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Results from set-operations on Fuzzy soft sets

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Abstract: *In this paper considering a class of Fuzzy-Soft Sets, seven set operations are defined and several relations arising from these set-operations are established using matrix representation of fuzzy soft sets.*

Key words: *Fuzzy Soft Sets, equality, operations, Fuzzy Soft matrix.*

I. Introduction

Recent advances present phenomena, in many areas including engineering, social and medical sciences that are neither deterministic nor stochastic in nature. These cannot be characterized in terms of classical set theory. As such fundamental extensions and generalizations of sets in mathematics have been proposed.

Zadeh [1], in 1965, introduced the theory of fuzzy sets for dealing with imprecise phenomena. These were further generalized by Atanassov [2,3], to what has come to be known as 'Intuitionistic fuzzy sets', to characterize a broader class of vague phenomena. Molodstov [4], in 1999, on the other hand, introduced the concept of 'Soft set' associating characteristics or parameters in considering subsets of a set.

Maji, et. al [5], inducing the concept of fuzzyness on soft-sets, introduced the concept of Fuzzy Soft Sets. The hybrid 'Fuzzy Soft Set theory' has attracted the attention of researchers for its further study and applications. Yong Yang and Chenli Ji [6], using matrix representation of Fuzzy Soft Sets considered applications. The notion of Fuzzy Soft matrices has been further extended in [7] and applied in certain decision making problems.

While set-operations, refer Verma & Sharma [8], on intitutionistic fuzzy sets have been studied, for mathematical viability and usefulness, there is a need to examine and to study these over fuzzy-soft-sets. In this paper we define seven operations analogous to [8] on fuzzy soft sets in terms of their matrices and prove various different relations amongst these operations.

II. Preliminaries

In this section we give definitions and notions, refer [7], used in following work.

Definition 1: Fuzzy Soft Set - Let X be an initial universal set and E be a set of parameters. Let $\tilde{P}$ (x) denotes the power set of all Fuzzy Subsets Sets of X. Let A $\subseteq$E. A pair (F, A) is called Fuzzy Soft Set over X. where F is a mapping given by F : A $\rightarrow \tilde{P}(X)$.

Definition 2: - Fuzzy Soft Class - The pair (X, E) denotes the collection of all Fuzzy Soft Sets on X with attributes from E and is called Fuzzy Soft Class.

Definition 3: Fuzzy Soft Matrices

Let X = {x_1, x_2, ... x_m} be the universal set and E = {e_1, e_2, ...e_n} be the set of parameters. Let A $\subseteq$E and (F, A) be a Fuzzy Soft Set in the Fuzzy Soft Class (X, E). Then we represent the Fuzzy Soft Set (F, A) in the matrix form as:

$$A_{mn} = \lfloor a_{ij} \rfloor_{mxn} \text{ or simply by } A = \lfloor a_{ij} \rfloor$$

where

$$a_{ij} = \begin{cases} \mu_j(x_i) & if \quad e_j \in A \\ 0 & if \quad e_j \notin A \end{cases}.$$

Here $\mu_j(x_i)$ represent the membership of x_i in the Fuzzy Set F(e_j). We would identify a Fuzzy Soft Set with its Fuzzy Soft matrix and vice versa. The set of all mxn Fuzzy Soft Matrices will be denoted by FSM_{mxn} over X.

Definition 4: Set of Operations on FSM_{mxn}

Let A = $\lfloor a_{ij} \rfloor_{mxn}$ and B = $\lfloor b_{ij} \rfloor_{mxn}$ be two Fuzzy Soft matrices over the universal set X.

Some operations on FSM_{mxn} are defined as follows:

(1) $A \cup B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = \max\left(a_{ij}, b_{ij}\right)$, for all i and j

(2) $A \cap B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = \min\left(a_{ij}, b_{ij}\right)$, for all i and j

(3) $A \lozenge B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = a_{ij} + b_{ij} - a_{ij}\, b_{ij}$, for all i and j

(4) $A \,.\, B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = a_{ij}\ b_{ij}$, for all i and j

(5) $A \,@\, B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = \frac{1}{2}\left(a_{ij} + b_{ij}\right)$, for all i and j

(6) $A \,\$\, B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = \sqrt{a_{ij}\ b_{ij}}$, for all i and j

(7) $A \,\#\, B = C = \lfloor c_{ij} \rfloor_{mxn}$ where $c_{ij} = \dfrac{2a_{ij}\ b_{ij}}{a_{ij} + b_{ij}}$, for all i and j,

for which we will accept that if $a_{ij} = b_{ij} = 0$ then $\dfrac{a_{ij}\ b_{ij}}{a_{ij} + b_{ij}} = 0$.

