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Time Dependent Quadratic Demand Inventory Models when Delay in Payments is Acceptable

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Abstract: *An EOQ model is constructed for deteriorating items with time dependent quadratic demand rate. It is assumed that the deterioration rate is constant and the supplier offers his retailer the credit period to settle the account of the procurement units. To solve the model it is further assumed that shortages are not allowed and the replenishment rate is instantaneous. We have presented the models under two different scenarios, viz., (i)the offered credit period is less than or equal to the cycle time and (ii) the offered credit period by the supplier to the retailer for settling the account is greater than cycle time. The objective is to minimize the retailers total inventory cost. Salvage value is also taken to see its effect on the total cost. A numerical example is given to study the effect of allowable credit period and the total cost of the retailer.*

Key words: *Quadratic demand, perishable, constant deterioration, trade credit, holding cost.*

I. Introduction

It is well known that 'trade credit policy' is the most effective way of a supplier to encourage retailer to buy more goods and to attract more retailers. Trade credit can also be used as a multi-faceted marketing management (or relationship management) tool which gives some information to the market or to a buyer about the firm or its products or its future plans. The EOQ model developed by Wilson was based on the assumption that the retailer will pay for the items as soon as it is received by the system. In reality the supplier may offer some credit period to the retailer to settle the accounts in a reasonable time period. Thus the delay in payment can be treated as a kind of price discount to the retailer.

The relationship between inventory policy and credit policy in the context of the classical lot size model was studied by Haley and Higgins (1973). Chapman et al. (1984) developed an economic order quantity model which considers possible credit periods allowable by suppliers. This model is shown to be very sensitive to the length of the permissible credit period and to the relationship between the credit period and inventory level. Davis and Gaither (1985) developed optimal order quantities for firms that are offered a one time opportunity to delay payment for an order of a commodity. A mathematical model is developed by Goyal (1985) when supplier announces credit period in settling the account, so that no interest charges are payable from the outstanding amount if the account is settled within the allowable delay period. Shah et al. (1988) extended the above model by allowing shortages. Mandal and Phaujdar (1989a, b) have studied Goyal (1985) model by including interest earned from the sales revenue on the stock remaining beyond the settlement period. Carlson and Rousseau (1989) examined EOQ under date terms supplier credit by partitioning carrying cost into financial cost and variable holding costs. Chung and Huang (2003) extended Goyal (1985) model when replenishment rate is finite. Dallenbach (1986, 1988), Ward and Chapman (1987), Chapman and Ward (1988) argued that the usual assumptions as to the incidence and the value of the inventory investment opportunity cost made by the traditional inventory theory are correct and also established that if trade credit surplus is taken into account, the optimal ordering quantities decreases rather than increase. Chung (1998) established the convexity of the total annual variable cost function for optimal economic order quantity under conditions of permissible delay in payments. Jamal et al. (2000) discussed the problem in which the retailer can pay the supplier either at the end of credit period or later incurring interest charges on the unpaid balance for the overdue period. Sarker et al. (2001) obtained optimal payment time under permissible delay in payments when units in an inventory are subject to deterioration. Abad and Jaggi (2003) considered the seller-buyer channel in which the end demand is price sensitive and the suppler offers trade credit to the buyer. Shinn and Hwang (2003) dealt with the problem of determining the retailer's optimal price and order size simultaneously under the condition of order size dependent delay in payments. It is assumed that the length of the credit period is a function of the retailer's order size and also the demand rate is a function of the selling price. Chung et al. (2005) determined the economic order quantity under conditions of permissible delay in payments where the delay in payments depends on the quantity ordered when the order quantity is less than the quantity at which the delay in payments is permitted, the payment for the item must be made immediately. Otherwise, the fixed credit period is allowed. Huang (2007) examined optimal retailer's replenishment decisions in the EOQ model under two levels of trade credit policy by assuming that the supplier would offer the retailer partially permissible delay in payments when the order quantity is smaller than a predetermined quantity. Teng et. al. (2007) derived retailer's optimal ordering policies with trade credit financing.

