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On Characterizations of NANO RGB-Closed Sets in NANO Topological Spaces

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ABSTRACT: *The purpose of this paper is to establish and derive the theorems which exhibit the characterization of nano rgb-closed sets in nano topological space and obtain some of their interesting properties. We also use this notion to consider new weak form of continuities with these sets.*
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I. INTRODUCTION

Levine [9] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. This concept was found to be useful to develope many results in general topology. The notion of nano topology was introduced by Lellis Thivagar [11] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of nano open sets namely nano α -open sets, nano semi open sets and nano pre open sets [11]. Extensive research on generalizing closedness in nano topological spaces was done in recent years by many mathematicians [5, 6, 16].

The aim of this paper is to continue the study of nano generalized closed sets in nano topological spaces. In particular, we present the notion of nano regular generalized b-closed sets (briefly, nano rgb-closed sets) and obtain their characterizations with counter examples. Also we establish various forms of continuities associated to nano regular generalized closed sets.

II. PRELIMINARIES

Definition 2.1[17]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X⊆ U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x∈U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2[11]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X⊆U. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 2.3[12]: If $\tau_R(X)$ is the nano topology on U with respect to X, then the set

B = {U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition 2.4[12]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where X$\subseteq$U and if A$\subseteq$U, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A).

That is, Nint(A) is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.5[11]: Let $(U, \tau_R(X))$ be a nano topological space and A⊆U. Then A is said to be

(i) nano semi-open if A $\subseteq$ Ncl(Nint(A))

(ii) nano pre-open if A $\subseteq$ Nint(Ncl(A))

(iii) nano α -open if A $\subseteq$ Nint(Ncl(Nint(A)))

(iv) nano semi pre-open if A $\subseteq$ Ncl(Nint(Ncl(A)))

(v) nano b-open if A ⊂Ncl(NintA) $\cup$ Nint(NclA).

NSO(U, X), NPO(U, X), NαO(U, X), NSPO(U, X) and NBO(U, X) respectively denote the families of all nano semi-open, nano pre-open, nano α-open, nano semi pre-open and nano b- open subsets of U.

Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U. A is said to be nano semi closed, nano pre-closed, nano α-closed, nano semi pre closed and nano b-closed if its complement is respectively nano semi-open, nano pre-open, nano α –open, nano semi pre open and nano regular open.

Definition 2.6: A subset A of a nano topological space $(U, \tau_R(X))$is called

(1) nano generalized closed (briefly, nano g-closed)[5], if Ncl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.

(2) nano semi-generalized closed (briefly, nano sg-closed)[6], if Nscl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano semi-open in U.

(3) nano α -generalized closed (briefly, nano α g-closed)[16], if N α cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.

(4) nano generalized α -closed (briefly, nano g α -closed)[16], if N α cl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano α -open in U.

Definition 2.7[12]: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be

(i) nano continuous if f^{-1}(B) is nano open in U for every nano-open set B in V.

(ii) nano α- continuous if f^{-1}(B) is nano α-open in U for every nano-open set B in V.

(iii) nano semi-continuous if f^{-1}(B) is nano semi-open in U for every nano-open set B in V.

(iv) nano pre-continuous if f^{-1}(B) is nano pre-open in U for every nano-open set B in V.

III. Nano RGB-closed sets

In this section, we investigate the class of nano regular generalized b-closed sets and study some of their characterizations.

Definition 3.1: A subset A of a nano topological space $(U, \tau_R(X))$ is said to be nano regular generalized closed sets (briefly Nrgb-closed) if Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is regular open in U.

The collection of all nano rgb-closed subsets of U is denoted by NrgbC(U, X).

Theorem 3.1: Every nano b-closed set is nano rgb-closed.

Proof: If A is nano b-closed in U and G is nano regular open in U such that A $\subseteq$ G, Nbcl(A) = A $\subseteq$ G. Therefore A is nano rgb-closed.

Remark 3.1: Converse of the above theorem need not be true which can be seen from the following example.

Example 3.1: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ , {a}, {b, d}, {a, b, d}}. Here the sets {a, b}, {a, d} are nano rgb-closed but not nano b-closed.

