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Discrete Model of Two Predators competing for One Prey

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ABSTRACT: *This paper investigates the dynamical behavior of a discrete model of one prey two predator systems. The equilibrium points and their stability are analyzed. Time series plots are obtained for different sets of parameter values. Also bifurcation diagrams are plotted to show dynamical behavior of the system in selected range of growth parameter.*

Keywords: *Discrete Model, Prey - Predator System, Equilibrium Points, Local Stability, Time Plots, Bifurcation.*

I. INTRODUCTION

Ecology is the study of inter-relationships between organisms and environment. It is natural that two or more species living in a common habitat interact in different ways. The application of mathematics to ecology dates back to the book "An Essay on the Principle of Population" by Malthus. In recent decades, many researchers [1,2] have focused on the ecological models with three and more species to understand complex dynamical behaviors of ecological systems in the real world. They have demonstrated very complex dynamic phenomena of those models, including cycles, periodic doubling and chaos [6,7]. The discrete time models which are usually described by difference equations produce much richer patterns [3, 4, 5]. Discrete time models are ideally suited to describe the population dynamics of species, which are characterized by discrete generations.

II. MATHEMATICAL MODEL

In this paper, we consider the one prey- two predator systems describing the interactions among three species by the following system of difference equations:

$$\begin{aligned} x(n+1) &= rx(n)(1-x(n)) - \frac{ax(n)y(n)}{1+x(n)} - \frac{bx(n)z(n)}{1+x(n)} \\ y(n+1) &= y(n)(1-c) + \frac{ax(n)y(n)}{1+x(n)} \\ z(n+1) &= z(n)(1-d) + \frac{bx(n)z(n)}{1+x(n)} \end{aligned} \tag{1}$$

where $x(n)$ Prey, $y(n)$ and $z(n)$ are two predator respectively, and all parameters are positive constants. The parameter a is the intrinsic growth rates of the prey population, b denotes the death rate of the mid-predator, c denotes the death rate of the top predator, d is the rate of conversion of a consumed prey to a mid-predator, e is the rate of conversion of a consumed mid-predator to a top predator.

III. EXISTENCE OF EQUILIBRIUM

The equilibrium points of (1) are the solution of the equations

$$x = rx(1-x) - \frac{axy}{1+x} - \frac{bxz}{1+x}$$

$$y = y(1-c) + \frac{axy}{1+x}$$

$$z = z(1-d) + \frac{bxz}{1+x}$$

The equilibrium points are $E_0 = (0,0,0)$, $E_1 = \left(1-\frac{1}{r}, 0, 0\right)$, $E_2 = \left(\frac{d}{-d+b}, 0, \frac{d(1-2r)+(1+r)b}{(b-d)^2}\right)$.

IV. Dynamical Behavior Of The Model

In this section, we study the local behavior of the system (1) around the equilibrium points. The stability of the system (1) is carried out by computing the Jacobian matrix corresponding to each equilibrium point. The Jacobian matrix J for the system (1) is

$$J(x,y,z) = \begin{bmatrix} r(1-2x) - \frac{ay+bz}{1+x} + \frac{axy+bxz}{(1+x)^2} & \frac{-ax}{1+x} & \frac{-bx}{1+x} \\ \frac{ay}{1+x} + \frac{axy}{(1+x)^2} & 1-c+\frac{ax}{1+x} & 0 \\ \frac{bz}{1+x} \frac{bxz}{(1+x)^2} & 0 & 1-d+\frac{bx}{1+x} \end{bmatrix} \quad (2)$$

Theorem1. The equilibrium point E_0 is locally asymptotically stable if $0<r<1$, $0<c<2$ and $0<d<2$ otherwise unstable equilibrium point.