The literature is replete in the field of trade-credit. Previously several economic order quantity inventory models were developed with trade-credit and a very few production inventory models were developed under allowable delay in payment. All these works were based on the assumption that the demand rate is either linear or exponential function of time. Several authors argued that, in realistic terms, the demand need not follow either linear or exponential trend. So, it is reasonable to assume that the demand rate, in certain commodities, is due to seasonal variations may follow quadratic function of time [i.e., $D(t) = a + bt + ct^2$; $a \geq 0, b \neq 0, c \neq 0$]. The functional form of time-dependent quadratic demand explains the accelerated (retarded) growth/decline in the demand patterns which may arise due to seasonal demand rate (Khanra and Chaudhuri (2003). We may explain different types of realistic demand patterns depending on the signs of *b* and *c*. Bhandari and Sharma (2000) have studied a single period inventory problem with quadratic demand distribution under the influence of marketing policies. Khanra and Chaudhuri (2003) have discussed an order-level inventory problem with the demand rate represented by a continuous quadratic function of time. It is well known that the demand for spare parts of new aero planes, computer chips of advanced computer machines, etc. increase very rapidly while the demands for spares of the obsolete aero planes, computers etc. decrease very rapidly with time. This type of phenomena can well be addressed by inventory models with quadratic demand rate. Sana and Chaudhuri (2004) have developed a stock-review inventory model for perishable items with uniform replenishment rate and stock-dependent demand. Recently, Ghosh and Chaudhuri (2004) have developed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand and shortages in inventory. They have used a two-parameter Weibull distribution to represent the time to deterioration. Venkateswarlu and Mohan (2011) have developed inventory models for deteriorating items with time dependent quadratic demand and salvage value. Recently Venkateswarlu and Mohan (2013) studied inventory model for time varying deterioration and price dependent quadratic demand with salvage value.

In literature we seldom find on the inventory models with trade credit policy for perishable items with time dependent quadratic demand rate. Thus, in this paper, we wish to develop a mathematical model when the units in an inventory are subjected to a constant deterioration rate and the demand rate follows a time dependent quadratic function. It is assumed that the supplier offers a credit period to the retailer to settle the account. We have also considered the salvage value for deteriorating units of the inventory. Sensitivity analysis is presented with a numerical example.

II. Assumptions and Notations

The following assumptions are used to develop the model:

- The system deal with a single item
- The demand rate R is time dependent quadratic demand
- The replenishment rate is infinite.
- The lead time is zero and shortages are not allowed.
- The salvage value, γp $(0 \leq \gamma < 1)$ is associated to deteriorated units during the cycle time. Here p is the purchase cost of an item.

The following notations are used to develop the model:

- The Demand rate $R(t)$ at time t is assumed to be $R(t) = a + bt + ct^2$ $a \geq 0, b \neq 0, c \neq 0$. Here *a* is the initial rate of demand, *b* is the initial rate of change of the demand and *c* is the acceleration of demand rate.
- $\theta (0 < \theta < 1)$ is the constant rate of deterioration.
- A is the ordering cost per order.
- S is the selling price per item $(S > p)$.
- Q(t) is the ordering quantity at time t=0
- h is per unit holding cost excluding interest charges per unit per year.
- I_e is the interest earned per year.
- I_c is the interest charged per stocks per year.

- M is the permissible delay in settling in the accounts, 0<M<1.
- T is the interval between two successive orders
- K(T) is the total cost per unit time.

III. Formulation and Solution of the Model

The objective of the model is to determine the total cost of the system and the demand rate of items is time dependent quadratic function with constant rate of deterioration.

Let *I(t)* be the inventory level at time *t* ($0 \le t \le T$). The inventory depletes due to deterioration and the demand, and then the differential equation which describes the inventory level at time t is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -R(t),\ 0 \le t \le T \tag{1}$$

$$where \quad R(t) = (a + bt + ct^2)$$

and (i) $I(T)=0$ when $t=T$, (ii) $I(0)=Q$. (2)

The solution of equation (1) using the boundary condition $I(T)=0$ is given by

$$I(t) = a(T-t) + (a\theta + b)(\frac{T^2}{2} - \frac{t^2}{2}) + (b\theta + c)(\frac{T^3}{3} - \frac{t^3}{3}) + c\theta(\frac{T^4}{4} - \frac{t^4}{4}) - (\theta t)\left[a(T-t) + b(\frac{T^2}{2} - \frac{t^2}{2}) + c(\frac{T^3}{3} - \frac{t^3}{3})\right] \tag{3}$$

where we taken series expansion and ignored the second and higher powers of θ as θ is small.
Since $I(0)=Q$, we obtain

$$Q(t) = aT + (a\theta + b)(\frac{T^2}{2}) + (b\theta + c)(\frac{T^3}{3}) + c\theta(\frac{T^4}{4}) \tag{4}$$

