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Contra α* Continuous Functions in Topological Spaces

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Abstract: *In [3], Dontchev introduced and investigated a new notion of continuity called contra-continuity. Recently, Jafari and Noiri [5] introduced new generalization of contra-continuity called contra α continuity . The aim of this paper is to introduce and study the concept of a contra α * continuous and almost contra α * continuous functions are introduced*

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I. Introduction

Dontchev[3] introduced the notion of contra continuity. Later Jafari and Noiri introduced and investigated the concept of contra α continuous and discussed its properties. Recently, S.Pious Missier and P. Anbarasi Rodrigo [9] have introduced the concept of α * -open sets and studied their properties. In this paper we introduce and investigate the contra α * continuous functions and almost contra α * continuous functions and discuss some of its properties.

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space X is said to be a α ***open** [9] if A $\subseteq$ int* (cl (int* (A))).

Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a α **** continuous*** [10] if f^{-1}(O) is a α *open set of (X, τ) for every open set O of (Y, σ).

Definition 2.3: A map f*:* (X, τ) ⟶ (Y, σ) is said to be ***perfectly*** α **** continuous*** [11] if the inverse image of every α *open set in (Y, σ) is both open and closed in (X, τ) .

Definition 2.4: A function f*:* (X, τ) ⟶(Y, σ) is said to be α ****Irresolute*** **[**10**]** if f^{-1}(O) is a α *open in (X, τ) for every α *open set O in (Y, σ).

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra g continuous*** [2] if f^{-1}(O) is a **g -** ***closed*** set [6] of (X, τ) for every open set O of (Y, σ).

Definition 2.6: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra continuous*** [3]if f^{-1}(O) is a closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.7: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra*** α ***continuous*** [5] if f^{-1}(O) is a α closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.8: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra semi continuous*** **[**4**]** if f^{-1}(O) is a semi closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.9: A function f: (X, τ) ⟶ (Y, σ) is called a ***contra g*** α ***continuous*** **[**1**]** if f^{-1}(O) is a g α -closed set of (X, τ) for every open set O of (Y, σ).

Definition 2.10: A Topological space X is said to be α ***$T_{1/2}$ space*** *or* α ****space*** [9] if every α * open set of X is open in X.

Definition 2.11: A Topological space X is said to be a ***locally indiscrete*** [7] if each open subset of X is closed in X.

Definition 2.12: Let A be a subset of a topological space (X, τ). The set ∩{U ∈ τ | A ⊂ U} is called the ***Kernel of A*** [7] and is denoted by ker(A).

Lemma 2.13: [6] The following properties hold for subsets A,B of a space X:

1. x ∈ ker(A) if and only if A ∩ F≠ ϕ ,for any F ∈ C(X, x);

2. A ⊂ ker(A) and A = ker(A) if A is open in X;
3. If A ⊂ B, then ker(A) ⊂ ker(B).

Theorem 2.14: [10] Let f: (X, τ) ⟶(Y, σ) be a function. Then the following are equivalent:

1. f is α * continuous
2. The inverse image of closed set in Y is α * closed in X.
3. f (α * cl (A)) ⊆ f(cl(A)) for every subset A in X.
4. α * cl(f^{-1} (G)) ⊆ f^{-1} (cl(G)) for every subset G of Y.
5. f^{-1}(int (G)) ⊆ α * int (f^{-1}(G)) for every subset G of Y.

Theorem 2.15[9]**:**

(i) Every open set is α *- open and every closed set is α *-closed set
(ii) Every α -open set is α *-open and every α -closed set is α *-closed.
(iii) Every g-open set is α *-open and every g-closed set is α *-closed.

III. Contra α * continuous functions

Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is called **Contra α * continuous functions** if f^{-1} (O) is α * closed in (X, τ) for every open set O in (Y, σ).

Example 3.2: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{b},{c},{ab},{ac},{bc},{abc}, X} and σ = {ϕ, {a},{ab},{abc},Y}. α *C(X, τ) = {ϕ ,{d},{ad},{bd},{cd},{abd},{acd},{bcd},X}Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a = f(d) , f(c) = b , f(a) = c. clearly, f is contra α * continuous .