III. Main Results

Before starting discussion of the main results we prove some rather simple inequalities to be used in the subsequent work.

$$a_{ij} + b_{ij} \geq 2\sqrt{a_{ij}\ b_{ij}} \geq 2a_{ij}\ b_{ij} \tag{2.1}$$

$$\Rightarrow\quad a_{ij} + b_{ij} - a_{ij}\, b_{ij} \geq a_{ij}\ b_{ij} \tag{2.2}$$

$$\Rightarrow\quad a_{ij} + b_{ij} - 2a_{ij}\, b_{ij} \geq 0$$

$$\Rightarrow\quad 2\left(a_{ij} + b_{ij} - a_{ij}\, b_{ij}\right) \geq a_{ij} + b_{ij}$$

$$\Rightarrow\quad \left(a_{ij} + b_{ij} - a_{ij}\, b_{ij}\right) \geq \frac{1}{2}(a_{ij} + b_{ij}) \tag{2.3}$$

Next

$$a_{ij} + b_{ij} - a_{ij}\, b_{ij} - \sqrt{a_{ij}\ b_{ij}}$$

$$\geq 2\sqrt{a_{ij}\ b_{ij}} - a_{ij}\, b_{ij} - \sqrt{a_{ij}\ b_{ij}} \qquad \text{(on using 2.1)}$$

$$= \sqrt{a_{ij}\ b_{ij}} - a_{ij}\, b_{ij} \geq 0$$

$$\Rightarrow\quad a_{ij} + b_{ij} - a_{ij}\, b_{ij} \geq \sqrt{a_{ij}\ b_{ij}}) \tag{2.4}$$

Also

$$a_{ij} + b_{ij} - a_{ij}\, b_{ij} - \frac{2a_{ij}\ b_{ij}}{a_{ij} + b_{ij}}$$

$$= \frac{(a_{ij})^2(1 - b_{ij}) + (b_{ij})^2(1 - a_{ij})}{a_{ij} + b_{ij}} \geq 0$$

$$\Rightarrow\quad a_{ij} + b_{ij} - a_{ij}\, b_{ij} \geq \frac{2a_{ij}\ b_{ij}}{a_{ij} + b_{ij}}. \tag{2.5}$$

Further

$$\frac{2a_{ij}\ b_{ij}}{a_{ij} + b_{ij}} - a_{ij}\, b_{ij} = \frac{a_{ij}\ b_{ij}(2 - a_{ij} - b_{ij})}{a_{ij} + b_{ij}} \geq 0.$$

Thus

$$\frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}} \geq a_{ij}\ b_{ij}\,. \qquad (2.6)$$

Theorem If A = $\lfloor a_{ij} \rfloor$ and B = $\lfloor b_{ij} \rfloor$ are any two FSM_{mxn}, then

(1) (A @ B) $ (A # B) = A $ B
(2) (A ◊ B) ∩ (A . B) = A . B , (A ◊ B) ∪ (A . B) = A ◊ B
(3) (A ◊ B) ∩ (A @ B) = A @ B , (A ◊ B) ∪ (A @ B) = A ◊ B
(4) (A . B) ∩ (A @ B) = A . B , (A . B) ∪ (A @ B) = A @ B
(5) (A ◊ B) ∩ (A $ B) = A $ B, (A ◊ B) ∪ (A $ B) = (A ◊ B)
(6) (A . B) ∩ (A $ B) = A . B , (A . B) ∪ (A $ B) = A $ B
(7) (A ◊ B) ∩ (A # B) = A # B , (A ◊ B) ∪ (A # B) = (A ◊ B)
(8) (A . B) ∩ (A # B) = A . B , (A . B) ∪ (A # B) = (A # B)

Proof of the Theorem: Using definitions, we have:

(1) (A @ B) $ (A # B)

$$= \left[\frac{a_{ij}+b_{ij}}{2}\right] \$ \left[\frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right]$$

$$= \left[\sqrt{\frac{a_{ij}+b_{ij}}{2}\cdot\frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}}\right]$$

$$= \left\lfloor\sqrt{a_{ij}\ b_{ij}}\right\rfloor = \text{A \$ B}$$

(2) (A ◊ B) ∩ (A . B)

$$= \lfloor a_{ij}+b_{ij}-a_{ij}\,b_{ij}\rfloor \cap \lfloor a_{ij}\ b_{ij}\rfloor$$

$$= \lfloor \min\,(a_{ij}+b_{ij}-a_{ij}\,b_{ij},\,a_{ij}\,b_{ij})\rfloor$$

$$= \lfloor a_{ij}\,b_{ij}\rfloor \qquad \text{(on using 2.2)}$$

= A . B

and

(A ◊ B) ∪ (A . B)

$$= \lfloor \max\,(a_{ij}+b_{ij}-a_{ij}\,b_{ij},\,a_{ij}\,b_{ij})\rfloor$$

$$= \lfloor a_{ij}+b_{ij}-a_{ij}\,b_{ij}\rfloor$$

= A ◊ B

(3) (A ◊ B) ∩ (A @ B)

$$= \lfloor a_{ij}+b_{ij}-a_{ij}\,b_{ij}\rfloor \cap \left[\frac{a_{ij}+b_{ij}}{2}\right]$$

$$= \left[\min\left(a_{ij}+b_{ij}\ -\ a_{ij}\,b_{ij},\ \frac{a_{ij}+b_{ij}}{2}\right)\right]$$

$$= \left[\frac{a_{ij}+b_{ij}}{2}\right] \qquad \text{on using (2.3)}$$

= A @ B

and

(A ◊ B) ∪ (A @ B)

$$= \left[\max\left(a_{ij}+b_{ij}\ -\ a_{ij}\,b_{ij},\ \frac{a_{ij}+b_{ij}}{2}\right)\right]$$

$$= \lfloor a_{ij}+b_{ij}\ -\ a_{ij}\,b_{ij}\rfloor \qquad \text{on using (2.3)}$$

