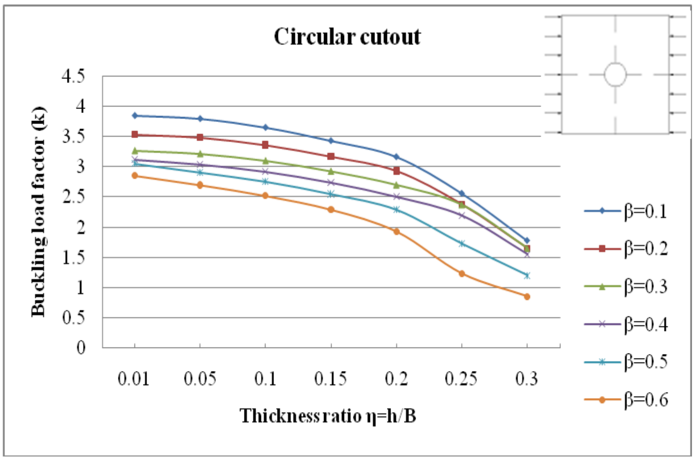


Fig.3: Variation of k with □ and □ foruniaxial compression.

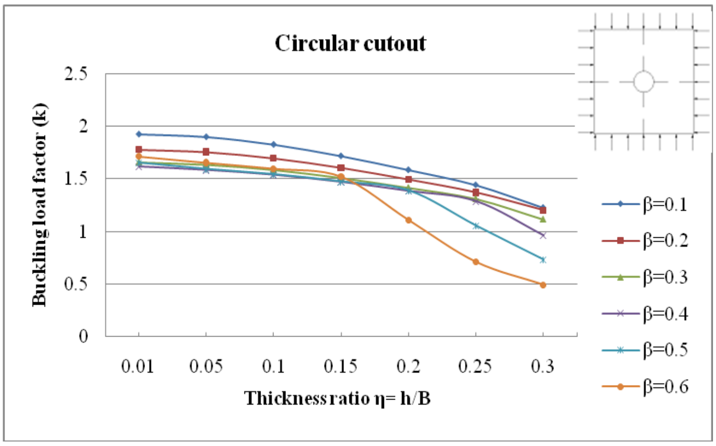


Fig.4 : Variation of k with □ and □ for biaxial compression

In Fig.3, buckling load factor k decreases gradually up to the thickness ratio η < 0.2 for all circular cutout ratios β and when η > 0.2, k reduction is in higher magnitude i.e., in the range of 25% to 52% as η increases from 0.01 to 0.3 for each cutout ratio. Fig.4 represents the variation of buckling load factor k for square plate having concentric circular cutout subjected to biaxial loading. In case of biaxial loading, k value is found to be almost 50% to that of uniaxial compression value. This may be due to stiffening of the plate in both x and y-directions. It is noticed that k always decreases with increase in η and β and this decrease is more steeper for β > 0.3 and η > 0.15, when β = 0.6 and η = 0.3, 60% of reduction in k is observed to that of β = 0.1.