Theorem 3.3: Every contra continuous function is a contra α * continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a function. Let O be an open set in (Y, σ). Since, f is contra continuous , then f^{-1} (O) is closed in (X, τ) . Hence by thm [2.15] , f^{-1} (O) is α * closed in (X, τ) . Therefore, f is contra α * continuous.

Remark 3.4 : The converse of the above theorem need not be true.

Example 3.5: : Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{ab},{abc}, X}, τ^c = {ϕ,{d},{cd},{bcd},X} and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {b} , {c} , {d} , {ad} ,{bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a = f(d) , f(b) = b , f(c) = d. clearly, f is contra α * continuous , but f^{-1}({a}) = {ad} is not closed in X.

Theorem 3.6: Every contra g continuous map is contra α * continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a contra g continuous map and O be any open set in Y. Since, f is contra g continuous, then f^{-1} (O) is g - closed in (X, τ) . Hence by thm [2.15] , f^{-1} (O) is α * closed in (X, τ) . Therefore, f is contra α * continuous.

Remark 3.7: The converse of the above theorem need not be true.

Example 3.8: : Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{ab},{abc}, X}, g-closed (X, τ) = {ϕ,{d},{ad} , {bd} {cd}, {abd} ,{bcd}, {acd} ,X} and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {b} , {c} , {d} , {ad} ,{bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a = f(d) , f(b) = b , f(c) = d. clearly, f is contra α * continuous , but f^{-1} ({b}) = {b} is not g-closed in X.

Theorem 3.9: Every contra α continuous map is contra α * continuous.

Proof: Let f: (X, τ) ⟶ (Y, σ) be a contra α continuous map and O be any open set in Y. Since, f is contra α continuous, then f^{-1} (O) is α - closed in (X, τ) . Hence by thm [2.15] , f^{-1} (O) is α * closed in (X, τ) . Therefore, f is contra α * continuous.

Remark 3.10: The converse of the above theorem need not be true.

Example 3.11: Let X = Y= {a, b, c, d}, τ = {ϕ,{a}, {b},{ab}, {bc} ,{abc}, X}, and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {c} , {d} , {ad} ,{bd}, {cd} , {abd} , {acd} , {bcd} , X }. α C(X, τ) = {ϕ , {c} , {d} , {ad} , {cd} , {acd} , {bcd} , X}.Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a = f(d) , f(c) = b , f(a) = d. clearly, f is contra α * continuous , but f^{-1} ({a}) = {bd} is not α -closed in X.

Remark 3.12: The concept of contra semi continuous and contra α * continuous are independent.

Example 3.13: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{ab},{abc}, X}, SC(X, τ) = {ϕ,{b} , {c} ,{d},{bc} , {bd} {cd}, {bcd},X} and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {b} , {c} , {d} , {ad} ,{bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a = f(d) , f(b) = b , f(c) = d. clearly, f is contra α * continuous , but f^{-1} ({a}) = {ad} is not semi-closed in X.

Example 3.14: Let X = Y= {a, b, c, d}, τ = {ϕ,{a}, {b} , {c} ,{ab}, {ac} , {bc} ,{abc}, X}, SC(X, τ) = {ϕ, {a} ,{b} , {c} ,{d}, {ab} , {bc} , {ac} , {ad} , {bd} {cd}, {abd} , {acd}, {bcd},X} and σ = {ϕ, {a},{ab},{ac},{abc},Y}. α *C(X, τ) = {ϕ , {b} , {c} , {d} , {ad} ,{bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X }. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a, f(b) = b , f(c) = d., f(d) = c. Clearly, f is semi closed, but f^{-1} ({a}) = {a} is not α *closed in X.

Remark 3.15: The concept of contra gα continuous and contra α * continuous are independent.

Example 3.16: Let X = Y = {a, b, c, d}, τ = {ϕ,{a}, {b},{ab}, {bc} ,{abc}, X}, and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {c} , {d} , {ad} ,{bd}, {cd} , {abd} , {acd} , {bcd} , X }. gα - Closed(X, τ) = {ϕ , {c} , {d} , {ad} , {cd} , {acd} , {bcd} , X}.Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a = f(d) , f(c) = b , f(a) = d. clearly, f is contra α * continuous , but f^{-1} ({a}) = {bd} is not gα -closed in X. Therefore, f is not contra gα continuous.

