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Performance analysis of Hybrid Transform, Hybrid Wavelet and Multi-Resolution Hybrid Wavelet for Image Data Compression

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Abstract: *Compression of digital images play vital role in transmission of multimedia data. This paper presents application of hybrid wavelet transform in image compression. Multi-resolution property of Wavelet transform helps to analyze the information contents of image effectively. This property has been used in image compression application. Hybrid wavelet transform is generated using two different component transforms. Various sizes of these component transforms can be used. In this hybrid wavelet, global and local properties of component transforms are incorporated and hence are called bi-resolution analysis. Different levels of resolutions can also be included in generated hybrid transform. Hence It is called multi-resolution analysis and is applied on images. At each level of resolution number of components can be changed. It provides great flexibility to generate hybrid transform matrix. Image is compressed using hybrid wavelet, hybrid wavelet with multi-resolution and hybrid transform. Their performance is compared and it has been observed that hybrid wavelet transform gives lower error values than multi-resolution analysis and hybrid transform. Along with Root mean Square Error (RMSE), Mean Absolute Error (MAE) and Average Fractional Change in Pixel Value (AFCPV) is used to measure error. AFCPV gives better perception to image quality as it is a fractional change in pixel values. Lower the value of AFCPV better is the image quality.*

Keywords: *AFCPV, Compression Ratio, Hybrid Transform, Hybrid Wavelet transform, Multi-Resolution Transform.*

I. Introduction

In recent years, tremendous increase in internet usage has resulted in increased use of digital and multimedia data. Images are crucial part of this digital data as they represent the information effectively. Efficient storage manipulation and transmission of these digital images is equally important. Image compression serves this purpose. Aim of any compression technique is to eliminate redundant and irrelevant information while preserving the significant information [1]. This is possible as in case of images neighboring pixels are highly correlated and hence contain redundant information. Redundancy reduction removes the details in image which are not noticed by human visual system [2]. Data compression methods are usually classified into two categories: lossless compression and lossy compression. Image data compression is generally lossy compression. Lossy compression provides higher compression than lossless compression but decoded image approximately matches to original image [3]. Compression ratio is basic performance measurement criteria. It is defined as ratio of original data size and compressed data size. Higher compression ratio results in lower image quality and vice versa. Transform based coding is widely used for image compression in which image is transformed from spatial domain to frequency domain. Transforms have property to concentrate useful information into few low frequency coefficients. Initially Fourier transform was used for image compression. In Fourier transform local properties of signal are not detected easily. To overcome this drawback short time Fourier Transform (STFT) was introduced which is also called as window transform. It gives local properties at the cost of global properties [4]. The length of window limits the resolution in frequency. Discrete cosine Transform (DCT) [5] is popular transform used for image compression due to its good energy compaction property. Many compression systems use block based DCT where image is divided into blocks of uniform size and transform is applied on individual block. It does not take into account the discontinuities across the boundaries and results in degraded image. This is called as blocking effect [6]. Wavelets provide solution to these problems [7]. Wavelet transform gives time and frequency representation simultaneously [8]. Wavelet transform has higher energy compaction property. It is applied on entire image rather than on blocked image hence it eliminates blocking effect. Multi-resolution representation of an image is another most important characteristic of wavelet transform. Information contents of the image depend on the local variations of image intensity. Multi-resolution representation provides a hierarchical framework for interpreting the image information. The wavelets can be scaled and shifted to analyze the spatial frequency contents of an image at different resolutions and positions. This paper focuses on use of multi-resolution property in image compression.

Remaining sections of this paper are organized as follows: section II contains review of literature, proposed method is presented in section III, results of experimental work are discussed in section IV and conclusion is outlined in section V.

II. REVIEW OF LITERATURE

So far wavelets of Haar transform have been studied because it is simple and fast. Modified fast Haar wavelet transform (MFHWT) has been discussed by Chang P. et al. [9]. It uses one dimensional approach and FHT is used to find N/2 detail coefficients at each level for a signal of length N. Extension of this work has been proposed by Anuj Bharadwaj and Rashid Ali [10]. It works for 2D images with the addition of considering the detail coefficients for N/2 elements at each level. Comparison of different wavelets in medical image compression is discussed in [**Error! Bookmark not defined.**]. It compares performance of Daubechies, Coeflits, Haar and biorthogonal transform. Image compression using Walsh transform has been studied in [11]. Image compression using approximate matching and run length coding is proposed in [12].

Compression using biorthogonal wavelet transform based on lifting scheme with SPIHT is proposed by Hong Liu et al. [13]. Wavelet based image compression using zero tree data structure is proposed by Tham Jo Yiew [14]. Wavelet transform decomposes the image hierarchically into oriented sub bands and then encodes the wavelet coefficients using zero tree data structure. Quad EZW method is further used to decompose higher frequency sub bands. Also blocks of wavelet coefficients are shifted with optimum translation factors prior to application of inverse wavelet transform to obtain better PSNR and reconstructed image quality. Column, row and full wavelet transform based image compression and comparison of their performances has been proposed by Kekre et al. [15]. In their paper simpler method of wavelet generation [16] has been used to generate the wavelet transform from orthogonal transform. Column wavelet transform can be used for compression to save computational overhead in full wavelet transform. Compressions using real Fourier transform and its wavelet transform has been presented in [17]. Wavelets of DCT, Discrete Sine Transform (DST), Hartley transform and Real-DFT have been used for image compression by Kekre et al. [18] by varying the sizes of their component orthogonal transforms. Different size combinations of component transforms give variation in error values. Proper component sizes can be selected to get acceptable image quality. Use of Kekre wavelet, Slant wavelet and Walsh wavelet is proposed in [19] by H.B. Kekre, Tanuja Sarode and Prachi Natu.

