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Momentum and Thermal Slip Flow of MHD Casson Fluid over a Stretching Sheet with Viscous Dissipation

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Abstract: *The two-dimensional magneto hydrodynamic (MHD) stagnation-point flow of electrically conducting non-Newtonian Casson fluid and heat transfer towards a stretching sheet in the presence of viscous dissipation, momentum and thermal slip flow is considered. Implementing similarity transformations, the governing momentum, and energy equations are transformed to self-similar nonlinear ODEs and numerical computations are performed to solve those. The investigation reveals many important aspects of flow and heat transfer. If velocity ratio parameter (C) and momentum slip parameter (S), magnetic parameter (M) increase, then the velocity boundary layer thickness becomes thinner. On the other hand, for Casson fluid it is found that the velocity boundary layer thickness is larger compared to that of Newtonian fluid. The magnitude of wall skin-friction coefficient reduces with Casson parameter (β) and also wall skin-friction coefficient increases with S for C<1 whereas decreases with S for C>1. The velocity ratio parameter, Casson parameter, and magnetic parameter also have major effects on temperature distribution. The heat transfer rate is enhanced with increasing values of velocity ratio parameter. The rate of heat transfer is reduces with increasing thermal slip parameter b. Moreover, the presence of viscous dissipation reduces temperature and thermal boundary layer thickness.*

Keywords: *MHD, Casson Fluid, Heat Transfer, Stretching Sheet, Viscous Dissipation, Momentum Slip, Thermal Slip.*

I. Introduction

Non-Newtonian fluids are widely used in industries such as chemicals, cosmetics, pharmaceuticals, food and oil & gas [1]. Due to their numerous applications several scientists and engineers are working on them. Despite of the fact non-Newtonian fluids are not as easy as Newtonian fluids. It is due to the fact that in non- Newtonian fluids there does not exist a single constitutive relation that can be used to explain all of them. Therefore several constitutive equations or models are introduced to study their characteristics. The different non-Newtonian models include power law [2], second grade [3], Jeffrey [4], Maxwell [5], viscoplastic [6], Bingham plastic [7], Brinkman type [8], Oldroyd-B [9] and Walters-B [10] models. However, there is another model known as Casson model which is recently the most popular one. Casson [11] was the first who introduce this model for the prediction of the flow behaviour of pigment oil suspensions of the printing ink type. Later on, several researchers studied Casson fluid for different flow situations and configurations. Hussanan et al. [12] studied the unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. Eldabe and Elmohands [13] have studied the Casson fluid for the flow between two rotating cylinders and Boyd et al. [14] investigated the Casson fluid flow for the steady and oscillatory blood flow. Mustafa et al. [15] analyzed the unsteady flow of a Casson fluid for a flat plate. Recently, Mukhopadhyay and Gorla [16] conclude that momentum boundary layer thickness decreases with increasing Casson parameter.

In fluid dynamics the effects of external magnetic field on magnetohydrodynamic (MHD) flow over a stretching sheet are very important due to its applications in many engineering problems, such as glass manufacturing, geophysics, paper production, and purification of crude oil. The flow due to stretching of a flat surface was first investigated by Crane [17]. Pavlov [18] studied the effect of external magnetic field on the MHD flow over a stretching sheet. Andersson [19] discussed the MHD flow of viscous fluid on a stretching sheet and Mukhopadhyay et al. [20] presented the MHD flow and heat transfer over a stretching sheet with variable fluid viscosity. On the other hand, Fang and Zhang [21] reported the exact solution of MHD flow due to a shrinking sheet with wall mass suction. Nadeem et al. [22] examined the magnetohydrodynamic (MHD) boundary layer flow of a Casson fluid over an exponentially penetrable shrinking sheet. Bhattacharyya [23] conclude that the velocity boundary layer thickness for Casson fluid is larger than that of Newtonian fluid and also magnitude of wall skin-friction coefficient decreases with Casson parameter.

Many researchers have studied, both analytically and numerically, slip boundary layer problems over different surface configurations. Wang [24] investigated the effect of surface slip and suction on viscous flow over a stretching sheet. Sajid et al. [25] analyzed the stretching flow with general slip condition. Sahoo [26] investigated the flow and heat transfer solution for third grade fluid with partial slip boundary condition. Bhattacharyya et al. [27] analyzed the boundary layer force convection flow and heat transfer past a porous plate embedded in the porous medium with first order velocity and temperature slip effect. Noghrehabadi et al. [28] analyzed the effect of partial slip on flow and heat transfer of nanofluids past a stretching sheet. Zheng et al. [29] analysed the effect of velocity slip with temperature jump on MHD flow and heat transfer over a porous shrinking sheet. However, in all of these papers, only the first order Maxwell slip condition was considered. Recently, Wu [30] proposed a new second order slip velocity model. Fang et al. [31] analyzed the effect of second order slip on viscous fluid flow over a shrinking sheet. Fang and Aziz [32] studied the flow of a viscous fluid with a second order slip over a stretching sheet without considering the heat transfer aspect. Nandeppanavar et al. [33] studied the second order slip flow and heat transfer over a stretching sheet. Gbadeyan and Dada [34] studied the effects of radiation and heat transfer on a MHD non-Newtonian unsteady flow in a porous medium with slip condition. Piotr Bogusławmucha [35] studied on Navier–Stokes equations with slip boundary conditions in an infinite pipe. Sharma and Ishak [36] investigated the second order slip flow of Cu-water Nanofluid over a stretching sheet With Heat Transfer