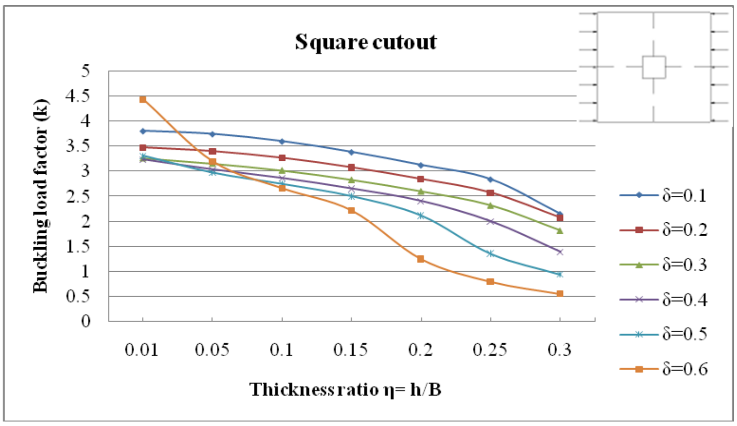


Fig. 5: Variation of k with □ and □ for uniaxial compression

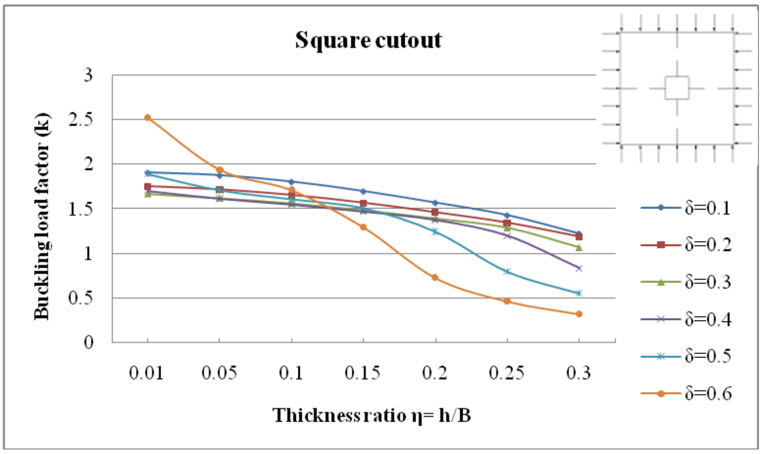


Fig. 6: Variation of k with □ and □ for biaxial compression

Fig.5, represents the variation of buckling load factor k for square plate with concentric square cutout ratios δ, subjected to uniaxial loading. Here, the value of k is 10% less than that of circular cutout and k value decreases as δ and η increases, except at δ=0.6 for η=0.01, where an increase of 16.33% in k value can be observed compared to δ=0.1. This doesnot necessarily mean that the actual increase in buckling strength of the plate because the strength of the plate corresponds to less removal of plate material. When the cutout ratio becomes large and $\eta > 0.15$, the reduction is more rapid and it is up to 50% for $\delta \leq 0.3$ for all values of η and it is more than 50% for $\delta > 0.3$. Fig.6, represents the variation of buckling load factor k for square plate having concentric square cutout, subjected to biaxial loading. Here, k value is 50% compared to uniaxial loading case. It is noticed that the variation of k is very less for δ up to 0.3 with respect to η values, but high variation is noticed when $\delta > 0.3$. Decrease in buckling load factor k is unpredictable at δ=0.6 and k reduction is noticed in the magnitude of 54%, 67% and 74% when η=0.2, 0.25 and 0.3 respectively.

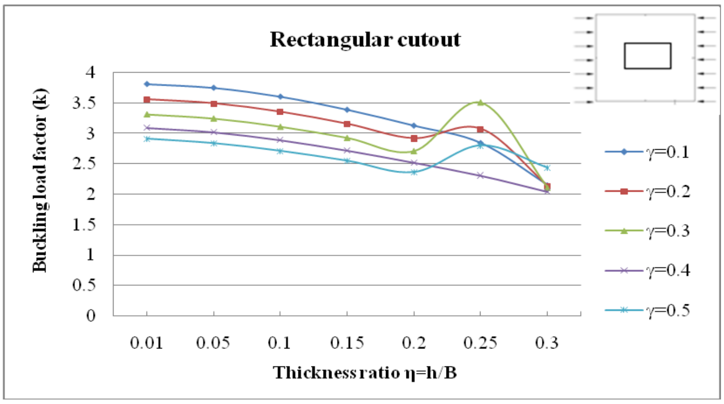


Fig.7: Variation of k with □ and □ for uniaxial compression

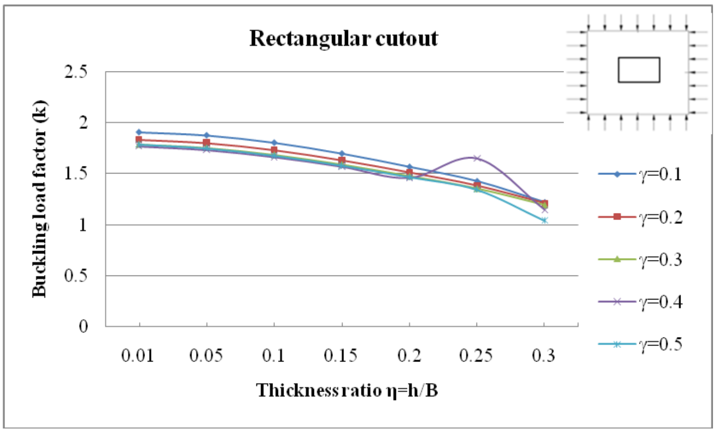


Fig.8 : Variation of k with □ and □ for biaxial compression.

