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International
Journal
Of Modern Engineering Research (IJMER)

On πgθ-Homeomorphisms in Topological Spaces

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Abstract: *In this paper, we introduce the concepts of πgθ-closed map, πgθ-open map, πgθ-homeomorphisms and πgθc-homeomorphisms and study their properties. Also, we discuss its relationship with other types of functions.*
***Mathematics Subject Classification**: 54E55*
Key words: *πgθ-closed maps, πgθ-M-closed maps, πgθ-homeomorphisms and πgθc- homeomorphisms.*

I. Introduction

Velicko[27] introduced the notions of θ-open sets. Dontchev and Maki [11] alone have explored the concept of θ-generalized closed sets. Regular open sets have been introduced and investigated by Stone[26]. The finite union of regular open sets is π-open and subsequently the complement of π-open set is said to be π-closed, which has been highlighted by Zaitsev[28]. Maki et al[20] introduced generalized homeomorphisms (briefly g-homeomorphisms) and gc-homeomorphisms in topological spaces. Devi et al[9] introduced generalized-semi-homeomorphisms(briefly gs-homeomorphisms) and gsc-homeomorphisms in topological spaces.

In this paper, we first introduce a new class of closed maps called πgθ-closed maps and we study some of the properties of πgθ-homeomorphisms.

II. Preliminaries

Throughout this paper (X,τ) and (Y,σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned . For a subset A of a space (X,τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X,τ) and (Y,σ) will be replaced by X and Y if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of space (X,τ) is called
(1) a pre- open set[21] if $A\subset$ int(cl(A)) and a pre closed set if cl(int(A)) $\subset$ A;
(2) a semi-open set[4] if $A\subset$cl(int(A)) and semi-closed if int(cl(A)) $\subset$ A;
(3) a regular open set [15]if A= int (cl(A)) and a regular closed set if A = cl(int(A)).

Definition 2.2: A subset A of a space (X,τ) is called
(1) generalized closed (briefly g-closed) [17] if cl(A)$\subset$U whenever A$\subset$U and U is open in (X,τ).
(2) a semi-generalized closed (briefly sg-closed)[6] if scl(A)$\subset$U whenever A$\subset$U and U is semi-open in (X,τ).
(3) a generalized semi-closed (briefly gs-closed)[6] if scl(A)$\subset$U whenever A$\subset$U and U is open in (X,τ).
(4) a α-generalized closed (briefly αg-closed)[20] if αcl(A)$\subset$U whenever A$\subset$U and U is open in (X,τ).
(5) a generalized α-closed (briefly gα-closed)[20] if αcl(A)$\subset$U whenever A$\subset$U and U is α-open in (X,τ).
(6) a θ-generalized closed (briefly, θg-closed)[11] if $cl_\theta(A)\subset$U whenever A$\subset$U and U is open in (X,τ).
(7) a π generalized closed(briefly πg-closed)[12] if cl(A)$\subset$U, whenever A$\subset$U and U is π-open in (X,τ).
(8) a π generalized α-closed (briefly πgα-closed)[15] if αcl(A)$\subset$U, whenever A$\subset$U and U is π-open in (X,τ).
(9) a π generalized semi-closed (briefly πgs-closed)[5] if scl(A)$\subset$U,whenever A$\subset$U and U is π-open in (X,τ).
(10) a π generalized b-closed (briefly πgb-closed)[25] if bcl(A)$\subset$U, whenever A$\subset$U and U is π-open in (X,τ).

(11) a π generalized pre-closed(briefly πgp-closed)[24] if pcl(A) ⊂ U,whenever A ⊂ U and U is π-open in (X,τ).

(12) a π generalized θ-closed(briefly πgθ-closed)[24] if $cl_\theta(A) \subset U$,whenever A ⊂ U and U is π-open in (X,τ).

The compliment of g-closed (resp. sg-closed, gs-closed, αg-closed, gα-closed, θg-closed, πg-closed, πgα-closed, πgs-closed, πgb-closed , πgp-closed,πgθ-closed) is g-open (resp. sg-open , gs- open , αg-open, gα-open , θg-open , πg-open , πgα-open , πgs-open , πgθ-open , πgp-open,πgθ-open).

