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Investigate the Route of Chaos Induced Instabilities in Power System Network

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Abstract: *In this paper possible causes of various instability and chances of system break down in a power system network are investigated based on theory of nonlinear dynamics applied to a Power system network. Here a simple three bus power system model is used for the analysis. First the routes to chaotic oscillation through various oscillatory modes are completely determined. Then it is shown that chaotic oscillation eventually leads to system break-down characterized by collapse of system voltage and large deviation in Generator rotor angle (angle divergence), also known as chaos induced instability. It has been shown that chaos and chaos induced instability in Power system take place due to the variation in system parameters and the inherent nonlinear nature of the power system network. The relation between chaotic oscillation and various system instabilities are discussed here. Using the simple power system model, here it is shown that how chaos leads to voltage collapse and angle divergence, taken place simultaneously when the stability condition of the chaotic oscillation are broken. All nonlinear analysis is implemented using MATLAB. It is indicated that there is a maximum lodability point after which the system enters into instability modes. All these studies are helpful to understand the mechanism of various instability modes and to find out effective anti-chaos strategies to prevent power system instability.*

Keywords: *Angle divergence, bifurcation, chaos, Period Doubling, Power system, Voltage collapse.*

I. INTRODUCTION

A power system is inherently of nonlinear nature i.e. the power system dynamics is described by a set of nonlinear equations obtained from system modeling and parameters. To a large extent, this is also due to the fact that most of the major power system breakdowns are caused by problems related to the system dynamic responses. It is believed that new types of instabilities emerge as the system approaches the stability limit which cannot be explain completely or accurately using linear analysis. So, theory of nonlinear dynamics is used to find out the proper explanation of those instabilities. Being an inherently nonlinear system, power system undergoes changes in state either qualitatively or quantitatively with the changes of one or more system parameters. This phenomenon is known as **Bifurcation**. Sometimes variation in parameter may result in complicated behavior which is random and indeterministic, known as chaos. This chaotic oscillation may lead to various instabilities. Most common types of system instabilities, which occur when the system is heavily loaded, are voltage collapse and angle divergence which eventually cause system breakdown.

Voltage collapse in electric power systems has recently received significant attention by researchers. A number of physical mechanisms have been identified which possibly leading to voltage collapse. In several papers [4-10] voltage collapse was viewed as an instability which coincides with the disappearance of the steady state operating point as a system parameter, such as a reactive power demand is quasistatically varied, which is known as fold or saddle node bifurcation of the nominal equilibrium point. Dobson and Chiang [1] first studied and analyzed voltage collapse, and concluded that this phenomenon occurs at a hypothesized static bifurcation of equilibrium points taking place as system loading is increased. The static bifurcation mechanism for voltage collapse postulated in [1] was investigated in [2] and in [3]. It is therefore not surprising that saddle node bifurcation is being studied as a possible route to voltage collapse [4].

Another possibility is that steady state operating point loses stability before the saddle node bifurcation. If this occurs in a given system, stability of the nominal equilibrium point may be lost prior to static/saddle node bifurcation point through a Hopf bifurcation. Study of Hopf bifurcation in power system was done in details in [2, 3, 4, and 5]. Except these, other type's bifurcation occurs in power system like Torus bifurcation [4], cyclic fold bifurcation, period doubling bifurcation [3, 2].

In this paper a complete and detail bifurcation analysis has been done which shows different behavioral changes (Bifurcation) with the slow and gradual variation of load reactive power. It has been shown here how the stable oscillatory behavior of the power system model tends to chaotic instability through period doubling bifurcation. Period doubling bifurcation (PDB) is the most important route to chaos in power system model which is analyzed here with great emphasis. Except continuation method, a detail and explicit picture of PDB has been developed which clearly shows the internal behavior changes of the proposed system which eventually leads to chaos. This paper, for the first time, proposes that Voltage collapse and angle divergence phenomenon which make the system unstable, occurs simultaneously. Also this paper gives an indication on maximum lodability point after which system tends towards instability.

