

Calculating Voltage Instability Using Index Analysis in Radial Distribution System

J.K Garg¹, Pallavi Swami²

Abstract: This paper presents analysis of voltage stability index by a simple and efficient load flow method to find out the magnitude of voltage at each node in radial distribution system in that network. It shows the value of voltage stability index at each node in radial distribution network and predicts which node is more sensitive to voltage collapse. This paper also presents the effect on voltage stability index with variation in active power, reactive power, active and reactive power both. The voltage and VSI and effect of load variation on VSI for 33-node system & 28-node system are calculated in this paper with results shown.

Index Terms: Voltage Stability Index, Load Flow Method, Voltage Collapse.

I. Introduction

The distribution system provides connections for power transfer between transmission and the consumer point. It is essential to carry out the load flow analysis of a network in order to evaluate the performance of a power distribution network during operation and to examine the effectiveness of proposed alterations to a system in the planning stages. This paper presents two issue in RDS (a) load flow method (b) Voltage stability index. The load flow studies are normally carried out to determine: (a) node voltage and branch current, (b) the flow of active and reactive power in network branches, (c) no circuits are overloaded, and the node voltages are within acceptable limits, (e) effect of loss of circuits under emergency conditions, (f) optimum system loading condition, (g) optimum system losses.

Radial Distribution System(RDS) is that distribution system , which have separate feeders radiating from a single substation and feed the distribution at only one end . The radial distribution system have high R/X ratio. Due to this reason, conventional Newton-Raphson method and Fast Decoupled load flow method fails to converge in many cases. Kersting & Mendive (1) and Kersting(2) have developed load flow techniques based on ladder theory whereas Steven et al (3) modified it and proved faster than earlier methods. However, it fails to converge in 5 out of 12 case studies. Baran & Wu (4) have developed a load flow method based on Newton Raphson method, but it requires a Jacobian matrix, a series of matrix multiplication, and at least one matrix inversion. Hence, it is considered numerically cumbersome and computationally inefficient. The choice of solution method for particular application is difficult. It requires a careful analysis of comparative merits and demerits of those methods available. A new power flow method for radial distribution networks with improved converge characteristics have been reported in (5) which is passed on polynomial equation on forward process and backward ladder equation for each branch of RDS. The author of (6) utilizes forward and backward sweep algorithm based on Kirchoff's current and voltage law for evaluating the node voltage iteratively which avoids repetitive computation at each branch and thus makes it computationally efficient and resulted in improved speed. B. Venkatesh (7) formulates a set of equations to describe radial distribution system by high R/X ratio. The benefits of this set of equation are that it is insensitive to the R/X ratio of distribution system and possesses an ability to seek an accurate solution.

First part of this paper makes an effort to modify the load flow algorithm developed in (8) for balanced three-phase RDS to make it computationally efficient. This method can also be used to calculate the energy loss in each branch of Radial Distribution System. The proposed method is simple and mathematically less complex due to non-involvement of matrices.

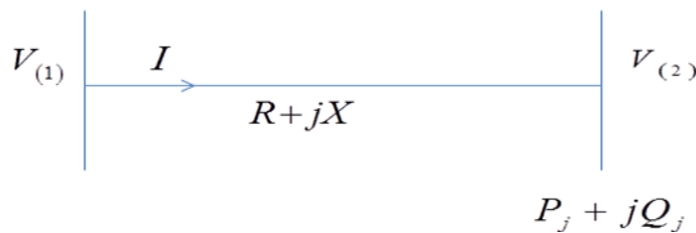
Voltage Stability Index is presented in the second part of this paper. Voltage Stability Index is numerical solution which helps the operator to monitor how close the system is to collapse. The purpose of finding Voltage Stability Index is to find most sensitive node of system that will predict the node which is more sensitive to voltage collapse. The problem of voltage stability & voltage collapse have increased because of the increased loading, exploitation and improved optimization operation of power transmission system. The problem of voltage collapse is the inability of power system to supply the reactive power or by an excessive absorption of reactive power by the system itself. Thus the voltage collapse is a reactive problem and it is strongly affected by the load behavior. Voltage collapse has become an increasing threat to power system security and reliability. One of the serious consequences of voltage stability problem is a system blackout. A fast