Proof: The equilibrium point $(0,0,0)$ in the system (1) is asymptotically stable. The Jacobian matrix for E_0 is evaluated as

$$J(E_0) = \begin{bmatrix} r & 0 & 0 \\ 0 & 1-c & 0 \\ 0 & 0 & 1-d \end{bmatrix}$$

The Eigen values are $\lambda_{1,2,3} = \{r, 1-c, 1-d\}$

If the equilibrium point E_0 satisfying the existence condition then it is most have one the following possible types:

(1) Sink if $0<r<1$, $0<c<2$ and $0<d<2$.

(2) Source if $r>1$, $c>2$ and $d>2$.

(3) Non-hyperbolic if $r=0$, $c=2$ and $d=2$.

(4) Saddle otherwise.

Theorem 2. The equilibrium point E_1 is locally asymptotically stable if $1<r<3$, $\frac{a(r-1)}{2r-1} < c < 2 + \frac{a(r-1)}{2r-1}$ and $\frac{b(r-1)}{2r-1} < d < 2 + \frac{b(r-1)}{2r-1}$, otherwise unstable equilibrium point.

Proof:

Corresponding to the equilibrium point E_1 , the Jacobian is

$$J(E_1)=\begin{bmatrix} 2-r & -\frac{a(r-1)}{2r-1} & -\frac{b(r-1)}{2r-1} \\ 0 & 1-c+\frac{a(r-1)}{2r-1} & 0 \\ 0 & 0 & 1-d+\frac{b(r-1)}{2r-1} \end{bmatrix}$$

The eigen values are $\lambda_{1,2,3}=2-r,1-c+\frac{a(r-1)}{2r-1},1-d+\frac{b(r-1)}{2r-1}$

If the equilibrium point E_1 satisfying the existence condition then it is most have one the following possible types:

(1) Sink if $1<r<3$, $\frac{a(r-1)}{2r-1}<c<2+\frac{a(r-1)}{2r-1}$ and $\frac{b(r-1)}{2r-1}<d<2+\frac{b(r-1)}{2r-1}$.

(2) Source if $r>3$, $\frac{a(r-1)}{2r-1}>c>2+\frac{a(r-1)}{2r-1}$ and $\frac{b(r-1)}{2r-1}>d$ or $d>2+\frac{b(r-1)}{2r-1}$.

(3) Non-hyperbolic if $a=1$ and $c=\frac{a(r-1)}{2r-1}$ or $c=2+\frac{a(r-1)}{2r-1}$ and $d=\frac{b(r-1)}{2r-1}$ or $d=2+\frac{b(r-1)}{2r-1}$.

(4) Saddle otherwise.

Theorem 3. The equilibrium point E_2 is locally asymptotically stable if and only if A , C , and $AB-C$ are positive.

Proof: The Jacobian matrix J at E_2 has of the form

$$J(E_2)=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (3)$$

$$a_{11}=\frac{r(b-2d)-rd}{b-d}-\left(\frac{d(-2r+1)+(1+r)b}{b}\right),\ a_{12}=\frac{-ad}{b};\quad a_{13}=-d$$

$$a_{21}=0\ ,\ a_{22}=1-c+\frac{ad}{b}\ ,\ a_{23}=0\ ,\ a_{31}=\frac{d(-2r+1)+(1+r)b}{b}\ ,\ a_{32}=0\ ,\ a_{33}=1$$

Its characteristic equation is

$$\lambda^3+A\lambda^2+B\lambda+C=0 \qquad (4)$$

with $A=-(a_{11}+a_{22}+a_{33})$, $B=a_{11}a_{33}+a_{22}a_{33}+a_{11}a_{22}-a_{23}a_{32}$ and $C=(a_{23}a_{32}-a_{22}a_{33})a_{11}$. By the Routh-Hurwitz criterion, E_4 is locally asymptotically stable if and only if A, C , and $AB-C$ are positive.

V. Numerical Simulations

Figure 1: The parameter values $r=0.99$, $a=0.95$, $b=0.9$, $c=0.97$, $d=0.79$, the system (1) has an equilibrium point E_0 in the Time Series Plot and Phase Portrait which is globally asymptotically stable.

