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On πGr-Separation Axioms

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Abstract: *In the present paper , we introduce and study the concept of πgr- T_i- space (for i =0,1,2) and obtain the characterization of πgr –regular space , πgr- normal space by using the notion of πgr-open sets. Further, some of their properties and results are discussed.*

Key Words: *πgr-T_0-space, πgr-T_1- space, πgr-T_2 –space, πgr- Normal, πgr- Regular.*

AMS Subject Classification: *54A05, 54D10, 54D15.*

I. Introduction

In 1970, Levine[8] introduced the concept of generalized closed set and discussed the properties of closed and open maps, compactness, normal and separation axioms. Later in 1996 Andrijivic [1]gave a new type of generalized closed set in topological space called b closed sets. The concept of regular continuous functions was introduced by Arya.S.P and Gupta.R [3]. Later Palaniappan.N and Rao. K.C[12] studied the concept of regular generalized continuous functions. Also, the concept of generalized regular closed sets in topological space was introduced by Bhattacharya.S[4,5].Zaitsev [14]defined the concept of π-closed sets and a class of topological spaces called quasi- normal spaces. Dontchev and Noiri [6] defined the notion of πg-closed sets and used this notion to obtain a characterization , preservation theorem for quasi- normal spaces. Maheswari and Prasad[10,11] first defined the notion of S-normal spaces by replacing open sets in the definition of normal spaces by semi-open sets due to Levine[9]. In 1973, Singal and Singal [13] introduced a weak form of normal spaces called mildly normal spaces.!n 1990, Arya and Nour[2] studied the characterizations of s-normal spaces. In 2012, Jeyanthi.V and Janaki.C[7] introduced πgr-closed sets in topological spaces.

The purpose of this paper is to introduce and study πgr-separation axioms in topological spaces. Further we introduced the concepts of πgr-regular space,πgr-Normal Space and study their behaviour.

II. Preliminaries

Throughout this paper (X,τ) , (Y,σ) (or simply X, Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated.

For a subset A of a topological space X, the closure and interior of A with respect to τ are denoted by Cl(A) and Int(A) respectively.

Definition: 2.1

A subset A of X is said to be regular open [12]if A=int(cl(A)) and its complement is regular closed.

The finite union of regular open set is π-open set[6,14] and its complement is π-closed set. The union of all regular open sets contained in A is called rint(A)[regular interior of A] and the intersection of regular closed sets containing A is called rcl(A)[regular closure of A]

Definition: 2.2

A subset A of X is called πgr-closed[7] if rcl(A) ⊂U whenever A⊂U and U is π-open,. The complement of πgr-closed set is πgr-open set. The family of of all πgr-closed subsets of X is denoted by πGRC(X) and πgr-open subsets of X is denoted by πGRO(X)

Definition: 2.3

The intersection of all πgr-closed containing a set A is called πgr-closure of A and is denoted by πgr-Cl(A). The union of πgr-open sets contained in A is called πgr-interior of A and is denoted by πgr-int(A).

Definition: 2.4

A function f: (X,τ)→(Y,σ) is called

1. Continuous [9]if $f^{-1}(V)$ is closed in X for every closed set V in Y.
2. Regular continuous (r-continuous) [3]if $f^{-1}(V)$ is regular-closed in X for every closed set V in Y.
3. An R-map[6] if $f^{-1}(V)$ is regular closed in X for every regular closed set V of Y.
4. πgr-continuous[7] if $f^{-1}(V)$ is πgr-closed in X for every closed set V in Y.
5. πgr-irresolute[7] if $f^{-1}(V)$ is πgr-closed in X for every πgr -closed set V in Y.

Definition: 2.5
A space X is called a πgr-$T_{1/2}$ space [7]if every πgr-closed set is regular closed.
Definition: 2.6
A map f: X→Y is called
1. Closed [9]if f(U) is Y for every closed set U of X.
2. R-closed (i.e,regular closed) [12]if f(U) is regular closed in Y for every closed set U of X
3. rc-preserving [6]if f(U) is regular closed in Y for every regular closed set U of X

Definition: 2.7
A map f: X→Y is called
1. πgr-open map f(V) is πgr-open in Y for every open set V in X.
2. strongly πgr-open map (M-πgr-open)if f(V) is πgr-open in Y for every πgr-open set V in X.
3. Quasi πgr-open if f(V) is open in Y for every πgr-open set V in X.
4. Almost πgr-open map if f(V) is πgr-open in Y for every regular open set V in X.

