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Three Policies for the Duplication System with Preventive Matntenance and Repair

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ABSTRACT:- This paper deals with three different policies for preventive maintenance have been considered for the standby duplex system with preventive maintenance and repair. The life time of each unit, the prepare time ,the inspection time and the time from inspection beginning to inspection completion is a random variable and has an arbitrary distribution function. The system is analyzed by the semi-Markov process technique, to solving the equations by using Laplace transformation for integral equations to find mean life time for each policy, then we conclude that it is better to use preventive maintenance under policy III, because the mean life time of the system under this policy is greater than the mean life times for the system under policies I and II.

KEYWORDS:- Reliability, Mean life time, Integral equations.

I. INTRODUCTION

In this paper three different policies for preventive maintenance have been considered for the standby duplex system with preventive maintenance and repair.

Gnedenko [3] has mentioned that there were some faults in the investigation in [1] of the standby duplex system under policy I.

In this paper we investigate policy I giving its final correct form. The mean life time of the duplex system under each policy has been given. Moreover we have compared between these three policies and we have found numerically and theoretically that the third policy is the optimal policy.

II. THE DUPLICATION SYSTEM UNDER THREE DIFFERENT PREVENTIVE MAINTENANCE POLICIES

We have three different preventive maintenance policies. The behavior of the system under each policy is considered to be as follows:

Under type I policy, the operative unit undergoes inspection, when its inspection time is due, only if the other unit is in standby stat; the other unit is switched on to continue the job, but if the inspection time of the operative unit is due while the other unit is under repair or under inspection, the operative unit goes to inspection after the repair of the failed unit or after the inspection of the unit under inspection is completed.

Under preventive maintenance type II policy, the operative unit undergoes inspection, when its inspection time comes, regardless of the state of the other unit.

Under preventive maintenance type III policy, the operative unit undergoes inspection, when its inspection time is due, only if the other unit is in standby state, the other unit is switched on the continue the job, but if the inspection time of the operative unit comes while the other unit is under repair or inspection the inspection of the operative unit is not made even after completing the repair or the inspection of the other unit.

The difference between policy I and policy III is that if the inspection time of the operative unit is due when the other unit is under repair or inspection then, under policy I the operative unit undergoes inspection after the completion of the repair or the inspection of the other unit, but under policy III the operative unit does not go inspection and continues operating until its failure.

To study the duplication system under each of these policies a number of assumptions are imposed on the system. These assumptions are:

1. The life time of each unit is a random variable and has an arbitrary distribution function F (.).
2. The repair time of each unit is a random variable and has an arbitrary distribution function G (.).
3. The inspection time of each unit is a random variable and has an arbitrary distribution function U (.).
4. The time from inspection beginning to inspection completion is a random variable and has an arbitrary distribution function V (.).
5. The repair or the inspection of unit completely restores all the initial properties of the unit.

6. The switch over times, from failure to repair, from repair completion to standby state from standby state to operative state, and from inspection state to operative or standby state, of each unit are all assumed to be negligible.
7. The time distribution G (.) is ≤ the time distribution V (.).
8. As seen as the main unit fails, the standby unit immediately assumes the lead of the failed unit, the repair of the failed unit or the inspection begins immediately.

To obtain the reliability function and the mean life time of the system under each policy, we use the following terminology:

R (t) : is the reliability function of the system at time t, where initially at t= 0 the main unit and the standby unit are completely new, i.e. the main unit starts to do the job and the other is standby.

$R_1(t)$: is the reliability function of the system at time t, where initially at t= 0, one unit is under repair and the other unit is operating.

$R_2(t)$: is the reliability function of the system at time t, where initially at t= 0, one unit is under inspection and the other is operating.

III. THE DUPLICATION SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE UNDER POLICY I.

Three integral equation for R (t), $R_1(t)$, and $R_2(t)$ can be established as follows:

1- Starting at time t=0 with an operative unit and a standby unit, we can consider the following three exhaustive and exclusive events:

i. The operative unit has not failed till time t and its inspection time has not come during this period. The probability of this event is $\overline{F}(t)\ \overline{U}(t)$.

ii. The operative unit fails at time x (x<t) and its inspection time does not come till this time x and then a system with failed unit at time X works the time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{U}(x)\ R_1(t-x)dF(x).$$

iii. The operative unit is called for inspection at time x, x<t and it does not fail till the time x, the system with one unit under inspection works the time (t-x) without failure. The probability of this event is:

$$\int_0^t \overline{F}(x)\ R_2(t-x)dU(x).$$

Therefore, the probability that the system will continue working till time t without failure, where initially we start with the two new units, is given by

$$R(t)=\overline{F}(t)\overline{U}(t)+\int_0^t \overline{F}(x)\,R_2(t-x)\,dU(x)+\int_0^t R_1(t-x)\overline{U}(x)dF(x). \tag{3.1}$$