The following theorem can also be proved in a similar way.

Theorem 3.2: Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U, then every nano closed, nano regular closed, nano α -closed , nano semi-closed, nano pre closed, nano g-closed, nano gb-closed, nano rg-closed, nano sg-closed, nano gs-closed, nano α g-closed, nano g α -closed sets are nano rgb-closed.

Remark 3.2: However the converse of the above theorem need not be true can be seen from the following example.

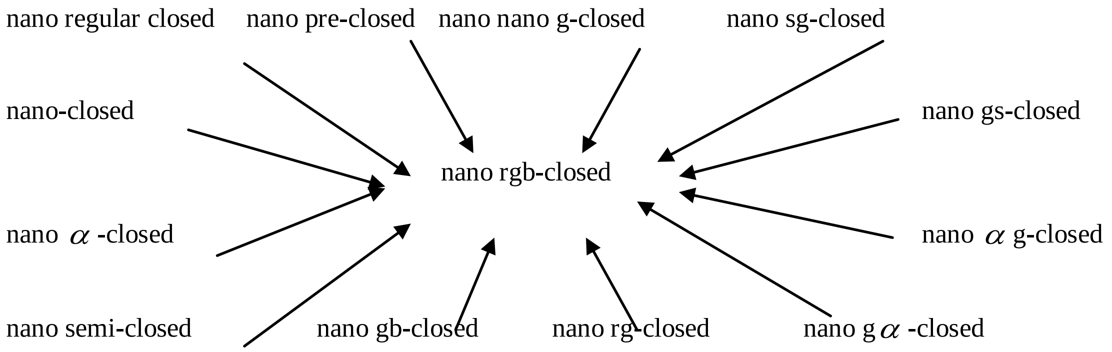
Example 3.1 shows that the sets {b}, {d} are nano rgb-closed but not nano closed and nano rg-closed, the sets {a}, {b} are nano rgb-closed but not nano regular closed, the sets {a}, {b, d} are nano rgb-closed but not nano pre-closed, the sets {a, c}, {a, c, d} are nano rgb-closed but not nano gb-closed, the sets {a, c}, {a, d} are nano rgb-closed but not nano α g-closed and {a, d}, {a, b, d} are nano rgb-closed but not nano g α -closed.

Example 3.2: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {b, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}}. Here the sets {b}, {d} are nano rgb-closed but not nano gs-closed and the sets {a, b}, {a, d} are nano rgb-closed but not nano sg-closed.

Example 3.3: Let U = {x, y, z} with U/R = {{x}, {y, z}} and X = {x, z}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {x}, {y, z}}. Here the sets {y}, {z} are nano rgb-closed but not nano α -closed and the sets {x, y}, {x, z} are nano rgb-closed but not nano semi closed.

Example 3.4: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Here the sets {c}, {d} are nano rgb-closed but not nano g-closed.

We have the following implications for properties of subsets but none of its reverse implication is true.



Theorem 3.3: A set A is nano rgb-closed in (U, X). Then Nbcl(A)-A has no non-empty nano regular-closed set.

Proof: Let A be nano rgb-closed in (U, X), and F be nano regular-closed subset of Nbcl(A)-A. That is, F $\subseteq$ Nbcl(A)-A. which implies F $\subseteq$ Nbcl(A) $\cap$ A^c. Then F $\subseteq$ Nbcl(A) and F $\subseteq A^c$. F $\subseteq A^c$ implies A $\subseteq F^c$ where F is a nano regular-open set. Since A is nano rgb-closed, Nbcl(A) $\subseteq$ F^c. That is, F $\subseteq$ $[\text{Nbcl(A)}]^c$. Thus, F $\subseteq$ Nbcl(A) $\cap$ $[\text{Nbcl(A)}]^c$ = ϕ. Therefore F = ϕ.

Theorem 3.4: Let A be nano rgb-closed set. Then A is nano b-closed if and only if Nbcl(A)-A is nano regular closed.

Proof: Let A be nano rgb-closed set. If A is nano b-closed, then we have Nbcl(A)-A = ϕ , which is a nano regular closed set. Conversely, let Nbcl(A)-A be nano regular-closed. Then by Theorem 3.3, Nbcl(A)-A does not contain any non-empty nano regular closed set. Thus, Nbcl(A)-A = ϕ . That is, Nbcl(A) = A. Therefore A is nano b-closed.

