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Design a PID Controller by Using Ziegler's Nichols Method via a Mixed Model Order Reduction Techniques

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ABSTRACT : *In this paper presents, a tuned parameters PID controller is designed for the original higher order system through its lower order reduced model. The PID parameters are obtained by using classical control technique and these parameters are tuned through Ziegler's Nichols method using MATLAB software. The designing of PID controller can improve settling time and steady state response. Designing of a controller or compensator for the original higher order system is complex and costly. So to avoid the complexity, first controller is designed for the reduced order model of the original system and then this designed controller is connected to the original system. Defiantly, this Designed PID controller can improve the time response characteristics of the original higher order system. So in this paper for determining the reduced order model, a mixed method is used. The combination of PSO and Polynomial are the methods used to reduce the high order system to low order system. PSO technique is used for determining the denominator coefficients and Polynomial method is used for obtaining the numerator coefficients. The original and reduced order systems without PID controller and with PID controller are tested with numerical examples.*

Key Words: *PID controller, Ziegler's Nichols method, PSO optimization, polynomial technique, order reduction, transfer function, settling time, steady state error.*

I. Introduction

The order reduction techniques play a very important role in the control systems. The design of controllers and compensator for original system is costly and tedious. . In order to perform simulation analysis or control design on those higher order models one will face many difficulties. Designing of compensator and controller to the reduced order system is easy and simple. So to avoid the above problem order reduction is necessary.

There are many order reduction methods proposed in both time domain and frequency domain for reducing order from the higher order system to lower order system. One of the order reduction method is pade approximation, Shamash [1] proposed this method. The disadvantage of the pade method is that once the reduced order model is unstable, if the poles of the original higher order systems are more, then reduced order model is checked the stability for every time. Another method is Routh approximation, Hutton and Friendland [2] proposed this method. The drawback of the rough approximation method is no guarantee of the stability of the reduced order system.

To reduce the order from higher order to lower order system Evolutionary techniques [6] are used. So many Evolutionary techniques are there, among all the Evolutionary techniques Particle swarm optimization technique is most useful technique. Particle swarm optimization technique is similar to the Genetic algorithm technique. The difference between the PSO and Genetic algorithm is the PSO is an easy and simple algorithm. In this paper a tuned parameter PID controller is designed for the original higher order system through its lower order system. By designing the PID controller, we have to improve the settling time and steady state error of the original and reduced order systems. The reduce order model is obtained the mixed methods. The mixed methods are a PSO and Polynomial method.

II. PROCEDURE FOR PSO AND POLYNOMIAL METHODS

II.1. Procedure for PSO:

The original higher order system transfer function of order 'n' is

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0+a_1s+a_2s^2+\cdots\ldots\ldots a_{m-1}S^{n-1}}{b_0+b_1s+b_2s^2+\cdots\ldots\ldots b_n s^n} \quad (1)$$

The reduced order system transfer function of order 'k' is

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{d_0+d_1s+d_2s^2+\cdots\ldots\ldots d_{k-1}s^{k-1}}{e_0+e_1s+e_2s^2+\cdots\ldots\ldots e_k s^k} \quad (2)$$

The PSO method is the one very important optimization method for solving the optimization problems. It is the swarm intelligence technique and computer oriented algorithm. The particles are the very important for the PSO algorithm, these particles are randomly generated from initial population. These particles are moving in the search space according to updating its own position and velocity. Every particle in the algorithm is affected by its personal best (pbest) that is also called as local best and global best (g best) [13] for obtaining the best position in the search space. In this PSO algorithm for every iteration the values of personal best and global best are calculated and by using these values position and velocity of the particles are updated. For each iteration the best position values are called personal best (pbest) and among all the iteration the best fitness value in the swarm or population is called global best (gbest).the velocity and position values are updated by using below formulae.

$$v_{id}^{k+1} = v_{id}^{k} + c_1 * r_1 * \left(p_{id}^{k} - x_{id}^{k}\right) + c_2 * r_2 * (p_{gd}^{k} - x_{id}^{k}) \quad (3)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k} \quad (4)$$