Use of hybrid techniques for image compression has also been done. Hybrid techniques combine two or more traditional approaches so that features of two individual techniques can be combined to get better performance. Transform coding combined with predictive coding was used by Clarke [20]. Recently hybrid DCT-VQ based approach for efficient compression of color images has been proposed by Arup Kumar Pal et al. [21]. Image compression based on DCT based compressive sensing and vector quantization is proposed by Dipti Bhatnagar and Sumeet Budhiraja [22]. It combines the advantages of compressive sensing and JPEG image compression standard. Compression based on discrete wavelet transform with arithmetic coding is proposed by Deepika Sunoriya et al. [23]. It applies Walsh Wavelet on blocks of image and then compression is achieved by arithmetic coding. Hybrid Wavelet transform using Kekre Transform [24] has been recently proposed [25] in which different orthogonal transforms can be combined with Kekre Transform to generate hybrid wavelet transform. Column hybrid wavelet transform can be used to save computations at small reduction in compression ratio.

III. PROPOSED TECHNIQUE

In this paper, Kekre transform (DKT) and Hartley transform is used to generate hybrid wavelet transform. By varying the global properties in hybrid wavelet transform hybrid transform is generated and used to compare its performance with hybrid wavelet. Also hybrid wavelet with multi-resolution is generated and its performance is compared with other two.

3.1 Hybrid Wavelet Transform

Hybrid wavelet transform matrix is generated using two different orthogonal component transform matrices. If A is a matrix of size pxp and B is matrix of size qxq then hybrid wavelet transform generated from A and B is of size nxn=pqxpq [**Error! Bookmark not defined.**]. In this paper Discrete Kekre transform (DKT) and Hartley transform (DHT) are used as component transforms to generate DKT-DHT hybrid wavelet transform.

3.2 Kekre Transform

Kekre Transform is an orthogonal transform in which all diagonal elements and the upper diagonal elements are one. Lower diagonal elements except the one exactly below the diagonal are zero. All other orthogonal transforms need to be in power of two. But this restriction is not there in Kekre transform.

3.3 Hartley Transform

Hartley transform is closely related to the Fourier transform. It was proposed as an alternative to the Fourier transform by R. V. L. Hartley in 1942, [**Error! Bookmark not defined.**] and is one of many known Fourier-related transforms. Compared to the Fourier transform, the Hartley transform has the advantages of transforming real functions to real functions (as opposed to requiring complex numbers) and of being its own inverse.

3.4 Generation of Hybrid Wavelet Transform

Hybrid wavelet is generated using Kronecker product as it reduces computations to great extent. Kronecker product is given as:

$$A \otimes B = a_{ij} [B] \qquad (1)$$

Where a_{ij} is individual element in matrix A.

Generation of hybrid wavelet is given by following Kronecker product:

$$T_{AB} = \begin{pmatrix} A_p \otimes B_q (1) \\ I_p \otimes B_q (2) \\ I_p \otimes B_q (3) \\ . \\ . \\ . \\ . \\ I_p \otimes B_q (n) \end{pmatrix} \qquad (2)$$

Here A and B are component matrices of size pxp and qxq respectively. B_q (1) indicates first row of matrix B, B_q (2) is second row of matrix B and so on. $A_p \otimes B_q$ (1) gives first p rows of pqxpq matrix which represents global properties of hybrid wavelet transform. Identity matrix I is used to translate the rows of matrix B. Thus matrix B contributes to local properties. In hybrid wavelet, properties of two different component transforms are combined which gives better results than the results obtained in single transform. Such hybrid wavelet transform gives bi-resolution because global and local properties are incorporated in transformation matrix T_{AB}. It means that it will analyze an image at global and local level of resolution. Generation of above hybrid wavelet matrix can be made more flexible to include analysis of image at various levels. Various levels of analysis can be introduced between global and local and hence called as semi global properties. It helps to analyze image at various resolutions. Hybrid wavelet transformation matrix which gives semi global properties is represented using Kronecker product in eq. (3) [**Error! Bookmark not defined.**]. In below matrix, Kronecker product in first row represents first p rows of hybrid wavelet. These rows represent global properties. Here r_0, $r_1, \ldots, r_{n-1}$ are divisors of p except 1 and p. Scaling operation is done using lower order matrices of A and shifting operation is done using Identity matrix 'I'. As size of matrix A decreases, size of 'I' increases by same scale. A and B are selected as two different matrices. Number of rows of matrix B selected for Kronecker product in each row can be varied by varying the values of $I_1, I_2, \ldots, I_n$ in each row of Kronecker product below such that size of T_{AB} remains pqxpq.

$$T_{AB} = \begin{pmatrix} A_p \otimes B_q (0{:}i_1) \\ I_{r0} \otimes (A_{p/r0} \otimes B_q (i_1+1{:}i_2)) \\ I_{r1} \otimes (A_{p/r1} \otimes B_q (i_2+1{:}i_3)) \\ . \\ . \\ . \\ . \\ . \\ I_{rn-1} \otimes (A_{p/rn-1} \otimes B_q (i_{n-2}+1{:}i_{n-1})) \\ I_p \otimes (B_q(i_n{:}q)) \end{pmatrix} \begin{matrix} \text{Global} \\ \text{Semiglobal 1} \\ \text{Semiglobal 2} \\ . \\ . \\ . \\ . \\ . \\ . \\ \text{Local} \end{matrix} \qquad (3)$$

3.5 Performance Measurement Criteria

3.5.1 Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is a frequently used measure of the difference between values predicted by a model and the values actually observed. It is calculated as:

$$\text{RMSE}= \sqrt{\frac{\sum_{i=1}^{p}\sum_{j=1}^{q}(x_{ij}-y_{ij})^2}{p*q}} \tag{4}$$

Basically, the RMSD represents the sample standard deviation of the differences between predicted values and observed values. Here x_{ij} is reconstructed image, y_{ij} is original image and p is total number of rows and q is total number of columns in image.