Theorem 3.5: Let A be nano regular open nano rgb-closed set. Then, A $\cap$ F is nano rgb-closed whenever F $\in$ NbC(U, X).

Proof: Let A be nano regular open and nano rgb-closed, then Nbcl(A) $\subseteq$ A, but A $\subseteq$ Nbcl(A). Thus, A is nano b-closed in U. Hence A $\cap$ F is nano b- closed in U which implies that A $\cap$ F is nano rbg-closed in U.

Theorem 3.6: Let B $\subseteq$ A $\subseteq$ U where A is a nano rgb-closed and nano regular open set. Then B is nano rgb-closed relative to A if and only if B is nano rgb-closed in U.

Proof: We first note that since B $\subseteq$ A and A is both a nano rgb-closed and nano regular open set, then Nbcl(A) $\subseteq$ A and thus Nbcl(B) $\subseteq$ Nbcl(A) $\subseteq$ A. Now from the fact that A $\cap$ Nbcl(B) = $Nbcl_A$(B), we have Nbcl(B) = $Nbcl_A$(B) $\subseteq$ A. If B is nano rgb-closed relative to A and G is nano regular open subset of U such that B $\subseteq$ G, then B = B $\cap$ A $\subseteq$ G $\cap$ A where G $\cap$ A is nano regular open in A. Hence as B is nano rgb-closed relative to A, Nbcl(B) = $Nbcl_A$(B) $\subseteq$ G $\cap$ A $\subseteq$ G. Therefore B is nano rgb-closed in U.

Conversely if B is nano rgb-closed in U and G is an nano regular open subset of A such that B $\subseteq$ G, then G = N $\cap$ A for some nano regular open subset N of U. As B $\subseteq$ N and B is nano rgb-closed in U, Nbcl(B) $\subseteq$ N . Thus $Nbcl_A$(B) = Nbcl(B) $\cap$ A $\subseteq$ N $\cap$ A = G . Therefore B is nano rgb-closed relative to A.

Theorem 3.7: If A is nano rgb-closed set and B is any set such that A $\subseteq$ B $\subseteq$ Nbcl(A), then B is nano rgb-closed set.

Proof: Let B $\subseteq$ G, where G is a nano regular open set. Since A is nano rgb-closed and A $\subseteq$ G, then Nbcl(A) $\subseteq$ G and Nbcl(B) $\subseteq$ Nbcl(Nbcl(A)) = Nbcl(A) $\subseteq$ G. Therefore, Nbcl(B) $\subseteq$ G and hence B is a nano rgb-closed set.

Theorem 3.8: If A and B are nano rgb-closed sets, then A $\cup$ B is nano rgb-closed.

Proof: Let G be a nano regular open set in U such that A $\cup$ B $\subseteq$ G. Then, A and B $\subseteq$ G. Since A and B are nano rgb-closed and G is a nano regular open set containing A and B we have Nbcl(A) $\subseteq$ G and Nbcl(B) $\subseteq$ G. Therefore, Nbcl(A $\cup$ B) = Nbcl(A) $\cup$ Nbcl(B) $\subseteq$ G. That is Nbcl(A $\cup$ B) $\subseteq$ G. Hence A $\cup$ B is nano rgb-closed.

Remark 3.3: Intersection of two nano rgb-closed sets need not be nano rgb-closed which is shown by the following example.

Example 3.5: Let U = {a, b, c, d} with U/R = {{a, d},{b}, {c}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {c}, {a, d}, {a, c, d}}. Here the sets {a, c}, {c, d} are nano rgb-closed but {a, c} $\cap$ {c, d} = {c} is not nano rgb-closed set.

Theorem 3.9: Let U and V be two nano topological spaces, A $\subseteq$ V $\subseteq$ U, and A be nano rgb-closed in U. Then A is nano rgb-closed in V.