Example 3.17: Let X = Y = {a, b, c, d}, τ = {ϕ,{a}, {b},{ab},{abc}, X}, and σ = {ϕ, {a},{b},{ab},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {c} , {d} , {ad} ,{bd}, {cd} , {abd} , {acd} , {bcd} , X }. gα - Closed(X, τ) = {ϕ , {c} , {d} , {ac} , {ad} , {bc} , {bd} , {cd} , {acd} , {bcd} , X}.Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = a = f(c) , f(d) = b , f(b) = d. clearly, f is contra gα continuous , but f^{-1} ({a}) = {ac} is not α *- closed in X. Therefore, f is not contra α * continuous.

Remark 3.18: The concept of α * continuous and contra α * continuous are independent.

Example 3.19: Let X = Y= {a, b, c, d}, τ = { ϕ , {a} , {b} , {ab} , {abc} , X } and σ = { ϕ , {a} , {b} {ab},{bc},{abc},Y}, α *O (X,τ)={ϕ, {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , {abd} , X} α *C (X, τ) = { ϕ , {c} , {d} , {ad} ,{bd} , {cd} , {abd} ,{acd} , {bcd}, X} . Let f: (X, τ) ⟶(Y, σ) be a map defined by f (c) = a, f(a) = f(b) = b,f(d) = c , clearly, f is α * continuous but f is not contra α * - continuous because f^{-1} ({b}) = {ab} is not α * - closed.

Example 3.20: Let X = Y = {a, b, c, d}, τ = {ϕ,{a}, {b},{ab}, {bc} ,{abc}, X}, and σ = {ϕ, {a},{b},{c},{ab},{ac},{bc},{abc},Y}. α *C(X, τ) = {ϕ , {c} , {d} , {ad} ,{bd}, {cd} , {abd} , {acd} , {bcd} , X }. α *O(X, τ) = {ϕ , {a} , {b} , {c} , {ab} , {ac} , {bc} , {abc} , {abd} , X}.Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a = f(d) , f(c) = b , f(a) = d. clearly, f is contra α * continuous , but f^{-1} ({a}) = {bd} is not α *- open in X. Therefore, f is not α * continuous.

Remark 3.21: The composition of two contra α * continuous need not be contra α * continuous.

Example 3.22: Consider X = Y = Z = {a, b, c }, τ = {ϕ,{a},{ab},{ac}, X} and σ = {ϕ, {a},{b} ,{ab},Y}, η ={ϕ,{a},{ab},Z}, α *C(X, τ) = {ϕ,{b},{c},{bc},X}, α *C (Y, σ) = {ϕ , {b}, {c},{ac},{bc}, Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(b) = a , f(c) = b, f(a) = c. Clearly, f is contra α *continuous. Consider the map g: Y ⟶ Z defined g(b) = a, g(c) = b, g(b) = c , clearly g is contra α *continuous. But g ∘f : X ⟶ Z is not a contra α *continuous, $(g \circ f)^{-1}$ ({ab}) = f^{-1} (g^{-1}{ab}) = f^{-1} (bc) = ac which is not a α *closed in X.

Theorem 3.23: Let f: (X, τ) ⟶ (Y, σ) be a map. The following are equivalent.

1. f is contra α * continuous.
2. The inverse image of a closed set F of Y is α * open in X

Proof: let F be a closed set in Y . Then Y\F is an open set in Y. By the assumption of (1), f^{-1}(Y\F) = X \ f^{-1} (F) is α * closed in X . It implies that f^{-1} (F) is α * open in X. Converse is similar.

Theorem 3.24: The following are equivalent for a function f: (X, τ) ⟶ (Y, σ). Assume that α *O(X, τ) (resp α *C (X, τ)) is closed under any union (resp; intersection).

1. f is contra α * continuous.
2. The inverse image of a closed set F of Y is α * open in X
3. For each x ∈ X and each closed set B in Y with f(x) ∈ B , there exists an α * open set A in X such that x ∈ A and f(A) ⊂B
4. f(α *cl (A))⊂ker (f(A)) for every subset A of X.
5. α *cl (f^{-1} (B)) ⊂f^{-1} (ker B) for every subset B of Y.