Definition 2.3 A map f: (X,τ) → (Y,σ) is said to be
(1) generalized-continuous (briefly g-continuous)[7] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V in (Y,σ).
(2) semi generalized-continuous (briefly sg-continuous)[8] if $f^{-1}(V)$ is sg-closed in (X,τ) for every closed set V in (Y,σ).
(3) generalized semi-continuous (briefly gs-continuous)[8] if $f^{-1}(V)$ is gs-closed in (X,τ) for every closed set V in (Y,σ).
(4) generalized-α-continuous (briefly gα-continuous) [18] if $f^{-1}(V)$ is gα-closed in (X,τ) for every closed set V in (Y,σ).
(5) α-generalized-continuous (briefly αg-continuous)[18] if $f^{-1}(V)$ is αg-closed in (X, τ) for every closed set V in (Y,σ).
(6) θ-continuous[11] if $f^{-1}(V)$ is θ-closed in (X, τ) for every closed set V in (Y,σ).
(7) θ-generalized-continuous (briefly θg-continuous)[14] if $f^{-1}(V)$ is θg-closed in (X, τ) for every closed set V in (Y,σ).
(8) πgθ-open[1] if for each open set V in (X,τ), f(V) is πgθ-open in (Y,σ).
(9) πgθ-closed [1] if for each closed set V in (X,τ), f(V) is πgθ-closed in (Y,σ).
(10) πgθ-continuous[1] if $f^{-1}(V)$ is πgθ-closed in (X,τ) for every closed set V in (Y,σ).
(11) πgθ-irresolute[1] if $f^{-1}(V)$ is πgθ-closed in (X,τ) for every πgθ-closed set V in (Y,σ).
(12) A space X is called πgθ-T½-space[1] if every πgθ-closed set is θ-closed.

Definition 2.4 A bijection f: (X,τ) → (Y,σ) is said to be
(1) g-homeomorphism[20] if f is g-open and g-continuous.
(2) sg-homeomorphism[20] if f is sg-open and sg-continuous.
(3) gs-homeomorphism[20] if f is gs-open and gs-continuous.
(4) gα-homeomorphism[18] if both f and f^{-1} are gα-continuous in (X,τ).
(5) αg-homeomorphism[18] if both f and f^{-1} are αg-continuous in (X,τ).
(6) θ-homeomorphism[23] if f is both θ-continuous and θ-open.
(7) θg-homeomorphism[14] if f is both θg-continuous and θg-open.
(8) πg-homeomorphism[12] if f is both πg-continuous and πg-open.
(9) πgb-homeomorphism[12] if f is both πgb-continuous and πgb-open.
(10) πgα-homeomorphism[15] if f is both πgα-continuous and πgα-open.
(11) πgs-homeomorphism[5] if f is both πgs-continuous and πgs-open.
(12) πgp-homeomorphism[5] if f is both πgp-continuous and πgp-open.

III. Πgθ-Closed Maps

In this section, we introduce the notions of πgθ-closed maps, πgθ-open maps in topological spaces and obtain certain characterizations of these maps.

Definition 3.1 The map f: (X,τ) →(Y,σ) is called πgθ-closed if the image of every closed set in (X,τ) is πgθ-closed in (Y,σ).

Proposition 3.2 If a mapping f: (X,τ)→(Y,σ) is πgθ-closed, then πgθ-cl(f(A)) ⊂ f(cl(A)) for every subset A of (X,τ).

Proof: Let A ⊂ X. Since f is πgθ-closed, f(cl(A)) is πgθ-closed in (Y,σ). Now f(A) ⊂ f(cl(A)). Also f(A) ⊂ πgθ- cl(f(A)). By definition, we have πgθ-cl(f(A)) ⊂ f(cl(A)).

Remark 3.3 The converse of the proposition 3.2 need not be true as seen in the following example.

Example 3.4 Let X=Y={a,b,c,d}.Let f: (X,τ)→(Y,σ) be an identity map with τ ={ϕ,{a},{c},{a,c},{b,d},{a,b,d},{b,c,d},X}, σ ={ ϕ,{a},{c},{a,c},{a,c,d},X} be their topologies. For every subset A of (X,τ), πgθ-cl(f(A)) ⊂ f(cl(A)), but f is not a πgθ-closed map.

Theorem 3.5 A map f: (X,τ) →(Y,σ) is πgθ-closed if and only if for each subset S of (Y,σ) and for each open set U containing $f^{-1}(S)$ there exists a πgθ-open set V of (Y,σ) such that S ⊂ V and $f^{-1}(V) \subset U$.