2- Starting at time t=0 with an operative unit and the other unit under repair process, we can consider the following five mutually exhaustive and exclusive events:

i. The operative unit works smoothly to the time t without failure and without inspection. The probability of this event is

$$\overline{F}(t)\overline{U}(t).$$

ii. The operative unit has not failed till time t, and its inspection comes before time t but the repair of the failed unit at time t=0 ends after time t therefor the inspection of the operative unit is not made. The probability of this event is

$$\overline{F}(t)\ \ U(t)\ \ \overline{G}(t).$$

iii. The operative unit fails at time x(x<t) (with the probability dF(x)),after the repair completion of the failed unit (with probability G(x)) and its inspection time has not come till time x with the probability $\overline{U}(x)$ then the system with failed unit at time x works smoothly the time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{U}(x)G(x)R_1(t-x)dF(x).$$

iv. The operative unit has been called for inspection at time x(x<t) (with the probability d U(x)) and it has not failed till time x (with the probability $\overline{F}(x)$) after the repair completion of the failed unit (with the probability G(x)), then, the system with one unit under inspection at time x works smoothly the time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{F}(x)G(x)R_2(t-x)dU(x).$$

v. The operative unit has not failed till the time x and its inspection time comes at time x (x<t) before the repair completion and the failed unit ends before the time t and therefor the operative unit goes to inspection after the repair completion works the time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{F}(x)U(x)R_2(t-x)dG(x).$$

Hence for the previous events the probability that the system is working continuously till time t without failure, where initially there was a unit under repair is given by

$$R_1(t)=\overline{F}(t)\overline{U}(t)+\overline{F}(t)U(t)\overline{G}(t)+\int_0^t \overline{F}(x)\overline{G}(x)R_2(t-x)dU(x)+$$
$$\int_0^t \overline{U}(x)G(x)R_1(t-x)dF(x)+\int_0^t \overline{F}(x)U(x)R_2(t-x)dG(x) \qquad (3.2)$$

3- Similarly, starting at time t=0 with an operative unit and the other unit is under inspection activity, five events may be considered:

i. The operative unit has not failed and its inspection time has not come till the time t. the probability of this event is

$$\overline{F}(t)\overline{U}(t).$$

ii. The operative unit has not failed till time t , and its inspection is due before time t and the inspection of the maintained unit at time t=0 ends after time t; therefore , the inspection of the operative unit is not made. The probability of this event is

$$\overline{F}(t)U(t)\overline{V}(t)$$

iii. The operative unit fails at time x(x<t) after the inspection completion of the maintained unit and its inspection time has not come till the time x ; and so the system with one failed unit at time x works the time (t-

x) without failure. The probability of this event is

$$\int_0^t \overline{U}(x)V(x)R_1(t-x)dF(x) \text{ .}$$

iv. After the inspection completion of the maintained unit, the operative unit is called for inspection at time x (x<t) and it does not fail till the time x and so the system with one unit under inspection at time x works the remaining time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{F}(x)V(x)R_2(t-x)dU(x) \text{ .}$$

v. The operative unit is called for inspection at time x (x<t) before the inspection compaction completion of the maintained unit and it does not fail till time x, the inspection of the maintained unit ends before the time t and as soon as the inspection of the maintained unit ends the operative unit goes to inspection and the system with a unit under inspection works the time (t-x) without failure. The probability of this event is

$$\int_0^t \overline{F}(x)U(x)R_2(t-x)dV(x) .$$

Hence, for the previous events the probability that the system is working continuously till time t without failure, where initially there was a unit under inspection activity, is given by

$$R_2(t)=\overline{F}(t)\overline{U}(t)+\overline{F}(t)U(t)\overline{V}(t)+\int_0^t \overline{U}(x)V(x)R_1(t-x)dF(x)+$$
$$+\int_0^t \overline{F}(x)V(x)R_2(t-x)dU(x)+\int_0^t \overline{F}(x)U(x)R_2(t-x)dV(x) \tag{3.3}$$

To solve equations (3.1), (3.2) and (3.3) we introduce the following Laplace transforms:

$$R^*(s)=\int_0^\infty e^{-st} R(t)dt \ , \quad R_i^*(s)=\int_0^\infty e^{-st} R_i(t)dt \ , \ i=1,2$$

$$a(s)=\int_0^\infty e^{-st} \ \overline{F}(t)\overline{U}(t)dt \ , \ b(s)=\int_0^\infty e^{-st} \ \overline{F}(t)U(t)\overline{G}(t)dt$$

$$c(s)=\int_0^\infty e^{-st} \ \overline{F}(t)\overline{V}(t)U(t)dt \ ,$$

$$b_1(s)=\int_0^{\infty} e^{-st}G(t)\overline{U}(t)dF(t)\,,\; b_2(s)=\int_0^{\infty} e^{-st}V(t)\overline{U}(t)dF(t)$$

$$c_1(s)=\int_0^{\infty} e^{-st}G(t)\overline{F}(t)dU(t)\,,\; c_2(s)=\int_0^{\infty} e^{-st}V(t)\overline{F}(t)dU(t)$$

$$d_1(s)=\int_0^{\infty} e^{-st}\,\overline{U}(t)dF(t)\qquad,\; d_2(s)=\int_0^{\infty} e^{-st}\,\overline{F}(t)dU(t)$$

$$e_1(s)=\int_0^{\infty} e^{-st}\,\overline{F}(t)U(t)dG(t)\;\;,\; e_2(s)=\int_0^{\infty} e^{-st}\,\overline{F}(t)U(t)dV(t)$$