Where c_1 and c_2 are the cognition and social parameters and r_1 and r_2 are the random parameters. c_1 and c_2 values generally taken as '2' these parameters are either variable or constant. The ranges of random parameters are in between 0 and 1. And p_{id} and p_{gd} are personal best and global best values. For balancing the personal best or local best and global best values inertia weight 'w' is introduced.

$$w = w_f + \left(w_f - w_i\right)(\max it - it)/it \quad (5)$$

Then the above equation 3 can be written as

$$v_{id}^{k+1} = w * v_{id}^{k} + c_1 * r_1 * \left(p_{id}^{k} - x_{id}^{k}\right) + c_2 * r_2 * (p_{gd}^{k} - x_{id}^{k}) \quad (6)$$

II.2. Procedure for Polynomial method:

For finding the values of the numerator coefficients polynomial method is used. The numerator coefficients are obtained by equating the original higher order system and reduce order system.

$$\frac{a_0+a_1s+a_2s^2+\cdots\ldots\ldots a_{n-1}s^{n-1}}{b_0+b_1s+b_2s^2+\cdots\ldots\ldots b_ns^n} = \frac{d_0+d_1s+d_2s^2+\cdots\ldots\ldots d_{k-1}s^{k-1}}{e_0+e_1s+e_2s^2+\cdots\ldots\ldots e_ks^k} \quad (7)$$

Then equate the power's of 's' on both sides

$$a_0e_0 = b_0d_o \quad (8)$$

$$a_0e_1 + a_1e_0 = b_0d_1 + b_1d_0 \quad (9)$$

$$a_0e_2 + a_1e_1 + a_2e_0 = b_0d_2 + b_1d_1b_2d_0 \quad (10)$$

$$a_{n-1}e_k = b_nd_{k-1} \quad (11)$$

By solving the above equations we get the values of d_0,d_1 and d_2,….d_q

III. PID Controller

The combinations of suitable parameters that are proportional, integral and derivative controller can improve the all aspects of system performances. The prortional controller settle the gain, but produces a steady state error. The integral controller decreases or avoid the steady state error The derivative controller reduces the rate of error. By using this PID controller we can improve the settling time and steady state errors.

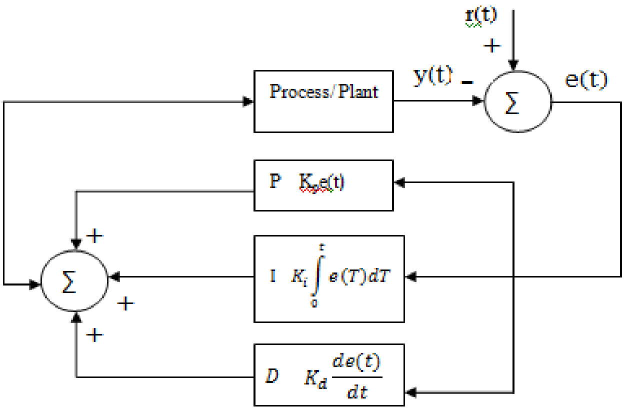

Fig-1: PID controller Block diagram

In order to obtain the output of the above controller, the P, I and D controller terms are added. Then the final expression of PID controller is,

$$u(t) = K_pe(t) + K_i\int_0^t e(T)dT + K_d\frac{d}{dt}e(t) \quad (12)$$

In this paper, PID controller is designed and tuned by using Ziegler's Nichols method. The PID controller parameters are tuned by classical control technique through Ziegler's Nichols method in MATLAB software. The tuned parameters of PID controller using Ziegler's Nichols method are shown in below table.