Proof:

(1) ⟹ (3) Let x ∈ X and B be a closed set in Y with f(x) ∈ B. By (1), it follows that f^{-1} (Y\B) = X\ f^{-1} (B) is α * closed and so f^{-1} (B) is α * open. Take A = f^{-1} (B). We obtain that x ∈ A and f(A) ⊂B.

(3) ⟹ (2) Let B be a closed set in Y with x ∈ f^{-1} (B). Since, f(x) ∈ B, by (3), there exist an α * open set A in X containing x such that f(A) ⊂B. It follows that x ∈ A ⊂f^{-1} (B). Hence, f^{-1} (B) is α * open.

(2) ⟹(1) Follows from the previous theorem

(2) ⟹(4) Let A be any subset of X. Let y ∉ker (f(A)). Then there exists a closed set F containing y such that f(A) ∩F = ϕ. Hence, we have A ∩f^{-1} (F) = ϕ and α *cl (A) ∩f^{-1} (F) = ϕ. Hence, we obtain f(α *cl (A)) ∩F = ϕ and y ∉ f(α *cl (A)). Thus, f(α *cl (A))⊂ker (f(A)).

(4) ⟹(5) Let B be any subset of Y. By (4) and lemma [2.13] f(α *cl (f^{-1} (B))) ⊂ (ker B) and α *cl (f^{-1} (B)) ⊂ f^{-1} (ker B)

(5) $\Rightarrow$ (1) Let B be any open set in Y. By (5) and lemma [2.13] α *cl (f^{-1} (B)) $\subset f^{-1}$ (ker B) = f^{-1} (B) and α *cl (f^{-1} (B)) = f^{-1} (B). We obtain that f^{-1} (B) is α * closed in X.

Theorem 3.25: If f: (X, τ) ⟶ (Y, σ) is α *irresolute and g: (Y, σ) ⟶ (Z, η) is contra α * continuous , then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra α * continuous.

Proof: Let O be any open set in (Z, η). Since, g is contra α * continuous, then g^{-1}(O) is α * closed in (Y, σ) and since f is α *irresolute , then f^{-1}(g^{-1} (O)) is α * closed in (X, τ) . Therefore, g ∘ f is contra α * continuous.

Theorem 3.26: If f: (X, τ) ⟶ (Y, σ) is contra α * continuous g: (Y, σ) ⟶ (Z, η) is continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra α * continuous.

Proof: Let O be any open set in (Z, η). Since, g is continuous, then g^{-1}(O) is open in (Y, σ) and since f is contra α * continuous, then f^{-1}(g^{-1} (O)) is α * closed in (X, τ). Therefore, g ∘ f is contra α * continuous.

Theorem 3.27: If f: (X, τ) ⟶ (Y, σ) is contra α continuous g: (Y, σ) ⟶ (Z, η) is continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra α * continuous.

Proof: Let O be any open set in (Z, η). Since, g is continuous, then g^{-1}(O) is open in (Y, σ) and since f is contra α continuous, then f^{-1}(g^{-1} (O)) is α closed in (X, τ).Hence by thm [2.15], every α closed set is α * closed. We have f^{-1}(g^{-1} (O)) is α * closed in (X, τ). Therefore, g ∘ f is contra α * continuous.

Theorem 3.28: If f: (X, τ) ⟶ (Y, σ) is contra α * continuous g: (Y, σ) ⟶ (Z, η) is g-continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra α * continuous.

Proof: Let O be any open set in (Z, η). Since, g is g- continuous, then g^{-1}(O) is g- open in (Y, σ) and since f is contra α * continuous, then f^{-1}(g^{-1} (O)) is α * closed in (X, τ). Therefore, g ∘ f is contra α * continuous.

Theorem 3.29: If f: (X, τ) ⟶ (Y, σ) is strongly α * continuous and g: (Y, σ) ⟶ (Z, η) is contra α * continuous , then their composition g ∘ f: (X, τ) ⟶ (Z, η) is contra continuous.