The numerical solution of the basic nonlinear differential equations of the proposed model of power system network and load are implemented using the **MATLAB** environment [16] and assembly language programming.

II. Brief Review Of Nonlinear Theory [17,18]

A typical nonlinear system with state x can often be expressed as-

$$\dot{x} = f(x, \mu)\ ;\ x \in \Re^{n},\ \ \mu \in \Re^{p} \text{ ------------- (2.1)}$$

The corresponding properties of such a system are:

- The solution of (2.1) is called trajectory. With initial condition $x(t_0) = x_0$, the solution is given by $x(t) = \lambda_t(x_0)$.
- Four steady state behaviors are associated with the nonlinear system. Equilibrium points, Periodic solutions, quasi periodic solution and chaos.
- The equilibrium points mean the solutions of the nonlinear equation $f(x, \lambda)= 0$. Alternatively an equilibrium point is a degenerate trajectory which stays in the equilibrium point for all time. It is asymptotically stable if all the Eigen values of its corresponding Jacobian matrix have negative real part. Power system is generally operate on a stable equilibrium point. $\lambda_t(x^*)$ is a periodic solution if $\lambda_t(x_*) = \lambda_{t+T}(x^*)$ for all t and some minimal period $T>0$. It represents a limit cycle which is a self sustained and bounded oscillation, and, is stable or unstable depending upon its characteristics multiplier..
- Finally, **chaos** is a random, indeterministic phenomenon exhibits stable, bounded but aperiodic behavior. While equilibrium points are zero dimensional and periodic solutions are one-dimensional, chaos is more complex and having fractional dimension.

2.1. Bifurcation Theory:

Bifurcation Theory is used to interpret the way in which qualitative changes occur in the system as one or more parameters are varied. A power system is modeled in the typical form of a nonlinear dynamic system with state x:

$$\dot{x} = f(x\ ,\ \mu)\ ;\ x \in \Re^{n},\ \mu \in \Re^{p} \text{ ------------- (2.1.1)}$$

μ represents the vector of the system parameters that can be varied during the analysis.

Bifurcations in dynamical system mean "qualitative" changes of the asymptotic behavior of the system trajectory (2.1.1) which is obtained by varying the μ components.

At a value of $\mu = \mu_c$ the vector field f loses its structural stability, is called the **Bifurcation Point** and μ_c ,the Bifurcation value. This simply means that the Phase portraits for $\mu < \mu_c$ and for $\mu > \mu_c$ are different. In power system's nonlinear model there are following types of bifurcations taken place depending upon the Jacobian ($\Im$) of the system (2.1.1) –

- **Saddle-node bifurcation (SNB)**:- A saddle-node bifurcation is a local bifurcation in which two fixed point (or equilibrium) of a dynamical system collide and annihilate each other, at this point the Jacobian has a zero Eigen value and no other Eigen value with zero real part.
- **Hopf bifurcation (HB):-** A Hopf-bifurcation is a local bifurcation in which a fixed point of a dynamic system loses stability as a pair of the complex conjugate Eigen values of the linearized system around the fixed point cross the imaginary axis of the complex plane. If has $\Im$ a pair of complex conjugate Eigen values on the imaginary axis where all other Eigen values are off the imaginary axis, then Hopf Bifurcation results the emergence of a family of periodic solution in the vicinity of μ_c. If the periodic solution is unstable then it is called "subcritical" and "supercritical" when stable.
- **Period Doubling Bifurcation (PDB)**:- During this kind bifurcation with the change in multiplier a new periodic solution or orbit emerges from the previous/existing solution with periodicity approx. Twice that of the previous one. If there is a sequence of such bifurcation which accumulate at a critical value $\mu = \mu_c$,then

period almost becomes infinite, which means we get a aperiodic but bounded solution to the system which is called "Chaos". Actually this is one special type of Hopf **Bifurcation.**

III. POWER SYSTEM MODEL FOR NONLINEAR ANALYSIS

For applying theory of nonlinear dynamics, a simple power system model [1] is considered here. This model is widely used for nonlinear behavior study of power system [2, 3, 4, and 14]. In this power system model generator is represented by classical model. Here the system is represented by a set of four ordinary non linear differential equation. In this paper, classical model is implemented for a three BUS power system network as follows.

