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Investigation of the Behaviour for Reinforced Concrete Beam Using Non Linear Three Dimensional Finite Elements

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Abstract: *This study presents theoretical investigation that reinforced concrete and composite construction might be suitably combined to give a new structural material : composite reinforced concrete. To study theoretically the composite beam, non-linear three-dimensional finite elements have been used to analyze the tested beam.*

The 8-node brick elements in (ANSYS) are used to represent the concrete, the steel bars are modelled as discrete axial members connected with concrete elements at shared nodes assuming perfect bond between the concrete and the steel. The results obtained by finite element solution showed good agreement with experimental results.

The main objective of the present investigation is to carry out a nonlinear analysis of reinforced concrete beams resting on elastic foundation. Material nonlinearities due to cracking of concrete, plastic flow, crushing of concrete and yielding of reinforcement are considered. Foundation representation is assumed linear using Winkler model.

The reinforced concrete beam is modelled by using three dimensional finite elements with steel bars as smeared layers. The examples have been chosen in order to demonstrate the applicability of the modified computer program (Dynamic Analysis of Reinforced Concrete Beams on Elastic Foundations DARCEF) by comparing the predicted behaviour with that from other experimental and analytical observations. The program modified in the present research work is capable of simulating the behaviour of reinforced concrete beams resting of Winkler foundation and subjected to different types of loading. The program solutions obtained for different reinforced concrete beams resting on elastic foundations are in good agreement with the available results. Maximum percentage difference in deflection is 15 %

Keywords: *Investigation, Behaviour, Reinforced Concrete Beam , Non Linear Three Dimensional Finite Element*

I. Introduction

The idea for a new form of construction emerged from two separate research investigations. One of these on composite construction with deep haunches, the other was on the use of very high strength steels in reinforced concrete. Both these separate modes of construction, although as yet little used in practice, will undoubtedly develop further in their own right. However, they have some disadvantages 2which will mitigate against their development, but by altering slightly the form of the deep haunch, the disadvantages of both can be largely overcome. The resulting form of construction is known as composite reinforced concrete.

The normal form of composite construction is shown. The main advantage of using deep haunches is the considerable economy that can be affected in the amount of steelwork. This can be 40% of that used in normal composite construction, even for the same overall depth. The deep haunch can be formed easily by precast units or similar spanning between the steel beams and the problem of deep haunches composite beams developed form the desirability to use this method of construction.

Concrete Problem of reinforced concrete beams on elastic foundations with both compressional and frictional resistances are good examples to extend the applications to foundations. In these problems, the compressional resistances of the elastic medium to the bottom face of the beams are determined by considering the linear relationships of the normal displacement to the compressional reactions (Winkler model).

The frictional restraints are modeled by considering the linear relation to the horizontal displacement (Winkler model). Winkler (1867) proposed the first model of beam on an elastic foundation based on pure bending beam theory, later Pasternak in 1954 proposed the shear model based on the assumption of pure shear of the beam. Both of these two models take an extreme point of view on the deformation behavior of the beam. Cheung and Nag (1968) studied the effects of separation of contact surfaces due to uplift forces. In addition, they have enabled the prediction of the bending and torsion moments in the plate sections by adopting three degrees of freedom per node. Bowles (1974) developed a computer program to carry out the analysis of beams on elastic foundation by using the finite element method, in which Winkler model is adopted to represent the elastic foundation. Selvadurai (1979) presented a theoretical analysis of the interaction between a rigid circular foundation resting on elastic half space and a distributed external load of finite extent which acts at an exterior region of the half space. Yankelevsky et al. (1988) presented an iterative procedure for the analysis of beams resting on nonlinear elastic foundation based on the exact solution for beams on a linear elastic foundation. Yin (2000) derived the governing ordinary differential equation for a reinforced Timoshenko beam on an elastic foundation. Guo and Wietsman (2002) made an analytical method, accompanied by a numerical scheme, to evaluate the response of beams on the space-varying foundation modulus, which is called the modulus of subgrade reaction (Kz=Kz(x)).

