

Dynamic Structural Optimization with Frequency Constraints

Qasim H. Bader

(Department Mechanical Engineering, College of Engineering, University of Babylon, Iraq)

Abstract: The present work introduces a new method of structural optimization based on frequency constraints by using FEM in conjunction with Evolution optimization. First able as in any structure using this type of optimization the structure is meshed by very small element and the Eigen value (natural frequencies) of the whole structure is find. Then for each element removal the same eigenvalue analysis is done and a new natural frequency is calculated and assigned to an element removed. The sensitivity of structure to this removal of material for each element will be calculated. The question now what will do to control the natural frequency of the structure based on this removal of each element alone. The answer for this question and based upon the sensitivity value, all natural frequencies can be shift towards the required value, this include increasing or decreasing the frequency value and also increasing or decreasing the gap between any pair of natural frequencies.

Keywords: Structural Optimization, Plate, Frequency Constraints, FEM

I. INTRODUCTION

Design optimization has long been sought by engineer. The established classic method of optimization demanded large scale computing facilities met only by the main frame computers till 1980. Two important development of the last three decades hsr changed the course of events. First the large capacity computers and rapid-processing computing capability. Second, the method of optimization has been joined by new approaches based on concepts of evolutionary or genetic progression.

The optimum design of any structure including frequency constraints is very importance, especially in mechanical engineering like aeronautical industries. Also the structure response depends strongly upon the first six natural frequencies of structure. In practice we will be care to avoid a resonance phenomenon or excessive vibration when the excitation forces are near to one of the natural frequencies of the structure.

In the last two decades, there are many researches dealing with the structural optimization based on frequency constraints. Suzuki and Kikuchi[1], One of the most challenging tasks in the field of structural optimization, where the topology of a structure is not constant and internal holes can be created during the optimization process. An important recent development in layout and shape optimization is the homogenization method in which a material with voids is introduced and the optimization is seeking the optimal porosity of such media. Pan & Wang[2], in this paper adaptive genetic algorithm (AGA) is applied to topology optimization of structure with frequency constraints.

Qasim H. Bader[3], introduce an Evolutionary structural optimization for static analysis for both isotropic and orthotropic shell structure. X.Y. Yong, presents an efficient harmony search (HS) based algorithm for solving the shape optimization subjected to multiple natural frequencies constraints[4].

Reference [5] introduced the same object to achieving the optimum design of beam based on frequency constraints. In this works the optimum design for structure is found and the approximation is done to calculate the value of sensitivity number. A new technique for multiple constraints environments by using of evolutionary structural optimization [6]. Grandhi, presents the same topic for frequency constraints. In this works it is based on changing the thickness of the parts only (i.e. the size of the structure)[7].

Tenk and Hagiwara investigated a vibration shape and topology optimization method for dynamic problems by using homogenization method[8].

In this paper the optimization technique used can applied to a wide range of problems and easy to use in conjunction with ANSYS program as a solution program. The benefit of this method is can be overcomes all difficult may be happen in the previous techniques and also can achieve the required result easy with low cost and without approximation.

II. OPTIMIZATION CRITERIA

By using an evolution technique, the material removal form the structure is firstly done. The problem here is where we can remove the element and how. The sensitivity equation is required to decide the location of element removal and must join the element removal with the required response. Usually this equation is complicated especially for dynamic analysis.

In this paper a sensitivity equation can be calculated in simple way which indicate the change happen in the value of frequency due to effect of remove material on the frequencies of structure Xie[3]. After finished mesh the structure by fine element, the natural frequencies of the structure is calculated based upon the eigenvalue problem.

$$([\mathbf{K}] - \omega_i^2 [\mathbf{M}])\{\mathbf{u}_i\} = \{\mathbf{0}\} \quad (1)$$

Where $[\mathbf{K}]$ is the global stiffness matrix, $[\mathbf{M}]$ is the global mass matrix, ω_i is the natural frequency and $\{\mathbf{u}_i\}$ is the eigenvector corresponding to natural frequencies.