A simple 3-BUS power system shown in **fig.1** and its equivalent circuit in **fig.2**, this model consists of an infinite bus on the left, a load bus on the center and a generator bus on the right. $\mathbf{Y_0 \angle (-\theta_0 - \frac{\pi}{2})}$ and $\mathbf{Y_m \angle(-\theta_m - \frac{\pi}{2})}$ are the admittances of the transmission lines. One of the generator buses treated as slack bus and the other is described by swing equation. The concept of an infinite/slack bus refers to a particular node of the system with enough capacity to absorb any mismatch in the power balance equations. Thus, it can be considered as fictitious generator with constant voltage magnitude $\mathbf{E}_0$ and phase δ_0 (usually $\mathbf{E_0 = 1}$ and $\mathbf{\delta_0 = 0}$). On the other hand, the generator has constant voltage magnitude $\mathbf{E_m}$ but the angle δ_m varies according to the so-called swing equation.

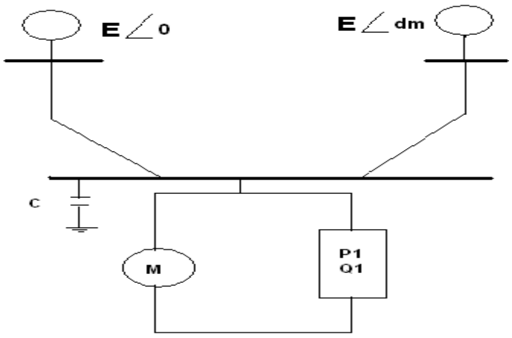


Figure 1: 3-bus power system model

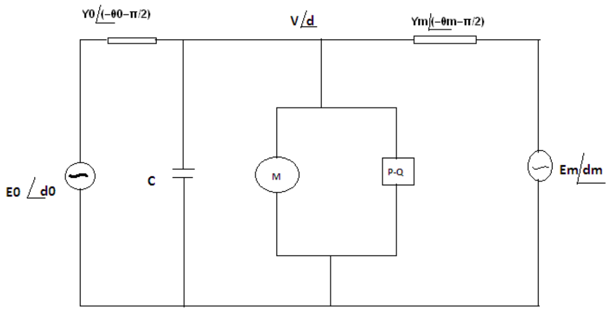


Figure 2: equivalent circuit

So, the set of four dynamic equations is developed for this model as –

$$\dot{\delta}_m = \omega \qquad (3.1)$$

$$M\dot{\omega} = -d_m\omega + E_m Y_m \sin(\theta_m) + E_m V Y_m \sin(\delta - \delta_m + \theta_m)] \qquad (3.2)$$

$$k_{qw}\dot{\delta} = -k_{qv2}V^2 - k_{qv}V - Q_0 - Q_1 + Q(\delta_m, \delta, V) \qquad (3.3)$$

$$Tk_{qw}k_{qv2}\dot{V} = k_{pw}k_{qv2}V^2 + (k_{pw}k_{qv} - k_{qw}k_{pv})V + k_{qw}[P(\delta_m, \delta, V) - P_0 - P_1] - k_{pw}[Q(\delta_m, \delta, V) - Q_0 - Q_1] \qquad (3.4)$$

Where, δ_m = Generator rotor angle, δ = Generator load angle, ω= Angular frequency, **V**= Magnitude of Generator load voltage.