Definition: 2.8
A space X is said to be R-regular [10] if for each closed set F and each point x∉F, there exists disjoint regular open sets U and V such that x∈U and F⊂V.
Definition: 2.9
A space X is said to be R-Normal[11,13] (Mildly Normal)if for every pair of disjoint regular closed sets E and F of X, there exists disjoint open sets U and V such that E⊂U and F⊂ V.

III. πGr Separation Axioms

In this section, we introduce and study πgr-separation axioms and obtain some of its properties.
Definition: 3.1
A space X is said to be πgr-T_o -space if for each pair of distinct points x and y of X, there exists a πgr-open set containing one point but not the other.
Theorem: 3.2
A space X is πgr-T_o -space iff πgr-closures of a distinct points are distinct.
Proof: Let x and y be distinct points of X. Since X is a πgr-T_o-space, there exists a πgr-open set G such that x∈G and y∉G.

Consequently , X–G is a πgr-closed set containing y but not x. But πgr-cl(y) is the intersection of all πgr-closed sets containing y. Hence y∈πgr-cl(y), but x∉πgr-cl(y) as x∉X-G. Therefore, πgr-cl(x) ≠πgr-cl(y).

Conversely, let πgr-cl(x) ≠πgr-cl(y) for x≠y.
Then there exists atleast one point z∈X such that z∉ πgr-cl(y).

We have to prove x∉ πgr-cl(y), because if x∈ πgr-cl(y), then {x}⊂ πgr-cl(y)

⟹ πgr-cl(x)⊂πgr-cl(y).So, z∈ πgr-cl(y), which is a contradiction. Hence x∉ πgr-cl(y).⟹x∈X-πgr-cl(y), which is a πgr-open set containing x but not y. Hence X is a πgr-T_o-space.
Theorem: 3.3
If f:X→Y is a bijection, strongly-πgr-open and X is a πgr-T_o-space, then Y is also πgr-T_o-space.
Proof: Let y_1 and y_2 be two distinct points of Y. Since f is bijective, there exists points x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$.Since X is a πgr-T_o-space, there exists a πgr-open set G such that x_1 ∈G and x_2∉G. Therefore, $y_1 = f(x_1)$ ∈f(G), $y_2 = f(x_2)$∉f(G). Since f is strongly πgr-open function , f(G) is πgr-open in Y. Thus, there exists a πgr-open set f(G) in Y such that y_1∈f(G) and y_2∉πgr-T_o-space.
Definition: 3.4
A space X is said to be πgr-T_1-space if for any pair of distinct points x and y, there exists πgr-open sets G and H such that x∈G,y∉G and x∉H, y∈H.
Theorem: 3.5
A space X is πgr-T_1-space iff singletons are πgr-closed sets.
Proof: Let X be a πgr-T_1-space and x∈X .Let y ∈X−{x}. Then for x≠y, there exists πgr-open set U_y such that y∈U_y and x∉U_y.
Conversely, y∈U_y ⊂X-{x}.
That is X−{x} = $\cup\{U_y$: y∈X−{x}},which is πgr-open set.

Hence {x} is πgr-closed set.

Conversely, suppose {x} is πgr-closed set for every $x\in X$. Let $x,y \in X$ with $x\neq y$. Now, $x\neq y \Rightarrow y\in X-\{x\}$. Hence $X-\{x\}$ is πgr-open set containing y but not x. Similarly, $X-\{y\}$ is πgr-open set containing x but not y. Therefore, X is a πgr-T_1-space.

Theorem: 3.6

If $f: X\rightarrow Y$ is strongly πgr -open bijective map and X is πgr -T_1 -space, then Y is πgr- T_1- space.

Proof: Let $f: X\rightarrow Y$ be bijective and strongly-πgr-open function. Let X be a πgr-T_1-space and y_1,y_2 be any two distinct points of Y.