(3.4)

Taking the Laplace transform of (3.1), (3.2) and (3.3) and applying (3.4) we get:

$$R^*(s)=a(s)+d_2(s)R_2^*(s)+d_1(s)R_1^*(s)$$

$$R_1^*(s)=a(s)+b(s)+c_2(s)R_2^*(s)+b_1(s)R_1^*(s)+e_1(s)R_2^*(s)$$

$$=a(s)+b_1(s)R_1^*(s)+[c_1(s)+c_1(s)]\;R_2^*(s)$$

$$R_2^*(s)=a(s)+c(s)+b_2(s)R_1^*(s)+c_2(s)R_2^*(s)+e_2(s)R_2^*(s)$$

$$=b(s)+b_2(s)R_1^*(s)+[c_2(s)+e_2(s)]R_2^*(s) \tag{3.5}$$

Solving (3.5) in three unknowns, $R^*(s)\,,\,R_1^*(s)\;\; and \;\; R_2^*(s)$ we get.

$$R_1^*(s)=\frac{A(s)[1-c_2(s)-e_2(s)]+B(s)[c_1(s)+e_1(s)]}{[1-b_1(s)][1-c_2(s)-e_2(s)]-b_2(s)[c_1(s)+e_1(s)]}$$

$$R_2^*(s)=\frac{A(s)b_2(s)+B(s)[1-b_1(s)]}{[1-b_1(s)][1-c_2(s)-e_2(s)]-b_2(s)[c_1(s)+e_1(s)]}$$

(3.6)

Therefore

$$R^*(s)=a(s)\{d_1(s)[A(s)(1-c_2(s)-e_2(s))$$
$$+B(s)(c_1(s)+e_1(s))]$$
$$+d_2(s)[A(s)b_2(s)+B(s)(1-b_1(s))]\}$$
$$/[(1-b_1(s))(1-c_2(s)-e_2(s))-b_2(s)(c_1(s)+e_1(s))]$$

where;

$$A(s)=a(s)+b(s) \qquad and \qquad B(s)=a(s)+c(s).$$

The mean life time of the system (the mean time to system failure period) is given by

$$T_1=R^*(0)=\int_0^\infty R(t)\,dt$$

$$=a(0)+\frac{\left\{d_1(0)\begin{bmatrix}A(0)(1-c_2(0)-a_2(0))\\+B(0)(c_1(0)+e_1(0))\end{bmatrix}+d_2(0)[A(0)b_2(0)+B(0)(1-b_1(0))]\right\}}{[(1-b_1(0))(1-c_2(0)-e_2(0))-b_2(0)(c_1(0)+e_1(o))]} \tag{3.7}$$

where , $A(0)=a(0)+b(0)\ \ ,B(0)=a(0)+c(0)$.

IV. THE DUPLICATION SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE UNDER POLICY II.

Following the same analysis in type I policy we obtain the following three integral equations for the reliability distribution

$$R(t)\ ,\ R_1(t)\ and\ \ R_2(t)$$

$$R(t)=\overline{F}(t)\overline{U}(t)+\int_0^t \overline{U}(x)\,R_1(t-x)\,dF(x)+\int_0^t \overline{F}(x)\,R_2(t-x)\,dU(x)$$

$$R_1(t)=\overline{F}(t)\overline{U}(t)+\int_0^t G(x)\,\overline{U}(x)\,R_1(t-x)dF(x)+$$
$$\int_0^t G(x)\,\overline{F}(x)\,R_2(t-x)dU(x)$$

$$R_2(t) = \overline{F}(t)\,\overline{U}(t) + \int_0^t V(x)\,\overline{U}(x)\,R_1(t-x)\,dF(x) + \int_0^t V(x)\,\overline{F}(x)\,R_2(t-x)\,dU(x) \tag{4.1}$$

The terms in equations (4.1) are all established and discussed in the previous section.
Taking the Laplace transform of (4.1) and applying (3.4) we get

$$\begin{aligned} R^*(s) &= a(s) + d_1(s)\,R_1^*(s) + d_2(s)\,R_2^*(s), \\ R_1^*(s) &= a(s) + b_1(s)\,R_1^*(s) + e_1(s)\,R_2^*(s), \\ R_2^*(s) &= a(s) + b_2(s)\,R_1^*(s) + e_2(s)\,R_2^*(s). \end{aligned} \tag{4.2}$$