The load bus, with voltage magnitude **V** and phase δ, consists of an induction motor, a generic load P-Q and a capacitor C. The dynamics of this part is derived from a power balance at the bus. Considering an empirical model for the induction motor [9] and a static load P-Q, the active and reactive power supplied to the load is –

$$P\,(\delta_m, \delta, V) = P_0 + k_{pw} d\delta + k_{pv}\,(V + TdV) + P_1 \qquad (3.5)$$

$$Q\,(\delta_m, \delta, V) = Q_0 + k_{qw} d\delta + k_{qv} V + k_{qv2} V^2 + Q_1 \qquad (3.6)$$

Where $\mathbf{T}$, $\mathbf{k_{pw}}$, $\mathbf{k_{pv}}$, $\mathbf{k_{qw}}$, $\mathbf{k_{qv}}$ and $\mathbf{k_{qv2}}$ are constants of the motor, $\mathbf{P_0}$, $\mathbf{Q_0}$ and $\mathbf{P_1}$, $\mathbf{Q_1}$ are the static active and reactive power drained by the motor and by the load P-Q, respectively.

Here in this power system network the reactive power demand at the load bus $\mathbf{Q_1}$ is chosen as bifurcation parameter. Therefore the model has the form-

$$\dot{x} = f\,(x, \lambda)$$

where $\mathbf{x} = [\delta_m, \omega, \delta, V]^T$ is the state vector and $\lambda = [Q_1]^T$ is the bifurcation parameter vector so that increasing Q_1 corresponds to increase in load reactive power.

IV. NUMERICAL SOLUTION- SIMULATION & RESULT

Here the numerical solution of the nonlinear Eqn. **(3.1 - 3.4)** has been carried out using **MATLAB** simulation, the result of which can be sub divided into following two parts-
1) Bifurcation analysis and Chaos, 2) Chaos induced instability and system collapse.

4.1. Bifurcation and chaos:

In this paper continuous Bifurcation diagram is plotted using **MATLAB** continuation software **'MATCONT'** to indicate different types of Bifurcation mentioned in **II** along with the respective bifurcation values in the proposed Power system model described by Eqns**. (3.1-3.6).**

Fig.3 shows the complete bifurcation diagram using the continuation method depicting the variation of system voltage **V (p.u)** with $\mathbf{Q_1}$. As the power system network is inherently nonlinear system it's dynamic equations are also nonlinear so it's solution exhibits different dynamic behavior with changes in system parameters, which is completely identified in the bifurcation diagram in Fig 3.

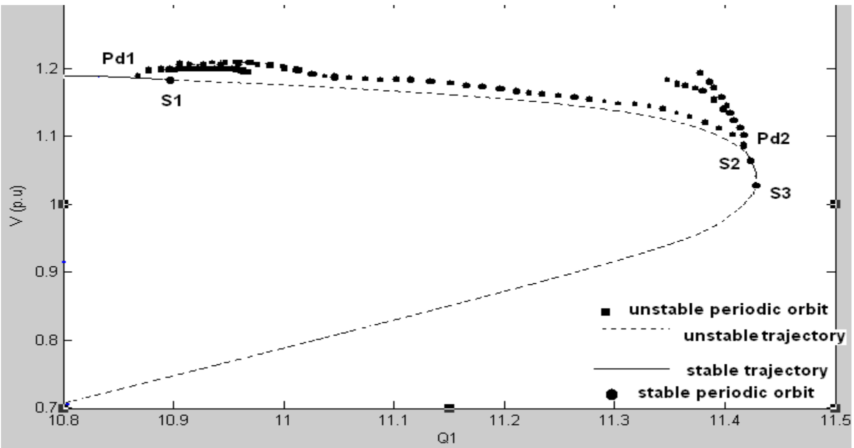


Figure 3: b**ifurcation diagram using continuous method showing various type of dynamic behavior**

Suppose the power system model describe in section **3.1** is operating at a stable equilibrium point. Now Q_1 is slowly increased while other parameter remains fixed. At each parameter step, the system eigen values are calculated. In the course of computing stationary branch, three critical points **S1,S2 & S3** are detected as shown in the Fig.3, at which system changes it's stability. In Fig.3 the solid and dotted lines represent the stable and the unstable stationary trajectory. The stable trajectory becomes unstable at a **subcritical Hopf bifurcation** (**S1**) for $\mathbf{Q_1}$**=10.868** and regain stability at **supercritical Hopf bifurcation** (**S2**) at $\mathbf{Q_1}$**=11.407.** The Eigen values of two critical points **S1** and **S2** are calculated numerically as **(0.0000 ± j3.4234, -125.2174, -18.2564)** and **(0.0000 ± j2.8957, -92.1458, -2.3652)** respectively i.e. at **S1** and **S2**, a pair of complex conjugate Eigen values cross the imaginary axes. So the real part of the complex conjugate Eigen values of the system becomes positive through **S1** and negative through **S2**. After a short stable region, one real Eigen value becomes positive at **S3**. This last critical point **S3** is called **Saddle node bifurcation** point where one real Eigen value becomes zero. After that system becomes completely unstable.