3.5.2Mean Absolute Error

It is a quantity used to measure how close the predicted values are to the actual outcomes. The mean absolute error is given by:

$$\text{MAE}=\frac{\sum_{i=1}^{p}\sum_{j=1}^{q}(|x_{ij}-y_{ij}|)}{p*q} \tag{5}$$

Where x_{ij}= original Image , y_{ij}=Reconstructed image, p= Number of rows and q=Number of columns.

3.5.3Average Fractional Change in Pixel value (AFCPV)

This parameter is used to measure perceptibility of compressed image to human eye. Higher perceptibility is observed when AFCPV has lower value. It is calculated as:

$$\text{AFCPV}= \frac{\sum_{i=1}^{p}\sum_{j=1}^{q}(|x_{ij}-y_{ij}|)/x_{ij}}{p*q} \tag{6}$$

where x_{ij}= original Image, y_{ij}=Reconstructed image, p= Number of rows and q=Number of columns.

3.6 General Steps applied for Image Compression

Using eq. (2) bi-resolution analysis of image is obtained using hybrid wavelet transform. To get multi-resolution analysis, hybrid wavelet transform is generated using eq. (3) and then following steps are repeated. Variation in results can also be observed by considering all global features of an image and no semi global and local properties. In that case hybrid wavelet matrix is generated using full Kronecker product of component transforms and is called as hybrid transform matrix.

Following are the general steps applied for image compression.

1. Generate hybrid wavelet transform of size equal to image size. Let the generated hybrid wavelet transform is T_{AB}.
2. Transform individual plane of an image.
3. Calculate energy of individual coefficient and Find highest energy coefficients in transformed image.
4. Retain high energy coefficients (low frequency coefficients) and discard those with low energy.
5. Reconstruct image using high energy coefficients obtained in above step.
6. Calculate Root Mean Square Error (RMSE), MAE and AFCPV between original image and reconstructed image for various compression ratios up to 32.
7. Repeat above steps using hybrid wavelet with multi-resolution analysis and also using Hybrid transform.

IV. RESULTS AND DISCUSSIONS

Test set of images used for experimental purpose is shown in Fig. 1. Execution is performed on AMD dual core processor with 4 GB RAM using Matlab 7.0. Kekre Transform is selected as base transform and Hartley transform is used to contribute to local features. Hybrid wavelet (bi resolution analysis), hybrid wavelet with multi-resolution analysis and Hybrid transform is applied on images. Their performances are compared using three different parameters like: Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Average Fractional Change in Pixel value (AFCPV).