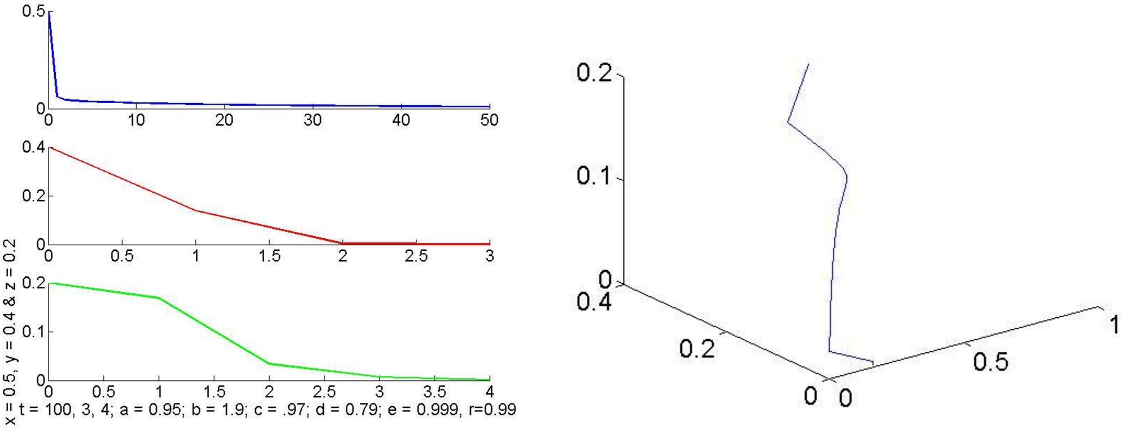


Fig 1. Time Series Plot and Phase Portrait at E_0

Figure 2: The parameter values r=2.89, a=0.9, b=1.35, c=0.99, d=1.09, the system (1) has a equilibrium point E_1 in the Time Series Plot and Phase Portrait which is globally asymptotically stable.

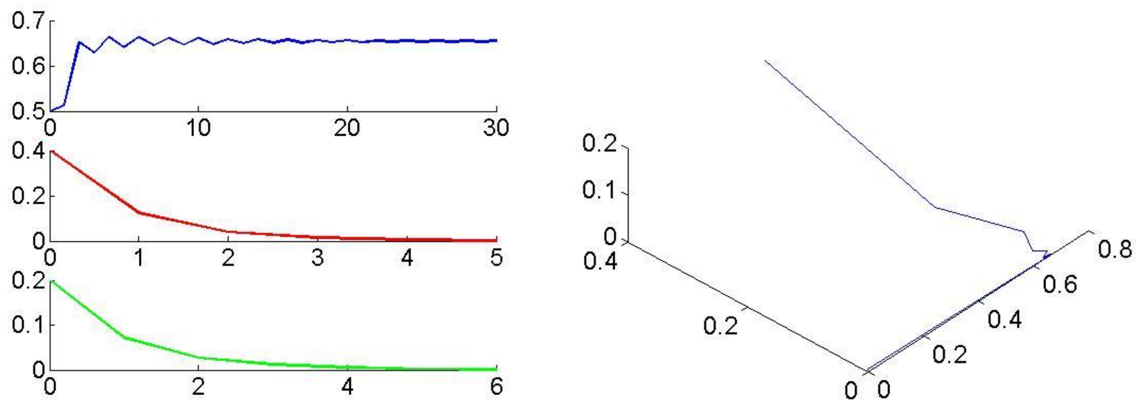


Fig 2. Time Series Plot and Phase Portrait at E_1

Figure 3: The parameter values r=1.98, a=1.78, b=0.41, c=0.94, d=0.09, the system (1) has a equilibrium point E_2 in the Time Series Plot and Phase Portrait which is globally asymptotically stable.