At point **S1** and **S2**, the possibility of oscillatory dynamic behavior is investigated by applying Hopf bifurcation theory. At this two points periodic branches are calculated. The periodic branch emanating from **S1** up to **Pd1** is unstable because it is subcritical. Then at **Pd1** periodic limit cycle emanates and gains stability. With further increase of $\mathbf{Q_1}$ period doubling bifurcation occurs at **Pd1** where the previous periodic solution is bifurcate to a new periodic solution or orbit with periodicity approx. twice that of the previous one. If one keep tracing an old periodic orbits, it again passes through the unstable periodic region between **Pd1** and **S2** and reaches **S2, the second super critical Hopf bifurcation point.** Now if we decrease the value of Q1, it reaches second period doubling point Pd2. the periodic orbits gain stability again in the supercritical region between **Pd2** and **S2**. After **Pd2** if we decrease the value of $\mathbf{Q_1}$, system enters into a series of period doubling bifurcation. All these limit cycles represent periodic oscillation in system behavior with different frequency.

After a short stable region beyond **S2** system become unstable at **S3**. If the reactive power of the load $\mathbf{Q_1}$ is increased beyond this value, the system looses it's stability and leading to system collapse.

4.2. Chaos via sequence of Period doubling*:*

According to bifurcation theory one way to chaotic motion is through a sequence of period doubling bifurcations. Therefore, we give special attention to two Period doubling points **Pd1** and **Pd2**, in this case and tried to show explicitly what happened after these two points with the variation of $\mathbf{Q_1}$ with the detail bifurcation diagram plotted in the neighborhood of these two points. First, consider the period doubling bifurcation point **Pd1** at the left side of main bifurcation diagram of Fig.3. With the further increase of $\mathbf{Q_1}$ a new periodic orbit with periodicity two is emerged, and then the periodic orbit of period four and so on. After a certain value of $\mathbf{Q_1}$ the system finally reaches to a state which is completely indeterministic and having infinite or fractional no of period or become aperiodic but exhibits a bounded oscillation. Dynamic behavior of the power system model at this region is very complex and unpredictable and there is no obvious relation between cause and effect. At this region the power system network shows completely erratic and random behavior. This bounded but random oscillation is known as **Chaos**. It can be termed as **Left side chaos (LSC)** as it occurs left part of continuous Bifurcation diagram of **Fig.3**. This **Left Side Period doubling Bifurcation** leading to Chaos is shown explicitly in **Fig.4**.

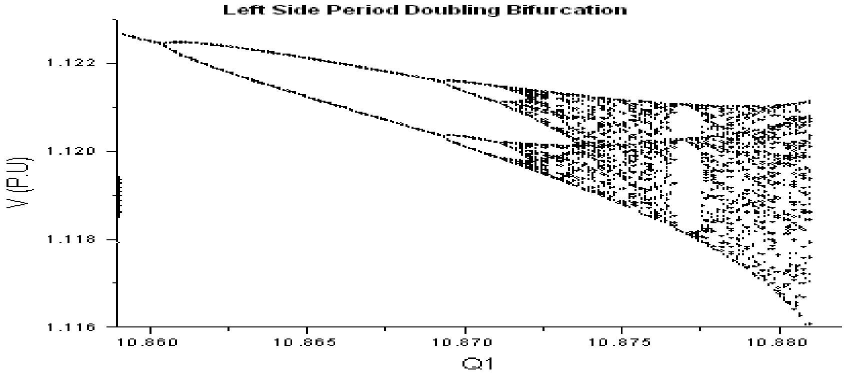


Figure 4: **bifurcation diagram showing left side period doubling bifurcation leading to chaos**

In the right hand side of the periodic branch, the same phenomenon takes place but here the state enters into period doubling bifurcation at **Pd2** towards the chaos with the decrease of values of $\mathbf{Q_1}$ from its value at **Pd2**. This chaos can be termed as **Right side chaos (RSC)**. This **right Side Period doubling Bifurcation** leading to Chaos is shown explicitly in **Fig.5**.