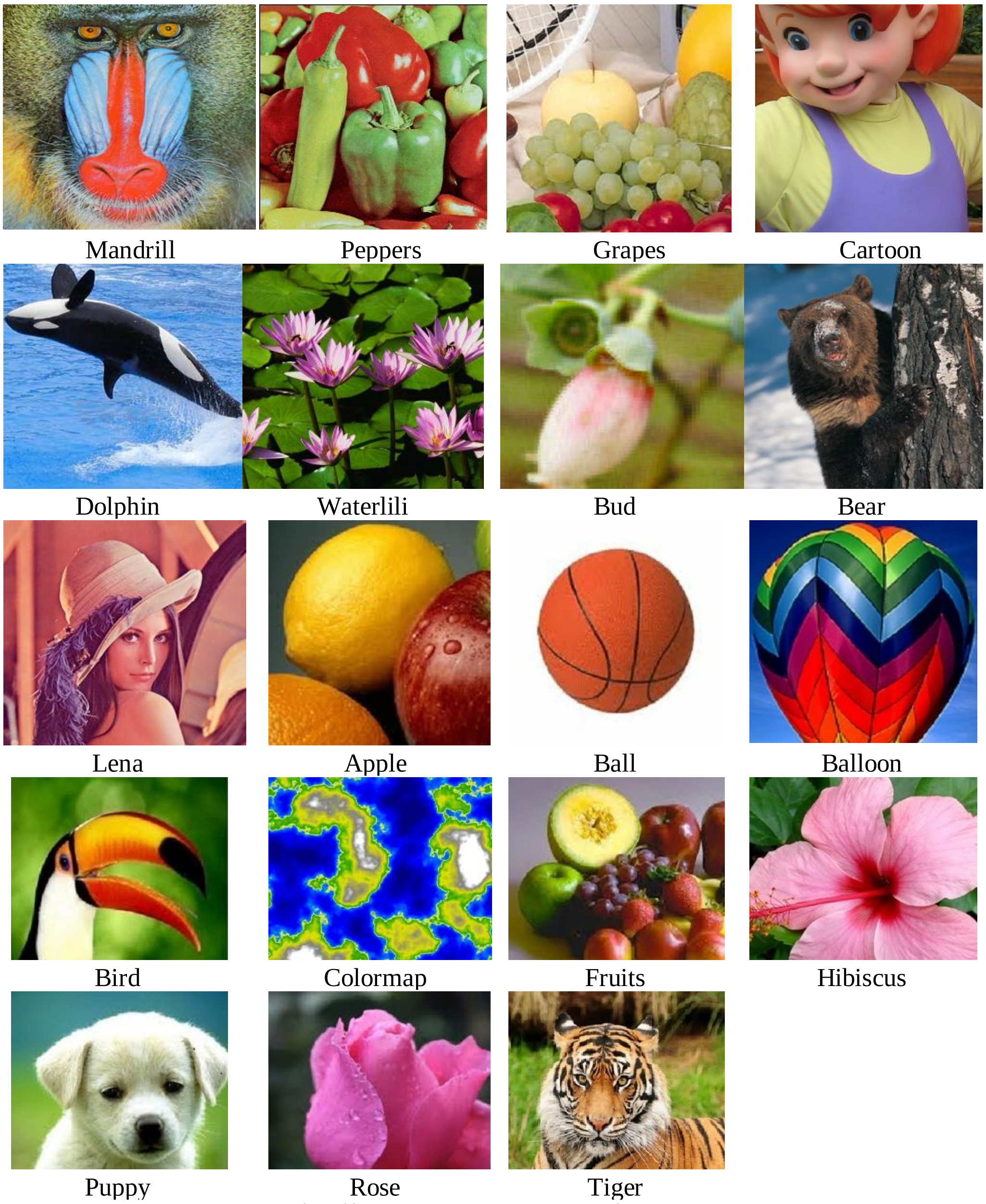


Fig. 1 Test set of Different Color images used for Experimental Purpose

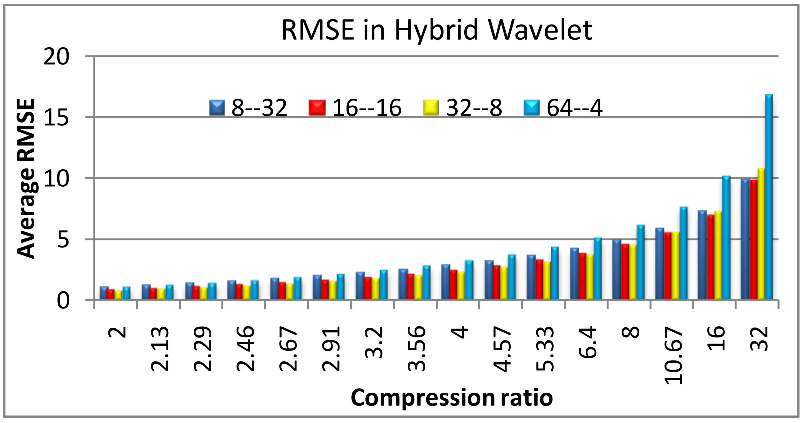


Fig. 2 Average RMSE against compression ratio using DKT-Hartley hybrid wavelet transform by varying the size of component transforms

Fig. 2 shows plot of RMSE vs. Compression ratio using DKT-Hartley hybrid wavelet transform. Here hybrid wavelet is generated using Kronecker product in eq. (1). Hence it gives bi-resolution analysis i.e. contain global and local properties. Size of component transforms is varied to get transform matrix of size 256x256. In above graph 8--32 indicate that first figure '8' is size of base transform i.e. Kekre transform and second figure '32' is size of Hartley transform. Similarly other size combinations like 16--16, 32--8 and 64--4 were tried. Up to compression ratio 16, size 16--16 and 32--8 give nearly equal values of RMSE but for higher compression ratio of 32, lesser error is given by 16--16 size DKT-Hartley hybrid wavelet.

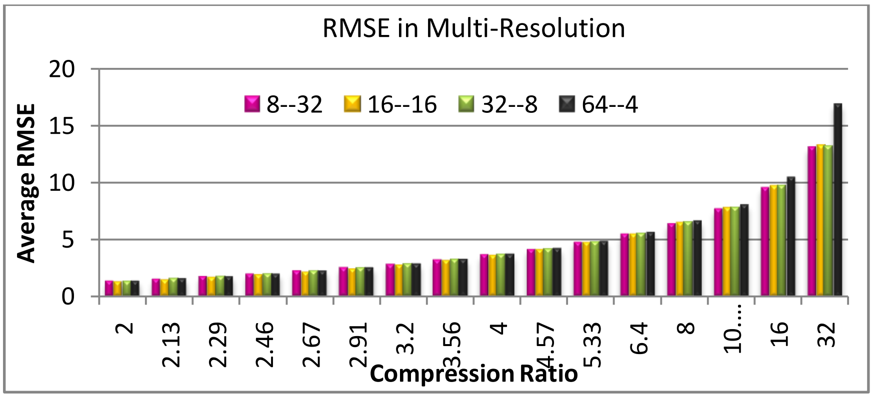


Fig. 3 Average RMSE against compression ratio using multi resolution analysis of DKT-Hartley hybrid wavelet transform by varying the size of component transforms

Fig. 3 shows graph of RMSE against Compression ratio using multi-resolution analysis of DKT-Hartley hybrid wavelet. Transform matrix is generated using eq. (2) to include semi global properties along with global and local properties. Size of component transforms is varied to select one which gives minimum RMSE. All size combinations except 64--4, give almost equal error.