Proof: Let G_1 be nano regular open relative to V such that A $\subseteq$ G_1. Then G_1= V $\cap$ G, where G is nano regular open in U and A $\subseteq$ V $\cap$ G. Therefore A $\subseteq$ G. That is G is nano regular open set containing A. Since A is nano rgb-closed in U, Nbcl(A) $\subseteq$ G. Therefore V $\cap$ Nbcl(A) $\subseteq$ V $\cap$ G. That is $Nbcl_V$(A) $\subseteq$ G_1 for every nano regular open set G_1in V such that A $\subseteq$ G_1. Hence A is nano rgb-closed relative to V.

Definition 3.2: A subset A of a nano topological space U is called nano regular generalized b-open (simply Nrgb-open) if A^c is nano rgb-closed.
The collection of all nano rgb-open subsets of U is denoted by NrgbO(U, X).

Example 3.6: Let U = {a, b, c, d} with U/R = {{a, d}, {b}, {c}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {c}, {a, d}, {a, c, d}}. Let A = {a, c, d}, then A^c = {b} is nano rgb-closed, since U is the only nano regular open set containing A^c. Therefore A is nano rgb-open. If A = {a, b, d}, then A^c = {c} is not nano rgb-closed, since Nbcl(A^c) = Nbcl({c}) = {b, c} and {b, c} $\not\subset$ {c}, a regular open set such that $A^c \subseteq$ G. Therefore A is not nano rgb-open.

Remark 3.4: $x \in$ Nrgbcl(A) if and only if G $\cap$ A $\neq \phi$ for every rgb-open set containing x.

Remark 3.5: For subsets A, B of a nano topological space $(U, \tau_R(X))$

(i) U – Nrgbint(A) = Nrgbcl(U-A)
(ii) U – Nrgbcl(A) = Nrgbint(U-A)

Theorem 3.10: Every nano b-open set nano rgb- open.
Proof follows from the Theorem 3.1**.**

Theorem 3.11: A subset A $\subseteq$ U is nano rgb-open if and only if F $\subseteq$ Nbint(A) whenever F is nano regular closed set and F $\subseteq$ A.

Proof: Let A be nano rgb-open set and suppose F $\subseteq$ A where F is nano regular closed. Then U-A is a nano rgb-closed set contained in the nano regular open set U-F. Hence Nbcl(U-A) $\subseteq$ U-F and U-Nbint(A) $\subseteq$ U-F. Thus F $\subseteq$ Nbint(A). Conversely, if F is a nano regular closed set with F $\subseteq$ Nbint(A) and F $\subseteq$ A, then U-Nbint(A) $\subseteq$ U-F. Thus Nbcl(U-A) $\subseteq$ U-F. Hence U-A is a nano rgb-closed set and A is a nano rgb-open set.

Theorem 3.12: If A and B are nano rgb-open, then A $\cap$ B is nano rgb-open.

Proof: Let A and B are nano rgb-open sets, then A^c and B^c are nano rgb-closed. Hence $A^c \cup B^c (A \cap B)^c$ is nano rgb-closed and thus A $\cap$ B is nano rgb-open.

Remark 3.6: The Union of nano rgb-open sets need not be rgb-open which is proved from the following example.

Example 3.7: Let U = {a, b, c, d} with U/R = {{a, d}, {b}, {c}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {c}, {a, d}, {a, c, d}}. Here the sets {d}, {a, b} are nano rgb-open sets but {d} $\cup$ {a, b} = {a, b, d} is not nano rgb-open.

Theorem 3.15: If Nbint(A) $\subseteq$ B $\subseteq$ A and if A is nano rgb-open then B is also nano rgb-open.

Proof: Let Nbint(A) $\subseteq$ B $\subseteq$ A, then $A^c \subseteq B^c \subseteq$ Nbcl(A^c) where A^c is nano rgb-closed set and hence B^c is also nano rgb-closed by Theorem 3.7**.** Therefore B is nano rgb-open.

IV. nano rgb-continuous and rgb-irresolute function

In this section we define nano rgb-continuous function and study some of their characterizations.

Definition 4.1: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(X))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is said to be nano regular generalized b-continuous (nano rgb-continuous) if the inverse image of every closed set in V is nano rgb-closed in U.