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II. Study Area

2.1 COMPOSITE BEAMS

Many researchers studied the behavior of simply supported composite beams. Some of these models, which are comprehensive and worth evaluating, are reviewed herein. In 1975, Johnson has derived a differential equation for Newmark. The equilibrium and compatibility equations are reduced to a single
second order differential equation in terms of interface slip instead of axial forces. The solution of which lead to slip values at the interface along the beam span, after satisfying the suitable boundary conditions. In 1985, Roberts presented an elastic analysis for composite beams with partial interaction assuming a linear shear connector behavior while the normal stiffness of the connectors was taken infinity. The basic equilibrium and compatibility equations were expressed in terms of four independent displacements, which are the horizontal and vertical displacements in each component of the composite section. These equations were solved using finite difference representation of various derivatives.

2.1.1 Nonlinear Finite Element (by using ANSYS program)

In 2006 Ali Hameed Aziz has investigated experimentally the strength, deflection and ductility of reinforced concrete I-beams made ofnormal or hybrid reinforced concrete under two point monotonic loads. The experimental work includes the following four main categories:

i- Flexural behaviour of I-beams cast monolithically.

ii- Flexural behaviour of I-beams cast in two-step procedure (with one construction joint).

iii- Flexural behaviour of I-beams cast in three-step procedure (with two construction joints).

iv- Shear behaviour of I-beams cast monolithically, but, the web is made with concrete different from the concrete of compression and tension flanges. To implement a nonlinear finite element procedure to analyze all tested beams.

III. Material Modelling

3.1 Numerical modelling of concrete

The numerical modelling of concrete, which is used in the nonlinear finite element program, includes the following:

- A stress-strain model to represent the behaviour of concrete in compression.
- Failure criteria to simulate cracking and crushing types of fracture.
- A suitable crack representation model.
- A post-cracking stress-strain relationship.

3.2 Stress-strain models for concrete in compression

Because of the complex behaviour of reinforcement and concrete under general load, a mathematical description of the material properties of concrete is established by using suitable constitutive equations. Several models have been proposed to define the complicate stress-strain behavior of concrete in compression under various stress states (Chen 1982). These models can be generally classified into:

- Elasticity based models.
- Plasticity based models.

3.3 Modelling of concrete fracture

Fracture of concrete can be classified as crushing or cracking. The crushing type of fracture is defined by progressive degradation of the material internal structure under compressive stresses. The cracking type of fracture is characterized by a general growth of micro-cracks which join together to form a failure plane. In general, two different methods were employed to represent the cracks in the finite element analysis of concrete structures. These are the discrete crack representation and the smeared cracks representation. In the first model, the cracks are simulated by physical separation or disconnection by the displacements at nodal points for adjoining element. The major difficulty in this approach is that the location and orientation of the cracks are not known in advance.

In the second approach, the cracked concrete is assumed to remain a continuum and implies an infinite number of parallel fissures across that part of a finite element. The cracks are assumed to be distributed, or smeared, over the volume of the sampling point under consideration.

3.4 Post-cracking behaviour

Many experimental studies show that when the concrete cracks, the tensile stresses normal to the cracked plane are gradually released as the crack width increases. This response can be generally modelled in the finite element analysis using the tension-stiffening model. When the tensile stress reaches the limiting tensile strength, primary cracks will be formed. The properties of cracks, and their number and extent, depend on the size, position and orientation of the reinforcing bars with respect to the plane of cracks.

3.5 Yield criterion

As mentioned earlier, a yield criterion must be independent of the coordinates system in which the stress state is defined. Therefore, it must be a function of stress invariants only. Under multiaxial state of stresses, the yield criterion for concrete is assumed to depend on three stress invariants. However, a yield criterion, which depends on two stress invariants, has been proved to be adequate for most practical situations. The yield criterion incorporated in the present model is expressed as (Chen 1982):

$$f(\{\sigma\}) = f(I_1, J_2) = (\alpha I_1 + 3\beta J_2)^{1/2} = \sigma_o \quad (1)$$

The stress (σ_0) is the equivalent effective stress at the onset of plastic deformation which can be determined from uniaxial compression test as:

$$\sigma_o = C_P . f_c' \quad (3)$$

The coefficient **(***Cp***)** is the plasticity coefficient, which is used to mark the initiation of plasticity deformation. The material parameters αand ßcan be determined from the uniaxial compression test and biaxial test under equal compressive stresses. The parameters αand ßhave been found to be (0.35468) and (1.35468) respectively and equation (1) can be rewritten as:

$$f(\{\sigma\}) = (2c\sigma_o I_1 + 3\beta J_2)^{1/2} = \sigma_o \quad (4)$$

Equation (4) can be solved for (σ_0) as:

$$f(\{\sigma\}) = cI_1 + \{(cI_1)^2 + 3\beta J_2\}^{1/2} = \sigma_o \quad (6)$$

The hardening rule is necessary to describe the motion of the loading surfaces during plastic deformation. In the present study an isotropic hardening rule is adopted. Therefore, equation (6) can be expressed for the subsequent loading functions as:

$$f(\{\sigma\}) = cI_1 + \{(cI_1)^2 + 3\beta J_2\}^{1/2} = \sigma^* \quad (7)$$

In the present model, the stress-strain curve is assumed to have a parabolic relationship for both normal and high strength concrete. This parabolic curve represents the work-hardening stage of behavior, when the peak compressive stress is reached, a perfectly plastic response is assumed to occur. The equivalent uniaxial stress-strain curve in the various stages of behavior, which is given:

1. During the elastic stage, when $\sigma^* \leq C_p . f_c'$

$$\sigma^* = E . \varepsilon_c \quad (10)$$

2. After the initial yield, and up to the peak concrete compressive strength, when

$\varepsilon_c \geq \varepsilon_o'$

In the present study, values of 0.3 and 0.5 are assumed for plastic coefficient Cp for the normal and high strength concrete respectively, and the plastic yielding begin at a stress level of Cp fc '(Al-Ani, 2000). If Cp=1.0, then the elastic perfectly plastic behaviour is specified.

IV. Finite Element Method

4.1. Finite element representation

The finite element idealization of reinforced concrete is necessary for the application to reinforced concrete beams under static loads. In the present study, concrete is simulated by brick elements, and reinforcement bars are modeled as a smeared layer (Al-Obaidie 2005).

4.2 Concrete representation

The 20-node isoparametric quadratic brick element is used in the present study to represent the concrete.

4.3 Displacement representation

The displacement field within the element is defined in terms of the shape functions and displacement values at the nodes. Each nodal point has three degrees of freedom u, v and w along the Cartesian coordinates x, y and z, respectively. Therefore, for each element the displacement vector is:

$$\{a\} = \{u_1, v_1, w_1, u_2, v_2, w_2, ..., u_{20}, v_{20}, w_{20}\} \qquad (14)$$

The strains are given by :

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \sum_{i=1}^{20} \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & 0 & \frac{\partial N_i}{\partial z} \end{bmatrix} . \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \qquad (15)$$

or

$$\{\varepsilon\} = [B]\{a\} \qquad (16)$$

where, [B] is the strain displacement matrix. The stresses are given by :

$$\{\sigma\} = [D] \cdot \{\varepsilon\} = [D][B]\{a\} \qquad (17)$$

where, [D] is the stress-strain elastic relation matrix given by:

$$D_1 = \frac{E \cdot (1-\nu)}{(1+\nu)\cdot(1-2\nu)} \;,\; D_2 = \frac{E \cdot \nu}{(1+\nu)\cdot(1-2\nu)} \text{ and } G = \frac{E}{2(1+\nu)}$$

in which E is the Young's modulus, vis the Poisson's ratio and G is the shear modulus.

4.4 Reinforcement representation

There are three alternative representations, which have been widely used to simulate the reinforcement in connection with the finite element formulation. These representations are (Scordelis 1971):

a- Discrete representation.
b- Distributed representation.
c-Embedded representation.

In the present study, the distributed representation is used. For this representation, the steel bars are assumed to be distributed over the concrete element in any direction in a layer with uniaxial properties. A composite concrete-reinforcement constitutive relation is used in this case. To derive such a relation, perfect bond is assumed between concrete and steel.

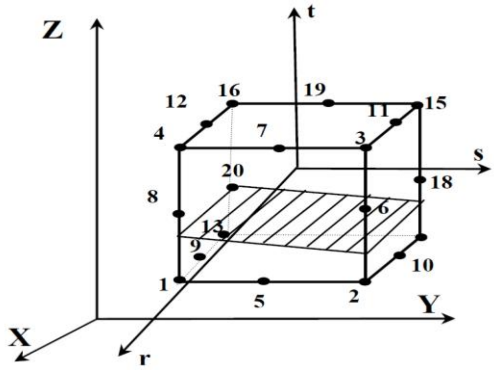

Fig. 4.1 Distributed representation of reinforcement.