Solving the system of equations (4.2) in the three unknowns $R^*(s),\ R_1^*(s)$ *and* $R_2^*(s)$. we have

$$\begin{aligned} R_1^*(s) &= a(s)\left[1 - c_2(s) + c_1(s)\right] / D_2(s) \\ R_2^*(s) &= a(s)\left[1 - b_1(s) + b_2(s)\right] / D_2(s) \\ R^*(s) &= a(s)\left\{1 + [D_2(s) + d_1(s)\left[1 - e_2(s) + c_1(s)\right] + \right. \\ &\quad \left. + d_2(s)\left[1 - b_1(s) + b_2(s)\right]\right\} / D_2(s)] \end{aligned}$$

(4.3)

Where, $D_2(s) = \left[1 - b_1(s)\right]\left[1 - c_2(s)\right] - b_2(s)c_1(s)$

The mean life time of the system (the mean time to system failure period) is given by

$$T_2 = R^*(0) = \int_0^\infty R(t)dt = a(0)\left\{1 + \frac{d_1(0)\left[1 - c_2(0) + c_1(0)\right] + d_2(0)\left[1 - b_1(0) + b_2(0)\right]}{D_2(0)}\right\} .$$

(4.4)

V. THE DUPLICATION SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE UNDER POLICY III.

This policy is the new policy in this paper. Under type III policy we can obtain the following integral equations for the reliability distributions $R(t), R_1(t)$ *and* $R_2(t)$.

$$R(t) = \overline{F}(t)\overline{U}(t) + \int_0^t \overline{U}(x) R_1(t-x) dF(x) + \int_0^t \overline{F}(x) R_2(t-x) dU(x) \tag{5.1}$$

$$\begin{aligned} R_1(t) = {} & \overline{F}(t)\overline{U}(t) + \overline{F}(t)\overline{G}(t)U(t) + \overline{F}(t)\int_0^t U(x)dG(x) + \\ & \int_0^t \overline{U}(x)G(x)R_1(t-x)dF(x) + \int_0^t \overline{F}(x)G(x)R_2(t-x)dU(x) + \\ & \int_0^t R_1(t-x)\int_0^x U(y)dG(y)dF(x) \end{aligned} \tag{5.2}$$

$$\begin{aligned} R_2(t) = {} & \overline{F}(t)\overline{U}(t) + \overline{F}(t)\overline{V}(t)U(t) + \overline{F}(t)\int_0^t U(x)dV(x) + \\ & \int_0^t \overline{U}(x)V(x)R_1(t-x)dF(x) + \int_0^t \overline{F}(x)V(x)R_2(t-x)dU(x) + \\ & \int_0^t R_1(t-x)\int_0^x U(y)dV(y)dF(x) \end{aligned}$$

) (5.3)

The explanation for equation (5.1) is the same as that for equation (3.1).
For equation (5.2), the third term is that: the operative unit has not failed till time t and its inspection time comes at time x (x<t) before the repair completion of the failed unit, the repair of the failed unit ends before the time t; therefore the inspection of the operative unit is not done till the time t. the probability of this event is

$$\overline{F}(t)\int_0^t U(x)\,dG(x).$$

The last term of equation (5.2)is that; the operative unit is called for inspection at time y (y< x< t) before the repair completion of the failed unit and so on, the inspection of the operative unit is not done and is left to operate, the repair of the failed unit ends before time x and the operative unit fails at time x, and then the system with one failed unit at time x works the time (t-x) without failure. The probability of this event is

$$\int_0^t R_1(t-x)\int_0^x U(y)\,dG(y)\,dF(x).$$

The explanation of the other term in (5.2) is given in policy I .

For the analysis of equation (5.3): we have the following. The third term means that: the operative unit has not failed till the time t and its inspection time comes at time x (x<t) before the inspection completion of the maintained unit ends before time t therefore the inspection of the operative unit is not done.
The last term means that : the operative unit is called for inspection at time y(y< x < t) before the inspection completion of the maintained unit and therefore the inspection of the operative unit is not done and the unit is left to operate , the inspection of the maintained unit ends before the time x and the operative unit fails at time x and then the system with one failed unit at time x works the time (t-x) without failure.