method for finding the maximum load, especially the reactive power demand, at a particular bus through thevenin's equivalent circuit before reaching the voltage stability limit is developed in (9) for general power system. Voltage instability in power network is a phenomenon of highly non-linear nature posing operational as well as prediction problem in power system control. The voltage instability is a local phenomenon, variables and network parameters contain sufficient information to assess proximity to instability. Hence a direct analytical approach to voltage instability assessment for radial network is presented in (10). Voltage collapse is characterized by slow variations in system operating point due to increase in loads in such a way that the voltage magnitude gradually decreases until a sharp accelerated change occurs. The voltage stability analysis, using different methods are described in(11) which presents a comparative study and analysis of performance of some on line static voltage collapse indices. Line indices provide accurate information with regard to stability condition of lines. In(12) a new and simple voltage stability margin of a radial distribution system to get a better estimation of distance to voltage collapse is developed.

This paper also presents the effect on VSI and voltage collapse point with increase of active power, reactive power and both active & reactive power. The results for 33-node system & 28 node system with magnitude of voltage, VSI, effect on VSI are shown along with their graphs.

II. Load Flow Calculation

Load flow calculation is an important and basic tool in the field of power system engineering. It is used in planning and designing stages as well operation stages of the power system. Some applications especially in the field of optimization of power system need fast converging load flow solution. To calculate the magnitude of voltage in a network, consider a line connected between two nodes as shown in figure given below.



Where $V^{(1)}$ is sending end of jth branch and $V^{(2)}$ is the receiving end of jth branch.

For flat voltage profile, consider $V^{(1)} = 1$ p.u.

Where R = resistance of branch between node 1 and 2.

X = Reactance of branch between node 1 and 2.

P_j = Total active power loads beyond the node 2.

Q_j = Total reactive power loads beyond the node 2.

$$A_j = (P_j R + Q_j X) - \frac{1}{2} V_1^2$$

$$B_j = \sqrt{A_j^2 - (P_j^2 + Q_j^2)(R^2 + X^2)}$$

$$V_2 = \sqrt{B_j - A_j}$$

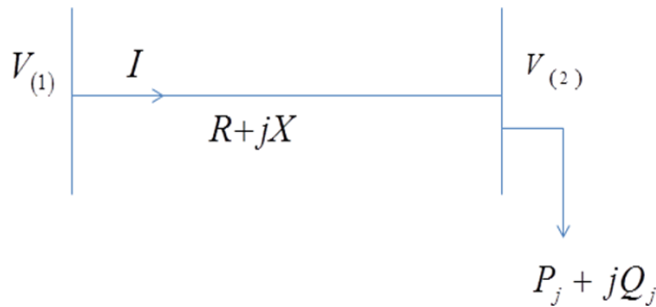
$$P_{loss(j)} = \frac{(P_j^2 + Q_j^2)}{V_2^2} R$$

$$Q_{loss(j)} = \frac{(P_j^2 + Q_j^2)}{V_2^2} X$$

P_{loss} =real power loss of branch

Q_{loss} =reactive power loss of branch

Voltage stability index:-



The proposed voltage stability index will be formulated in this section. The sending end voltage can be written as

$$\begin{aligned}
 V_1 &= V_2 + I(R + jX) \\
 &= V_2 + \frac{S_j^*}{V_2^*} (R + jX) \\
 &= V_2 + \frac{(P_j - jQ_j)}{V_2^*} (R + jX) \\
 &= \frac{|V_2|^2 + P_j R + Q_j X + j(P_j X - Q_j R)}{V_2^*} \quad \dots 1
 \end{aligned}$$

Now substitute the voltage by its magnitude, equation 1 can be written as

$$\begin{aligned}
 |V_1| &= \sqrt{\frac{(|V_2|^2 + P_j R + Q_j X)^2 + (P_j X - Q_j R)^2}{|V_2|}} \\
 |V_1||V_2| &= \sqrt{V_2^4 + (P_j R + Q_j X)^2 + 2|V_2|^2(P_j R + Q_j X) + (P_j X - Q_j R)^2} \quad \dots 2
 \end{aligned}$$