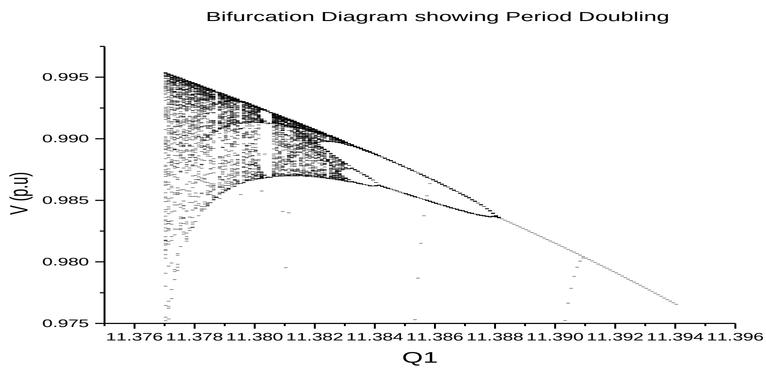


Figure 5: **bifurcation diagram showing right side period doubling bifurcation leading to chaos**

Table I: values Q_1 (bifurcation parameter) at different bifurcation point

Value of Q1	Nature of Bifurcation
10.946	Subcritical Hopf Bifurcation (S1)
11.407	Supercritical Hopf Bifurcation (S2)
10.870	Period-1 LHS
10.880	Period-2 LHS
10.883	Period-4 LHS
11.392	Period 1RHS
11.388	Period 2 RHS
11.384	Period 4 RHS

4.3. Phase Plots and Time plots:

Below the various phase plots are shown which describes the various states of the system dynamics with different values of Q_1, during PDB leading to chaos through both **Left side PDB** and **Right side PDB.** Also the time plots are shown for different values of Q_1.These plots are obtained from the numerical solution of the Eqn**. (3.1.1-3.1.4)** which are numerically integrated with initial condition **(0.3, 1.5, 0.2, 0.97)** and **(0.315, 0.150, 0.150, 0.98)** using **Runge-Kutta method.** Here Chaos is observed for **Q_1=10.894**, called **Left side chaos** and **Q_1=11.383**, called **Right side chaos**.

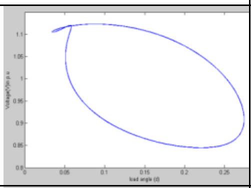
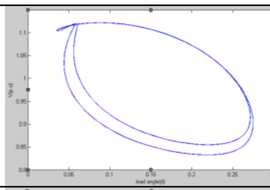
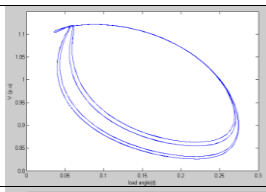
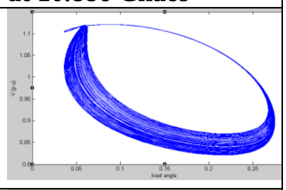
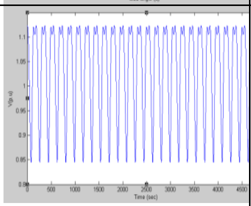
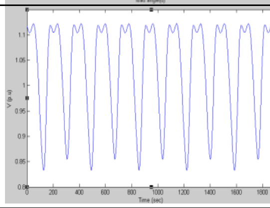
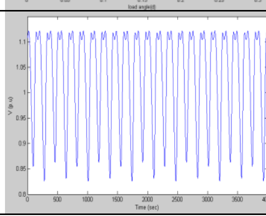
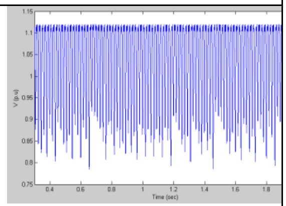
Q_1	**at 10.870 Period-1**	**at 10.880 Period- 2**	**at 10.883 Period- 4**	**at 10.886 Chaos**
Phase plots (V vs. □)				
Time Plots (V vs. t)				