The analysis of the other terms in equation (5.3) is given in policy I above.
To solve the integral equations (5.1), (5.2), and (5.3) we introduce the Laplace transforms:

$$\hat{b}(s) = \int_0^\infty e^{-sy}\,\overline{F}(t)\int_0^t U(x)dG(x)dt\,,$$

$$\hat{c}(s) = \int_0^\infty e^{-st}\,\overline{F}(t)\int_0^t U(x)dV(x)dt\,,$$

$$\alpha_1(s) = \int_0^\infty e^{-st}\int_0^t U(y)dG(y)dF(t)\,,$$

$$\alpha_2(s) = \int_0^\infty e^{-st}\int_0^t U(y)dV(y)dF(t) \quad . \tag{5.4}$$

Taking the Laplace transform (5.1), (5.2) and (5.3), applying (3.4) and (5.4), we get,

$$R^*(s) = a(s) + d_1(s)R_1^*(s) + d_2(s)R_2^*(s) \tag{5.5}$$

$$R_1^*(s) = a(s)+b(s)+\hat{b}(s)+b_1(s)R_1^*(s)+c_1(s)R_2^*(s)+\alpha_1(s)R_1^*(s)$$
$$= a_1(s) + \left[b_1(s) + \alpha_1(s)\right]R_1^*(s) + c_1(s)R_2^*(s) \tag{5.6}$$

$$R_2^*(s)=a(s)+c(s)+\hat{c}(s)+b_2(s)R_1^*(s)+c_2(s)R_2^*(s)+\alpha_2(s)R_1^*(s)$$
$$= a_2(s) + \left[b_2(s) + \alpha_2(s)\right]R_1^*(s) + c_2(s)R_2^*(s) \tag{5.7}$$

Where; $a_1(s) = a(s) + b(s) + \hat{b}(s)\,,\; a_2(s) = a(s) + c(s) + \hat{c}(s)\,.$

Solving (5.5) , (5.6) and (5.7) in the three unknowns $R_1^*(s), R_1^*(s), R_2^*(s)$, we get

$$R_1^*(s) = \left\{a_1(s)\left[1 - c_2(s)\right] + a_2(s)c_1(s)\right\}/D_3(s)$$

$$R_2^*(s) = \left\{a_1(s)\left[\alpha_2(s) + b_2(s)\right] + a_2(s)\left[1 - \alpha_1(s) - b_1(s)\right]\right\}/D_3(s)$$

$$R^*(s) = a(s) + \left\{a_1(s)\left\{d_1(s)\left[1 - c_2(s)\right] + d_2(s)\left[\alpha_2(s) + b_2(s)\right]\right\} + a_2(s)\left\{d_1(s)c_1(s) + d_2(s)\left[1 - \alpha_1(s) - b_1(s)\right]\right\}\right\}/D_3(s) \tag{5.8}$$

where;

$$D_3(s)=\left[1 - \alpha_1(s) - b_1(s)\right]\left[1 - c_2(s)\right] - \left[\alpha_2(s) + b_2(s)\right]c_1(s)$$

The mean life time of the system is given by

$$T_3 = R^*(0) = \int_0^\infty R(t)dt =$$

$$a(0) + \left\{ a_1(0)\left[d_1(0)\left(1 - c_2(0)\right) + d_2(0)\left(\alpha_2(0) + b_2(0)\right)\right] + \right.$$
$$\left. + a_2(0)\left[d_1(0)c_1(0) + d_2(0)\left(1 - \alpha_1(0) - b_1(0)\right)\right]\right\} \Big/ D_3 \tag{5.9}$$

If there is no preventive maintenance but only repair, we have

$$a(0) = \int_0^\infty \overline{F}(t)dt\ ,\ b_1(0) = \int_0^\infty G(t)dF(t)$$

$$d_1(0) = 1,\ b(0) = c(0) = b_2(0) = c_1(0) = e_2(0) = d_2(0) = 0$$

$$\hat{b}(0) = \hat{c}(0) = \alpha_1(0) = \alpha_2(0) = 0.$$

So that the mean life time of the duplication system with repair only is given by

$$T_1 = T_2 = T_3 = T' = a(0) + \frac{a(0)}{1 - b_1(0)} \tag{5.10}$$

where ; $$a(0) = \int_0^\infty \overline{F}(t)dt, b_1(0) = \int_0^\infty \overline{G}(t)dF(t).$$

This result is the result obtained in [3] when the duplication system is with repair only.

6. COMPARISON BETWEEN THE THREE POLICIES

In the above analysis we have considered that the inspection time is a random variable. But in most practical cases the inspection time is a constant random variable, i.e.

$$U(t) = \begin{cases} 1 & for \quad t \geq T \\ 0 & for \quad t < T\ . \end{cases}$$

In this case the mean life time of the system under policy I is given by

$$T_1 = a + F(T)\ .\ \frac{A_1}{D_1} + \overline{F}(T)\ .\ \frac{A_2}{D_2} \tag{6.1}$$

where; $a = \int_0^T \overline{F}(x)dx$.