Since f is bijective, there exists distinct points x_1, x_2 of X such that $y_1=f(x_1)$ and $y_2=f(x_2)$. Now, X being a πgr-T_1-space, there exists πgr-open sets G and H such that $x_1 \in G, x_2\notin G$ and $x_1\notin H$, $x_2\in H$. Since $y_1=f(x_1) \in f(G)$ but $y_2=f(x_2)\notin f(G)$ and $y_2= f(x_2)\in f(H)$ and $y_1 = f(x_1) \notin f(H)$.

Now, f being strongly-πgr-open, f(G) and f(H) are πgr-open subsets of Y such that $y_1 \in f(G)$ but $y_2\notin f(G)$ and $y_2\in f(H)$ and $y_1\notin f(H)$. Hence Y is πgr-T_1-space.

Theorem: 3.7

If $f: X\rightarrow Y$ is πgr -continuous injection and Y is T_1 , then X is πgr- T_1- space.

Proof: Let $f: X\rightarrow Y$ be πgr- continuous injection and Y be T_1 . For any two distinct point x_1, x_2 of X, there exists distinct points y_1,y_2 of Y such that $y_1 = f(x_1)$ and $y_2= f(x_2)$.

Since Y is T_1- space, there exists open sets U and V in Y such that $y_1 \in U$ and $y_2 \notin U$ and $y_1 \notin V$, $y_2 \in V$.

i.e. $x_1\in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2\in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$

Since f is πgr - continuous, $f^{-1}(U)$, $f^{-1}(V)$ are πgr -open sets in X.

Thus for two distinct points x_1,x_2 of X, there exists πgr - open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$, $x_2 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$.

Therefore, X is πgr -T_1 - space.

Theorem : 3.8

If $f: X \rightarrow Y$ be πgr –irresolute function, and Y is πgr - T_1 - space, there X is πgr -T_1 -space.

Proof: Let x_1, x_2 be distinct points in X. Since f in injective, there exists distinct points y_1, y_2 of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$.

Since Y is πgr -T_1 -space, there exists πgr- open sets U and V in Y such that $y_1 \in U$ and $y_2 \notin U$ and $y_1 \notin V$, $y_2 \in V$.

i.e. $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$.

Since f is πgr- irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are πgr - open sets in X.

Thus, for two distinct points x_1,x_2 of X, there exists πgr- open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$.

Hence X is πgr - T_1 - space.

Definition: 3.9

A space X is said to be πgr-T_2-space, if for any pair of distinct points x and y, there exists disjoint πgr-open sets G and H such that $x\in G$ and $y\in H$.

Theorem: 3.10

If $f: X \rightarrow Y$ be πgr -continuous injection, and Y is T_2-space, then X is πgr -T_2 -space.

Proof: Let $f: X \rightarrow Y$ be the πgr continuous injection and Y be T_2. For any two distinct points x_1 and x_2 of X, there exists distinct points y_1, y_2 of Y such that $y_1 = f(x_1)$, $y_2 = f(x_2)$.Since Y is T_2 -space, there exists disjoint open sets U and V in Y such that $y_1\in U$ and $y_2\in V$.

i.e. $x_1 \in f^{-1}(U)$, $x_2 \in f^{-1}(V)$.

Since f is πgr - continuous, $f^{-1}(U)$ & $f^{-1}(V)$ are πgr - open sets in X.

Further f is injective, $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U\cap V) = f^{-1}(\phi)= \phi$.

Thus, for two disjoint points x_1, x_2 of X, there exists disjoint πgr-open sets $f^{-1}(U)$ & $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$.Hence X is πgr-T_2-space.

Theorem: 3.11

If $f: X \rightarrow Y$ be the πgr irresolute injective function and Y is πgr- T_2 -space, then X is πgr-T_2-space.

Proof : Let x_1,x_2 be any two distinct points in X. Since f in injective, there exists distinct points y_1,y_2 of Y such that $y_1= f(x_1)$ and $y_2 = f(x_2)$.

Since Y is πgr-T_2, there exist disjoint πgr-open sets U and V in Y such that $y_1\in U$ and $y_2\in V$.

i.e, $x_1 \in f^{-1}(U)$, $x_2 \in f^{-1}(V)$.

Since f is πgr-irresolute injective, $f^{-1}(U)$, $f^{-1}(V)$ are disjoint πgr-open sets in X.

Thus, for two distinct points x_1,x_2 of X, there exists disjoint πgr-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1\in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$.

Hence X is πgr-T_2-space.