Proof: Let O any open set in (Z, η). Since, g is contra α * continuous, then g^{-1}(O) is α * closed in (Y, σ) and since f is strongly α * continuous, f^{-1}(g^{-1} (O)) is closed in (X, τ). Therefore, g ∘ f is contra continuous.

Theorem 3.30: If f: (X, τ) ⟶ (Y, σ) is perfectly α * continuous, and g: (Y, σ) ⟶ (Z, η) is contra α * continuous, then their composition g ∘ f: (X, τ) ⟶ (Z, η) is .

Proof: Let O any open set in (Z, η). By thm [2.15] every open set is α * open set which implies O is α * open in (Z, η). Since, g is contra α * continuous, then g^{-1}(O) is α * closed in (Y, σ) and since f is perfectly α * continuous , then f^{-1} (g^{-1}(O)) is both open and closed in X, which implies (g ∘ f) $^{-1}$ (O) is both open and closed in X . Therefore, g ∘ f is perfectly α * continuous.

Theorem 3.31: Let f: (X, τ) ⟶ (Y, σ) is surjective α *irresolute and α * open and g: (Y, σ) ⟶ (Z, η) be any function. Then g ∘ f: (X, τ) ⟶ (Z, η) is contra α * continuous if and only if g is contra α * continuous.

Proof: The if part is easy to prove. To prove the only if part , let F be any closed set in (Z, η). Since g ∘ f is contra α * continuous, then f^{-1}(g^{-1} (F)) is α * open in (X, τ) and since f is α * open surjection, then f(f^{-1}(g^{-1} (F))) = g^{-1} (F) is α * open in (Y, σ). Therefore, g is contra α * continuous.

Theorem 3.32: Let f: (X, τ) ⟶ (Y, σ) be a function and X a α * $T_{1/2}$ space. Then the following are equivalent:

1. f is contra continuous
2. f is contra α * continuous

Proof:

(1) $\Rightarrow$ (2) Let O be any open set in (Y, σ). Since f is contra continuous, f^{-1} (O) is closed in (X, τ) and since Every closed set is α *closed, f^{-1} (O) is α *closed in (X, τ). Therefore, f is contra α * continuous.

(2) $\Rightarrow$ (1) Let O be any open set in (Y, σ). Since, f is contra α * continuous, f^{-1} (O) is α *closed in (X, τ) and since X is α * $T_{1/2}$ space, f^{-1} (O) is closed in (X, τ). Therefore, f is contra continuous.

Theorem 3.33: If f: (X, τ) ⟶ (Y, σ) is contra α * continuous and (Y, σ) is regular , then f is α *continuous.

Proof: Let x be an arbitrary point of X and O be any open set of Y containing f(x). Since Y is regular, there exists an open set U in Y containing f(x) such that cl(U) $\subset$ O. Since, f is contra α * continuous, so by thm[3.24] , there exists N $\in$ α *O (X, τ) , such that f(N) $\subset$ cl(U) . Then, f(N) $\subset$ O. Hence by thm[2.14], f is α *continuous.

Theorem 3.34: If f is α *continuous and if Y is locally indiscrete, then f is contra α *continuous.

Proof: Let O be an open set of Y. Since Y is locally discrete, O is closed. Since, f is α *continuous, $f^{-1}(O)$ is α * closed in X. Therefore, f is contra α *continuous.

Theorem 3.35: If a function f : (X, τ) ⟶(Y, σ) is continuous and X is a locally indiscrete space, then f is contra α *continuous.

Proof: Let O be any open set in (Y, σ). Since f is continuous f^{-1} (O) is open in X. and since X is locally discrete, f^{-1} (O) is closed in X. Every closed set is α * closed. f^{-1} (O) is α * closed in X. Therefore, f is contra α *continuous

Theorem 3.36: Let f: (X, τ) ⟶ (Y, σ) be a function and g: X ⟶ X × Y the graph function, given by g(x) = (x,f(x)) for every $x \in X$. Then f is contra α * continuous if g is contra α * continuous.

Proof: Let F be a closed subset of Y. Then X × F is a closed subset of X × Y. Since g is a contra α * continuous, then $g^{-1}(X \times F)$ is a α *open subset of X. Also, $g^{-1}(X \times F) = f^{-1}(F)$. Hence, f is contra α * continuous.