Rearranging equation 2, it will become

$$\begin{aligned}
 V_2^4 + (P_j R + Q_j X)^2 + (P_j X - Q_j R)^2 + 2|V_2|^2(P_j R + Q_j X) - |V_1|^2|V_2|^2 &= 0 \\
 V_2^4 + (P_j R + Q_j X)^2 + (P_j X - Q_j R)^2 + |V_2|^2[2(P_j R + Q_j X) - |V_1|^2] &= 0 \quad \dots 3
 \end{aligned}$$

The equation 3 is in form of $ax^2 + bx + c = 0$. To guarantee that 3 is solvable, the following inequality constraint should be satisfied

$$\begin{aligned}
 b^2 - 4ac &\geq 0 \\
 \text{i.e.,} \\
 V_1^4 - 4|V_1|^2(P_j R + Q_j X) - 4(P_j X - Q_j R)^2 &\geq 0 \quad \dots 4
 \end{aligned}$$

With the increase of receiving end power demand, the left hand side of equation 4 approaches zero, and the two bus network reaches its maximum power transfer limit. So the voltage stability index is

$$VSI = V_1^4 - 4|V_1|^2(P_j R + Q_j X) - 4(P_j X - Q_j R)^2 \quad \dots 5$$

III. Case study

In this paper we are testing the eq.4. by increasing the receiving end power demand.

Case(1):

When active & reactive power both increases with a multiplier K. Then eq.5. will be

$$VSI(P \ \& \ Q) = V_1^4 - 4|V_1|^2(KP_j R + KQ_j X) - 4(KP_j X - KQ_j R)^2$$

Case(2):

When only active power increases with the multiplier K. Then eq.5. will be

$$VSI(P) = V_1^4 - 4|V_1|^2 (KP_j R + Q_j X) - 4(KP_j X - Q_j R)^2$$

Case(3):

When only reactive power increases with the multiplier K. Then eq5. Will be

$$VSI(Q) = V_1^4 - 4|V_1|^2 (P_j R + KQ_j X) - 4(P_j X - KQ_j R)^2$$

IV. Results of System Study

The 33-bus RDS and 28-bus RDS has been considered for the study. The load data and transmission line details, are presented in table 5 in Appendix-C and Fig 1 along with a single line diagram for 33 node system, Table 6 in Appendix-C presents the details of data for 28-bus system along with single line diagram in fig4 . The details of results are presented in table 1 and table 2 in Appendix-A along with their graphical representation in fig 2 and fig 3 respectively for 33-bus system and details of results for 28-bus system are presented in table 3 and in table 4 in Appendix-B along with their graphical representation in fig 5 and fig 6.

NOTE: In Each Graph X-Axis Represents Nodes And Y-Axis Represents Voltage In Per Unit.

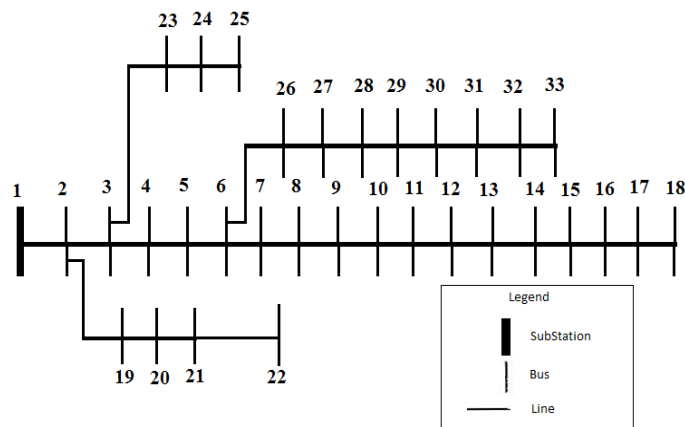
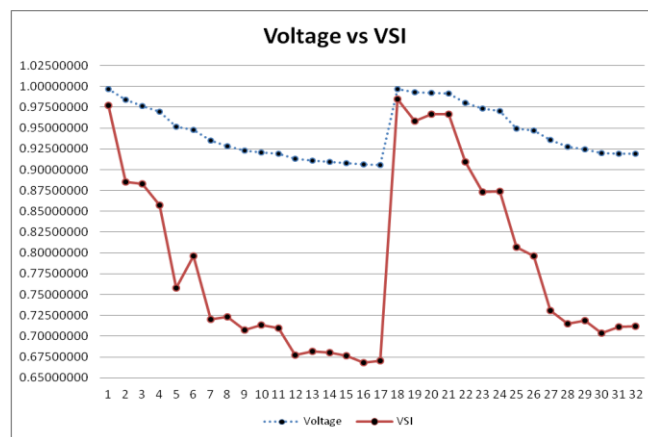