Theorem: 3.12

In any topological space, the following are equivalent.

1. X is πgr-T_2-space.
2. For each $x \neq y$, there exists a πgr-open set U such that $x \in U$ & $y \notin \pi$gr-cl(U)
3. For each $x \in X$, $\{x\} = \cap \{\pi$gr -cl(U): U is a πgr - open set in Z is $x \in U\}$.

Proof: (1) ⇒ (2): Assume (1) holds.

Let $x \in X$ and $x \neq y$, then there exists disjoint πgr-open sets U and V such that $x \in U$ and $y \in V$. Clearly, X–V is πgr-closed set. Since $U \cap V = \phi$, $U \subset X-V$.

Therefore, πgr-cl(U) $\subset$ πgr-cl(X–V)

Y $\notin$ X–V ⇒ y $\notin$ πgr-cl(X–V) and hence y $\notin$ πgr-cl(U), by the above argument.

(2)⇒ (3): For each $x \neq y$; there exists a πgr-open set U such that $x \in U$ and $y \notin \pi$gr-cl(U)

So, $y \notin \cap\{\pi$gr -cl(U): U is a πgr - open set in X and $x \in U\} = \{x\}$.

(3) ⇒ (1): Let $x,y \in X$ and $x \neq y$.

By hypothesis, there exists a πgr-open set U such that $x \in U$ and $y \notin \pi$gr-cl(U).

⇒ There exists a πgr-closed set V set $y \notin V$. Therefore, $y \in X-V$ and X–V is a πgr -open set.

Thus, there exists two disjoint πgr-open sets U and X–V such that $x \in U$ and $y \in X-V$.

Therefore, X is πgr-T_2-space.

IV. πGr- Regular Space

Definition: 4.1

A space X is said to be πgr-regular if for each closed set F and each point $x \notin F$, there exists disjoint πgr-open sets U and V such that $x \in U$ and $F \subset V$.

Theorem: 4.2

Every πgr-regular T_o - space is πgr-T_2.

Proof : Let $x,y \in X$ such that $x \neq y$.

Let X be a T_o-space and V be an open set which contains x but not y.

Then X-V is a closed set containing y but not x. Now, by πgr-regularity of X, there exists disjoint πgr-open sets U and W such that $x \in U$ and $X-V \subset W$.

Since $y \in X-V$, $y \in W$.

Thus, for $x,y \in X$ with $x \neq y$ there exists disjoint open sets U and W such that $x \in U$ and $y \in W$.

Hence X is πgr-T_2-space.

Theorem: 4.3

If f: X→Y is continuous bijective, πgr- open function and X is a regular space, then Y is πgr-regular.

Proof: Let F be a closed set in Y and $y \notin F$. Take y=f(x) for some $x \in X$.

Since f is continuous, $f^{-1}(F)$ is closed set in X such that $x \notin f^{-1}(F)$. (since $f(x) \notin F$)

Now, X is regular, there exists disjoint open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$.

i.e. $y=f(x) \in f(U)$ and $F \subset f(V)$.

Since f is πgr-open function, f(U) and f(V) are πgr-open sets in Y.

Since f is bijective, $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi$.

⇒ Y is πgr-regular.

Theorem: 4.4

If f : X→Y is regular continuous bijective, almost πgr-open function and X is R-regular space, then Y is πgr-regular.

Proof:

Let F be a closed set in Y and $y \notin F$.

Take y=f(x) for some $x \in X$.

Since f is regular continuous function , $f^{-1}(F)$ is regular closed in X and hence closed in X.

⇒ $x = f^{-1}(y) \notin f^{-1}(F)$.

Now, X is R-regular, there exists disjoint regular open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$.

i.e. $y=f(x) \in f(U)$ and $F \subset f(V)$.

Since f is almost πgr-open function f(U) and f(V) are πgr-open sets in Y and also f is bijective, $f(U) \cap f(V) =$ $f(U \cap V)$

$= f(\phi) = \phi$.

⇒ Y is πgr-regular.