Figure 6: phase plots and time plots showing left side PDB leading to chaos

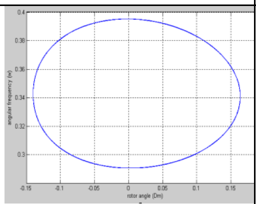
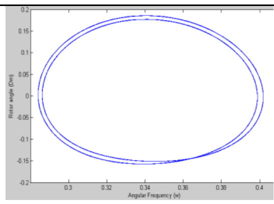
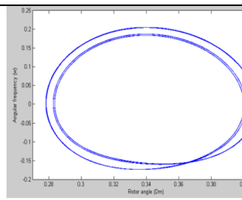
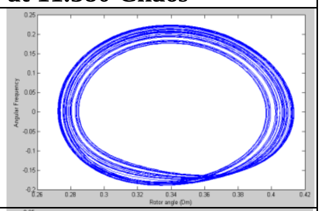
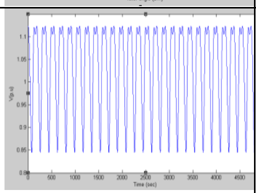
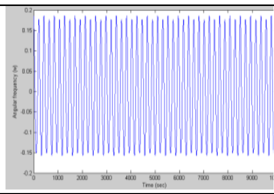
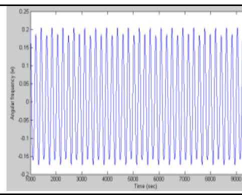
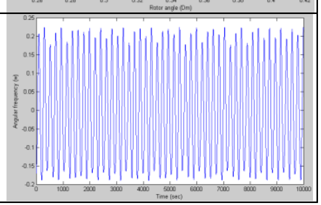
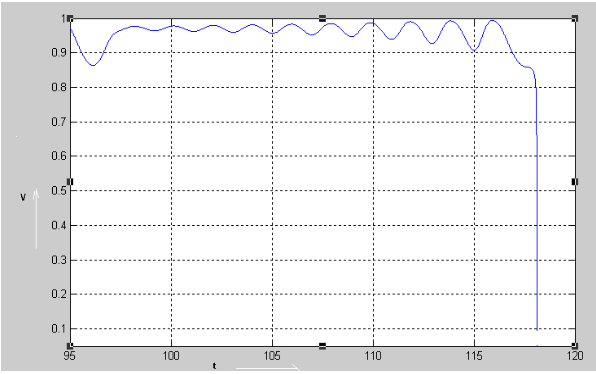
Q_1	**at 11.392 Period-1**	**at 11.388 Period- 2**	**at 11.384 Period- 4**	**at 11.380 Chaos**
Phase plots (V vs. □)				
Time Plots (V vs. t)				

Figure 7: phase plots and time plots showing right side PDB leading to chaos

4.5. Chaos induced instability and system collapse:

Though chaos exhibits aperiodic and random oscillatory behavior and completely indeterministic, it is a bounded and marginally stable region. But Chaos is very sensitive to initial condition and system parameter variation. Any small change to them can break their stable oscillation. Here we discuss what happen after stable chaotic oscillation is broken in power systems. It will be shown that chaos can lead to Voltage collapse and angle divergence simultaneously when value of Q_1 is increased beyond the value corresponds to Left side chaos and decreased below the value corresponds to Right side chaos, which makes a stable system into complete unstable and the system breakdown takes place.