Form equation (1), the natural frequency equation:

$$\omega_i = k_i/m_i \quad (2)$$

where ω_i is the eigenvalue of the whole structure for mode i without removing any material. k_i is the modal stiffness and is defined as:

$$k_i = \{\mathbf{u}_i\}^T [\mathbf{K}] \{\mathbf{u}_i\} \quad (3)$$

And m_i is the modal mass and is defined as:

$$m_i = \{\mathbf{u}_i\}^T [\mathbf{M}] \{\mathbf{u}_i\} \quad (4)$$

Now assume we remove one element e from the structure whose natural frequency for mode i is ω_i , the new natural frequency of the structure due to remove element e is for mode i is ω_{i_e} .

So the change between these values is known as sensitivity parameters and is defined as:

$$\beta_i^e = \omega_i - \omega_{i_e} \quad (5)$$

Where β_i^e is the sensitivity parameters for mode i due to remove element number e .

Equation (5) is used for optimization of mode i alone and the others i^{th} modes remain unchanged or kept at the same values.

Hence, when we need to optimize the structure including the relation between i and j modes, in this case the two direction sensitivity parameters needs to find and as follow.

$$\alpha_{ij}^e = \beta_i^e - \beta_j^e \quad (6)$$

Where α_{ij}^e is the two direction sensitivity parameters between mode i and j when element number e is removed.

III. OPTIMIZATION PROCEDURE

By using evolution procedure, the optimum design of the structure start with removing some element of the structure depends on the sensitivity value and also the direction required.

3.1 Change the value of mode i

- 1- Apply the finite element model with a fine mesh.
- 2- Apply the solution to the model to get the eigenvalue.
- 3- For each element removed, find the sensitivity parameters β_{i_e} as described in eq.(5).
- 4- Remove some element whose sensitivity parameters is positive (the bigger values) when we needs to **increase** the value of natural frequency or remove some element whose sensitivity parameters is negative (the lowest values) when we needs to **decrease** the value of mode i .
- 5- Step 2 to step 4 is repeated until the optimal value reached or until the percentage of element removal is reached. Now when the designer wants to keep the natural frequency for mode i at constant value and wish to decrease the total weight of the structure. In this case remove elements whose sensitivity value near to zero (i.e. the elements have no change on mode number i).

3.2 Change the gap between two modes i and j

In this case the two direction sensitivity parameter α_{ij}^e is used as described in equation (7). So this parameters indicate the difference between natural frequency i and natural frequency j due to remove element number e , and from this definition it is clear that when the designer needs to increase the gap between mode i and mode j , we will remove the element whose absolute value of α_{ij}^e is lowest. But when the designer wants to decrease the gap between the two modes i and j , then he must remove the elements whose absolute value of α_{ij}^e is highest.

3.3 Optimization criteria with multiple frequency constraints.

The optimum design of structure for this case including multiple frequency constraints is similar to one direction sensitivity parameters β_i^e described in eq. (5). Assume the objective is to increase the value of mode one and keep the other five mode as soon as possible with same values. In this case we will remove the elements whose positive value of β_1^e among β_2^e to β_6^e near to zero value. This means that the removal of element will increase mode number one and at the same time the effect of this removal on the others five modes is approximately zero. The ANSYS program is used as analytical program to calculate the natural frequency of each mode, and also the natural frequency due to removal of each element and for all modes. The advance programming language APDL was used to write the optimization program to works together with Ansys model. By this program the one dimension sensitivity parameters is calculated for each removal of element and for each mode, this represent a two dimensions array vector of $\{i^{th} \text{ by } NE\}$, where i represent the natural frequency for mode number i and NE represent the total element. The two dimensional sensitivity parameters represents a three dimensions array vector of $\{i^{th} \times j^{th} \times NE\}$. The remove of an element is done by assign the material number to zero or by killing element. This action will reduce the total global mass matrix and stiffness matrix due to remove these elements and also the time required for solution become smaller. The number of iterations depends on reaching the required value of objective function or the iteration umber is finished or the percentage of material reduction from the original structure is reached.