Remark 4.1: Since the complement of nano rgb-closed set is nano rgb-open, f is nano rgb-continuous if and only if the inverse image of every nano open set in V is nano rgb-open in U.

Theorem 4.1: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be nano topological spaces and a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$. Then, every nano b-continuous, nano continuous, nano α-continuous, nano semi continuous, nano pre continuous, nano g-continuous, nano sg-continuous functions are nano rgb-continuous.

Proof follows from the definition.

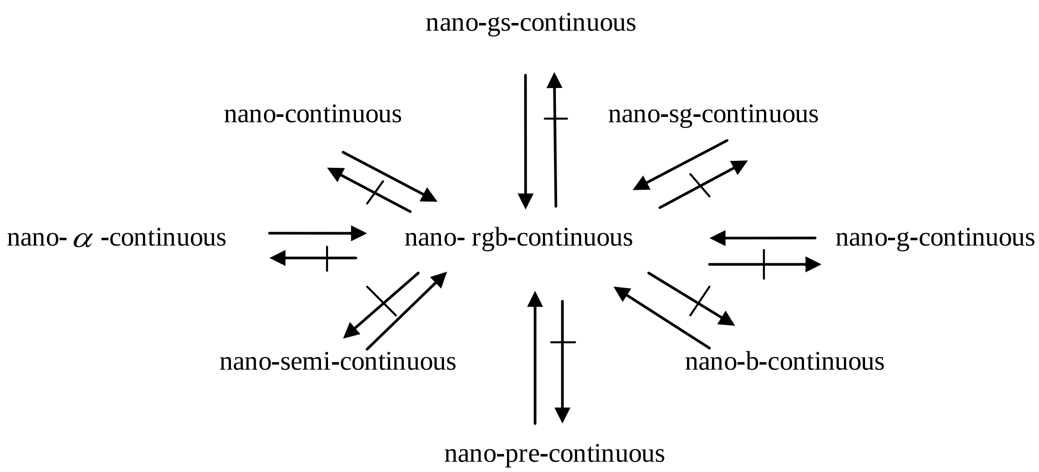
Remark 4.2: The converse of the above theorem need not be true which can be seen from the following examples.

Example 4.1: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_R(X)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f : U \rightarrow V$ as f(a) = x, f(b)= w, f(c) = y and f(d) = z. Then f is nano rgb-continuous but f is not nano continuous and nano semi continuous since $f^{-1}(\{x, w\})$ = {a, b} and $f^{-1}(\{y, z, w\})$ = {b, c, d} which are not nano closed and nano semi closed in U whereas {x, w}, {y, z, w} are nano closed in V. Thus a nano rgb-continuous function is not nano continuous and nano semi continuous.

Example 4.2: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f : U \rightarrow V$ as f(a) = w, f(b)= x, f(c) = y and f(d) = z. Then f is nano rgb-continuous but f is not nano b-continuous, g-continuous and nano pre continuous since $f^{-1}(\{y, z, w\})$ = {a, c, d} is not nano b-closed, nano g-closed and nano pre-closed in U. Thus a nano rgb-continuous function is not nano b-continuous, nano g-continuous and nano pre continuous.

Example 4.3: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {b, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f: U \to V$ as f(a) = x, f(b)= w, f(c) = y and f(d) = z. Then f is nano rgb-continuous but f is not nano sg-continuous since $f^{-1}(\{x, w\})$ = {a, b} and $f^{-1}(\{w\})$ = {b} which are not nano sg-closed in U whereas {x, w}, {w} are nano closed in V. Thus a nano rgb-continuous function is not nano sg-continuous.

The above results can be summarized in the following diagram.



Theorem 4.2: A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano rgb-continuous if and only if the inverse image of every nano closed set in V is nano rgb-closed in U.

Proof: Let f be nano rgb-continuous and F be nano closed in V. That is, V-F is nano open in V. Since f is nano rgb- continuous, $f^{-1}(V-F)$ is nano rgb-open in U. That is, $U - f^{-1}(F)$ is nano rgb-open in U. Therefore, $f^{-1}(F)$ is nano rgb-closed in U. Thus, the inverse image of every nano closed set in V is nano rgb-closed in U, if f is nano rgb-continuous on U. Conversely, let the inverse image of every nano closed be nano rgb-closed. Let G be nano open in V. Then V-G is nano closed in V. Then $f^{-1}(V-G)$ is nano rgb-closed in U. That is, $U - f^{-1}(G)$ is nano rgb-closed in U. Therefore, $f^{-1}(G)$ is nano rgb-open in U. Thus, the inverse image of every nano open set in V is nano rgb-open in U. That is f is nano rgb-continuous on U.