The number of deteriorated units D (T) during one cycle is given by

$$D(T) = Q - R(T)T = \frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} + \frac{c\theta T^4}{4} - \frac{bT^2}{2} - \frac{cT^3}{3} \tag{5}$$

The cost due to deterioration is given by

$$CD = pD(T) = p\left(\frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} + \frac{c\theta T^4}{4} - \frac{bT^2}{2} - \frac{cT^3}{3}\right) \tag{6}$$

The salvage value of deteriorated units is

$$SV = \gamma CD \tag{7}$$

The inventory holding cost during the cycle is

$$IHC = h\int_0^T I(t)dt$$

$$= h\left(\frac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right) \tag{8}$$

The Ordering cost is given by

$$OC = A \tag{9}$$

Following Nita Shah and Pandey (2008), we have considered the following two cases for interest charged and the interest earned:

Case-1: The offered credit period is less than or equal to the cycle time i.e., $M \le T$.

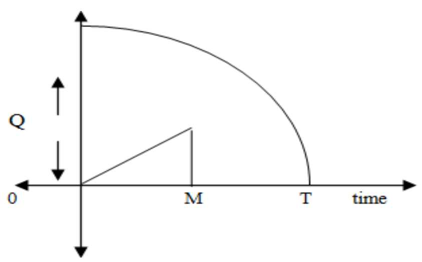

Figure-1

Case-2: The offered credit period for settling the account is greater than the cycle time i.e., $M > T$.

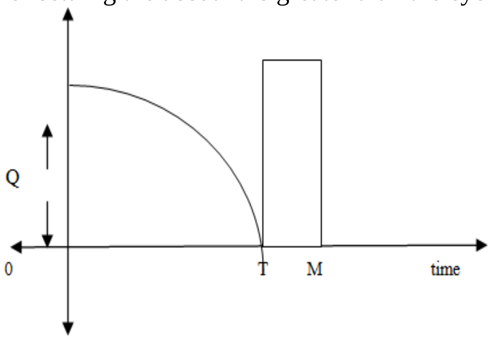

Figure-2

3.1 *Case-1*

The retailer can sale units during *[0, M]* at a sale price: '*S*' per unit which he can put an interest rate 'I_e'per unit per annum in an interest bearing account. So the total interest earned during *[0, M]* is

$$IE_1 = SI_e \int_0^M R(t)t\,dt = SI_e\left(\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4}\right) \tag{10}$$

Now, in the period *[M, T]*, the supplier will charge the interest to the retailer on the remaining stock at the rate 'I_c' per unit per annum. Hence, total interest charges payble by the retailer during *[M, T]* is

$$\begin{aligned} IC_1 &= SI_c \int_M^T I(t)dt \\ &= \frac{1}{120}(SI_c)(M-T)^2(60a + 10M^2c + 30T^2c + 20Mb \\ &\quad + 40Tb + 20T\theta a - 5M^2\theta - 2M^3\theta c + 15T^2\theta b + 12T^3\theta c \\ &\quad + 20M\theta a - 10MT\theta b - 6MT^2\theta c - 4M^2T\theta c) \end{aligned} \tag{11}$$

Now, the total cost $K_1(T)$ per time unit is

$$K_1(T) = \frac{1}{T}(OC + IHC + CD - IE_1 + IC_1 - SV)$$

$$= \frac{1}{T}\left(\begin{aligned} &A + h\left(\frac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right) \\ &+ (1-\gamma)p\left(\frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} + \frac{c\theta T^4}{4} - \frac{bT^2}{2} - \frac{cT^3}{3}\right) \\ &- SI_e\left(\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4}\right) \\ &+ \frac{1}{120}(SI_c)((M-T)^2(60a + 10M^2c + 30T^2c + 20Mb \\ &\quad + 40Tb + 20T\theta a - 5M^2\theta b - 2M^3\theta c + 15T^2\theta b + 12T^3\theta c \\ &\quad + 20M\theta a - 10MT\theta b - 6MT^2\theta c - 4M^2T\theta c) \end{aligned}\right) \tag{12}$$