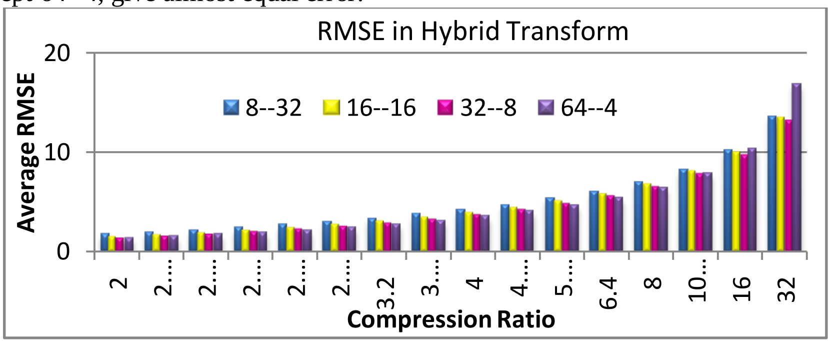


Fig. 4 Average RMSE against compression ratio using multi resolution analysis of DKT-Hartley hybrid transform by varying the size of component transforms

Fig. 4 shows RMSE vs. compression ratio using hybrid DKT-Hartley transform. Hybrid transform is Kronecker product of two component transforms. Hence it is the limiting case of hybrid wavelet where no local properties are included. DKT size 32x32 and Hartley size 8x8 gives lower RMSE at higher compression ratios like 16 and 32 than other combinations.

Fig. 5, 6 and 7 show plot of Mean Absolute Error (MAE) against compression ratio using hybrid wavelet, hybrid wavelet with multi-resolution analysis and hybrid transform respectively. In each of these cases, different size combinations of hybrid wavelet give better performance. In hybrid wavelet 8--32 sizes of components shows lowest MAE at compression ratio 32. In hybrid wavelet with multi-resolution analysis all combinations except 64-4 show nearly same performance whereas in hybrid transform, minimum MAE is obtained using DKT 32x32 and Hartley 8x8 size.

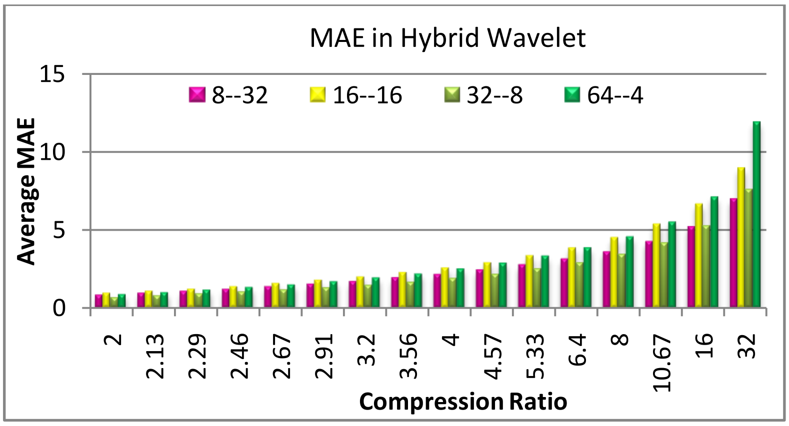


Fig. 5 Average MAE against compression ratio using DKT-Hartley hybrid wavelet transform by varying the size of component transforms

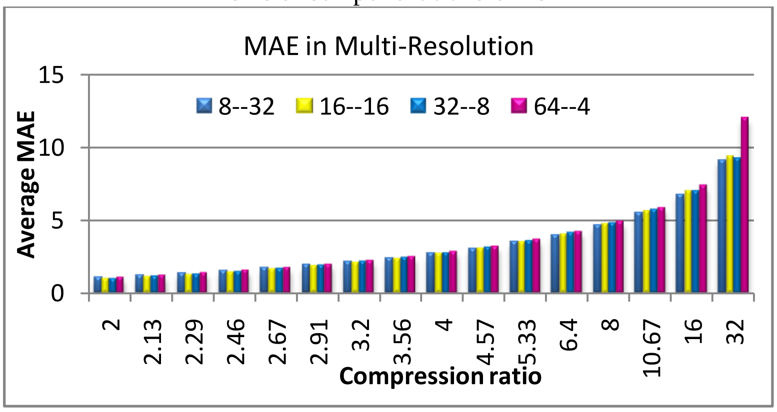


Fig. 6 Average MAE against compression ratio using multi resolution analysis of DKT-Hartley hybrid wavelet transform by varying the size of component transforms

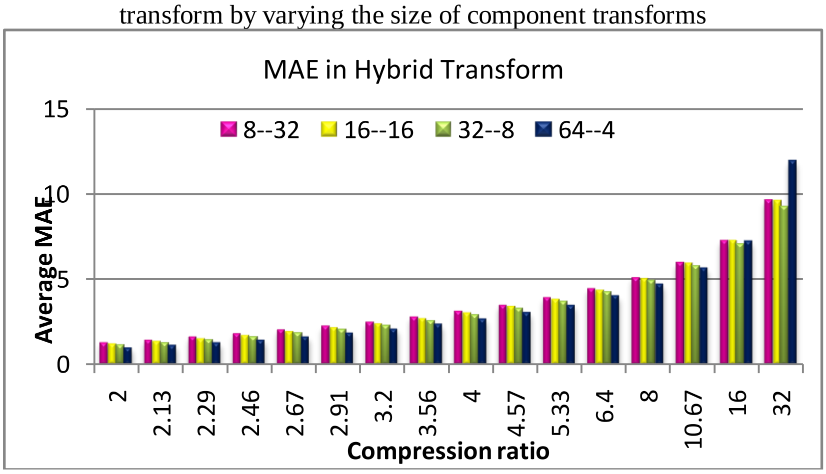


Fig. 7 Average MAE against compression ratio using multi resolution analysis of DKT-Hartley hybrid wavelet transform by varying the size of component transforms

Fig. 8, 9 and 10 show average fractional change in pixel value at various compression ratios. With increase in compression ratio, AFCPV increases. In each case it has been tried to select best suitable component size giving least value of AFCPV. As observed in fig. 8, hybrid wavelet 64-4 pair of DKT-Hartley gives lower AFCPV. But at higher compression ratio 32-8 size pair gives low AFCPV. For hybrid wavelet with multi-resolution analysis same observations are noted in Fig. 9 giving best size of component pair as DKT 32x32 and Hartley 8x8. Graph plotted for Hybrid transform in Fig. 10 also shows that DKT 32x32 and Hartley 8x8 is component size giving lowest value of AFCPV.