Table 1: Values P, PI, PID using Ziegler's Nichols method

Type of Controller	**K_p**	**K_i**	**K_d**
P	$0.5K_u$	∞	0
PI	$0.45K_u$	$\frac{1.2\,K_p}{T_u}$	0
PID	$0.60K_u$	$\frac{2\,K_p}{T_u}$	$\frac{K_p T_u}{8}$

IV. Numerical Examples

IV.1. Example:

IV.1.1. Determination of reduced order system:

Consider the original higher order transfer function of an order '5' [6]

$$G(s)= \frac{10s^4+82s^3+264s^2+396s+156}{2s^5+21s^4+84s^3+173s^2+148s+40} \quad (13)$$

Numerator coefficients are obtained by PSO algorithm. For implementing this several parameters are to be considered. The cognition c_1 and social parameters c_2 are '2' and the range of random parameters are in between 0 and 1.

Population size or Swarm size = 30

Coefficients to be finding = 2

Maximum number of iterations = 300

The obtained denominator polynomial for second order reduced model by using PSO is

$$D_2(s) = 2.36919 + 3.918816s + s^2 \quad (14)$$

For finding the values of the numerator coefficients polynomial method is used. The numerator coefficients are obtained by equating the original higher order system and reduce order system

$$\frac{10s^4+82s^3+264s^2+396s+156}{2s^5+21s^4+84s^3+173s^2+148s+40} = \frac{d_0+d_1s}{2.36919+3.91886s+s^2} \quad (15)$$

On equating the power's of 's' on both sides we get numerator coefficients and multiplying numerator with k to match the steady state response .

Therefore, the numerator polynomial by polynomial method is

$$N_2(s) = 9.255+5.717s \quad (16)$$

The obtained reduced order model by using PSO and Polynomial method is shown below.

$$R_2(s) = \frac{9.255+5.717s}{2.36919+3.91886s+s^2} \quad (17)$$

IV.1.2. PID Controller design using reduced order model:

For designing PID controller first reduced original higher order system into lower order system. The obtained reduced order model by using PSO and Polynomial method is shown below.

$$R_2(s) = \frac{9.255+5.717s}{2.36919+3.91886s+s^2} \quad (18)$$

For the above reduced order model the PID controller is designed. The tuning parameters of PID controller are obtained by using Ziegler's Nichols method:

$K_p = 2.24096$ $K_i = 14.006$ $K_d = 0$

The transfer function of PID controller is

$$G_d(s) = \frac{14.006+2.24096s}{s} \quad (19)$$

The reduced order system with PID controller is

$$R_{2C}(s) = \frac{12.81s^2+100.8s+129.6}{s^3+16.73s^2+103.2s+129.6} \quad (20)$$

The original high order system with PID controller is

$$G(s)= \frac{22.41s^5+323.8s^4+1740s^3+4585s^2+5895s+2185}{2s^6+43.41s^5+407.8s^4+1913s^3+4733s^2+5936s+2185} \quad (21)$$

Table 2: comparison of time respons characteristics with and without PID controller

	Settling time (Sec)	**Rise time (Sec)**	**Steady state value**
Original higher order system without PID	5.45	2.76	3.9
Original order system with PID controller	0.68	0.124	1
Reduced order system with PID	4.77	2.54	3.91
Reduced order system with PID controller	0.616	0.112	1