Since our objective is to minimize the total cost $K_1(T)$ per unit time, the necessary condition for the total cost to be minimum is $\frac{\partial}{\partial T}K_1(T) = 0$ i.e.,

$$\frac{\partial K_1(T)}{\partial T} = \left(\frac{1}{T}\right)(X_1 + X_2) - \left(\frac{1}{T^2}\right)(X_3 + X_4 - X_5 + X_6) \tag{13}$$

where

$$X_1 = \frac{h(120M^2b + 120M^3c + 120Ma + 120M^2a\theta + 60M^3\theta b + 60M^4\theta c)}{120}$$

$$X_2 = (1-\gamma)(p)(M^2\theta b - Mb - M^2c + M^3\theta c + M\theta a)$$

$$X_3 = A + h\left(\frac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right)$$

$$X_4 = (1-\gamma)p\left(\frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} + \frac{c\theta T^4}{4} - \frac{bT^2}{2} - \frac{cT^3}{3}\right)$$

$$X_5 = SI_e\left(\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4}\right)$$

$$X_6 = (\frac{1}{120})SI_c((M-T)^2(60a + 10M^2c + 30T^2c + 20Mb + 40Tb + \\ 20T\theta a - 5M^2\theta b - 2M^3\theta c + 15T^2\theta b + 12T^3\theta c + 20M\theta a - \\ 10MT\theta b - 6MT^2\theta c - 4M^2T\theta c)$$

From equation (13), we have

$$\left(\begin{array}{l} \left(\dfrac{-1}{T^2}\right)\left(\begin{array}{l} A + h\left(\dfrac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right) \\ + (1-\gamma)p\left(\dfrac{a\theta T^2}{2} + \dfrac{b\theta T^3}{3} + \dfrac{c\theta T^4}{4} - \dfrac{bT^2}{2} - \dfrac{cT^3}{3}\right) \\ - SI_e\left(\dfrac{aM^2}{2} + \dfrac{bM^3}{3} + \dfrac{cM^4}{4}\right) \\ + \dfrac{1}{120}(SI_c)(M-T)^2(60a + 10M^2c + 30T^2c + \\ 20Mb + 40Tb + 20T\theta a - 5M^2\theta b - 2M^3\theta c + 15T^2\theta b + 12T^3\theta c + \\ 20M\theta a - 10MT\theta b - 6MT^2\theta c - 4M^2T\theta c) \end{array}\right) \\ + \left(\dfrac{1}{T}\right)\left[\begin{array}{l} \dfrac{h(120M^2b + 120M^3c + 120Ma + 120M^2a\theta + 60M^3\theta b + 60M^4\theta c}{120} \\ + (1-\gamma)(p)(M^2\theta b - Mb - M^2c + M^3\theta c + M\theta a \end{array}\right] \end{array}\right) = 0 \qquad (14)$$

We solve the above equation for optimal T using MATHCAD. For this optimal T, the total cost is minimum only if $\frac{\partial^2 K_1(T)}{\partial T^2} > 0$.

3.1.1 Numerical Example

To demonstrate the effectiveness of the models developed, a numerical example is taken with the following values for the parameters:

a =500	b=5	c=0.1	h=5	S=30	$\gamma = 0.08$
T=0.5	I_c=0.15	I_e=0.12	$\theta = 0.05$	p=20	A=200

For the above example, it is found that the optimality conditions are satisfied in all the following four cases for all T viz.,

(i) $a > 0$, $b > 0$ and $c > 0$ (i.e., accelerated growth model)
(ii) $a > 0$, $b < 0$ and $c > 0$ (i.e., retarded growth model)
(iii) $a > 0$, $b < 0$ and $c < 0$ (i.e., accelerated decline model)
(iv) $a > 0$, $b > 0$ and $c < 0$ (i.e., retarded decline model)

The MATHCAD output is given in Table-1 through table-4 which shows the variations of the deterioration rate, θ and the delay period, M.