Proof: Necessity: Suppose that f is a πgθ-closed map. Let S ⊂ Y and U be an open subset of X such that $f^{-1}(S) \subset U$. Then $V=(f(U^c))$ is πgθ-closed set in Y. Therefore $Y-V=Y-f(U^c) = (f(U^c))^c$ is πgθ-open in Y. So $f^{-1}(V)= f^{-1}(Y)-f^{-1}(f(U^c)) = X- U^c = (U^c)^c =U$ such that $f^{-1}(V) \subset U$.

Sufficiency: Let S be a closed set of (X,τ). Then $f^{-1}((f(S))^c) \subset S^c$ and S^c is open . By assumption, there exists a πgθ-open set V of Y such that $(f(S))^c \subset V$ and $f^{-1}(V) \subset S^c$ and so $S \subset (f^{-1}(V))^c$. Hence $V^c \subset f(S) \subset f((f^{-1}(V))^c) \subset V^c$ which implies $f(S)= V^c$. Since V^c is πgθ-closed, f(S) is πgθ-closed and therefore f is πgθ-closed.

The following example shows that the composition of two πgθ-closed maps need not be πgθ-closed.

Example 3.6 Let X=Y=Z= {a,b,c,d.e}. Let f: (X,τ)→(Y,σ) and g: (Y,σ)→ (Z,η) be two identity maps with τ = {ϕ,{e},{a,b,c,d},X}, σ = { ϕ,{b},{c},{b,c}, {a,b},{a,b,c},X} and η = { ϕ,{a,b}, {c,d},{a,b,c,d},X} be their topologies. Then (gof) [{a,b,c,d}]= {a,b,c,d} is closed in (X,τ) but not πgθ-closed in (Z,η).

Proposition 3.7 Let f: (X,τ)→(Y,σ) be a closed map and g: (Y,σ)→(Z,η) be πgθ-closed map, then their composition gof: (X,τ)→(Z,η) is πgθ-closed.

Proof: Let A be a closed set in X. Since f is closed map, f(A) is closed in Y. Since g is πgθ-closed, g(f(A)) is πgθ-closed. Hence gof is a πgθ-closed map.

Remark 3.8 If f: (X,τ)→(Y,σ) is a πgθ- closed map and g: (Y,σ)→ (Z,η) is a closed map, then their composition need not be closed map as seen from the following example.

Example 3.9 Let X=Y=Z={a,b,c,d}. Let f: (X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) be two identity maps with τ ={ ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,c,d},X}, σ ={ϕ,{a},{c},{a,c},{a,b,c}, {a,c,d},X} and η ={ϕ,{a},{c},{a,c},{a,b},{a,b,c},{a,c,d},X}. f: (X,τ)→(Y,σ) is a πgθ-closed map and g: (Y,σ)→ (Z,η) is a closed map. But (gof): (X,τ) →(Z,η) is not a closed map because (gof)({a,d}) = {a,d} is not closed in (Z,η).

Theorem 3.10 Let f: (X,τ)→(Y,σ) and g: (Y,σ)→ (Z,η) be two mappings such that their composition gof: (X,τ)→(Z,η) be a πgθ-closed mapping. Then the following statements are true.

(i) If f is continuous and surjective, then g is πgθ-closed.
(ii) If g is πgθ-irresolute and injective, then f is πgθ-closed.

Proof: (i) Let A be a closed set of (Y,σ). Since f is continuous, $f^{-1}(A)$ is closed in (X,τ) and since gof is πgθ-closed, (gof)($f^{-1}(A)$) is πgθ-closed in (Z,η). That is g(A) is πgθ-closed in (Z,η). Therefore, g is a πgθ-closed map.

(ii)Let B be a closed set of (X,τ). Since gof is πgθ-closed, (gof)(B) is πgθ-closed in (Z,η). Since g is πgθ-irresolute, g^{-1}(gof)(B) is πgθ-closed in (Y,σ). Hence f(B) is πgθ-closed in (Y,σ),. Thus f is a πgθ-closed map.

Analogous to a πgθ-closed map, we define a πgθ-open map as follows:

Definition 3.11 A map f: (X,τ)→(Y,σ) is called πgθ-open if the image of every open set in (X,τ) is πgθ-open in (Y,σ).