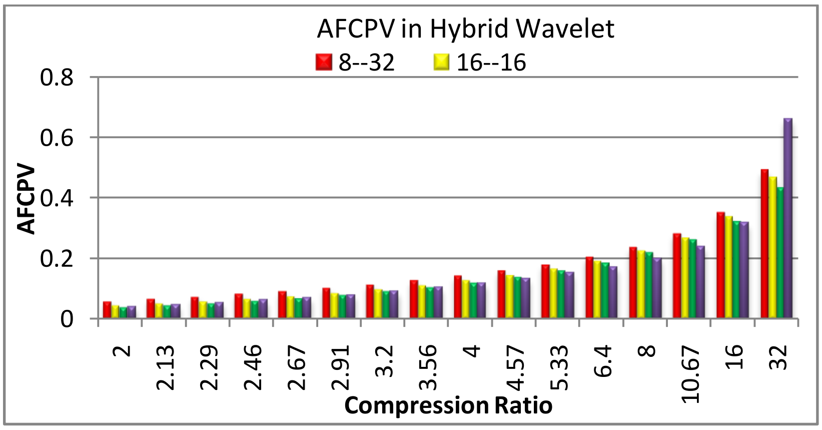


Fig. 8 AFCPV against compression ratio using DKT-Hartley hybrid wavelet transform by varying the size of component transforms

Fig. 9 AFCPV against compression ratio using multi resolution analysis of DKT-Hartley hybrid wavelet transform by varying the size of component transforms

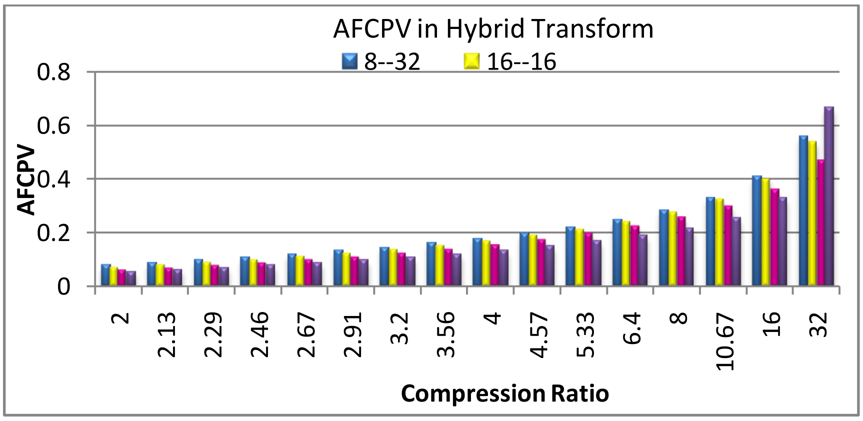


Fig. 10 AFCPV against compression ratio using DKT-Hartley hybrid transform by varying the size of component transforms

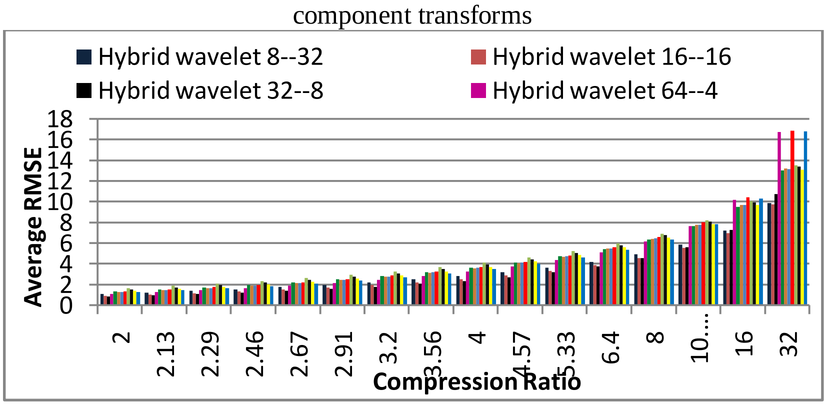


Fig. 11 Comparing RMSE of hybrid wavelet, multi resolution and hybrid transform using DKT-Hartley pair

Fig. 11 shows overall comparison of RMSE calculated in Hybrid wavelet transform, hybrid wavelet with multi-resolution analysis and hybrid transform with their possible sizes of component transforms. It has been observed that, values obtained in multi-resolution analysis are greater than those in hybrid wavelet transform. Further, values in hybrid transform are still greater than values in multi-resolution analysis for all sizes of component transforms except 64-4 pair. It indicates that, inclusion of semi global properties of image introduces more error in compressed image. Hybrid wavelet with DKT 16x16 and Hartley 16x16 gives lowest RMSE 9.79 among all. In multi-resolution analysis as size of base transform increases, error increases and reaches maximum value of 16.87.