From the output shown in table-1 to table-4, it is observed that the buyer's total cost decreases with the increase in delay period for a fixed value of deterioration rate. For example, if the deterioration rate is 0.05, the total cost K_1(T) decreases when the delay in payment increases from 15 days to 60 days in all the models. We may attribute this due to the interest earned by buyer who earns more revenue from the sold items. Further, across all the models, it can be noticed that the buyer's total cost increases when the rate of deterioration increases from 0.05 to 0.10.

Table-1: $a > 0$, $b > 0$ and $c > 0$ (i.e., accelerated growth model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$
1	0.05	0.28	1345.02	0.28	1256.13	0.28	1170.07	0.28	1086.76
2	0.06	0.28	1358.57	0.28	1269.65	0.28	1183.61	0.28	1100.37
3	0.07	0.27	1371.97	0.27	1283.02	0.28	1197.01	0.28	1113.84
4	0.08	0.27	1385.24	0.27	1296.27	0.27	1210.27	0.27	1127.17
5	0.09	0.27	1398.38	0.27	1310.36	0.27	1223.41	0.27	1140.38
6	0.10	0.27	1411.39	0.27	1322.36	0.27	1236.42	0.27	1153.46

Table-2: $a > 0$, $b < 0$ and $c > 0$ (i.e., retarded growth model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$
1	0.05	0.27	1367.99	0.28	1279.43	0.28	1193.75	0.28	1110.89
2	0.06	0.27	1381.29	0.27	1292.70	0.27	1207.05	0.28	1124.25
3	0.07	0.27	1394.46	0.27	1305.84	0.27	1220.20	0.27	1137.47
4	0.08	0.27	1407.50	0.27	1318.85	0.27	1233.24	0.27	1150.57
5	0.09	0.26	1420.41	0.27	1331.74	0.27	1246.15	0.27	1163.55
6	0.10	0.26	1433.21	0.26	1344.51	0.26	1258.94	0.27	1176.41

Table-3: $a > 0$, $b < 0$ and $c < 0$ (i.e., accelerated decline model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$
1	0.05	0.27	1368.07	0.28	1279.52	0.28	1193.84	0.28	1110.98
2	0.06	0.27	1381.37	0.27	1292.78	0.27	1207.13	0.28	1124.33
3	0.07	0.27	1394.54	0.27	1305.92	0.27	1220.29	0.27	1137.56
4	0.08	0.27	1407.58	0.27	1318.93	0.27	1233.32	0.27	1150.66
5	0.09	0.26	1420.49	0.27	1331.82	0.27	1246.23	0.27	1163.63
6	0.10	0.26	1433.28	0.26	1344.58	0.26	1259.02	0.27	1176.49

Table-4: $a > 0$, $b > 0$ and $c < 0$ (i.e., retarded decline model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$	T	$K_1(T)$
1	0.05	0.28	1345.02	0.28	1256.22	0.28	1170.15	0.28	1086.85
2	0.06	0.28	1358.65	0.28	1269.73	0.28	1183.69	0.28	1100.46
3	0.07	0.27	1372.05	0.27	1283.11	0.28	1197.09	0.28	1113.93
4	0.08	0.27	1385.32	0.27	1296.35	0.27	1210.36	0.27	1127.26
5	0.09	0.27	1398.46	0.27	1309.46	0.27	1223.49	0.27	1140.47
6	0.10	0.27	1411.46	0.27	1322.44	0.27	1236.50	0.27	1153.54

3.2 Case-2

In this case, the interest earned is

$$IE_2 = SI_e\left[\int_0^T R(t)tdt + R(T)T(M-T)\right]$$

$$= SI_e\left[aMT + bMT^2 + cMT^3 - \frac{aT^2}{2} - \frac{2bT^3}{3} - \frac{3cT^4}{4}\right] \quad (15)$$

and the interest charges is zero *i.e.,* $IC_2 = 0$

The total cost $K_2(T)$ per time unit is

$$K_2(T) = \frac{1}{T}\left(OC + IHC + CD - IE_2 + IC_2 - SV\right)$$

$$= \frac{1}{T}\left(\begin{array}{l} A + h\left(\dfrac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right) \\ +(1-\gamma)p\left(\dfrac{a\theta T^2}{2} + \dfrac{b\theta T^3}{3} + \dfrac{c\theta T^4}{4} - \dfrac{bT^2}{2} - \dfrac{cT^3}{3}\right) \\ -SI_e\left[aMT + bMT^2 + cMT^3 - \dfrac{aT^2}{2} - \dfrac{2bT^3}{3} - \dfrac{3cT^4}{4}\right] \end{array}\right) \quad (16)$$