IV. EXAMPLES

In this papers the two dimensional cantilever plate of 2 meters length and 1 meter width, the thickness of plate is 0.1 meter. Modulus of elasticity $E = 2.1e11 \text{ N/m}^2$ and poisons ratio $\nu = 0.3$ and density 7800 Kg/m^3 . The plate is fixed at one edge and free to move at the other three edges. The model is divided into 80×40 elements. The type of element used is quadrilateral element.

1. Increasing the value of mode number one.

The finite element model of the plate is shown in fig.1 including boundary conditions and mesh element. In this case we will try to increase the first natural frequency and keep the other two frequencies at the same value as soon as possible. Fig.2 shows the evaluation history of the first three modes of the plate. In this figure the iterations is terminated when the volume reduction was reached to 6% from the original volume of the structure. For each iteration the total element removed kept with 2 elements maximum and removes the element has the highest value of sensitivity numbers as described in equation 5.

Fig.3 shows the final design model corresponding to 4.4% element removal. At the end of this stage of optimization the first natural frequency if changed from 417 Mhz to 450 MHz with an increasing of 15%.

The result obtained here is similar to the result obtained by Xie& Steven [1].

2. Reducing the value of mode one

The same finite model is used as shown in fig.1, the main difference in this case when we required reducing the value of first natural frequency is the type of element removal depends on the sensitivity value which have. In this case the element has the minimum value of sensitivity parameters as described in equation 5 is removed.

Fig.4 shows the evolution history of optimization for the first 3-modes, the other two modes kept constant as soon as possible.

Fig.5 shows the new design of the cantilever plate corresponding to 3.5% of the total element removal. The first natural frequency changed from 417 MHz to 390 MHz with a decrease of 12%. The result obtained here is similar to the result obtained from Tnek& Hagiwara [8]by using homogenization method.

3. Changing the gap between chosen frequencies.

To increase the gap between the first natural frequency and the second natural frequency for the same model shown in fig.1, in this case the element removed which has the highest absolute value of two dimensional sensitivity parameters α_{ij}^e as described in eq.6.

Fig.6 shows the evolution history for the first two natural frequencies. For the total element removal of 6%, the final difference between first natural frequency and the second natural frequency by 24% from 220 Hz to 250 Hz.