Fig.7 represents the variation of buckling load factor k for square plate having concentric rectangular cutout γ along x-direction, subjected to uniaxial loading. Here, the decrease in k is gradual when compared to circular and square cutout and k always decreases as η and γ increases except for η lies between 0.2 to 0.25 and γ=0.2, 0.3 and 0.5, where a sudden variation in k value can be observed. Fig.8 and Fig.10 represents the variation of buckling load factor k for square plate having concentric rectangular cutout along x and y-directions γ and γ' respectively under biaxial loading. Here, the value of k is half of that of uniaxial loading case and the variation of k is very less as η and cutout ratios increases. In case of biaxial loading, rectangular cutout along x and y-directions, k reduces in slower manner. Fig.9 represents the variation of buckling load factor k for square plate having concentric rectangular cutout along y-direction subjected to uniaxial loading. Here, k increases with increase in the value of γ' and for η up to 0.15, beyond k reduces with the increase of γ'. It is noticed that as η increases k decreases and this decrease is significant when γ' > 0.3. The reduction of k is upto the range 43.5%, 51%, 60%, 69% and 76% with the increase of γ' from 0.1 to 0.5 with respect to η=0.01 - 0.3. It is also observed that, for small cutout ratios, there is a little effect of thickness ratio η upto 0.15. But when cutout ratio becomes larger, increasing the plate thickness ratio shows significant effect on the buckling strength of the plate.

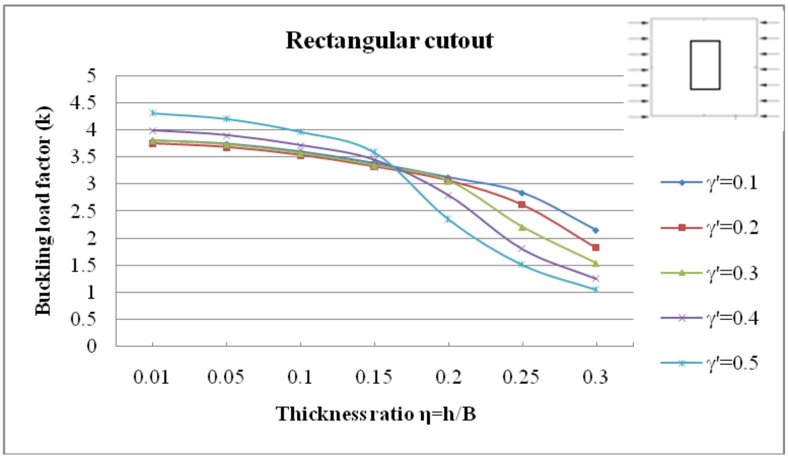


Fig.9 :Variation of k with □ and □' for uniaxial compression

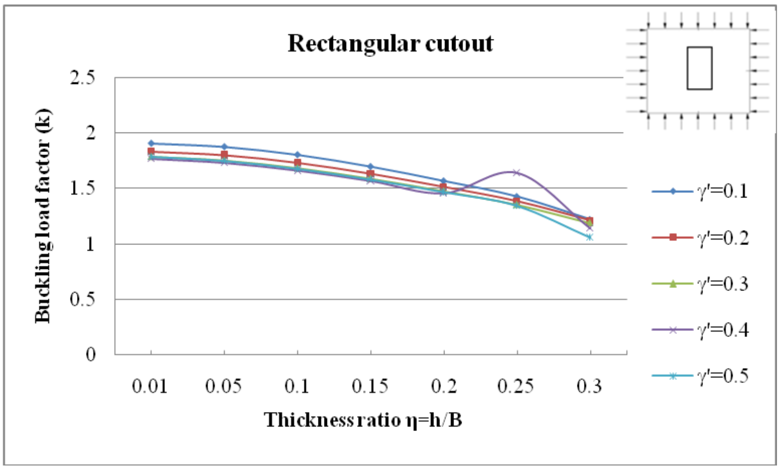


Fig.10 :Variation of k with □ and □' for biaxial compression

V. Conclusion

A study of buckling behavior of uniformly compressed isotropic square plate with different cutouts such as circular, square and rectangular are investigated using FEM. Simply supported square plate with wide ranges of concentric cutout ratios and plate thickness ratios are considered. Following conclusions are drawn.

1. Buckling is the critical mode of failure for the major portion of the compressed plate with cutout especially when its thickness is considerably small.
2. The cutouts have considerable influence on the buckling load factor k. The effect is larger in higher cutout ratios ≥ 0.3 and for thickness ratio ≥ 0.15.
3. Square plates with circular cutouts and square cutouts have similar buckling loads, but, plate with circular

cutout is more efficient compare to plate with square cutout.

4. In the plate with circular cutout and square cutout, buckling load factor k decreases with the increase of thickness ratio and cutout ratios. The value of k is less by 10% in case of plate having square cutout compared to circular cutout.
5. Plate with concentric rectangular cutout, along x-direction, decrease in the buckling load factor k of the plate is 34-43%.
6. In case of biaxial loading, k value is found to be nearly 50% to that of uniaxial compression value.

In summary, buckling of simply supported plate with concentric cutout having different plate thickness ratio is important for optimizing the engineering design. The present study offers useful information for the researchers.

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