The element stiffness matrix of the beam-foundation system can be calculated from the following expression:

$$[K] = [K_b] + [K_f] \qquad (24)$$

A modified computer program (DARCEF) (Dynamic Analysis of Reinforced Concrete Beams on Elastic Foundations) (Al-Obaidie 2005) of the original program (DARC3) (Dynamic Analysis of Reinforced Concrete in 3D) (Hinton 1988) is developed for solving the problems of reinforced concrete beams resting on elastic foundations. The Winkler model for compressional and frictional resistances is added to the original program to represent the foundation.

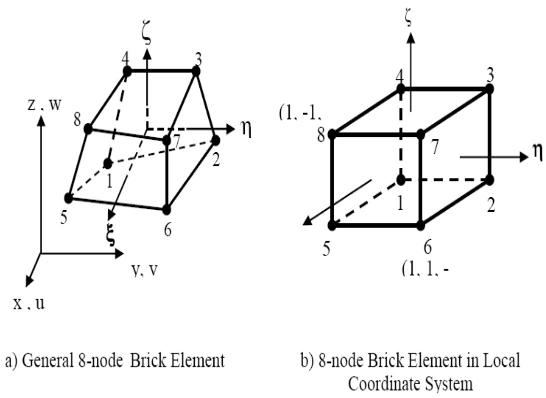

Figure 4.2 Three Dimensional 8-node Brick Element

V. Results

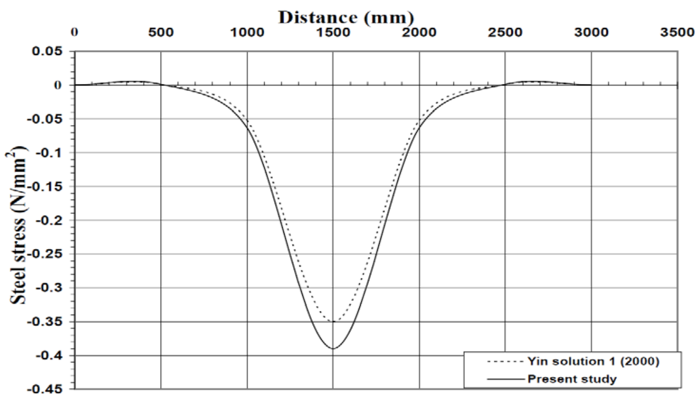

Fig. 4.3. Steel stress compared with previous study.

Distance (mm)	Deflections by (Yin 2000) solution 1 (mm)	Deflections by (Yin 2000) solution 2 (mm)	Present study (mm)
0	0	0	0
500	-0.05	-0.048	-0.053
1000	-1.7	-1.6	-1.9
1500	-3	-3.6	-2.98

Table 4.4. Values of deflections at mid span for half of simply supported beam resting on an elastic foundation in comparison with previous solutions.

VI. Conclusions

The Main conclusion to be drawn from this investigation is that composite reinforced concrete is a viable structural form. Flexural cracks up to the working load stage remain very fine and the calculation of their width is unlikely to be necessary in design. The arrangement of reinforcement and steel channel is ideally suited for the use of very high strength reinforcing steels, and reinforcement stresses over 120000 lb/sq. can be used at the ultimate load while still satisfying the serviceability requirements at working load. There are no cracks at the bottom of the beam because of the channel. There will be cracks in the concrete web, but these should remain fine, and they will not be visible.

- The (DARCEF) program used in the present research work is capable of simulating the behavior of reinforced concrete beams resting on elastic foundations and subjected to different types of loading. The program solutions obtained for different reinforced concrete beams resting on elastic foundations are in good agreement with the available results. Maximum percentage difference in deflection is 15%.
- The maximum deflection will decrease at a decreasing rate as the beam width is increased. It was found that by increasing the width of the beam from (150 to 450 mm), the maximum deflection for the beam is decreased by (31.25%).
- The maximum steel stress will decrease at a decreasing rate as the beam width is increased. It was found that by increasing the width of the beam from (150 to 450 mm), the maximum steel stress for the beam is decreased by (21.21%).
- The maximum steel stress will decrease at a decreasing rate as the beam depth is increased. It was found that by increasing the depth of the beam from (300 to 900 mm), the maximum steel stress for the beam is decreased by (12.12%).
- The maximum deflection will decrease as the vertical subgrade reaction is increased. Also, the maximum deflection will decrease as the horizontal subgrade reaction is increased. It was found that by increasing the vertical and horizontal subgrade reactions for the beam from (0.128 N/mm3 to 1.024 N/mm3), the maximum deflection is decreased by (51%) and (9%) respectively.

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