$$A_1 = [a + \int_T^\infty \overline{F}(x)\overline{G}(x)dx\ ][1 - \overline{F}(T)V\ (T) - \int_T^\infty \overline{F}(x)dV\ (x)] +$$

$$+[a+\int_T^\infty \overline{F}(x)\overline{V}(x)dx][\overline{F}(T)G(T)+\int_T^\infty \overline{F}(x)dG(x)],$$

$$A_2=[a+\int_T^\infty \overline{F}(x)\overline{G}(x)dx]\int_0^\infty V(x)dF(x)+[a+\int_T^\infty \overline{F}(x)\overline{V}(x)dx][1-\int_0^T G(x)dF(x)],$$

$$D=[1-\int_0^T G(x)dF(x)][1-\overline{F}(T)V(T)-\int_T^\infty \overline{F}(x)dV(x)]$$

$$-\int_0^T V(x)dF(x)[\overline{F}(T)G(T)+\int_T^\infty \overline{F}(x)dG(x)].$$

The mean life time of the system under policy II is given by

$$T_2=a[1+\{1+P(T)F(T)G(T)-V(T)+P(T)\int_0^T(V(x)-G(x)dF(x))/D_4\}$$

where;
$$a=\int_0^T \overline{F}(x)dx,P(T)=\overline{F}(T)=1-F(T), \tag{6.2}$$

$$D_4=[1-\int_0^T G(x)dF(x)][1-P(T)V(T)]-P(T)G(T)\int_0^T V(x)dF(x).$$

The mean life time of the system under policy III is given by

$$T_3=\lambda^{-1}(1+\gamma_1^{-1})+[(\gamma_1+G)(a\gamma_1-\lambda^{-1}(d_1\gamma_1+d_2\gamma_2))]/D_5 \tag{6.3}$$

Where;

$$D_5=\left[\gamma_1+d_2(\gamma_2G-\gamma_1V)\gamma_1\right],$$

$$\lambda^{-1}=\int_0^\infty \overline{F}(x)\,dx,\ a=\int_0^T \overline{F}(x)dx,\ d_1=F(T),$$

$$d_2=\overline{F}(T),G=G(T),V=V(T),$$

$$\gamma_1=\int_0^\infty \overline{G}(x)dF(x),\gamma_2=\int_0^\infty \overline{V}(x)dF(x).$$

In this case we have worked out a numerical comparison between the three policies defined as follows: For $F(x)=1-(1+\lambda x)e^{-\lambda x},\quad G(x)=1-e^{-\beta x},\quad V(x)=1-e^{-\gamma x},$

We have

$$a=\frac{2}{\lambda}-e^{-\lambda T}(T+\frac{2}{\lambda}),$$

$$\int_T^\infty \overline{F}(x)\overline{G}(x)dx(x)=\frac{e^{-(\lambda+\beta)T}}{(\lambda+\beta)}\left[1+\lambda T+\frac{\lambda}{\lambda+\beta}\right].$$

$$\int_0^\infty \overline{F}(x)\overline{V}(x)dx=\frac{e^{-(\lambda+\gamma)T}}{(\lambda+\gamma)}\left[1+\lambda^T+\frac{\lambda}{(\lambda+\gamma)}\right].$$

$$d_1=F(T)=1-(1+\lambda T)e^{-\lambda T},\quad d_2=\overline{F}(t)=(1+\lambda t)e^{-\lambda t}$$

$$\overline{F}(T)V(T)=(1+\lambda T)e^{-\lambda T}\left(1-e^{-\gamma T}\right).$$

$$\int_T^\infty \overline{F}(x)dv(x)=\frac{\gamma}{(\lambda+\gamma)}e^{-(\lambda+\gamma)T}\left[1+\lambda T+\frac{\lambda}{(\lambda+\gamma)}\right],$$

$$\int_T^\infty \overline{F}(x)dG(x)=\frac{\beta}{(\lambda+\beta)}e^{-(\lambda+\beta)T}\left[1+\lambda T+\frac{\lambda}{(\lambda+\beta)}\right],$$

$$\overline{F}(T)G(T)=(1+\lambda T)e^{-\lambda T}\left(1-e^{-\beta T}\right),$$

$$\int_0^T G(x)dF(x)=[1-e^{-\lambda T}-Te^{-\lambda T}]-[\frac{\lambda^2}{(\lambda+\beta)^2}-\frac{\lambda^2}{(\lambda+\beta)^2}e^{-(\lambda+\beta)T}],$$

$$\int_0^T v(x)dF(x)=[1-e^{-\lambda T}-\lambda Te^{-\lambda T}]-[\frac{\lambda^2}{(\lambda+\gamma)^2}-$$

$$\frac{\lambda^2 e^{-(\lambda+\gamma)T}}{(\lambda+\gamma)^2}-\frac{\lambda^2 T}{(\lambda+\gamma)^2}e^{-(\lambda+\gamma)T}],$$

$$\int_0^T [v(x)-G(x)]dF(x)=[\frac{\lambda^2}{(\lambda+\beta)^2}-\frac{\lambda^2}{(\lambda+\beta)^2}e^{-(\lambda+\beta)T}-\frac{\lambda^2}{(\lambda+\beta)^2}e^{-(\lambda+\beta)T}]$$

$$-[\frac{\lambda^2}{(\lambda+\gamma)^2}-\frac{\lambda^2}{(\lambda+\gamma)^2}e^{-(\lambda+\gamma)T}-\frac{\lambda^2}{(\lambda+\gamma)^2}e^{-(\lambda+\gamma)T}],$$