In the following theorem, we establish a characterization of nano rgb-continuous functions in terms of nano rgb-closure.

Theorem 4.3: A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano rgb-continuous if and only if $f(Nrgbcl(A)) \subseteq Ncl(f(A))$ for every subset A of U.

Proof: Let f be nano rgb-continuous and A $\subseteq$ U. Then f(A) $\subseteq$ V. Since f is nano rgb-continuous and $Ncl(f(A))$ is nano closed in V, $f^{-1}(Ncl(f(A)))$ is nano rgb -closed in U. Since f(A) $\subseteq$ $Ncl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(Ncl(f(A)))$, then $Nrgbcl(A) \subseteq Nrgbcl[f^{-1}(Ncl(f(A)))]$ = $f^{-1}(Ncl(f(A)))$. Thus, $Nrgbcl(A) \subseteq f^{-1}(Ncl(f(A)))$. Therefore, $f(Nrgbcl(A)) \subseteq Ncl(f(A))$ for every subset A of U. Conversely, let $f(Nrgbcl(A)) \subseteq Ncl(f(A))$ for every subset A of U. If F is nano closed inV, since $f^{-1}(F) \subseteq$U, $f(Nrgbcl(f^{-1}(F))) \subseteq Ncl(f(f^{-1}(F)))$ = $Ncl(F)$= F. That is, $f(Nrgbcl(f^{-1}(F))) \subseteq$ F. Thus $Nrgbcl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq Nrgbcl(f^{-1}(F))$. Hence, $Nrgbcl(f^{-1}(F))$ = $f^{-1}(F)$ Therefore, $f^{-1}(F)$ is nano rgb-closed in U for every nano closed set F in V. That is, f is nano rgb-continuous.

Remark 4.3: If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano b-continuous, then $f(Nrgbcl(A))$ is not necessarily equal to $Ncl(f(A))$.

Example 4.4: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_R(X)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f:U\to V$ as f(a) = x, f(b)= w, f(c) = y and f(d) = z. Then f is nano rgb-continuous. Let A = {c} $\subseteq$ V. Then $f(Nrgbcl(A))$ = f({c}) = {y}. But, $Ncl(f(A))$ = Ncl({y}) = {y, z, w}. Thus, $f(Nrgbcl(A)) \neq Ncl(f(A))$, even though f is nano b-continuous.

In the following theorem, we characterize nano rgb-continuous functions in terms of inverse image of nano closure.

Theorem 4.4: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano rgb-continuous if and only if $Nrgbclf^{-1}(B)) \subseteq f^{-1}(Ncl(B))$.

Proof: If f is nano rgb-continuous and B$\subseteq$V, then $Ncl(B)$ is nano closed in V and hence $f^{-1}(Ncl(B))$ is nano rgb-closed in U. Therefore, $Nrgbcl[f^{-1}(Ncl(B))]$ = $f^{-1}(Ncl(B))$. Since B $\subseteq Ncl(B)$, $f^{-1}(B)$ $\subseteq$ $f^{-1}(Ncl(B))$. Therefore, $Nrgbcl(f^{-1}(B)) \subseteq Nrgbcl[f^{-1}(Ncl(B))]$ = $f^{-1}(Ncl(B))$. That is, $Nrgbclf^{-1}(B)) \subseteq f^{-1}(Ncl(B))$. Conversely, let $Nrgbclf^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every subset B of V. If B is nano closed in V, then $Ncl(B)$= B. By assumption, $Nrgbclf^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ = $f^{-1}(B)$. Thus $Nrgbclf^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq Nrgbclf^{-1}(B))$. Therefore $Nrgbclf^{-1}(B))$ = $f^{-1}(B)$. That is, $f^{-1}(B)$ is nano rgb-closed in U for every nano closed set B in V. Therefore f is nano rgb-continuous on U.