Definition 3.12 Let x be a point of (X,τ) and V be a subset of X. Then V is called a πgθ-neighbourhood U of x in (X,τ) if there exists a πgθ-open set U of (X,τ) such that x ∈ U ⊆ V.

By πGθO(X) we mean the family of all πgθ-open subsets of the space (X,τ).

Theorem 3.13 :Suppose πGθO(X) is closed under arbitrary unions. Let f: (X,τ)→(Y,σ) be a mapping. Then the following statements are equivalent:

(i) f is a πgθ-open mapping.
(ii) For a subset A of (X,τ), f(int(A)) ⊂ πgθ-int(f(A)).

(iii) For each $x \in X$ and for each neighbourhood U of x in (X,τ), there exists a πgθ-neighbourhood W of f(x) in (Y,σ) such that $W \subset f(U)$.

Proof. (i) ⟹ (ii): Suppose f is πgθ-open. Let $A \subset X$. Since int(A) is open in (X,τ), f(int(A)) is πgθ-open in (Y,σ). Hence $f(int(A)) \subseteq f(A)$ and we have , $f(int(A) \subseteq$ πgθ-int(f(A)).

(ii) ⟹ (iii): Suppose (ii) holds. Let $x \in X$ and U be an arbitrary neighbourhood of x in (X,τ). Then there exists an open set G such that $x \in G \subseteq U$. By assumption, f(G) = f(int(G)) ⊆ πgθ-int(f(G)). This implies f(G) = πgθ-int(f(G)). Therefore f(G)is πgθ-open in (Y,σ). Further, $f(x) \in f(G) \subseteq f(U)$ and so (iii) holds, by taking W=F(G).

(iii) ⟹ (i): Suppose (iii) holds. Let U be any open set in (X,τ), $x \in U$ and f(x) = y. Then for each $x \in U$, $y \in f(U)$, by assumption there exists a πgθ-neighbourhood W_y of y in (Y,σ) such that $W_y \subset f(U)$. Since W_y is a πgθ-neighbourhood of y , there exists a πgθ-open set V_y of (Y,σ) such that $y \in V_y \subseteq W_y$. therefore, f(U) = $\bigcup \{ V_y : y \in f(U)\}$ and f(U) is a πgθ-open set of (Y,σ). Thus f is πgθ-open mapping.

Theorem 3.14 A function f: (X,τ) →(Y,σ) is πgθ-open if and only if for any subset B of (Y,σ) and for any closed set S containing $f^{-1}(B)$, there exists a πgθ-closed set A of (Y,σ) containing B such that $f^{-1}(A) \subseteq S$.

Proof: Similar to Theorem 3.5.

Remark 3.15 A map f: (X,τ)→(Y,σ) is πgθ-open if and only if f^{-1}(πgθ-cl(B)) ⊂ cl(f^{-1}(B)) for every subset B of (Y,σ).

Proof: Necessity: Suppose that f is a πgθ-open map. Then for any $B \subset Y$, $f^{-1}(B) \subset cl(f^{-1}(B))$. By Theorem 3.14, there exists a πgθ-closed set A of (Y,σ) such that $B \subset A$ and $f^{-1}(A) \subset cl(f^{-1}(B))$. Therefore, f^{-1} (πgθ-cl(B) ⊂ $f^{-1}(A) \subset cl(f^{-1}(B))$, since A is a πgθ-closed set in (Y,σ).

Sufficiency: Let S be any subset of (Y,σ) and F be any closed set containing $f^{-1}(S)$. Put A = πgθ-cl(S). Then A is a πgθ-closed set and $S \subset A$. By assumption, $f^{-1}(A)$ = f^{-1}(πgθ-cl(S)) ⊂ cl($f^{-1}(S)) \subset F$ and therefore by Theorem 3.14, f is πgθ-open.

Definition 3.16 A map f:(X,τ)→(Y,σ) is said to be a M-πgθ-closed map if the image f(A) is πgθ-closed in (Y,σ) for every πgθ-closed set A of (X,τ).

Remark 3.17 M-πgθ-closed map is independent of a πgθ-closed map as seen in the following examples.