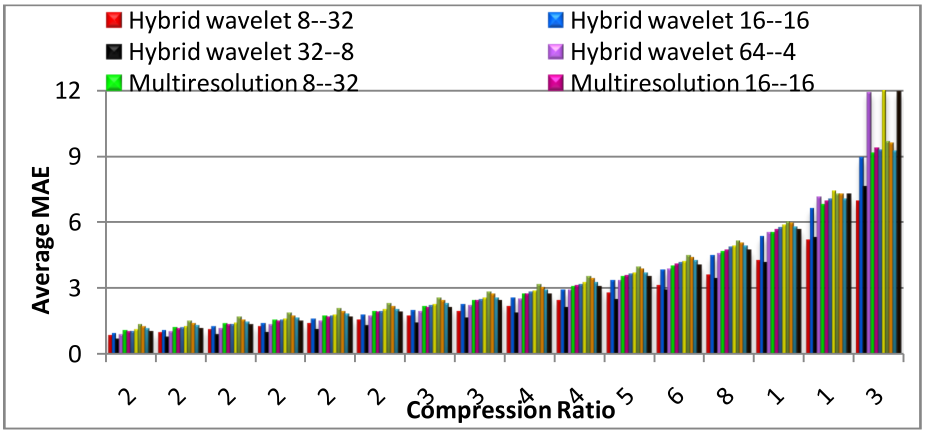


Fig. 12 Comparing MAE of hybrid wavelet, multi resolution and hybrid transform using DKT-Hartley pair

Fig. 12 shows overall comparison of MAE calculated in Hybrid wavelet transform, hybrid wavelet with Multi-resolution analysis and hybrid transform with their possible sizes of component transforms. Fig. 13 presents comparison of AFCPV in all possible cases of hybrid wavelet, its multi-resolution analysis and hybrid transform. Hybrid wavelet proves to be better than hybrid transform and multi-resolution analysis. 32x32 DKT and 8x8 Hartley gives lower value of AFCPV among all selected pairs.

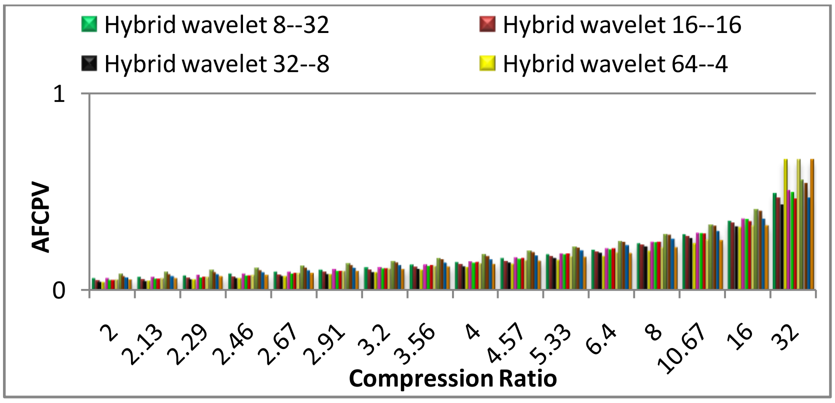


Fig. 13 Comparing AFCPV of hybrid wavelet, multi resolution and hybrid transform using DKT-Hartley pair.

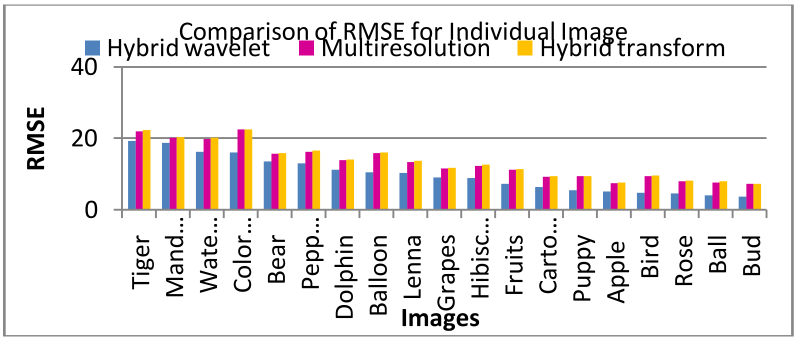


Fig. 14 RMSE of different images using three different techniques hybrid wavelet, multi resolution and hybrid transform

Fig. 14 shows RMSE values of individual image obtained by Hybrid wavelet, Multi-resolution analysis and hybrid transform respectively. For each image RMSE obtained using hybrid transform is greater than RMSE in Hybrid wavelet and multi-resolution technique. Same pattern is followed in MAE and AFCPV plot in Fig. 15 and Fig. 16. It has been observed that images containing more energy in high frequency components show higher error for same amount of data compression. Mandrill, Balloon, colormap, tiger show high error values.
Fig. 15 shows variation in MAE for three different techniques: Hybrid Wavelet, Multi-resolution analysis and hybrid transform. It has been observed that inclusion of semi global and global properties increases error for sharp image serroris more.

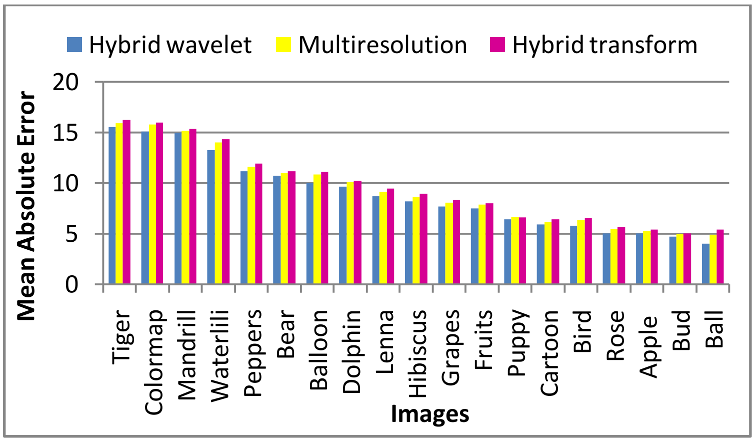


Fig. 15 MAE of different images using three different techniques hybrid wavelet, multi resolution and hybrid transform