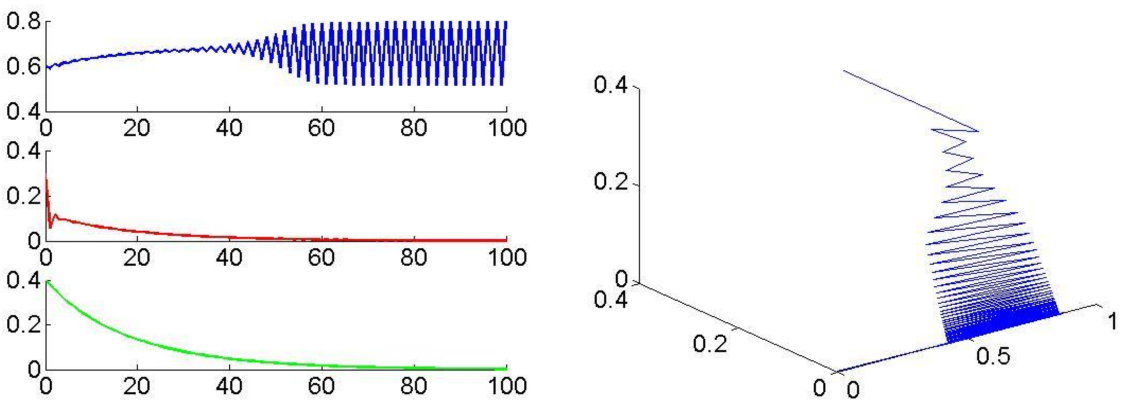


Fig 3. Time Series Plot and Phase Portrait at E_2

VI. BIFURCATION

Bifurcation is a change of the dynamical behaviours of the system as its parameters pass through a bifurcation (critical) value. Bifurcation usually occurs when the stability of an equilibrium changes. An examination of the bifurcation diagram of the system shows that a given qualitative change in the functioning of the ecosystem may be produced by changing different parameters. We restrict our analysis to events that occur when the parameter a is changed, and the other parameters are fixed at given values. In this section, we focus on exploring the possibility of chaotic behaviour for the prey population respectively.

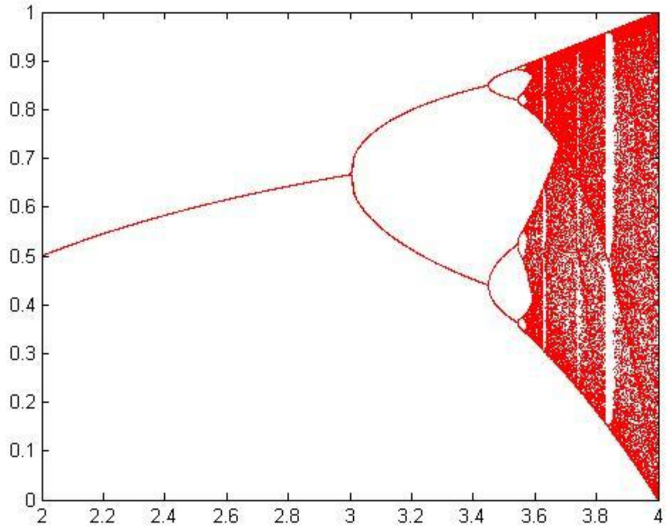
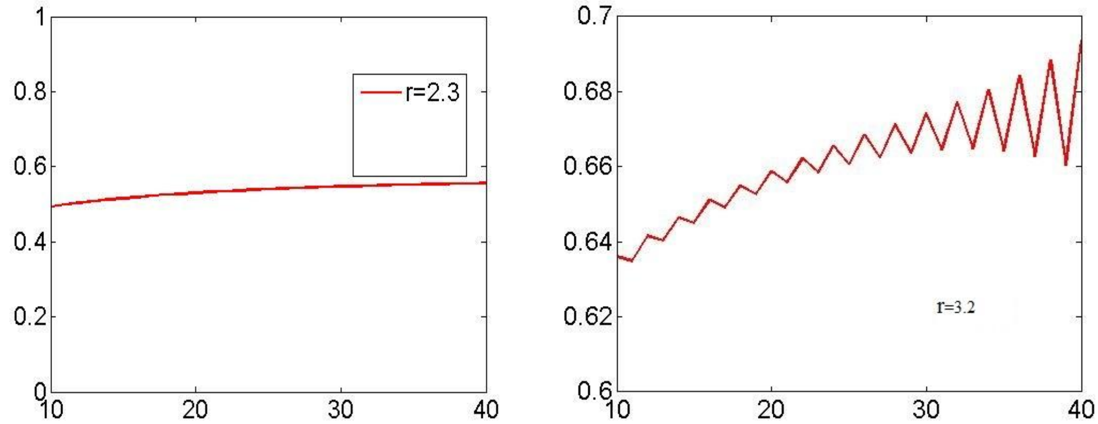