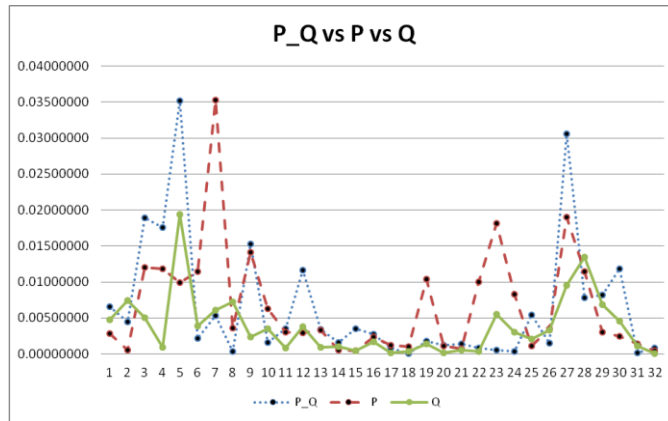
FIG 1: SINGLE LINE DIAGRAM FOR 33 NODE SYSTEM



X-Axis – Number of nodes

Y-Axis – Voltage vs VSI

FIG 2: The graph indicating that both voltage & VSI have same minimum & maximum



X-Axis – Number of nodes
Y-Axis – VSIP_Q vs VSIP vs VSIQ

FIG 3: This graph shows the value of (VSIP_Q) i.e. the values of vsi when we are using multiplier both with p,q to increase the value of receiving end power demand so that the left hand side eq. approaches to zero. Beyond these values at each node result becomes negative

minimum value is at node = 18

VSIP = Varying the equation of VSI by using multiplier with P
minimum value is at node = 15

VSIQ = Varying the equation of VSI by using multiplier with Q
minimum value is at node = 32

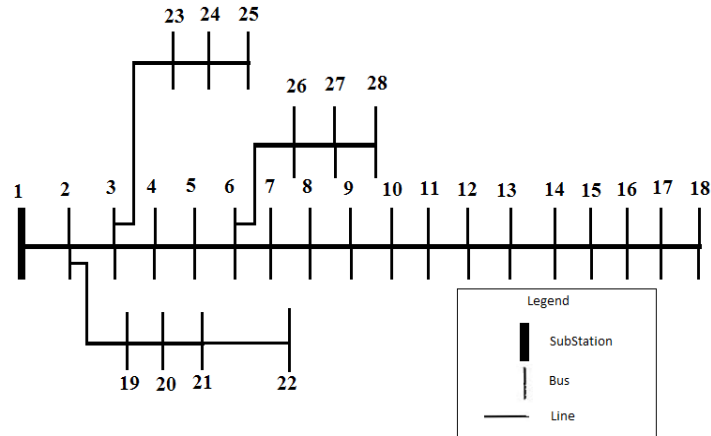
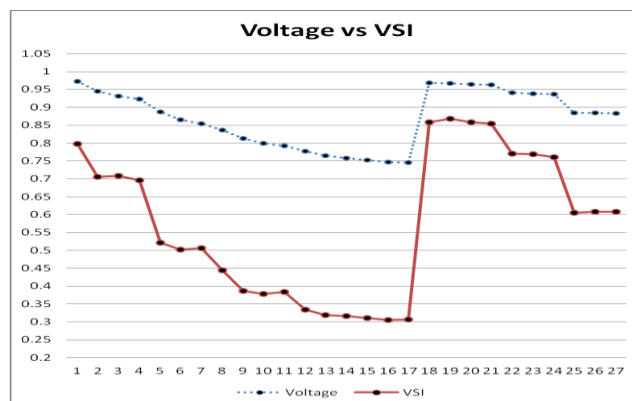
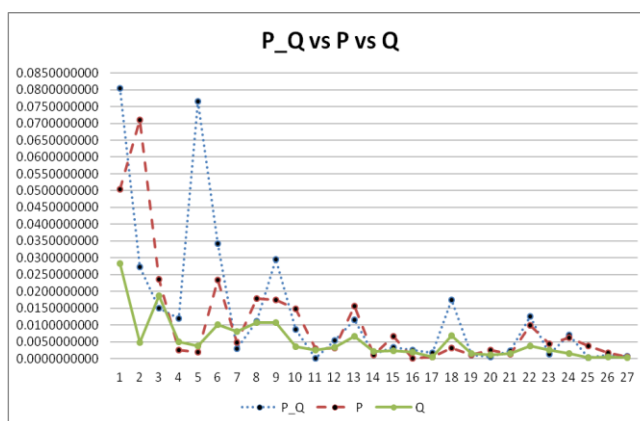