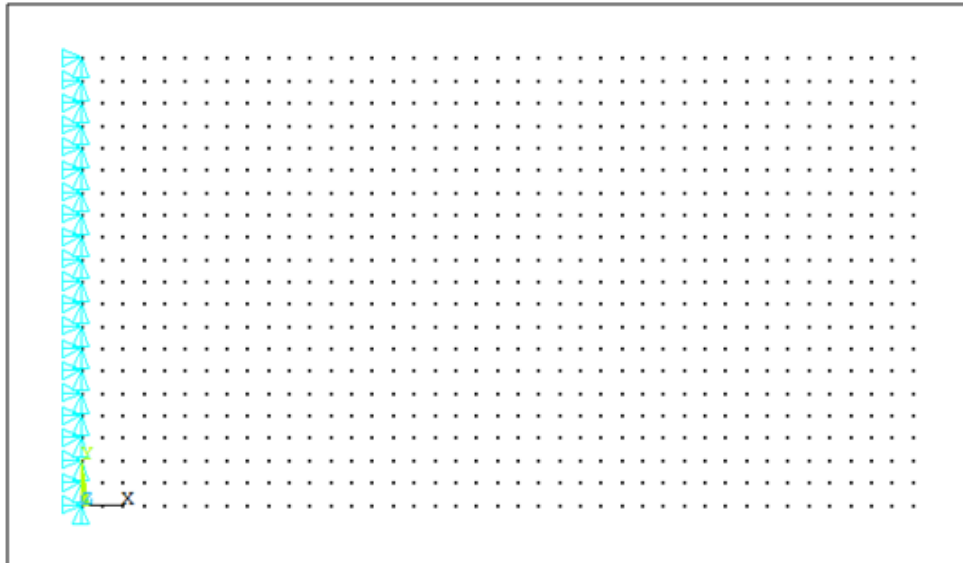


Fig.1 Finite element model mesh with boundary conditions

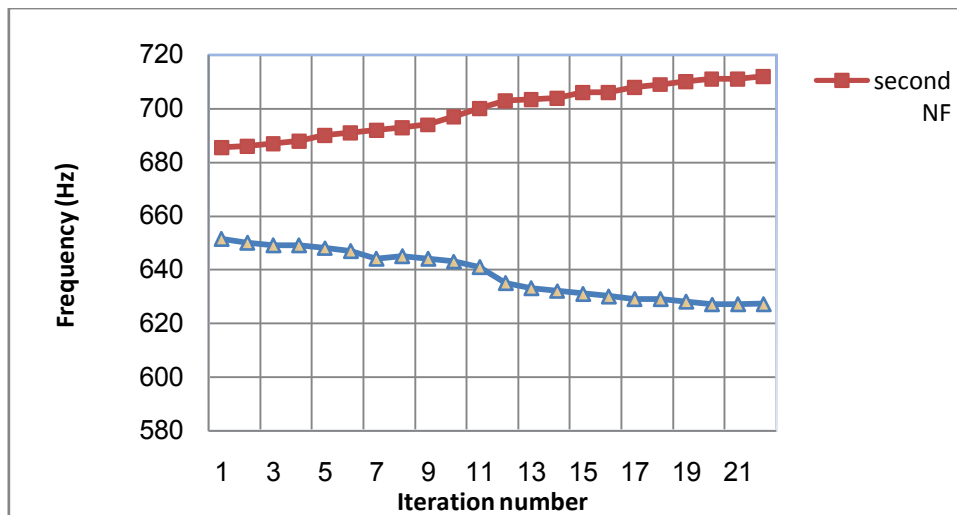


Fig.2 Natural frequencies versus iteration number

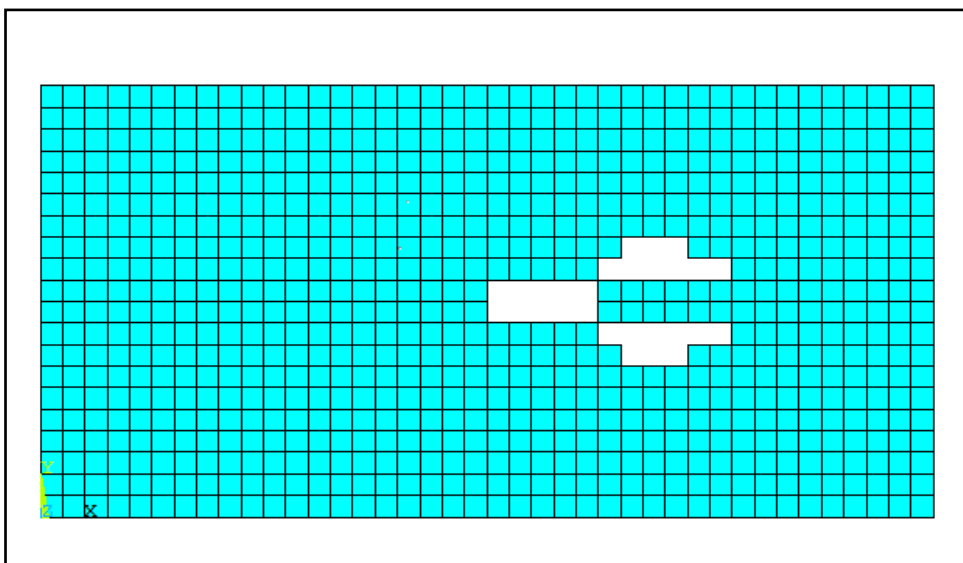


Fig.3 Final design model of 4.4% element removal

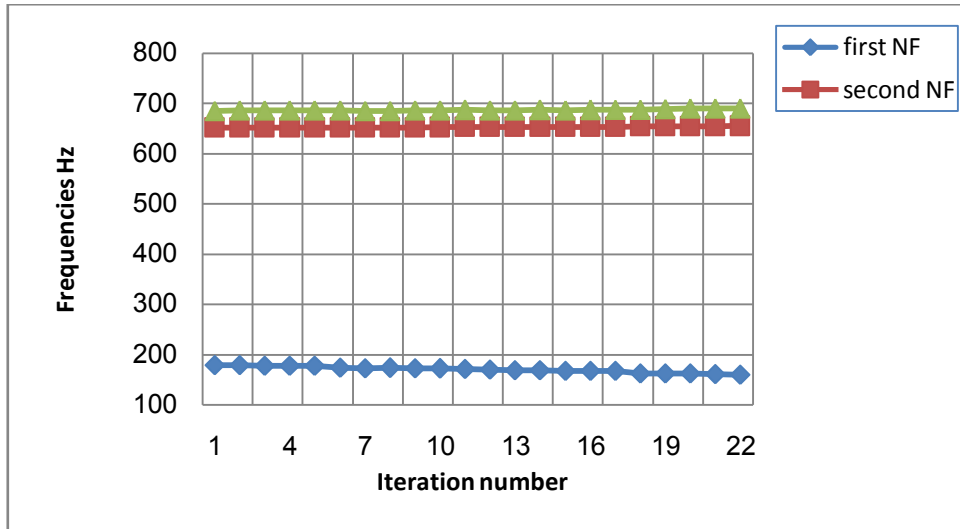


Fig.4 Evaluation history for the first three NF versus iteration number

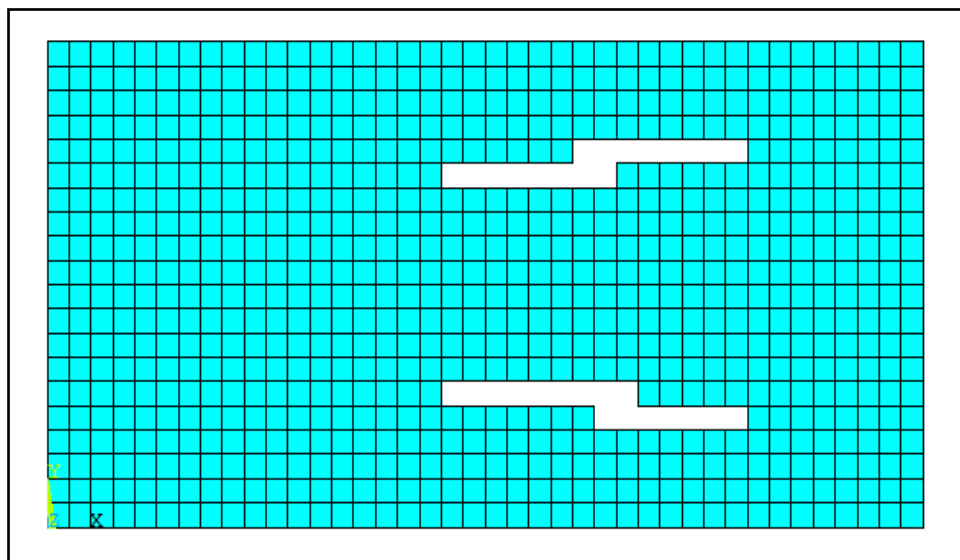


Fig.5 Final design model for 3.5% of element removal

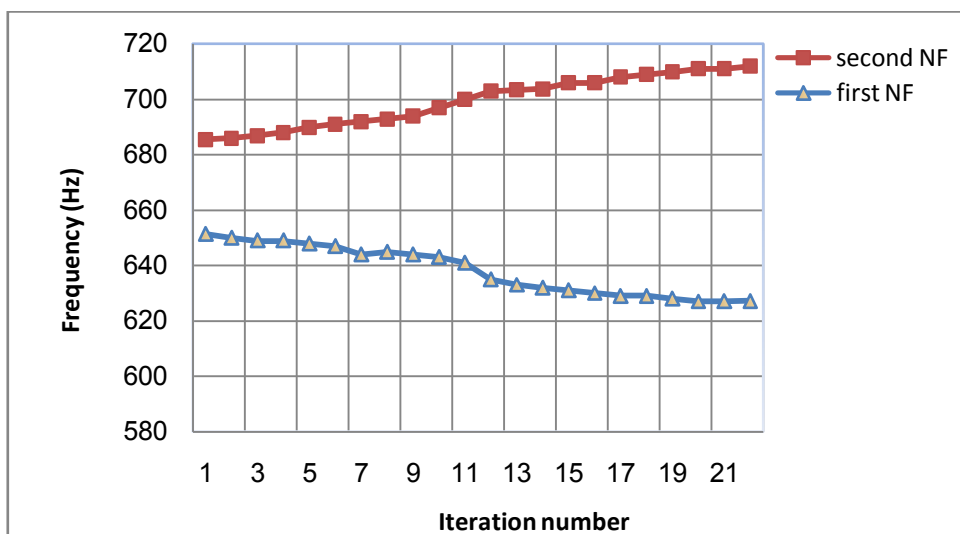


Fig.6 Evaluation history for the first two NF^s versus iteration number

V. CONCLUSION REMARKS

Due to existing the finite element solution and software and also to the rapid change in this software comparing to the optimization program and also because the complexity and difficult that producing from the traditional methods.

By this a new technique used (evolution structural optimization) and by use ANSYS Programming language (APDL) the optimization program can easily be written and works together with the ANSYS program as analytical program only.