Example 3.18 Let X ={a,b,c,d,e}=Y. Let f: (X,τ)→(Y,σ) be an identity map. Let τ ={ϕ,{b},{c}, {b,c},{a,b}, {a,b,c},X} and σ = { ϕ,{a,b},{c,d},{a,b,c,d},X} be their topologies. Then f is a πgθ-closed map. But not a M-πgθ-closed map, since f({a,d}) = {a,d} is not πgθ-closed in (Y,σ) where {a,d} is πgθ-closed in (X,τ).

Example 3.19 Let X={a,b,c,d,e}=Y. Let f:(X,τ)→(Y,σ) be an identity map. Let τ={ϕ,{a},{b}, {c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d},{a,b,c,e},X} and σ = { ϕ,{a,b},{c,d},{a,b,c,d},X} be their topologies. Then f is not a πgθ-closed map, since f({d})={d}is not πgθ-closed in (Y,σ) where {d} is closed in (X,τ). However f is a M-πgθ-closed map.

Theorem 3.20 If f:(X,τ)→(Y,σ) is a π-irresolute and pre-θ-closed map in X, then f(A) is πgθ-closed in Y for every πgθ-closed set A of X.

Proof: Let A be any πgθ-closed set of X and V be any π-open set of Y containing f(A). Then $A \subset f^{-1}(V)$ where $f^{-1}(V)$ is π-open in X .Since A is πgθ-closed, $cl_\theta(A) \subset f^{-1}(V)$ and hence $f(cl_\theta(A)) \subseteq V$. Since f is pre-θ-closed, $cl_\theta(f(cl_\theta(A))) \subset V$ and hence we obtain $cl_\theta(f(A)) \subset V$. Hence f(A) is πgθ-closed in Y.

IV. πgθ-Homeomorphisms

Definition 4.1: A bijection f:(X,τ)→(Y,σ) is called πgθ-homeomorphism if f is both πgθ-continuous and πgθ-open map.

Definition 4.2 A bijection f:(X,τ)→(Y,σ) is called πgθc-homeomorphism if both f and f^{-1} are πgθ-irresolute in (X,τ).

Theorem 4.3
1. Every θ- homeomorphism is πgθ- homeomorphism.
2. Every θg- homeomorphism is πgθ- homeomorphism.
3. Every πgθ-homeomorphism is πg-homeomorphism.
4. Every πgθ-homeomorphism is πgα-homeomorphism.
5. Every πgθ-homeomorphism is πgs-homeomorphism.
6. Every πgθ-homeomorphism is πgb-homeomorphism.
7. Every πgθ- homeomorphism is a πgp- homeomorphism.

Proof: Straight forward.

Remark4.4 The following examples show that the converse of Theorem 4.3 need not be true.

Example 4.5 Let X={a,b,c,d}=Y with topologies τ ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, {a,c,d}, X}, σ ={ϕ,{a},{a,b},{a,c},{a,b,c},{a,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map. Then f is not a θ-homeomorphism because f^{-1}({b})={b} is not θ-closed in X where {b} is closed in Y. However f is a πgθ-homeomorphism.

Example 4.6 Let X={a,b,c,d}=Y with topologies τ ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, {a,c,d}, X}, σ={ϕ {a},{a,b},{a,c},{a,b,c},{a,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map.Then f is not a θg-homeomorphism because f^{-1}({b})={b} is not θg-closed in X where {b} is closed in Y. However f is a πgθ-homeomorphism.

Example 4.7 Let X={a,b,c,d}=Y with topologies τ ={ ϕ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d},X}, σ = { ϕ,{a},{b},{a,b},{a,b,d},X} respectively. Let f:(X,τ) → (Y,σ) be the identity map. Then f is not a πgθ-homeomorphism because f^{-1}({c}) = {c} is not πgθ-closed in X, where {c} is closed in Y. However f is a πg-homeomorphism.

Example 4.8 Let X={a,b,c,d}=Y with topologies τ ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, {a,b,d},X}, σ ={ ϕ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d},X} respectively. Let f:(X,τ)→(Y,σ) be an identity map. Then f is not a πgθ- homeomorphism because f({c}) = {c} is not πgθ-closed in Y where {c} is closed in X. However f is a πgα-homeomorphism.

Example 4.9 Let X={a,b,c,d,e}=Y with topologies τ={ϕ,{a,b},{c,d},{a,b,c,d},X},σ ={ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d},{a,b,c,e}, X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is not a πgθ-homeomorphism because f^{-1}({d}) = {d} is not πgθ-closed in X where {d} is closed in Y. However f is a πgs-homeomorphism.