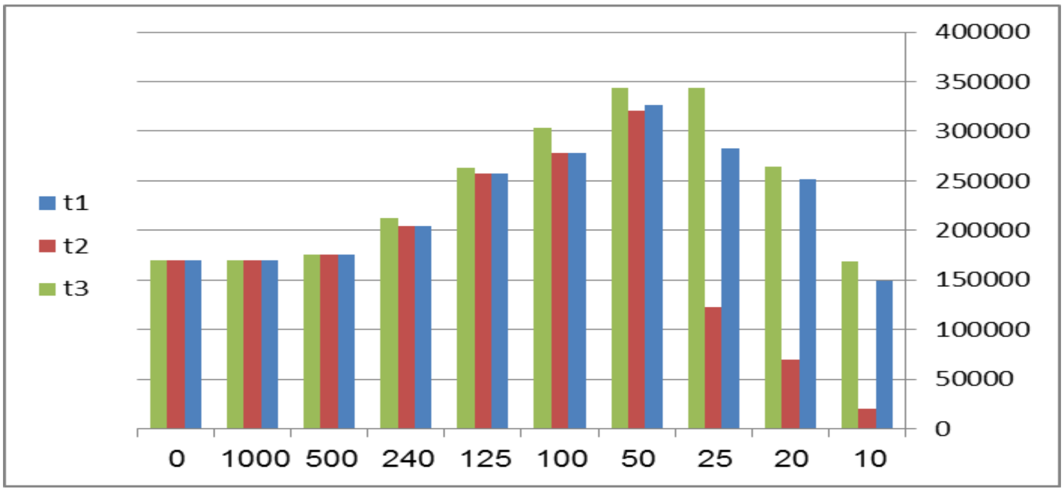
$$\lambda^{-1} = \frac{1}{\lambda}, \; G = G(T) = 1 - e^{-\beta T}, \; V = V(T),$$

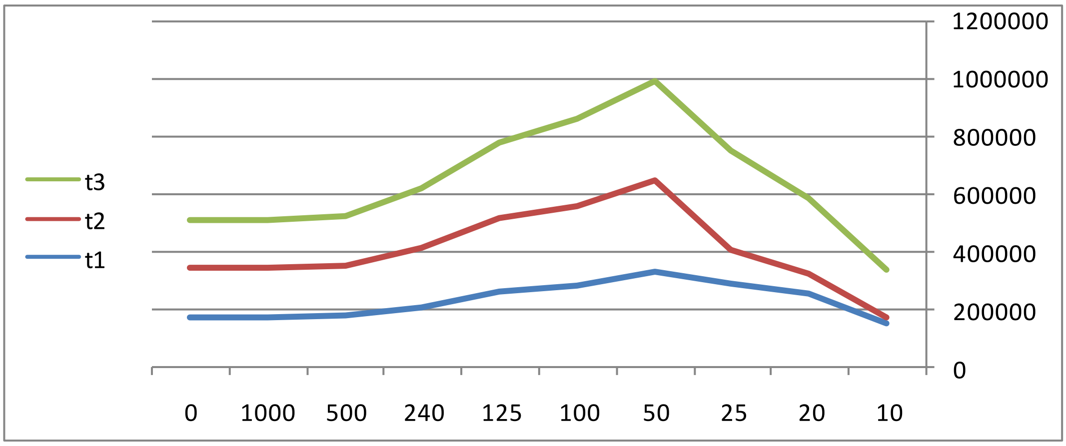
$$\gamma_1 = \frac{\lambda^2}{(\lambda + \beta)^2} \; , \quad \gamma_2 = \frac{\lambda^2}{(\lambda + \gamma)^2} \; .$$

Substituting these relations in equations (6.1), (6.2), and (6.3) we get the mean life time of the system under the three policies. For taking

$\lambda = 1/125$, $\beta = 1/5$, $\gamma = 1$ and taking random values of T . We obtain the following table:

Values of T	T_1	T_2	T_3
10	148822	19548	**169162**
20	251884	69994	**264557**
25	283031	122887	**344034**
50	326486	320964	**343693**
100	278309	278309	**303860**
125	256775	256775	**263501**
240	204406	204406	**212411**
500	175333	175333	**175643**
1000	169457	169457	**169468**
00	**169260**	**169260**	**169250**





This numerical comparison shows that policy III is the optical policy to be used practically, since $T_3 > T_1 > T_2$.

Now to ascertain the preceding result, we prove the following theorem.