So compare to Homogenization method by Hagiwara (8) this method has more simplicity and from using this method, the designer has the ability to control the dynamic characteristics of the structure, and comparing with new technique was introduce by Xie(3), this method is more accurate and precise due to use two dimensional sensitivity parameters when we needs to change the gap between any chose frequencies which is not used in the method of this author.

Also by this technique the new design topology of the structure can easily be calculated from the fine element removal with smooth change by the model.

Finally by using this technique in conjunction with any FEM solution no way to miss the goal required even the structure is complex or simple.

From the result mentioned to this research, for example the first natural frequency can be increased by 14% with removing of 3.5% volume, when the objective is to increase the first natural frequency and when needs to decrease the first natural frequency to 12%, the element removal of 4.4% is required.

Also when required to increase the gap between two natural frequency and kept the others near constant, the gap increase about 13% with removing of 6% from the original volume.

REFERENCES

- [1] Suzuki and Kikuchi., Homogenization method for shape and topology optimization, comp. meth. Appl. Mech. Eng. Vol. 93, pp291-318, 1991
- [2] Pan jin and Wang De-Yu, Topology optimization of truss structure with fundamental frequency and frequency domain response constraints,ActaMechanicasolidasinica ,Vol. 19, Sep. 2006, PP.231-240
- [3] Qasim H. Bader, Evaluation of evolutionary optimization technique of multi layered shell structure, PHD thesis, 2003, UOB
- [4] X.Y. Young, Topology optimization for structure with frequency constraints using evolutionary method. Journal of structural engineering, Vol.25, No.12,1999
- [5] Xie and Steven , " evolutionary structural optimization for dynamic problems, computers and structures , Vol. 58, no.6,pp 1067-1073, 1996
- [6] Steven &Querin , optimal design of multiple load case structures using an evolutionary procedure, Engineering computations , Vol. 11, 295-302, 1994
- [7] Grandihi, structural optimization with frequency constraints, AIAA, J.31, 2296-2303. 1993.BikashJoadder, Jagabandhu Shit, SanjibAcharyya, Fatigue Failure of Notched Specimen-A Strain-Life Approach, Materials Sciences and Applications, 2011.
- [8] Tenk and Hajiwara, Static and vibration shape and topology optimization using homogenization and mathematical programming, computers and meth. Applied mechanics engineering, 109, 143-154,1993.