FIG 4: SINGLE LINE DIAGRAM FOR 28 NODE SYSTEM



X-Axis – Number of nodes
Y-Axis – Voltage vs VSI

FIG 5: In this graph both voltage & VSI have same minimum & maximum points
 Minimum voltage is at node n= 17
 Maximum voltage is at node = 18



X-Axis – Number of nodes
 Y-Axis – VSIP_Q vs VSIP vs VSIQ

FIG 6: This graph shows the value of (VSIP_Q) i.e. the values of vsi when we are using multiplier both with p,q to increase the value of receiving end power demand so that the left hand side eq. approaches to zero. Beyond these values at each node result becomes negative.

Minimum value is at node = 11
 VSIP = Varying the equation of VSI by using multiplier with P
 Minimum value is at node = 16
 VSIQ = Varying the equation of VSI by using multiplier with Q
 Minimum value is at node = 25

NOTE: (minimum value node is the voltage collapse point)

V. Conclusion

This paper presents simple and efficient load flow method to find out the magnitude of voltage at each node in Radial Distribution System which is mathematically less complex. The method is tested on 33 node system and 28 node system and the results are shown along with their graphs. Hence minimum voltage node can be located.

A new voltage stability index for RDS is developed which indicates that the node having minimum voltage is highly susceptible to voltage collapse. This equation is tested on 33 node and 28 node system. The results obtained for 33 node system and 28 node systems show the value of VSI at each node and indicate the voltage collapse point. The effect of variation in receiving end power demand on VSI is also tested on both the RDS Systems for the cases when:

- a) Increase in both active and reactive power.
- b) Increase in only active power.
- c) Increase in only reactive power.

It is observed that for above three cases the value of VSI at each node beyond the calculated point will become negative means the system will terminate. It is concluded that with only increase in reactive power it will largely affect on voltage collapse point, as compared to increment in only active power and, with both active and reactive power taken together.

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APPENDIX-A

TABLE-1

Details of Magnitude of Voltage & Vsi at Each Node of 33 Node Systems

NODES	Voltage	VSI
1	0.99717062	0.97750862
2	0.98367898	0.88492637
3	0.97655889	0.88295501
4	0.96948503	0.85762879
5	0.95158811	0.75827594
6	0.94818000	0.79663604
7	0.93480205	0.71989823
8	0.92858852	0.72361773
9	0.92281261	0.70703595
10	0.92088209	0.71310664
11	0.91938468	0.70982353
12	0.91325271	0.67692071
13	0.91097514	0.68180369
14	0.90955564	0.68013802
15	0.90817935	0.67615419
16	0.90613619	0.66809386
17	0.90552406	0.67053897
18	0.99664426	0.98456008
19	0.99307008	0.95856285
20	0.99236598	0.96705921
21	0.99194108	0.96649308
22	0.98012458	0.90945010
23	0.97347089	0.87347550
24	0.97014866	0.87370057
25	0.94971573	0.80711546
26	0.94722385	0.79655438
27	0.93597383	0.73055688