Theorem 3.37: Let $\{X_i / i \in I\}$be any family of topological spaces. If $f : X \rightarrow \Pi X_i$ is a contra α * continuous function. then $\pi_i \circ f : X \rightarrow X_i$ is contra α * continuous for each i ∈ I, where π_i is the projection of ΠX_i onto X_i .

Proof: Suppose U_i is an arbitrary open sets in X_i for i ∈ I. Then $\pi_i^{-1}(U_i)$is open in ΠX_i. Since f is contra α * continuous, $f^{-1}(\pi_i^{-1}(U_i)) = (\pi_i \circ f)^{-1}(U_i)$ is α * closed in X. Therefore, $\pi_i \circ f$ is contra α * continuous.

For a map f: (X, τ) ⟶ (Y, σ), the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by G(f).

IV. Contra α * closed graph

Definition 4.1: The graph G(f) of a function f: (X, τ) ⟶ (Y, σ) is said to be ***Contra α * closed graph*** in X × Y if for each (x,y) ∈ (X × Y) - G(f) there exists U ∈ α *O (X, τ) and V∈ C (Y, y) such that (U × V) ∩ G(f) = ϕ.

Lemma 4.2: The graph G(f) of a function f: (X, τ) ⟶ (Y, σ) is α * closed in (X × Y) if and only if for each (x,y) ∈ (X × Y) - G(f) , there exists an U∈ α *O (X, x) and an open set V in Y containing y such that f(U) ∩ V = ϕ.

Proof: We shall prove that f(U) ∩ V = ϕ ⟺ (U × V) ∩ G(f) = ϕ. Let (U × V) ∩ G(f) ≠ ϕ. Then there exists (x,y) ∈ (X × Y) and (x,y) ∈ G(f). This implies that x ∈ U, y ∈ V and y = f(x) ∈ V. Therefore, f(U) ∩ V ≠ ϕ. Hence the result follows.

Theorem 4.3: If f: (X, τ) ⟶ (Y, σ) is contra α * continuous and Y is Urysohn, G(f) is a contra α * closed graph in X × Y.

Proof: Let (x,y) ∈ (X × Y) - G(f), then y ≠ f(x) and there exist open sets A and B such that f(x) ∈ A , y∈ B and cl(A) ∩ cl(B) = ϕ. Since f is contra α * continuous, there exist O∈ α *O (X, x) such that f(O) ⊂ cl(A). Therefore, we obtain f(O) ∩ cl(B) = ϕ. Hence by lemma [4.2], G(f) is contra α * closed graph in X × Y.

Theorem 4.4: If f: (X, τ) ⟶ (Y, σ) is α * continuous and Y is T_1, then G(f) is Contra α * closed graph in X × Y.

Proof: Let (x,y) ∈ (X × Y) - G(f), then y ≠ f(x) and since Y is T_1 there exists open set V of Y, such that f(x) ∈ V, y∉ V. Since f is α * continuous, there exist α * open set U of X containing x such that f(U) ⊂ V. Therefore, f(U) ∩ (Y – V) = ϕ and Y –V is a closed set in Y containing y. Hence by lemma [4.2], G(f) is Contra α * closed graph in X × Y.

Definition 4.5 A function f: (X, τ) ⟶ (Y, σ) is called ***almost contra α * continuous*** if $f^{-1}(O)$ is α *closed set in X for every regular open set O in Y.

Theorem 4.6 Every contra α *continuous function is almost contra α *continuous.

Proof: Let O be a regular open set in Y. Since, every regular open set is open which implies O is open in Y. Since f : (X, τ) ⟶ (Y, σ) is contra α *continuous then f^{-1} (O) is α * closed in X. Therefore, f is almost contra α *continuous.

Remark 4.7: The converse of the above theorem need not be true.

Example 4.8: Let X = Y = {a, b, c}, σ = {ϕ, {a},{b},{ab},Y}.RO(Y, σ) = { ϕ, {a},{b},Y}, α *C(X, τ) = {ϕ , {a},{b},{c} , {ab} ,{bc}, {ac} , X }. Let f : (X, τ) ⟶(Y, σ) be defined by f(a) = a , f(b) = b = f(c). clearly, f is contra α * continuous , but f^{-1} ({b}) = {bc} is not regular open in X. Therefore, f is not almost contra α * continuous.

