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International
Journal
Of Modern Engineering Research (IJMER)

A Finite Population Discrete Time Inventory System with Retrial Demands

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ABSTRACT: *In this paper, a discrete time inventory system with demands is generated by a finite number of homogeneous sources. The inter demand times of the primary arrival follows geometric distribution. The inventory is replenished according to an (s, S) policy and the lead times are assumed to follow a geometric distribution. The demands that occur during stock out period is permitted enter into the orbit. These orbiting demands retry for their demand after a random time, which is distributed as geometric. These orbiting demands send out signal to get their requirement and the time interval between two consecutive requests is distributed as geometric and the retrial policy is the multiplicative retrial policy. The joint probability distribution of the number of demands in the orbit and the inventory level is obtained in steady state case and some important system performance measures are derived in the steady state case.*

I. INTRODUCTION

Inventory models have been considered under continuous review as well as periodic review. In the recent past discrete time models have started receiving attention of researchers in the areas of queueing and telecommunications [2]. In discrete time setting, it is assumed that the time axis is calibrated into epochs by small units and that all the events are deemed to occur only at these epochs. With the advent of fast computing devices and efficient transaction reporting facilities, such epochs with small gaps can be conveniently assumed so that events can occur at these epochs.

In the case of inventory modelling under discrete times, the first paper was by Bar-Lev and Perry [5],who assumed that demands are non-negative integer value random variables and items have constant life times. Lian and Liu [8] developed a discrete time inventory model with geometrically distributed inter-demand times, bulk demands and constant life time for items. They assumed (0, S) ordering policy, with instantaneous supply which clears any backlog and restores the stock to the maximum capacity S. This assumption helped them to have fixed life time for all items. They derived the limiting distribution of inventory level through matrix-analytic method. D. L. Minh[10] developed the model of discrete time queue on the finite population with inter arrival time follows binomial distribution and general service times with first come first serve basis. Abboud [1] studied a discrete inventory model for production inventory systems with machine breakdowns. They assumed that the demand and production rates were constant and that the failure and repair times of each item were independently distributed as geometric. Lian et al. [9] developed a discrete time inventory system with discrete PH-renewal process for (batch) demand time points and assumed discrete- PH-distribution for life time of items. They also assumed zero lead time and that unsatisfied demand were completely backlogged.

I.Atentia and P.Moreno [4] developed a discrete time retrial queueing system with multiplicative repeated attempts. The authors used the retrial policy is the multiplicative one which analogous to the linear retrial policy in continuous-time[3]. The rest of the paper is organized as follows. In Section 2, we describe the problem under consideration and in the next section we model the problem mathematically. The steady-state analysis of the model is presented in Section 4 and some important system performance measures are derived in Section 5.

Notation:
- $[A]_{ij}$: entry at (i, j) th position of A
- 0 : zero vector
- I : identity matrix
- e : a column vector of 1's with appropriate dimension
- $\delta_{i,j}$: Kronecker delta function

II. PROBLEM FORMULATION

We consider a discrete time retrial inventory system where the time axis is divided into intervals of equal length, called slots (epochs). It is assumed that all system activities (arrivals, retrials and replenishment) occur at the slot boundaries, and therefore they may occur at the same time. The maximum capacity of the inventory is S.

- The arrival of demands generated from a homogeneous population of finite size(M). Each demands generates the arrival according to a Bernoulli stream with rate 1−p, thus 1−p is the probability that a demand arrives at a slot and p is the probability that an arrival does not take place in a slot. When the on-hand inventory level is more than one then the arriving demand is satisfied immediately.
- We adopt (s, S) ordering policy, that is, when the number of available items reaches the value s, an order for Q(= S−s > s) items is placed which is delivered after a lead time of geometric distribution with parameter q > 0. The condition S − s > s is assumed so that when the supply of an order is received during the stock out period, the inventory level would be brought above the reorder level. This provides an opportunity to place a reorder when a demand bring the inventory level down to the reorder level. Otherwise the inventory will have perpetual stock out.
- If the arrival finds the inventory level zero, he enters into the orbit of pending demands and he retry after a random amount of time. If more than one repeated demand retries at the same slot, any of them is randomly selected and the others must go back to the retrial group. The time between two successive repeated attempts is geometrically distributed with probability 1 − r

$1-r_1^{1-\delta_{0,k}}$, r_2^k ($((r_1,r_2)\varepsilon[0,1], r_1,r_2\neq 1)$) given that there are k demands in the orbit.