28	0.92782522	0.71504245
29	0.92428631	0.71865483
30	0.92013904	0.70389943
31	0.91922608	0.71114872
32	0.91894317	0.71222868

TABLE-2
Results for Effect of Change in Receiving End Power on Vsi Of 33 Node System

NODES	P_Q	P	Q
1	0.00651695	0.00277727	0.00472652
2	0.00440307	0.00050815	0.00737475
3	0.01889712	0.01199109	0.00504944
4	0.01759400	0.01182700	0.00089393
5	0.03515700	0.00990620	0.01939355
6	0.00213052	0.01144139	0.00390332
7	0.00533382	0.03528813	0.00611866
8	0.00036554	0.00356275	0.00723163
9	0.01528803	0.01415792	0.00233480
10	0.00158603	0.00622429	0.00345392
11	0.00346256	0.00299202	0.00079843
12	0.01158961	0.00287664	0.00375195
13	0.00342967	0.00332398	0.00088147
14	0.00158177	0.00056778	0.00104808
15	0.00345284	0.00040951	0.00039598
16	0.00268090	0.00230304	0.00171501
17	0.00082822	0.00119108	0.00009532
18	0.00003531	0.00101203	0.00037417
19	0.00176814	0.01037072	0.00137989
20	0.00114780	0.00109940	0.00014189
21	0.00135825	0.00069830	0.00051139
22	0.00077607	0.01000156	0.00028852
23	0.00050740	0.01814790	0.00547364
24	0.00029280	0.00827969	0.00300178
25	0.00540307	0.00106780	0.00210112
26	0.00145815	0.00350658	0.00328727
27	0.03063027	0.01901449	0.00955204
28	0.00783061	0.01140097	0.01344417
29	0.00814267	0.00305458	0.00688019
30	0.01185659	0.00244253	0.00450016
31	0.00017201	0.00137474	0.00108570
32	0.00080713	0.00045312	0.00005946

APPENDIX-B

TABLE-3

Details of Magnitude of Voltage & Vsi At Each Node Of 28-Bus System

NODES	Voltage	VSI
1	0.973127117	0.797673997
2	0.945389387	0.705037231
3	0.931467896	0.70777414
4	0.922580044	0.696541018
5	0.887322139	0.521350467
6	0.865082321	0.502450032
7	0.854435069	0.50641817
8	0.835783547	0.444384184
9	0.812771463	0.386954148
10	0.798668748	0.378121214
11	0.792867162	0.383615455
12	0.776906775	0.334355042
13	0.764546365	0.319556679
14	0.757251394	0.316145152
15	0.751991018	0.31083112
16	0.747473473	0.30461443
17	0.745942826	0.30707373
18	0.967768266	0.857739708
19	0.966399397	0.867278648
20	0.964555752	0.85896559
21	0.963013601	0.854552058
22	0.941146243	0.770414227
23	0.938686848	0.768258322
24	0.936187575	0.759956409
25	0.88474105	0.605573185
26	0.883922235	0.608196588
27	0.883512643	0.608197974

TABLE-4

Details of Results for Effect of Change in Receiving End Power on Vsi Of 28 Node Systems

NODES	P_Q	P	Q
1	0.0803396545	0.0503454	0.028211576
2	0.0271909944	0.070972628	0.004768841
3	0.0151153707	0.023538207	0.018743386
4	0.0119562662	0.002554366	0.005021475
5	0.0765888293	0.001911007	0.003775704

6	0.0341933061	0.023390061	0.010178326
7	0.0028865674	0.004837756	0.008043718
8	0.0111861563	0.017849625	0.01073459
9	0.0295596189	0.017449489	0.010636168
10	0.0087606930	0.014744798	0.003512189
11	0.0000412946	0.002937983	0.00251565
12	0.0053930670	0.003186235	0.00340622
13	0.0115142797	0.015728189	0.006737277
14	0.0012318403	0.001117193	0.002230367
15	0.0033455252	0.006706155	0.002361746
16	0.0025793336	0.000179466	0.001917427
17	0.0017861154	0.000539772	0.000524395
18	0.0174343501	0.003231937	0.00693262
19	0.0010999386	0.001148699	0.001480455
20	0.0005982575	0.002556389	0.001093885
21	0.0024153735	0.001253786	0.001583127
22	0.0126126113	0.009813919	0.003868215
23	0.0013314007	0.004435565	0.002614562
24	0.0069653688	0.006321335	0.001503738
25	0.0003562311	0.003731343	0.00029578
26	0.0010813418	0.001797301	0.000407927
27	0.0006234999	0.00045139	0.000365597