- **Voltage collapse and Angle Divergence:-**Many studies have observed this phenomenon [1-11] taking place individually. But in this paper, voltage collapse occurring along with Angle divergence simultaneously is reported for the first time in Power system dynamic stability study. During **Voltage collapse** the system voltage sharply decline to a very low value and possibly brings the blackouts and **Angle Divergence** is the phenomenon when the generator loosing synchronism i.e. rotor angle difference is much more than 2π. Here critical point at which voltage collapse and angle divergence phenomena take place simultaneously is **Q_1=10.890** after **Left side chaos** and **Q_1= 11.377** after **Right side chaos,** using same model, parameter values and same initial condition. These are shown below in Fig 8 & 9 where time plot of load Voltage and Generator angle are given for the above mentioned values of Q_1.For both the values of Q_1 we get almost same diagram. From the Fig 8 & 9 it is seen that voltage collapse and angle divergence appears after chaos is broken.



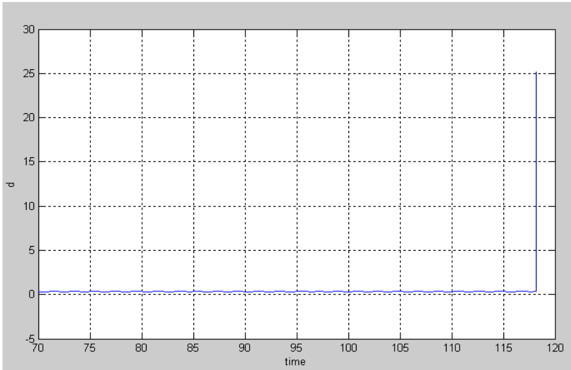


Figure 8: Voltage collapse at Q_1=10.890 and Q_1=11.377 Figure 9: angle divergence at Q_1=10.890 and Q_1=11.377

V. CONCLUSION

In this paper various nonlinear dynamical behavior of proposed power system model has been deeply studied using both "Matlab" and Assembly language Programming. Here cascaded period doubling bifurcation which is one of the most important routes to chaos and system instability has been observed in details. All previous studies used continuation method and compact continuation software's such as **AUTO [15]** for Bifurcation studies in power system network which predicted the occurrence of Period Doubling Bifurcation leading to chaotic phenomenon but were not able to show explicitly and elaborately, the structure of period doubling bifurcation leading to chaos. This limitation is overcome in this paper where bifurcation analysis is done using both MATLAB based continuation software MATCONT to get a continuation Bifurcation diagram and also get a expanded and elaborate diagram of Period doubling bifurcation which clearly shows various state changes of the proposed system with the variation of Q_1 which eventually leads to chaos. This is also described by various phase plots and time plots for different values of Q_1.The relationship between chaos and major instability modes in power system, such as Voltage collapse and Angle Divergence, has thoroughly been observed. Here it is shown for the first time that Chaos can induce voltage collapse and angle divergence simultaneously when the stability condition of the chaotic oscillation are broken. So it can be conclude that chaos is an inter mediate stage of the instability incident when changes in system parameter causes system breakdown. As chaos is very sensitive to initial condition and system parameters, any variation of them can make chaos to be annihilated and breaks in into instability. In a real power system all system parameters are fluctuating with changing operating condition and disturbances.

So chaos possibly exists in power system network as in prior stage of instability. When disturbance happens, power system comes into a transient stage. If the disturbance is small, HB may happen and stable oscillatory behavior follows. If the disturbance is prolonged, system may come into chaos. And, when the disturbance becomes larger, the chaos may be broken. Voltage collapse, angle instability or voltage collapse and angle divergence simultaneously may happen. If the disturbance is very large, system may directly come into the above three instability conditions over the stages of HB, chaos and chaos breaking. Here it is indicated that when there is a large disturbances in system parameter leading to system collapse, chaos is very likely to be an intermediate transient stage between a stable and unstable region a power system**.** All this studies are helpful in understanding, how various instabilities due to the interaction between system parameter itself take place and the possible routes through which the system moving towards breakdown through various stable and unstable oscillatory modes It also indicated that there is a maximum lodability point after which the system enters into instability modes. All these studies are helpful to understand the mechanism of various instability modes and to find out effective find appropriate measure to prevent nonlinearity induced instability in power system.

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