Unlike continuous review inventory systems, multiple events such as demand, supply and retrial from the orbit may occur between epochs n and n+1, n = 0, 1, 2, ... Hence we adopt the following convention:

If the events such as demand for an item, retrial from the orbit and supply of an order takes place at n (n = 1, 2, 3. . . .), it is assumed that first supply is received then demand occurs and finally retrial from the orbit takes place.

III. MODEL DESCRIPTION

Let Xn denote the number of demands waiting at the orbit, and Ln denote the inventory level at time n. From the assumptions made on the input and output processes, it can be shown that the stochastic process (X,L) = {(Xn,Ln), n ∈ N} is a Discrete Time Markov Chain with state space given by

E = {(i, j) : i = 0, 1, . . . ,M, j = 0, 1, . . . , S}.

The transition probability function is defined as for (i, j), (k, l) ∈ E,

p((i, j), (k, l)) = Pr[Xn+1 = k,Ln+1 = l|Xn = i,Ln = j].

The transition probability matrix P of this process,

P = ((p((i, j), (k, l)))), (i, j), (k, l) ∈ E

Then the transition probability matrix P can be viewed as,

$$P=\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ \cdots \\ M-1 \\ M \end{array}\begin{array}{c}\begin{array}{ccccccc} 0 & 1 & 2 & 3 & \cdots & M-1 & M \end{array}\\ \left|\begin{array}{ccccccc} A_0 & B_0 & & & & & \\ C_1 & A_1 & B_1 & & & & \\ & C_2 & A_2 & B_2 & & & \\ & & . & . & . & & \\ & & & . & . & . & \\ & & & & C_{M-1} & A_{M-1} & B_{M-1} \\ & & & & & C_M & A_M \end{array}\right|\end{array}$$

where,

$$[A_0]_{ij} = \begin{cases} qp^M & j = Q+i \quad i = 0,1,2,.........s \\ q(1-p^M) & j = Q+i-1 \quad i = 0,1,2,..........s \\ \bar{q}p^M & j = i \quad i = 0,1,2,............s \\ p^M & j = i \quad i = 0,1,2,...........s \\ 1-p^M & j = i-1 \quad i = s+1, s+2,.............S \\ 0 & otherwise \end{cases}$$

For k = 0, 1, . . . ,M − 1,

$$[B_k]_{ij} = \begin{cases} \bar{q}(1-p^{M-k}), j = 0 \quad , i = 0 \\ 0, otherwise \end{cases}$$

For k=1,2,3.........M

$$[A_k]_{ij} = \begin{cases} p^{M-k}qr_1r_2^k & j = Q+i \quad i = 0,1,2,.........s \\ (1-p^{M-k})qr_1r_2^k & j = Q+i-1 \quad i = 0,1,2,..........s \\ (1-\delta_{k,M})p^{M-k}\bar{q} + \delta_{k,M}\bar{q} & j = 0 \quad i = 0, \\ (1-p^{M-k})\bar{q} & j = i \quad i = 1 \\ p^{M-k}\bar{q}r_1r_2^k & j = i-1 \quad i = 1,2,.............s \\ (1-p^{M-k})\bar{q}r_1r_2^k & j = i-1 \quad i = 2,3,.............s \\ p^{M-k}r_1r_2^k & j = i \quad i = s+1, s+2,.............S \\ (1-p^{M-k})r_1r_2^k & j = i-1 \quad i = s+1, ,.............S \\ 0 & otherwise \end{cases}$$

$$[A_k]_{ij} = \begin{cases} p^{M-k}q(1-r_1r_2^k) & j = Q+i-1 \quad i = 0,1,2,.........s \\ (1-p^{M-k})q(1-r_1r_2^k) & j = Q+i-2 \quad i = 0,1,2,..........s \\ p^{M-k}\bar{q}(1-r_1r_2^k) & j = i-1 \quad i = 1,2,3,.........., s \\ (1-p^{M-k})\bar{q}(1-r_1r_2^k) & j = i-2 \quad i = 2,3,4........s \\ p^{M-k}(1-r_1r_2^k) & j = i-1 \quad i = s+1, s+2,.............S \\ (1-p^{M-k})(1-r_1r_2^k) & j = i-1 \quad i = s+1, s+2,.............S \\ 0 & otherwise \end{cases}$$

Here we notice that ,all the matrices A_k, B_k and c_k are square matrix of order S+1.