APPENDIX-C

DATA FOR 33 NODE SYSTEMS

Line no	Sending Bus	Receiving Bus	Resistance	Reactance	Load at Receiving End Bus	
					Real Power (kW)	Reactive Power (kVAr)
1	1 Mai	2	0.0922	0.0477	100.0	60.0

	n	SS				
2	2	3	0.4930	0.2511	90.0	80.0
3	3	4	0.3660	0.1864	120.0	30.0
4	4	5	0.3811	0.1941	60.0	20.0
5	5	6	0.1890	0.7070	60.0	100.0
6	6	7	0.1872	0.6188	200.0	100.0
7	7	8	1.7114	1.2351	200.0	20.0
8	8	9	1.0300	0.7400	60.0	20.0
9	9	10	1.0400	0.7400	60.0	30.0
10	10	11	0.1966	0.0650	45.0	35.0
11	11	12	0.3744	0.1238	60.0	35.0
12	12	13	1.4680	1.1550	60.0	35.0
13	13	14	0.5416	0.7129	120.0	80.0
14	14	15	0.5910	0.5260	60.0	10.0
15	15	16	0.7463	0.5450	60.0	20.0
16	16	17	1.2890	1.7210	60.0	20.0
17	17	18	0.7320	0.5740	90.0	40.0
18	2	19	0.1640	0.1565	90.0	40.0
19	19	20	1.5042	1.3554	90.0	40.0
20	20	21	0.4095	0.4784	90.0	40.0
21	21	22	0.7089	0.9373	90.0	40.0
22	3	23	0.4512	0.3083	90.0	50.0
23	23	24	0.8980	0.7091	420.0	200.0
24	24	25	0.8960	0.7011	420.0	200.0
25	6	26	0.2030	0.1034	60.0	25.0
26	26	27	0.2812	0.1447	60.0	25.0
27	27	28	1.0590	0.9337	60.0	20.0
28	28	29	0.8042	0.7006	120.0	70.0
29	29	30	0.5075	0.2585	200.0	600.0
30	30	31	0.9744	0.9630	150.0	70.0
31	31	32	0.3105	0.3619	210.0	100.0
32	32	33	0.3410	0.5302	60.0	40.0

TABLE-6
DATA FOR 28 NODE SYSTEMS

Line Number	Sending Bus	Receiving Bus	Resistance	Reactance	Load at Receiving End Bus	Reactive Power (kVAr)
					Real Power (kW)	
1	1	2	1.8	0.7	140	90
2	2	3	2.2	0.9	80	50
3	3	4	1.3	0.5	80	60
4	4	5	0.9	0.3	100	60
5	5	6	3.6	1.5	80	50
6	6	7	2.7	1.1	90	50
7	7	8	1.4	0.6	90	40
8	8	9	2.7	1.1	80	40
9	9	10	3.6	1.5	90	50
10	10	11	2.7	0.7	80	50
11	11	12	1.3	0.3	80	50
12	12	13	4.1	1.1	90	40

13	13	14	4.1	0.8	70	50
14	14	15	3	0.7	70	40
15	15	16	2.7	1.1	70	40
16	16	17	4.1	0.7	60	40
17	17	18	2.7	0.7	60	30
18	2	19	3.4	0.9	70	30
19	19	20	1.3	0.3	50	40
20	20	21	2.7	0.7	50	30
21	21	22	4.9	1.4	40	30
22	3	23	3.5	1	50	30
23	23	24	3	0.8	50	20
24	24	25	5.5	1.5	60	30
25	6	26	2.7	0.7	40	20
26	26	27	1.3	0.3	40	20
27	27	28	1.3	0.3	40	20