Theorem: 4.5
If f: $X \rightarrow Y$ is continuous, bijective,strongly πgr-open function(quasi πgr-open) and X is πgr-regular space, then Y is πgr-regular(regular).
Proof : Let F be a closed set in Y and $y \in F$.
Take $y=f(x)$ for some $x \in X$.
Since f is continuous bijective, $f^{-1}(F)$ is closed in X and $x \notin f^{-1}(F)$.
Now, since X is πgr-regular, there exists disjoint πgr-open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$.
i.e. $y=f(x) \in f(U)$ and $F \subset f(V)$.
Since f is strongly πgr-open (quasi πgr-open) and bijective, f(U) and f(V) are disjoint πgr-open(Open) sets in Y.
∴ Y is πgr-regular(regular).
Theorem: 4.6
If $f:X \rightarrow Y$ is πgr-continuous, closed, injection and Y is regular, then X is πgr-regular.
Proof: Let F be a closed in X and $x \notin F$.
Since f is closed injection, f(F) is closed set in Y such that $f(x) \notin f(F)$.
Now, Y is regular, there exists disjoint open sets G and H such that $f(x) \in G$ and $f(F) \subset H$.
This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$.
Since f is πgr-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$.
Hence X is πgr-regular.
Theorem : 4.7
If $f:X \rightarrow Y$ is almost πgr- continuous, closed injection and Y is R-regular, then X is πgr-regular.
Proof: Let F be a closed set in X and $x \notin F$. Since f is closed injection. f(F) is closed set in Y such that $f(x) \notin f(F)$.Now, Y is R-regular, there exists disjoint regular open sets G and H such that $f(x) \in G$ and $f(F) \subset H$.
$\Rightarrow x \in f^{-1}(G)$ & $F \subset f^{-1}(H)$
Since f is almost πgr-continuous, $f^{-1}(G)$ & $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$.
Hence X is πgr-regular.
Theorem: 4.8
If f: $X \rightarrow Y$ is πgr-irresolute, closed, injection and Y is πgr-regular,then X is πgr-regular.
Proof: Let F be a closed set in X and $x \notin F$.Since f is closed injection, f(F) is closed set in Y such that $f(x) \notin f(F)$.
Now, Y is πgr-regular, there exists disjoint πgr-open sets G and H such that $f(x) \in G$ and $f(F) \subset H$.
$\Rightarrow x \in f^{-1}(G)$ & $F \subset f^{-1}(H)$.
Since X is πgr-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$ and hence X is πgr-regular.

V. πGr-Normal Spaces

Definition: 5.1
A space X is said to be πgr-Normal if for every pair of disjoint closed sets E & F of X, there exists disjoint πgr-open sets U & V such that $E \subset U$ and $F \subset V$.
Theorem:5.2
The following statements are equivalent for a Topological space X:
1.X is πgr- normal.
2.For each closed set A and for each open set U containing A, there exists a πgr-open set V containing A such that πgr-cl(V) ⊂ U
3.For each pair of disjoint closed sets A and B, there exists πgr-open set U containing A such that πgr-cl(U)∩B = ϕ.
Proof:(1)⇒ (2): Let A be closed set and U be an open set containing A.
Then $A \cap (X-U) = \phi$ and therefore they are disjoint closed sets in X.
Since X is πgr-normal, there exists disjoint πgr-open sets V and W such that $A \subset V$, $X-U \subset W$. i.e. $X-W \subset U$.
Now, $V \cap W = \phi$, implies $V \subset X-W$
Therefore, πgr-cl(V)⊂ πgr-cl(X−W)=X−W, Because X−W is πgr-closed set.
Thus, A⊂ V ⊂ πgr-cl(V) ⊂ X−W ⊂ U.
i.e. A⊂V ⊂ πgr -cl(V) ⊂ U.
(2)⇒(3): Let A and B be disjoint closed sets in X, then $A \subset X-B$ and X−B is an open set containing A. By hypothesis, there exists a πgr-open set U such that $A \subset U$ and πgr-cl(U)⊂X−B, which implies πgr-cl(U)∩B = ϕ

(3)⇒(1): Let A and B be disjoint closed sets in X. By hypothesis (3), there exists a πgr-open set U such that $A\subset U$ and $\pi gr\text{-}cl(U)\cap B = \phi$ (or) $B\subset X - \pi gr\text{-}cl(U)$.
Now, U and $X-\pi gr\text{-}cl(U)$ are disjoint πgr-open sets such that $A \subset U$ and $B \subset X - \pi gr\text{-}cl(U)$.
Hence X is πgr-normal.