The following theorem establishes nano rgb-continuous functions in terms of inverse image of nano rgb-interior of a subset of V.

Theorem 4.5: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano rgb-continuous on U if and only if $f^{-1}(N\operatorname{int}(B)) \subseteq Nrgb\operatorname{int}(f^{-1}(B))$ for every subset B of V.

Proof: If f is nano rgb-continuous and B $\subseteq$V, then $N\operatorname{int}(B)$ is nano open in V and hence $f^{-1}(N\operatorname{int}(B))$ is nano rgb-open in U. Therefore, $Nrgb\operatorname{int}[f^{-1}(N\operatorname{int}(B))]$ = $f^{-1}(N\operatorname{int}(B))$. Also $N\operatorname{int}(B) \subseteq$ B implies that $f^{-1}(N\operatorname{int}(B)) \subseteq f^{-1}(B)$. Therefore, $Nrgb\operatorname{int}[f^{-1}(N\operatorname{int}(B))] \subseteq Nrgb\operatorname{int}(f^{-1}(B))$. That is, $f^{-1}(N\operatorname{int}(B)) \subseteq Nrgb\operatorname{int}(f^{-1}(B))$. Conversely, let $f^{-1}(N\operatorname{int}(B)) \subseteq Nrgb\operatorname{int}(f^{-1}(B))$ for every B $\subseteq$ V. If B is nano-open in V, then $N\operatorname{int}(B)$ = B. By assumption, $f^{-1}(N\operatorname{int}(B)) \subseteq Nrgb\operatorname{int}(f^{-1}(B))$. Thus, $f^{-1}(B) \subseteq Nrgb\operatorname{int}(f^{-1}(B))$. But $Nrgb\operatorname{int}(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore, $f^{-1}(B)$ = $Nrgb\operatorname{int}(f^{-1}(B))$. That is, $f^{-1}(B)$ is nano rgb-open in U for every nano open set B in V. Therefore, f is nano rgb-continuous on U.

Remark 4.4: Equality of the above theorems 4.4 and 4.5 does not hold in general can be seen from the following example.

Example 4.5: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_R(X)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f:U\to V$ as f(a) = x, f(b)= w, f(c) = y and f(d) = z. Then f is nano rgb-continuous.

(i) Let B = {z} $\subseteq$ V. Then $f^{-1}(Ncl(B))$ = $f^{-1}(y,z,w)$ = {b, c, d} and $Nrgbclf^{-1}(B))$ = $Nrgbclf^{-1}(B))$ = {d}. Therefore $Nrgbclf^{-1}(B)) \neq f^{-1}(Ncl(B))$.

(ii) Let B = {x, w}$\subseteq$V. Then $f^{-1}(N\operatorname{int}(B))$ = $f^{-1}(N\operatorname{int}(\{x,w\}))$ = {a, b} and

$Nrgb\text{int}(f^{-1}(B)) = Nrgb\text{int}(f^{-1}(\{x,w\})) = \phi$. Therefore $f^{-1}(N\text{int}(B)) \neq Nrgb\text{int}(f^{-1}(B))$.

Theorem 4.6: If $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ are nano topological spaces with respect to X $\subseteq$ U and Y $\subseteq$ V respectively, then any function $f: U \to V$, the following conditions are equivalent:

(a) $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano rgb-continuous.

(b) $f(Nrgbcl(A)) \subseteq Ncl(f(A))$ for every subset A of U.

(c) $Nrgbclf^{-1}(B)) \subseteq f^{-1}(Ncl(B))$ for every subset B of V.

Proof follows from the theorems 4.2, 4.3, 4.4.

Definition 4.2: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a function $f: U \to V$ is said to be nano regular generalized b-irresolute (nano rgb- irresolute) if the inverse image of every nano rgb-closed set in V is nano rgb-closed in U.

Example 4.4: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{x}, {y, z}, {w}}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {x, y, z}, {y, z}}.Define $f: U \to V$ as f(a) = x, f(b)= w, f(c) = y and f(d) = z. Then f is nano rgb-irresolute since the inverse image of every nano rgb-closed set in V is nano rgb-closed in U.