Since our object is to minimize the total cost $K_2(T)$ per unit time, the necessary condition for the total cost to be minimum is $\therefore \frac{\partial}{\partial T}K_2(T) = 0$ i.e.,

$$\frac{\partial}{\partial T}K_2(T) = -\frac{1}{T}(Y_1 + Y_2 - Y_3) - \frac{1}{T^2}(Y_4 + Y_5 + Y_6) \quad (17)$$

where

$$Y_1 = (1-\gamma)p\left(a\theta T - Tb - cT^2 + b\theta T^2 + c\theta T^3\right)$$

$$Y_2 = h\left(\frac{120bT^2 + 120cT^3 + 120aT + 120a\theta T^2 + 60b\theta T^3 + 60c\theta T^4}{120}\right)$$

$$Y_3 = SI_e\left[aM - 2bT^2 - 3cT^3 - aT + 2bMT + 3cMT^2\right]$$

$$Y_4 = A + h\left(\frac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right)$$

$$Y_5 = (1-\gamma)p\left(\frac{a\theta T^2}{2} + \frac{b\theta T^3}{3} + \frac{c\theta T^4}{4} - \frac{bT^2}{2} - \frac{cT^3}{3}\right)$$

$$Y_6 = SI_e\left[aMT + bMT^2 + cMT^3 - \frac{aT^2}{2} - \frac{2bT^3}{3} - \frac{3cT^4}{4}\right]$$

Now from equation (17),

$$T(Y_1 + Y_2 - Y_3) + (Y_4 + Y_5 + Y_6) = 0$$

$$i.e., \left\{\begin{array}{l} T\left(\begin{array}{l} (1-\gamma)p\left(a\theta T - Tb - cT^2 + b\theta T^2 + c\theta T^3\right) \\ +h\left(\dfrac{120bT^2 + 120cT^3 + 120aT + 120a\theta T^2 + 60b\theta T^3 + 60c\theta T^4}{120}\right) \\ -SI_e\left[aM - 2bT^2 - 3cT^3 - aT + 2bMT + 3cMT^2\right] \end{array}\right) \\ +\left(\begin{array}{l} A + h\left(\dfrac{60aT^2 + 40bT^3 + 30cT^4 + 40a\theta T^3 + 15b\theta T^4 + 12c\theta T^5}{120}\right) \\ +(1-\gamma)p\left(\dfrac{a\theta T^2}{2} + \dfrac{b\theta T^3}{3} + \dfrac{c\theta T^4}{4} - \dfrac{bT^2}{2} - \dfrac{cT^3}{3}\right) \\ +SI_e\left[aMT + bMT^2 + cMT^3 - \dfrac{aT^2}{2} - \dfrac{2bT^3}{3} - \dfrac{3cT^4}{4}\right] \end{array}\right) \end{array}\right\} = 0 \quad (18)$$

which minimises the $K_2(T)$ only if $\frac{\partial^2 K_2(T)}{\partial T^2} > 0$ for all values of T.

3.2.1 Numerical Example

Once again we consider the values of the parameters as given in 3.1.1. For these values, the output is presented in Table-5 through Table-8. It can be observed that the behaviour of these models, in the case of M > T, is quite similar to the results obtained as in the case of M < T. It is also observed that the total cost $K_2(T)$ is less than $K_1(T)$ when the delay period is increased from 15 days to 60 days. Thus it can be concluded that the total cost in both the cases is almost same when the delay period increases from 15 days to 60 days.