= (A ◊ B)

(4) (A . B) ∩ (A @ B)

$$= \left[a_{ij}\ b_{ij}\right] \cap \left[\frac{a_{ij}+b_{ij}}{2}\right]$$

$$= \left[\min\left(a_{ij}\, b_{ij}, \frac{a_{ij}+b_{ij}}{2}\right)\right]$$

$$= \left[a_{ij}\ b_{ij}\right] \qquad \text{on using (2.1)}$$

$$= \text{A . B}$$

and

$$(\text{A . B}) \cup (\text{A @ B}) = \left[\max\left(a_{ij}\, b_{ij}, \frac{a_{ij}+b_{ij}}{2}\right)\right]$$

$$= \left[\frac{a_{ij}+b_{ij}}{2}\right] \qquad \text{on using (2.1)}$$

$$= \text{A @ B}$$

(5) $$(\text{A} \lozenge \text{B}) \cap (\text{A \$ B}) = \left[a_{ij}+b_{ij} - a_{ij}\, b_{ij}\right] \cap \left[\sqrt{a_{ij}\, b_{ij}}\right]$$

$$= \left[\min\left(a_{ij}+b_{ij} - a_{ij}\ b_{ij}, \sqrt{a_{ij}\, b_{ij}}\right)\right]$$

$$= \left[\sqrt{a_{ij}\ b_{ij}}\right] \qquad \text{on using (2.4)}$$

$$= \text{A \$ B}$$

and

$$(\text{A} \lozenge \text{B}) \cup (\text{A \$ B}) = \left[\max\left(a_{ij}+b_{ij} - a_{ij}\ b_{ij}, \sqrt{a_{ij}\, b_{ij}}\right)\right]$$

$$= \left[a_{ij}+b_{ij} - a_{ij}\, b_{ij}\right] \qquad \text{on using (2.4)}$$

$$= \text{A} \lozenge \text{B}$$

(6) $$(\text{A . B}) \cap (\text{A \$ B}) = \left[a_{ij}\ \ b_{ij}\right] \cap \left[\sqrt{a_{ij}\ b_{ij}}\right]$$

$$= \left[\min\left(a_{ij}\ b_{ij}, \sqrt{a_{ij}\, b_{ij}}\right)\right]$$

$$= \left[a_{ij}\ b_{ij}\right] = \text{A . B}$$

and

$$(\text{A . B}) \cup (\text{A \$ B}) = \left[\max\left(a_{ij}\ b_{ij}, \sqrt{a_{ij}\, b_{ij}}\right)\right]$$

$$= \left[\sqrt{a_{ij}\, b_{ij}}\right] = A\,\$\,B$$

(7) $$(\text{A} \lozenge \text{B}) \cap (\text{A \# B}) = \left[a_{ij} + b_{ij} - a_{ij}\ b_{ij}\right] \cap \left[\frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right]$$

$$= \left[\min\left(a_{ij} + b_{ij} - a_{ij}\ b_{ij}, \frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right)\right]$$

$$= \left[\frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right] \qquad \text{on using (2.5)}$$

$$= \text{A \# B}$$

and

$$(\text{A} \lozenge \text{B}) \cup (\text{A \# B}) = \left[\max\left(a_{ij} + b_{ij} - a_{ij}\ b_{ij}, \frac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right)\right]$$

$$= \left[a_{ij} + b_{ij} - a_{ij}\ b_{ij}\right] \qquad \text{on using (2.5)}$$

$$= \text{A} \lozenge \text{B}$$

(8) (A . B) $\cap$ (A # B) = $\left[a_{ij}\ b_{ij}\right] \cap \left[\dfrac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right]$

= $\left[\min\left(a_{ij}\ b_{ij}\ ,\ \dfrac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right)\right]$

= $\left[a_{ij}\ b_{ij}\right]$ on using (2.6)

= A . B

and

(A . B) $\cup$ (A # B) = $\left[\max\left(a_{ij}\ b_{ij}\ ,\ \dfrac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right)\right]$

= $\left[\dfrac{2a_{ij}\ b_{ij}}{a_{ij}+b_{ij}}\right]$ on using (2.6)

= A # B

IV. Concluding Remarks

The results obtained in terms of various operations must go a long way in applications of Fuzzy Soft Sets.

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