Example 4.10 Let X={a,b,c,d}=Y with topologies τ={ϕ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d}, X}, σ ={ϕ,{a},{b},{a,b},{a,b,c},{a,b,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map. Then f is not a πgθ-homeomorphism because f^{-1}({c}) ={c} is not πgθ-closed in X where {c} is closed in Y. However f is a πgb-homeomorphism.

Example 4.11 Let X ={a,b,c,d,e}=Y with topologies τ ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{d,e}, {a,b,c},{a,d,e},{b,d,e},{c,d,e},{a,b,d,e},{a,c,d,e},{b,c,d,e},X}, σ = {ϕ,{a,b},{c,d},{a,b,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is not a πgθ-closed map, since f({a,b}) ={a,b} is not πgθ-closed in (Y,σ), where {a,b} is closed in (X,τ). However f is a πgp-homeomorphism.

Remark 4.12 The following examples show that πgθ-homeomorphism is independent of , g-homeomorphism, sg-homeomorphism, gs-homeomorphism, αg-homeomorphism and gα-homeomorphism.

Example 4.13 Let X={a,b,c,d}=Y with topologies τ={ϕ,{a},{b},{c}, {a,b},{a,c},{b,c}, {a,b,c}, {a,c,d},X}, σ ={ ϕ,{a},{a,b},{a,c},{a,b,c},{a,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f({a,d}) ={a,d} is not g-closed in (Y,σ) where {a,d} is closed in (X,τ). However f is a πgθ-homeomorphism.

Example 4.14 Let X={a,b,c,d}=Y with topologies τ={ ϕ,{a},{c},{a,c},{a,b},{a,b,c},{a,c,d}, X}, σ ={ϕ,{a},{a,b},{a,c},{a,b,c},{a,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map. Then f^{-1}({b}) ={b} is not πgθ-closed in X, where {b} is closed in Y. However f is g- homeomorphism.

Example 4.15 Let X={a,b,c,d,e}=Y with topologies τ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,c,e},{a,b,c,d},{a,b,c,e},X}, σ ={ϕ,{e},{a,b,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is neither αg nor gα-homeomorphisms because f^{-1}({a,b,c,d})={a,b,c,d} is neither αg nor gα-closed in X, where {a,b,c,d} is closed in Y. However f is a πgθ -homeomorphism.

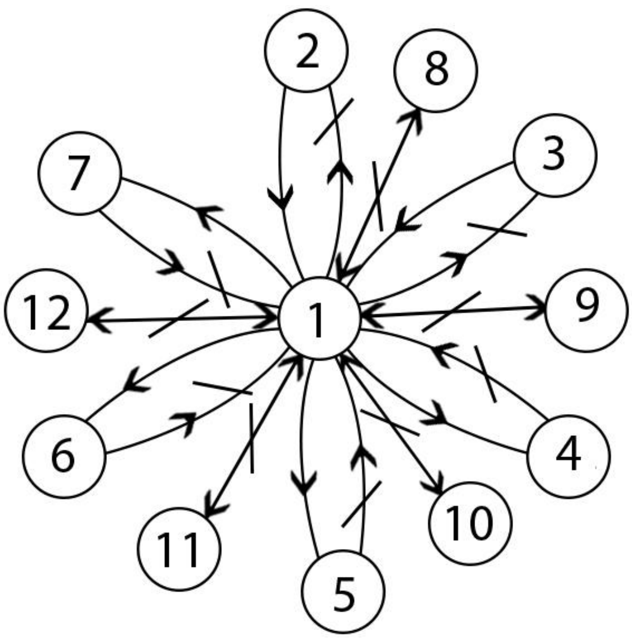
Example 4.16 Let X={a,b,c,d,e}=Y with topologies τ={ ϕ,{a,b},{c,d},{a,b,c,d},X}, σ ={ϕ,{e}, {a,b,c,d}, X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is not πgθ-homeomorphism because f^{-1}({a,b,c,d})={a,b,c,d} is not πgθ-closed in X, where {a,b,c,d} is closed in Y. However f is a gα-homeomorphism.

Example 4.17 Let X={a,b,c,d}=Y with topologies τ={ ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, {a,b,d},X}, σ ={ϕ,{a},{b},{a,b},{a,c},{a,b,c},{a,b,d}, X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is not πgθ-homeomorphism because f({c})={c} is not πgθ-closed in X, where {c} is closed in Y. However f is a αg-homeomorphism.