THEOREM: The preventive maintenance of the duplex system under policy III is the optimal policy to use, if the repair time distribution is less or equal to the inspection time distribution.

PROOF: for simplicity we denote $\psi^*(0) = \psi$, for any Laplace transform $\psi(s)$

We first prove that the mean life time $\boldsymbol{T}_1$ in (3.7) is greater than the mean life time $\boldsymbol{T}_2$ in (4.4) and next prove that the mean life time $\boldsymbol{T}_3$ in (5.9) is greater than the mean life time T_1.

First: Subtracting equation (4.4) from equation (3.7) we get:

$$T_1 - T_2 = \frac{(1-b_1)[e_1(1-c_2)+e_2c_1] + b_2c_1(1-e_1) + b_2c_1e_2}{[(1-b_1)(1-c_2) - b_2c_1][1-b_1)(1-c_2-e_2)-b_2(e_1+c_1)]} +$$

$$+ \frac{d_1[b(1-c_2-e_2) + c(c_1+e_1)] + d_2[b\,b_2+c(1-b_1)]}{(1-b_1)(1-c_2-e_2) - b_2(c_1+e_1)} =$$

$$= Q_1 + Q_2$$

It is clear that each of the terms in each numerator in Q_1 *and* Q_2 is positive, than it remains to prove that the denominators in Q_1 *and* Q_2 are also positive to insure that $T_1 - T_2 > 0$. Since $V(t) \geq G(t)$,

Then $b_2 > b_1$, $c_2 > c_1$ *and* $e_2 > e_1$,

Where ; $e_1 = 1 - \alpha - b_1 - c_1$, $e_2 = 1 - \mu - b_2 - c_2$,

$$\alpha = \int_0^\infty \overline{G}(x)dF(x) > \mu = \int_0^\infty \overline{V}(x)dF(x).$$

Hence, $(1-b_1)(1-c_2-e_2) - b_2(c_1+e_1) > (1-b_1)(1-c_2-e_2) -$

$$-b_2(c_2+e_2) = 1 - (b_2+c_2+e_2) = 1 - \mu > 0 .$$

Because $\mu < \int_0^\infty V(t)\,dF(t) < 1.$

Then we have

$$[(1-b_1)(1-c_2-e_2) - b_2(c_1+e_1)] > 0 .$$

Now, since

$$[(1-b_1)(1-c_2) - b_2c_1] > [(1-b_1)(1-c_2-e_2) - b_2(c_1+e_1)].$$

Then

$$[(1+b_1)(1-c_2) - b_2c_1] > 0$$

and consequently the denominators in Q_1 *and* Q_2 and in Q_2 are also positive. Consequently

$$T_1 - T_2 > 0 \tag{6.4}$$

Second: Subtracting equation (3.7) from equation (5.9), we get:

$$T_3 - T_1 = \frac{K_1}{K_2} - \frac{H_1}{H_2}$$

Where;

$$K_1 = a_1 [d_1(1-c_2) + d_2(\alpha_2 + b_2)] + a_2[d_1 c_1 + d_2(1-\alpha_2 - b_1)],$$

$$K_2 = (1-\alpha_1 - b_1)(1-c_2) - (\alpha_2 + b_2)c_1,$$

$$H_1 = d_1[A(1-c_2-e_2) + B(c_1+e_1)] + d_2[Ab_2 + B(1-b_1)],$$

and

$$H_2 = (1-b_1)(1-c_2-e_2) - b_2(c_1+e_1).$$

To prove that $T_3 > T_1$ it is sufficient to show that

$$K_1 > H_1 \text{ and } K_2 < H_2 .$$

since

$$b > c \text{ , } A > B \text{ and } \alpha_2 > \alpha_1$$

and hence

$$K_1 - H_1 = d_1(Ae_2 - Be_1) + d_2(A\alpha_2 - B\alpha_1) + \hat{b}[d_1(1-c_2) + \\ + d_2(\alpha_2 + b_2)] + \hat{e}[d_1 e_1 + d_2(1-\alpha_1 - b_1)] \tag{6.5}$$

It is clear that each term of the right hand side in (6.5) is positive and so $K_1 > H_1$ (6.6)

Again after some simple calculation we get:

$$H_2 - K_2 = \alpha_1(1-c_2) + (b_2\alpha - b_1\mu) + c_2(1-b_1) + b_2(1-c_1) + \\ + [\alpha_2 c_1 + b_1 + b_2 + \mu - 1] \tag{6.7}$$

It is clear that each term of the right hand side in (6.7) is positive and so

$$H_2 > K_2 . \tag{6.8}$$

From (6.6) and (6.8) , we get $T_3 - T_1 > 0$. (6.9)

From (6.4) and (6.9) , we have the inequalities $T_3 > T_1 > T_2$ (6.10)

We conclude that it is better to use preventive maintenance under policy III, because the mean life time of the system under this policy is greater than the mean life times for the system under policies I and II.

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