Theorem 4.7: A function $f: U \to V$ is nano rgb-irresolute if and only if for every nano rgb-open set F in V, $f^{-1}(F)$ is nano rgb-open in U.

Proof follows from the fact that the complement of a nano rgb-open set is nano rgb-closed and vice versa.

Theorem 4.8: A function $f: U \to V$ is nano rgb-irresolute, then f is nano rgb-continuous.

Proof: Since every nano closed set is nano rgb closed, the inverse image of every nano closed set in V is nano rgb closed in U, whenever the inverse image of every nano rgb-closed set is nano rgb-closed. Therefore, any nano rgb-irresolute function is nano rgb-continuous.

Remark 4.5: The converse of the above theorem need not be true shown in the following example.

Example 4.7: Let U = {a, b, c, d} with U/R = {{b}, {c}, {a, d}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {c}, {a, d}, {a, c, d}}. Let V = {x, y, z, w} with V/R' = {{y}, {w}, {x, z}}. Then $\tau_{R'}(Y)$ = {V, ϕ, {z}, {x, z}, {x, z, w}}.Define $f: U \to V$ as f(a) = x, f(b)= y, f(c) = z and f(d) = w. Then f is nano rgb-continuous. But f is not nano rgb-irresolute, since $f^{-1}(\{z\})$ = {c} which is not nano rgb-closed in V whereas {z} is nano rgb-closed in V. Thus a nano rgb-continuous function is not nano rgb-irresolute.

Theorem 4.9: If $f: U \to V$ is nano rgb-irresolute and $g: V \to W$ is nano rgb-continuous, then $g \circ f: U \to W$ is nano rgb-continuous.

Theorem 4.10: If $f: U \to V$ is nano rgb-continuous and $g: V \to W$ is nano continuous, then $g \circ f: U \to W$ is nano rgb-continuous.

The proof of the theorems 4.9 and 4.10 are obvious.

Theorem 4.11: If $f: U \to V$ is nano rgb-irresolute and $g: V \to W$ is nano b-continuous, then $g \circ f: U \to W$ is nano rgb-continuous.

Proof: Let G be nano open in W. Then $g^{-1}(G)$ is nano b- open in V, since g is nano b-continuous. Thus $g^{-1}(G)$ is nano rgb-open in V, since every nano b-open is nano rgb-open. Then $f^{-1}(g^{-1}(G)$ =

$(g \circ f)^{-1}(G)$ is nano rgb-open in U and hence $g \circ f$ is nano rgb-continuous. Similarly we can prove the following theorem.

Theorem 4.12: If $f : U \to V$ is nano rgb-irresolute and $g : V \to W$ is nano α -continuous, then $g \circ f : U \to W$ is nano rgb-continuous.

Theorem 4.13: If $f : U \to V$ is nano rgb-irresolute and $g : V \to W$ is nano semi-continuous, then $g \circ f : U \to W$ is nano rgb-continuous.

Theorem 4.14: If $f : U \to V$ is nano rgb-irresolute and $g : V \to W$ is nano pre-continuous, then $g \circ f : U \to W$ is nano rgb-continuous.

Theorem 4.15: If $f : U \to V$ is nano rgb-irresolute and $g : V \to W$ is nano g-continuous, then $g \circ f : U \to W$ is nano rgb-continuous.

Proof of the above theorems follows from the definition.

Theorem 4.16: Let $(U, \tau_R(X)), (V, \tau_{R'}(Y)), (W, \tau_{R''}(Z))$ be nano topological spaces. If $f : U \to V$ and $g : V \to W$ are nano rgb-irresolute, then $g \circ f : U \to W$ is nano rgb-irresolute.

Proof: Let G $\subseteq$ W is nano rgb-open, then $g^{-1}(G)$ is nano rgb-open in V, since g is nano rgb-irresolute. Since f is nano rgb-irresolute, $f^{-1}(g^{-1}(G)$ is nano rgb-open in U. Thus $(g \circ f)^{-1}(G)$ is nano rgb-open in U. Therefore, $g \circ f$ is nano rgb-irresolute.

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