Example 4.18 Let X={a,b,c,d}=Y with topologies τ={ ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, {a,c,d},X}, σ ={ϕ,{a},{a,b},{a,c},{a,b,c},{a,c,d}, X} respectively. Let f:(X,τ)→(Y,σ) be the identity map . Then f is neither sg nor gs- homeomorphism because f({a,d}) ={a,d} is neither sg nor gs -closed in Y, where {a,d} is closed in X. However f is a πgθ-homeomorphism.

Example 4.19 Let X={a,b,c,d}=Y with topologies τ={ ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}, X}, σ ={ ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c},{a,c,d},X}, resp. Let f:(X,τ)→(Y,σ) be the identity map. Then f is not a πgθ-homeomorphism because f^{-1}({b}) ={b} is not πgθ-closed in X, where {b} is closed in Y. However f is both sg and gs-homeomorphisms.

Remark 4.20 The above discussions are summarized in the following diagram.



1⟹ πgθ-homeomorphism
2⟹ θ-homeomorphism
3⟹ θg-homeomorphism
4⟹ πg-homeomorphism
5 ⟹ πgα-homeomorphism
6⟹ πgs-homeomorphism
7⟹ πgp-homeomorphism
8⟹ g-homeomorphism
9⟹ gs-homeomorphism
10⟹ sg-homeomorphism
11⟹ αg-homeomorphism
12⟹ gα-homeomorphism

Remark 4.21 Composition of two πgθ-homeomorphism need not be a πgθ-homeomorphism as shown in the following example.

Example 4.22 Let X=Y=Z={a,b,c,d.e}. Let f: (X,τ)→(Y,σ) and g: (Y,σ)→ (Z,η) be two identity maps. τ ={ ϕ,{e},{a,b,c,d},X}, σ ={ ϕ,{b},{c},{b,c}, {a,b},{a,b,c},X} and η ={ϕ, {a,b},{c,d}, {a,b,c,d},X} be their topologies. Then (gof) [{e}]={e} is open in X, but not πgθ-open in Z. This implies that gof is not πgθ-open and hence gof is not πgθ-homeomorphism.

Proposition 4.23 For any bijective map f:(X,τ)→(Y,σ) the following statements are equivalent.

(a) f^{-1}:(Y,σ)→(X,τ) is πgθ-continuous map.
(b) f is a πgθ-open map.
(c) f is a πgθ-closed map.

Proof: (a)⟹(b): Let U be an open set in (X,τ). Then X– U is closed in (X,τ). Since f^{-1} is πgθ-continuous $(f^{-1})^{-1}$(X– U) is πgθ-closed in (Y,σ). That is f(X– U)= Y– f(U) is πgθ-closed in (Y,σ). This implies f(U) is πgθ-open in (Y,σ). Hence f is πgθ-open map.

(b) ⟹(c): Let V be a closed set in (X,τ). Then X– V is open in (X,τ). Since f is πgθ-open, f(X– V) is πgθ-open in (Y,σ). That is Y– f(V) is πgθ-open in (Y,σ). This implies f(V) is πgθ-closed in (Y,σ). Hence f is πgθ-closed map.

(c) ⟹(a): Let U be closed set in (X,τ). Since f is πgθ-closed map, f(U) is πgθ-closed in (Y,σ). That is $(f^{-1})^{-1}$(U) is πgθ-closed in (Y,σ). Hence f^{-1} is πgθ-continuous.

Proposition 4.24 Let f:(X,τ)→(Y,σ) be a bijective and πgθ-continuous map. Then the following statements are equivalent.

(a) f is a πgθ-open map.
(b) f is a πgθ-homeomorphism.
(c) f is a πgθ-closed map.

Proof: Straight forward.

Remark 4.25 πgθc-homeomorphism is independent of a πgθ-homeomorphism as seen in the following examples.

Example 4.26 Let X={a,b,c,d,e}=Y with topologies τ={ϕ,{a,b},{c,d},{a,b,c,d},X}, σ ={ϕ,{b},{c},{b,c},{a,b},{a,b,c},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map. Then f is not πgθ-irresolute, because f^{-1}({d})={d} is not πgθ-closed in X where {d}is πgθ-closed in Y. However f is πgθ-homeomorphism.