Theorem 4.9: The following are equivalent for a function f: (X, τ) ⟶ (Y, σ)

1. f is almost contra α *continuous.
2. $f^{-1}(F)$ is α *open in X for every regular closed set F in Y.

3. for each x ∈ X and each regular open set F of Y containing f(x), there exists U ∈ α *O(X,x) such that f(U) ⊂ F
4. for each x ∈ X and each regular open set V of Y non – containing f(x), there exists an α * closed set K of X non – containing x such that $f^{-1}(V) \subset K$

Proof:

(1)⟺(2) Let F be any regular closed set of Y. Then (Y – F) is regular open and therefore $f^{-1}(Y – F) = X - f^{-1}(F) \in \alpha$ *C (X). Hence, $f^{-1}(F)$ is α * open in X. The converse part is obvious.

(2) ⟹(3) Let F be any regular closed set of Y containing f(x). Then $f^{-1}(F)$ is α * open in X and $x \in f^{-1}(F)$. Taking $U = f^{-1}(F)$ we get f(U) ⊂ F .

(3) ⟹(2) Let F be any regular closed set of Y and $x \in f^{-1}(F)$. Then there exists $U_x \in \alpha$ *O (X,x) such that $f(U_x) \subset F$ and so $U_x \subset f^{-1}(F)$ .Also, we have $f^{-1}(F) \subset \cup_{x \in f^{-1}(F)} U_x$. Hence, $f^{-1}(F)$ is α * open in X.

(3)⟺(4) Let V be any regular open set of Y non containing f(x). Then (Y – V) is regular closed set in Y containing f(x). Hence by (c) , there exists U∈ α *O (X,x) such that f(U) ⊂(Y – V). Hence, $U \subset f^{-1}(Y – V) \subset X - f^{-1}(V)$ and so $f^{-1}(V) \subset (X – U)$. Now, since U∈ α *O (X), (X-U) is α * closed set of X not containing x. The converse part is obvious.

Definition 4.7: A space X is said to be ***locally α * indiscrete*** if every α *open set of X is closed in X.

Theorem 4.8: A contra α *continuous function f : (X, τ) ⟶(Y, σ) is continuous when X is locally α * indiscrete.

Proof: Let O be an open set in Y. Since, f is contra α *continuous then $f^{-1}(O)$ is α * closed in X. Since, X is locally α * indiscrete which implies $f^{-1}(O)$ is open in X. Therefore, f is continuous.

Theorem 4.9: Let f : (X, τ) ⟶(Y, σ) is α *irresolute map with Y as locally α * indiscrete space and g: (Y, σ) ⟶ (Z, η) is contra α *continuous, then g ∘ f is α *continuous.

Proof: Let B be any closed set in Z. Since g is contra α *continuous, $g^{-1}(B)$ is α * open in Y. But Y is locally α * indiscrete, $g^{-1}(B)$ is closed in Y. Hence, $g^{-1}(B)$ is α *closed in Y. Since, f is α *irresolute, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is α *closed in X. Therefore, g ∘ f is α *continuous.

Definition 4.10: A map f : (X, τ) ⟶(Y, σ) is said to be ***pre α *open*** if the image of every α *open set of X is α *open in Y.

Theorem 4.11: Let f : (X, τ) ⟶(Y, σ) be surjective α * irresolute pre α *open and g: (Y, σ) ⟶ (Z, η) be any map. Then g ∘ f: (X, τ) ⟶ (Z, η) is contra α *continuous if and only if g is contra α *continuous.

Proof: The if part is easy to prove. To prove the " only if " part , let g ∘ f: (X, τ) ⟶ (Z, η) be contra α *continuous and let B be a closed subset of Z. Then $(g \circ f)^{-1}(B)$ is α *open in X which implies $f^{-1}(g^{-1}(B))$ is α *open in X. Since, f is pre α *open, $f(f^{-1}(g^{-1}(B)))$ is α *open of X. So, $g^{-1}(B)$ α *open in Y. Therefore, g is contra α *continuous.

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