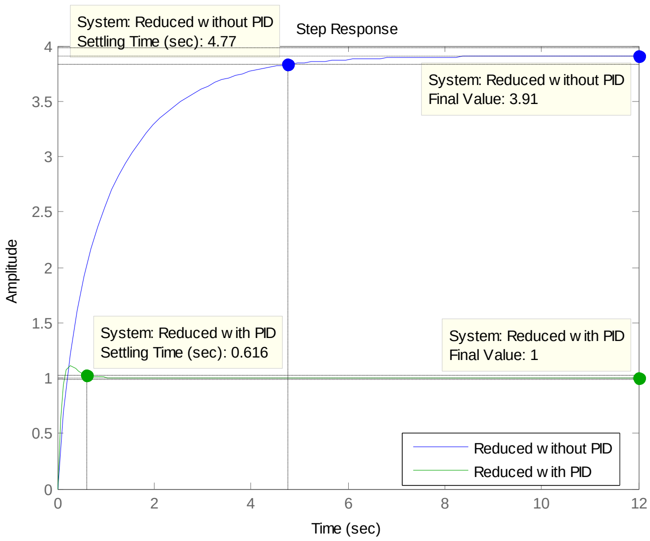

Fig-2: Step responses of reduced order model with and without PID controller

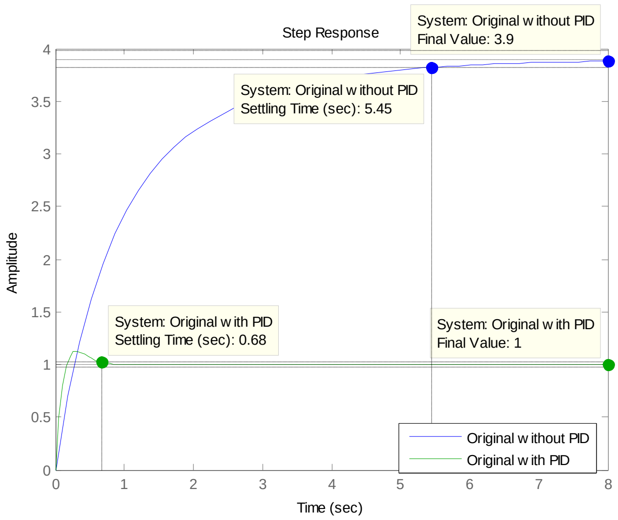

Fig-3: Step responses of original order model with and without PID controller.

IV.2. Example:

IV.2.1. Determination of reduced order system:

Consider the original higher order transfer function of an order '6' [4]

$$G(s) = \frac{3s^5+108s^4+1221s^3+5792s^2+11860s+8400}{s^6+41s^5+571s^4+3491s^3+10060s^2+13100s+6000} \quad (22)$$

Numerator coefficients are obtained by PSO algorithm. For implementing this several parameters are to be considered. The cognition c_1 and social parameters c_2 are '2' and the range of random parameters are in between 0 and 1.

Population size or Swarm size = 30

Coefficients to be finding = 2

Maximum number of iterations = 300

The obtained denominator polynomial for second order reduced model by using PSO is

$$D_2(s) = 8.714992 + 8.630098s + s^2 \quad (23)$$

For finding the values of the numerator coefficients polynomial method is used. The numerator coefficients are obtained by equating the original higher order system and reduce order system

$$\frac{3s^5+108s^4+1221s^3+5792s^2+11860s+8400}{s^6+41s^5+571s^4+3491s^3+10060s^2+13100s+6000} = \frac{d_0+d_1s}{8.714992+8.630098+s^2} \quad (24)$$

On equating the power's of 's' on both sides we get numerator coefficients and multiplying numerator with k to match the steady state response

Therefore, the numerator polynomial by polynomial method is

$$N_2(s) = 12.2039 + 3.355s \tag{25}$$

The obtained reduced order model by using PSO and Polynomial method is shown below.

$$R_2\,(s) = \frac{12.2039+3.355s}{8.714992+8.630098\,s+s^2} \tag{26}$$

IV.2.2. PID Controller design using reduced order model:

For designing PID controller first reduced original higher order system into lower order system. The obtained reduced order model by using PSO and Polynomial method is shown below.

$$R_2(s) = \frac{3.355s+12.2039}{s^2+8.630098\,s+8.714992} \tag{27}$$

For the above reduced order model the PID controller is designed. The tuning parameters of PID controller are obtained by using Ziegler's Nichols method:

$K_p = 6.46656$ $\quad K_i = 53.888$ $\quad K_d = 0$

The transfer function of PID controller is

$$G_d\,(s) = \frac{53.888+6.46656\,s}{s} \tag{28}$$

The reduced order system with PID controller is

$$R_{2C}(s) = \frac{21.7s^2+259.75s+657.6}{s^3+30.335\,s^2+268.4s+657.6} \tag{29}$$