Fig 4. Bifurcation diagram for prey population

Variation of prey population



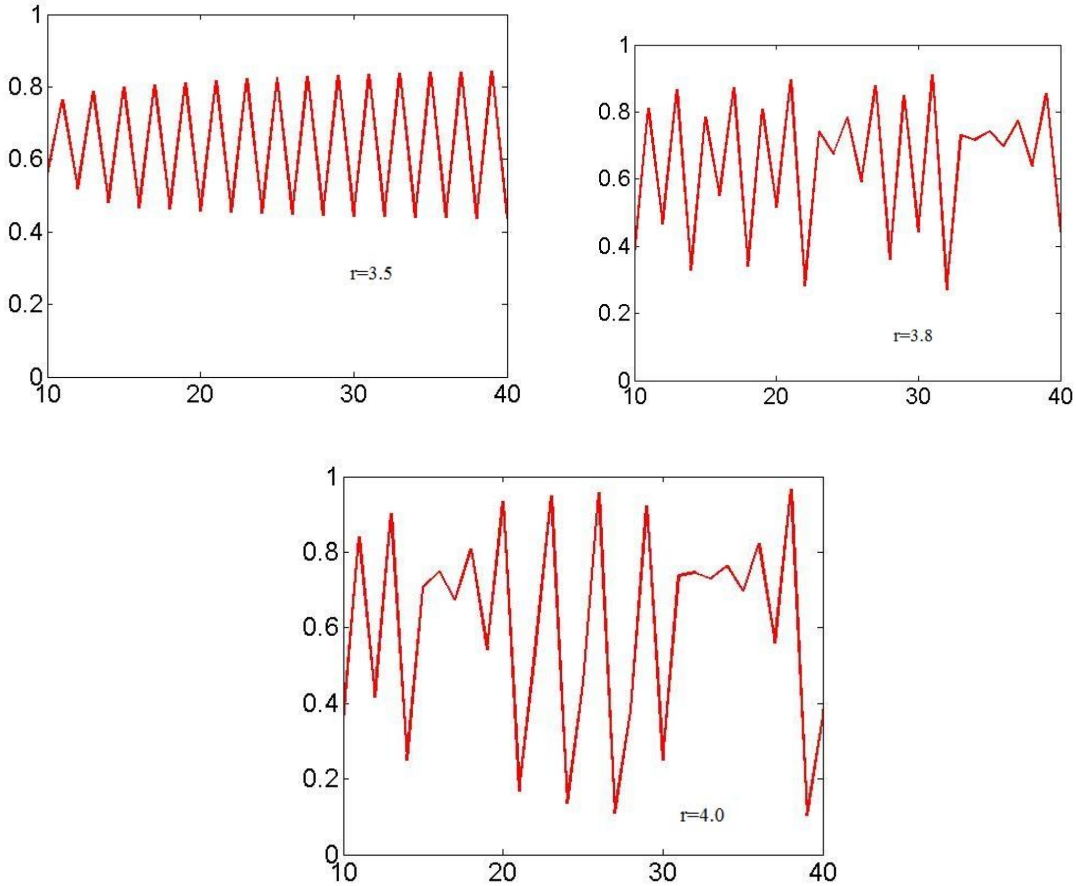


Fig 5. Variation of prey population

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