For T = M, we have

$$K_1(M) = K_2(M) = \frac{1}{M}(Z_1 - Z_2 - Z_3)$$

where

$$Z_1 = A + h\left(\frac{60aM^2 + 40bM^3 + 30cM^4 + 40a\theta M^3 + 15b\theta M^4 + 12c\theta M^5}{120}\right)$$

$$Z_2 = (1-\gamma)p\left(\frac{a\theta M^2}{2} + \frac{b\theta M^3}{3} + \frac{c\theta M^4}{4} - \frac{bM^2}{2} - \frac{cM^3}{3}\right)$$

$$Z_3 = SI_e\left[\frac{aM^2}{2} + \frac{bM^3}{3} + \frac{cM^4}{4}\right].$$

IV. Sensitivity Analysis

(a > 0, b > 0 and c > 0 i.e., accelerated growth model)

We now study the sensitivity of the models developed in 3.1.1 and 3.2.1 to examine the implications of underestimating and overestimating the parameters a, b, c, Ic , Ie , h and p on optimal value of cycle time and total cost of the system. Here we have taken the deterioration rate as θ=0.05 and the delay period is 30 days. The sensitive analysis is performed by changing each of the parameter by -20%, -10%, +10% and +20% taking one parameter at a time and keeping the remaining parameters are unchanged. The results are shown in Table-9 and Table-10.

The following interesting observations are made from the above tables:

Case-1($M \le T$ **)**

(i) Increase (decrease) in parameters a, I_c , h and p decreases (increases) the cycle time where as the total cost increases (decreases) with the increase (decrease) in these parameters. However the rate of increase/decrease is more pronounced in case of the changes made in the parameters 'a' and 'h' which indicate that the optimal values of cycle time and the total cost are less sensitive to 'I_c' and 'p' .

(ii) The effect of the parameters b, c and I_e on the optimum value of the cycle time and the total cost is similar but the rate of change is insignificant.

(iii) The salvage value of deteriorated items is not shown much effect on the optimal total cost of the system.

Case-2($M > T$ **)**

(iv) Increase (decrease) in parameters a, h and p decreases (increases) the cycle time where as the total cost increases (decreases) with the increase (decrease) in these parameters. However the rate of increase/decrease is more pronounced in case of the changes made in the parameters 'a' and 'h' and less sensitive to the changes in 'p'

(v) T and K_2(T) are less sensitive to the changes made in the parameter 'b' and moderately sensitive to I_e.

(vi) The effect of salvage value is not so significant on optimal policies.

V. Conclusions

The main objective of this study is the formulation of a deterministic inventory model for items which have constant deterioration rate and follows time dependent quadratic demand rate when supplier offers a specific credit period. The total cost of the system is calculated when shortages are not allowed. Salvage value is considered while calculating the total cost of the system. Sensitivity of the models is also discussed.

Table-5: $a > 0$, $b > 0$ and $c > 0$ (i.e., accelerated growth model)

S.No	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$
1	0.05	0.29	1297.63	0.29	1223.44	0.29	1149.25	0.29	1075.06
2	0.06	0.29	1311.65	0.29	1237.46	0.29	1163.27	0.29	1089.09
3	0.07	0.28	1325.51	0.29	1251.32	0.29	1177.14	0.29	1102.95
4	0.08	0.28	1339.22	0.28	1265.04	0.28	1190.86	0.28	1116.67
5	0.09	0.28	1352.79	0.28	1278.61	0.28	1204.43	0.28	1130.25
6	0.10	0.28	1366.22	0.28	1292.05	0.28	1217.87	0.28	1143.69

Table-6: $a > 0$, $b < 0$ and $c > 0$ (i.e., retarded growth model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$
1	0.05	0.29	1320.95	0.29	1247.19	0.29	1173.43	0.29	1099.66
2	0.06	0.28	1334.71	0.28	1260.95	0.28	1187.18	0.28	1113.42
3	0.07	0.28	1348.32	0.28	1274.56	0.28	1200.79	0.28	1127.03
4	0.08	0.28	1361.80	0.28	1288.03	0.28	1214.26	0.28	1140.49
5	0.09	0.28	1375.14	0.28	1301.36	0.28	1227.59	0.28	1153.82
6	0.10	0.27	1388.34	0.27	1314.57	0.27	1240.80	0.27	1167.03