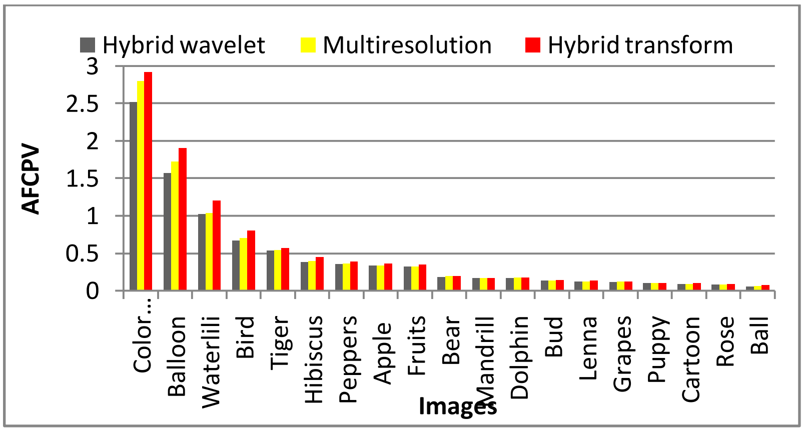


Fig. 16 AFCPV of different images using three different techniques hybrid wavelet, multi resolution and hybrid transform

Reconstructed 'Lena' image using component transform DKT 8x8, Hartley 32x32 at compression ratio 32		
Hybrid wavelet	Multi-Resolution	Hybrid Transform
		
RMSE= 10.56	RMSE= 13.08	RMSE= 13.67

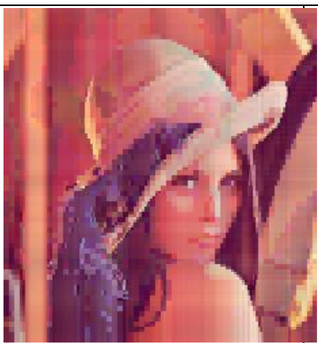
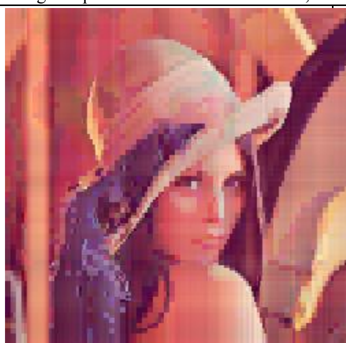
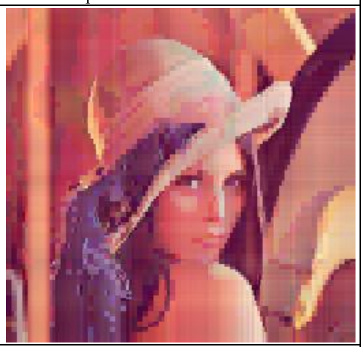
Reconstructed 'Lena' image using component transform DKT 16x16, Hartley 16x16 at compression ratio 32		
		
RMSE= 10.30	RMSE= 13.22	RMSE= 13.56
Reconstructed 'Lena' image using component transform DKT 32x32, Hartley 8x8 at compression ratio 32		
		
RMSE= 11.03	RMSE= 13.11	RMSE=13.17
Reconstructed 'Lena' image using component transform DKT 64x64, Hartley 4x4 at compression ratio 32		
		
RMSE= 15.41	RMSE= 15.50	RMSE= 15.46

Fig. 17 Reconstructed 'Lena' image using DKT-Hartley hybrid wavelet, multi resolution and hybrid transform with component sizes 8x8 and 32x32 i.e. (8-32), (16-16), (32-8) and (64-4) respectively

Fig. 17 shows reconstructed 'Lena' Image obtained at compression ratio 32 using three different techniques: hybrid wavelet, multi-resolution and hybrid transform. DKT-Hartley pair is used to generate hybrid wavelet transform. Component size is varied as 8-32, 16-16, 32-8 and 64-4. Blocking effect is observed in the reconstructed image but image quality is acceptable. For selected image 16-16 component size in hybrid wavelet gives lowest RMSE. Error is maximum in hybrid transform than all followed by multi-resolution analysis. Hence we can conclude that, inclusion of global and semi-global properties increases the error than hybrid wavelet with bi-resolution analysis.

V. CONCLUSION

This paper presents image compression technique based on hybrid wavelet transform with multi-resolution analysis. Kekre transform and Hartley transform are used to generate hybrid wavelet of size 256x256. Various combinations of these pairs (8-32, 16-16, 32-8 and 64-4 etc.) can be used to generate required size of transform matrix. In multi-resolution analysis semi global properties are introduced and their proportion in transformation matrix can be varied by changing the number of rows of second matrix. Since Kronecker product is used to generate the transformation matrix, this method is faster and simpler to implement. Inclusion of semi global properties increases error than usual bi resolution analysis. In hybrid transform where all global properties are present error is maximum. Along with RMSE, Mean Absolute Error (MAE) of these three techniques is compared as it gives better perception of image quality than RMSE. Average Fractional Change in Pixel Value (AFCPV) gives still better perceptibility than MAE as it is a ratio of change in pixel values. At component size 16-16 lowest average RMSE 9.79 is obtained in hybrid Wavelet. MAE and AFCPV are 8.91 and 0.468 at this size.

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