Example 4.27 Let X={a,b,c,d}=Y with topologies τ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c}, {a,b,c},X}, σ ={ϕ,{a},{b},{c},{a,b},{a,c},{b,c},{a,d},{a,b,c},{a,b,d},{a,c,d},X} respectively. Let f:(X,τ)→(Y,σ) be the identity map. Then f is not πgθ-continuous, because f^{-1}({b})={b} is not πgθ-closed in Y where {d}is closed in Y. However f is πgθc-homeomorphism.

Proposition 4.28 If f: (X,τ)→(Y,σ) and g:(Y,σ)→(Z,η) are πgθc-homeomorphisms, then gof : (X,τ)→(Z,η) is also a πgθc-homeomorphism.

Proof: Let U be a πgθ-open set in (Z,η). Now $(gof)^{-1}$(U)= f^{-1} (g^{-1} (U))= f^{-1}(V) where g^{-1} (U) =V. By hypothesis, V is πgθ-open in (Y,σ) and again by hypothesis, f^{-1}(V) is πgθ-open in (X,τ). Therefore (gof) is πgθ-irresolute. Also for a πgθ-open set A in (X,τ), we have (gof)(A)=g(f(A))=g(W) where W=f(A). By hypothesis, f(A) is πgθ-open in (Y,σ) and again by hypothesis. Therefore, g(A) is πgθ-open in (Z,η). Now $(gof)^{-1}$ is πgθ-irresolute. Hence (gof) is a πgθc-homeomorphism.

Proposition 4.29 The set πgθch(X,τ) is a group.

Proof: Define ψ: πgθch(X,τ) × πgθch(X,τ)→πgθch(X,τ) by ψ(f,g) = gof for every f,g∈ πgθch(X,τ). Then by proposition 4.28, (gof) ∈ πgθch(X,τ). Hence πgθch(X,τ) is closed. We know that the composition of maps is associative. The idendity map i:(X,τ) →(X,τ) is πgθc-homeomorphism and i ∈πgθch(X,τ). Also iof=foi=f for every f∈ πgθch(X,τ). For any f∈ πgθch(X,τ), fo f^{-1}=$f^{-1}$of= i. Hence inverse exists for each element of πgθch(X,τ). Thus πgθch(X,τ) is a group under composition of maps.

Proposition 4.30 Every πgθ-homeomorphism from a πgθ-T½-space into another πgθ-T½-space is a θ-homeomorphism.

Proof: Let f: (X,τ) →(Y,σ) be a πgθ-homeomorphism. Then f is bijective, πgθ-continuous and πgθ-open. Let U be an open set in X. Since f is πgθ-open, f(U) is πgθ-open in Y. Since (Y,σ) is a πgθ-T½-space, f(U) is θ-open in (Y,σ). This implies f is a θ-open map. Let V be closed in (Y,σ). Since f is πgθ- continuous and since (X,τ) is a πgθ-T½-space, f^{-1}(V) is θ-closed in (X,τ) . Therefore f is θ-continuous. Hence f is a θ-homeomorphism.

Definition4.31 A space X is called πgθ-space if every πgθ-closed set is closed.

Proposition 4.32 Every πgθ-homeomorphism from a πgθ-space into another πgθ-space is a πgθc-homeomorphism.

Proof: Let f: (X,τ)→(Y,σ) be a πgθ-homeomorphism. Then f is bijective, πgθ-continuous and πgθ-open. Let V be a πgθ-closed set in (Y,σ). Then V is closed in (Y,σ). Since f is πgθ- continuous, $f^{-1}(V)$ is πgθ- closed in (X,τ). Hence f is a πgθ-irresolute map. Let V be πgθ-open in (X,τ). Then V is open in (X,τ)). Since f is πgθ-open, f(V) is πgθ-open in (Y,σ). That is $(f^{-1})^{-1}(V)$ is πgθ- open in (Y,σ) and hence f^{-1} is πgθ-irresolute . Thus f is πgθc-homeomorphism.

V. Conclusion

This paper has attempted to compare πgθ-homeomorphism with the other homeomorphisms in topological spaces. It also states that the several definitions and results in this paper will result in obtaining several characterizations and also enable to study various properties. The future scope of study is the extension of πgθ-closed maps and πgθ-continuous maps in topological spaces.

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