Table-7: $a > 0$, $b < 0$ and $c < 0$ (i.e., accelerated decline model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$
1	0.05	0.29	1321.03	0.29	1247.27	0.29	1173.51	0.29	1099.75
2	0.06	0.28	1334.79	0.28	1261.03	0.28	1187.27	0.28	1113.51
3	0.07	0.28	1348.40	0.28	1274.64	0.28	1200.88	0.28	1127.11
4	0.08	0.28	1361.87	0.28	1288.11	0.28	1214.34	0.28	1140.58
5	0.09	0.28	1375.21	0.28	1301.44	0.28	1227.68	0.28	1153.91
6	0.10	0.27	1388.42	0.27	1314.65	0.27	1240.88	0.27	1167.11

Table-8: $a > 0$, $b > 0$ and $c < 0$ (i.e., retarded decline model)

S.No.	θ	M=15 days		M=30 days		M=45 days		M=60 days	
		T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$	T	$K_2(T)$
1	0.05	0.29	1297.72	0.29	1223.53	0.29	1149.34	0.29	1075.16
2	0.06	0.29	1311.73	0.29	1237.55	0.29	1163.36	0.29	1089.18
3	0.07	0.28	1325.59	0.29	1251.41	0.29	1177.23	0.29	1103.04
4	0.08	0.28	1339.30	0.28	1265.12	0.28	1190.94	0.28	1116.76
5	0.09	0.28	1352.87	0.28	1278.69	0.28	1204.51	0.28	1130.34
6	0.10	0.28	1366.30	0.28	1292.13	0.28	1217.95	0.28	1143.77

Case-1($M \leq T$): Table-9

S.No	Parameter	% Change	% Change in T	% Change in $K_2(T)$
1	a	-20%	11.6838	-9.5847
		-10%	5.1546	-4.6231
		10%	-4.8110	4.3345
		20%	-8.9347	8.4197
2	b	-20%	-0.3436	0.1950
		-10%	-0.3436	0.0975
		10%	0.0000	-0.0978
		20%	0.0000	-0.1955
3	c	-20%	-0.3436	0.1958
		-10%	-0.3436	0.1954
		10%	-0.3436	0.1947
		20%	-0.3436	0.1943
4	I_c	-20%	-0.3436	0.1950
		-10%	-0.3436	0.1950
		10%	-0.3436	0.1950
		20%	-0.3436	0.1950
5	I_e	-20%	3.7801	-1.7468
		-10%	1.7182	-0.7544
		10%	-2.0619	1.1039
		20%	-3.7801	1.9745
6	h	-20%	5.4983	-5.9713
		-10%	2.4055	-2.8441
		10%	-2.7491	3.1532
		20%	-5.1546	6.0364
7	p	-20%	0.3436	-0.7257
		-10%	0.0000	-0.2644
		10%	-0.6873	0.6527
		20%	-1.0309	1.1085
8	γ	-20%	-0.3436	0.2747
		-10%	-0.3436	0.2349
		10%	-0.3436	0.1551
		20%	-0.3436	0.1153

Case-2($M > T$ *): Table-10*

S.No	Parameter	% Change	% Change in T	% Change in $K_1(T)$
1	a	-20%	11.4695	-9.4032
		-10%	5.3763	-4.5309
		10%	-4.6595	4.2404
		20%	-8.9606	8.2299
2	b	-20%	-0.3584	0.1865
		-10%	-0.3584	0.0933
		10%	0.0000	-0.0935
		20%	0.0000	-0.1869
3	c	-20%	0.0000	-0.2381
		-10%	0.0000	0.0004
		10%	0.0000	-0.0003
		20%	0.0000	-0.0006
4	I_c	-20%	3.9427	-2.5888
		-10%	1.7921	-1.2700
		10%	-2.1505	1.2244
		20%	-3.9427	2.4064
5	I_e	-20%	0.0000	0.3472
		-10%	0.0000	0.1736
		10%	-0.3584	-0.1740
		20%	-0.3584	-0.3483
6	h	-20%	5.0179	-5.7520
		-10%	2.5090	-2.8386
		10%	-2.5090	2.7693
		20%	-4.6595	5.4743
7	p	-20%	0.7168	-0.8201
		-10%	0.3584	-0.4094
		10%	-0.3584	0.4079
		20%	-0.7168	0.8144
8	γ	-20%	-0.3584	0.0711
		-10%	-0.3584	0.0356
		10%	0.0000	-0.0356
		20%	0.0000	-0.0712

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