Definition: 5.3
A space X is said to be mildly πgr-Normal if for every pair of disjoint regular closed sets E & F of X, there exists disjoint πgr-open sets U & V such that $E\subset U$ and $F\subset V$.

Theorem: 5.4
If $f:X\to Y$ is continuous bijective, πgr-open function from a normal spaces X onto a space Y, then Y is πgr-normal.
Proof: Let E and F be disjoint closed sets in Y,
Since f is continuous bijective $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint closed sets in X.
Now, X is normal, there exists disjoint open sets U and V such that $f^{-1}(E) \subset U$, $f^{-1}(F)\subset V$.
i.e. $E\subset f(U)$, $f\subset f(V)$.
Since f is πgr-open function, f(U) and f(V) are πgr-open sets in Y and f is injective, $f(U)\cap f(V) = f(U\cap V) = f(\phi) = \phi$.Hence Y is πgr-Normal.

Theorem: 5.5
If $f:X\to Y$ is regular continuous bijective, almost πgr-open function from a mildly normal space X onto a space Y, then Y is πgr-normal.
Proof: Let E and F be disjoint closed sets in Y, Since f is regular continuous bijective $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint regular closed sets in X.
Now, X is mildly normal, there exists disjoint regular open sets U and V, such that $f^{-1}(E) \subset U$, $f^{-1}(F)\subset V$.
i.e. $E\subset f(U)$, $F\subset f(V)$.Since f is almost πgr-open function, f(U) & f(V) are πgr-open sets in Y and f is injective,
$f(U)\cap f(V) = f(U\cap V)$
$= f(\phi) = \phi$.
Thus, Y is πgr-Normal.

Theorem: 5.6
If $f:X\to Y$ is πgr-continuous, closed, bijective, and Y is normal, then X is πgr-normal.
Proof: Let E and F be disjoint closed sets in Y, since f is closed injection, f(E) and f(F) are disjoint closed sets in Y.
Now Y is normal, there exists disjoint open sets G and H such that $f(E)\subset G$, $f(F)\subset H$.
$\Rightarrow E\subset f^{-1}(G)$ & $F\subset f^{-1}(H)$.
Since f is πgr-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$.Hence X is πgr-Normal.

Theorem: 5.7
If $f:X\to Y$ is almost πgr-continuous, R-closed injective, and Y is R- normal, then , X is πgr-normal.
Proof: Let E and F be disjoint closed sets in Y. Since f is R-closed injection, f(E) and f(F) are disjoint regular closed sets in Y.
Now Y is Mildly Normal,(i.e, R- normal), there exists disjoint regular open sets G and H such that $f(E)\subset G$, $f(F)\subset H$.
$\Rightarrow E\subset f^{-1}(G)$ & $F\subset f^{-1}(H)$.
Since f is almost πgr-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$.
Hence X is πgr-Normal.

Theorem: 5.8
If $f:X\to Y$ is almost πgr-irresolute, R-closed injection, and Y is πgr - normal, then , X is πgr-normal.
Proof: Let E and F be disjoint closed sets in Y. Since f is R-closed injection, f(E) and f(F) are disjoint regular closed sets in Y.
Now Y is πgr -Normal, there exists disjoint πgr-open sets G and H such that $f(E)\subset G$, $f(F)\subset H$.
This implies $E\subset f^{-1}(G)$ and $F\subset f^{-1}(H)$.
Since f is πgr-irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are πgr-open sets in X.
Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$.
$\Rightarrow$ X is πgr-Normal.

Theorem: 5.9

If f:X→Y is continuous, bijective, M-πgr-open (quasi πgr-open) function from a πgr - normal space X onto a space Y, then Y is πgr-normal (normal).

Proof: Let E and H be disjoint closed sets in Y . Since f is continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint closed sets in X. Now, X is πgr-normal, there exists πgr-open sets U and U such that $f^{-1}(E) \subset U$ and $f^{-1}(F) \subset V$. That is $E\subset f(U)$ and $F\subset f(V)$. Since f is M-πgr-open(quasi πgr-open) function , f(U) and f(V) are πgr-open sets(open sets) in Y and f is bijective,

$f(U) \cap f(V) = f(U\cap V)=f(\phi) =\phi$.

Hence Y is πgr-normal (normal).

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