The original high order system with PID controller is

$$G(s)= \frac{19.4s^6+860.1s^5+13720\,s^4+103300\,s^3+3888000\,s^2+693400\,s+452700}{s^7+60.4s^6+1431\,s^5+17210\,s^4+113300+401900\,s^2+699400\,s+452700} \tag{30}$$

Table 2: comparison of time response characteristics with and without PID controller

	Settling time (Sec)	Rise time (Sec)	Steady state value
Original higher order system without PID	3.47	1.84	1.4
Original order system with PID controller	0.529	0.0839	1
Reduced order system with PID	3.16	1.74	1.4
Reduced order system with PID controller	0.517	0.0761	1

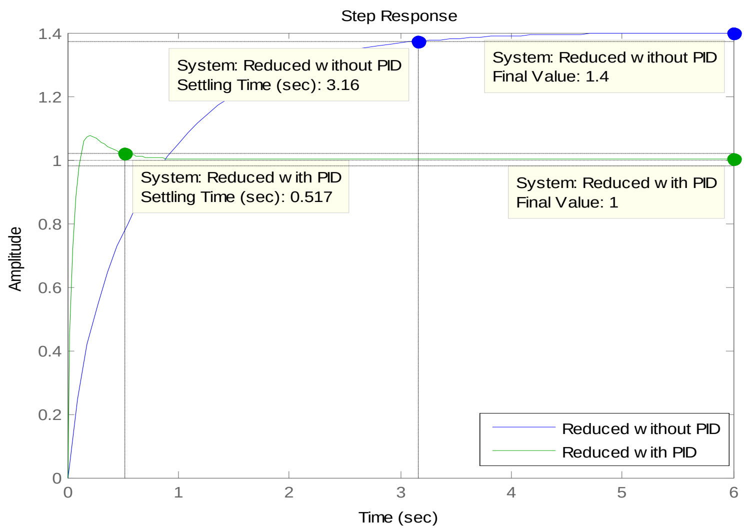

Fig-4: Step responses of reduced order model with and without PID controller.

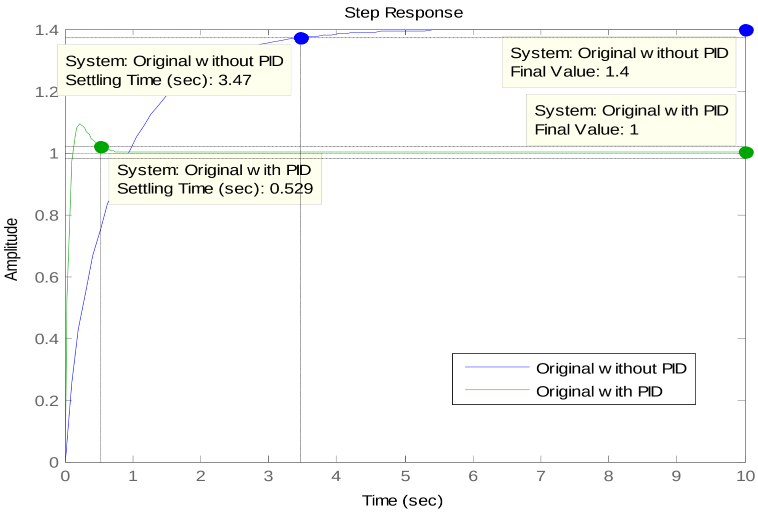

Fig-5: Step responses of original order model with and without PID controller

V. Conclusion

In this paper, a tuned parameters of PID controller is designed for original higher order system through its lower order reduced model. The PID parameters are obtained by using classical control technique and these parameters are tuned through Ziegler's Nichols method using MATLAB software. The designed PID controller is connected to original and reduced order systems. The design of PID controller improves the settling time and steady state error than the proposed reduced order method. The combination of PSO and Polynomial are the methods used to reduce the high order system to low order system. PSO technique is used for determining the denominator coefficients and Polynomial method is used for obtaining the numerator coefficients. The original and reduced order systems without PID controller and with PID controller are tested with numerical examples.

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