IV. Calculating Limiting Probabilities

It can be seen from the structure of P,the homogeneous Markov chain $\{(X_n, L_n), n \in \mathrm{N}\}$ on the finite state space is irreducible.Hence the limiting probability distribution

$$\pi^{(i,j)} = \lim_{n\to\infty} P_r[X_n = i, L_n = j] \mid X_0 = k, L_0 = l],$$

Where $\pi^{(i,j)}$ is the steady state probability for the state (i,j) exists and is independent of the initial state (k,l).Let Π be the steady state limiting probability vector of P.That is Π satisfies

$$\Pi P = \Pi$$

and

$$\Pi e = 1.$$

The vector Π can be represented by

$$\Pi = (\Pi^{<0>}, \Pi^{<1>}, \Pi^{<2>}, ...\Pi^{<M>})$$

$$\Pi^{<i>} = (\Pi^{(i,0)}, \Pi^{(i,1)}, \Pi^{(i,2)}, ...\Pi^{(i,S)}),$$ for i=0,1,2,.........M.

Now the structure of P shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the algorithm discussed by Gaver et al.[6] for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

Algorithm:

1. **Determine recursively the matrix Dn, 0 □ n □ M by using**

   $D_0 = A_0$

   $D_i = A_i + C_i(I - D_{i-1})-1B_0$, i = 1, 2, . . . ,M.

2. **Solve the system**

   $$\Pi^{<M>}(I - D_M) = 0$$

3. **Compute recursively the vector** $\Pi^{<i>}, i = M-1, ...0$ **using**

   $$\Pi^{<i>} = (\Pi^{<i+1>} C_{i+1} (I - D_i)^{-1}, i = M-1, ...0$$

4. **Normalize the vector** Π**, by using**

   $$\Pi e = 1.$$

V. System Performance Measures

In this section, we derive some system performance measures in the steady-state case.

5.1 Expected inventory level

Let ζ_i denote the expected inventory level in the steady-state. Then _i is given by

$$\zeta_i = \sum_{i=0}^{M} \sum_{j=1}^{S} j \pi^{(i,j)}$$

5.2 Expected reorder rate

Let ζ_r denote the expected reorder level in the steady-state. Then _r is given by

$$\zeta_r = (1 - P^M)\pi^{(0,s+1)} + \sum_{i=1}^{M} [(r_1 r_2^i (1 - P^{M-i}) + (1 - r_1 r_2^i) P^{M-i}))\pi^{(i,s+1)} + (1 - P^{M-i})(1 - r_1 r_2^i)\pi^{(i,s+2)}]$$

5.3 Expected number of customer in the orbit

Let ζ_0 denote the expected number of customer in the orbit in the steady-state. Then _o is given by

$$\zeta_0 = \sum_{i=1}^{M} i \Pi^{<i>} e = \sum_{i=1}^{M} \sum_{j=0}^{S} i \pi^{(i,j)}$$

5.4 Fraction of successful rate of retrials

The fraction of successful rate of retrials is given by

$$\zeta_{FR} = \frac{The\,successful\,rate\ of\ retrial}{The\,over\,all\ rate\ of\ retrial} = \frac{\sum_{i=1}^{M} \sum_{j=2}^{S} (1 - r_1 r_2^i) \pi^{(i,j)} + \sum_{i=1}^{M} p^{(M-i)} (1 - r_1 r_2^i) \pi^{(i,j)}}{\sum_{i-1}^{M} \sum_{j-0}^{S} (1 - r_1 r_2^i) \pi^{(i,j)}}$$

5.5 Cost function

The long-run expected cost rate for this model is defined to be

$$C(S,s,M) = c_h\zeta_i + c_s\zeta_r + c_w\zeta_0$$

where,

cs : Setup cost per order
ch : The inventory carrying cost per unit item per unit time
cw : Waiting cost of a customer in the orbit per unit time

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