Viscous dissipation effects are usually ignored in macro scale systems, in laminar flow in particular, except for very viscous liquids at comparatively high velocities. However, even for common liquids at laminar Reynolds numbers, frictional effects in micro scale systems may change the energy equation [37]. Koo and Kleinstreuer [38] have investigated the effects of viscous dissipation on the temperature field using dimensional analysis and experimentally validated computer simulations. Yazdi et al. [39] evaluates the slip MHD flow and heat transfer of an electrically conducting liquid over a permeable surface in the presence of the viscous dissipation effects under convective boundary conditions. Okoya [40] investigated the transition for a generalised couette flow of a reactive third-grade fluid with viscous dissipation. El-Aziz [41] conclude that the dimensionless temperature will increase when the fluid is being heated (Ec >0) but decrease when the fluid is being cooled (Ec <0). The dimensionless fluid temperature decreases with η monotonically for a positive Ec; while for a negative Eckert number θ initially decreases rapidly with η, attains a minimum value, and then increases more gradually to its free-stream value. Gangadhar [42] studied numerically the effects of internal heat generation and viscous dissipation on boundary layer flow over a vertical plate with a convective surface boundary condition. Salem [43] discussed coupled heat and mass transfer in Darcy-Forchheimer Mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation.

However, the interactions of magnetohydrodynamic (MHD) stagnation-point flow of electrically conducting non-Newtonian Casson fluid and heat transfer towards a stretching sheet in the presence of viscous dissipation, momentum and thermal slip flow is considered. The governing boundary layer equations have been transformed to a two-point boundary value problem in similarity variables and the resultant problem is solved numerically using bvp4c MATLAB solver. The effects of various governing parameters on the fluid velocity, temperature, Skin-friction and local Nusselt number are shown in figures and analyzed in detail.

II. Mathematical Formulation

Consider the steady two-dimensional incompressible flow of viscous and electrically conducting and Casson fluid bounded by a stretching sheet at $y = 0$, with the flow being confined in $y > 0$. It is also assumed that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as [44, 45]

$$\tau_{ij} = \begin{cases} \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right) 2e_{ij}, & \pi > \pi_c \\ \left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right) 2e_{ij}, & \pi < \pi_c \end{cases} \tag{2.1}$$

where μ_B is plastic dynamic viscosity of the non-Newtonian fluid, p_y is the yield stress of fluid, π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the $(i, j)^{\text{th}}$ component of the deformation rate, and π_c is critical value of π based on non-Newtonian model. The governing equations of motion and the energy equation may be written in usual notation as

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.2}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_s\frac{dU_s}{dx}+\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma H_0^2}{\rho}\left(u-U_s\right) \tag{2.3}$$

Energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.4}$$

The boundary conditions are

$$u=N_1\upsilon\frac{\partial u}{\partial y},v=0,T=T_w+D_1\frac{\partial T}{\partial y}\quad \text{at}\quad y=0$$

$$u\to U_s,T\to T_\infty \qquad \text{as}\quad y\to\infty \tag{2.5}$$

where u and v are the velocity components in x and y directions, respectively, $U_s = ax$ is the straining velocity of the stagnation-point flow with a (>0) being the straining constant, υ is the kinematic fluid viscosity, ρ is the fluid density, $\beta=\mu_B\sqrt{2\pi_c}/p_y$ is the non-Newtonian or Casson parameter, N_1 is the velocity slip factor, D_1 is the thermal slip factor, σ is the electrical conductivity of the fluid, T is the temperature, k is the thermal conductivity, c_p is the specific heat, and q_r is the radiative heat flux, T_w is the constant temperature at the sheet and T_∞ is the free stream temperature assumed to be constant, and H_0 is the strength of magnetic field applied in the y direction, with the induced magnetic field being neglected. Using the Rosseland approximation for radiation [46], $q_r=\frac{-4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y}$ is obtained, where σ^* the Stefan-Boltzmann constant is and k_1 is the absorption coefficient. We presume that the temperature variation within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher-order terms we get $T^4=4T_\infty^3T-3T_\infty^4$

Now (2.4) reduces to

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3k_1\rho c_p}\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.6}$$

We introduce also a stream functions ψ is defined by

$$u=\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\psi}{\partial x} \tag{2.7}$$

Equations (2.3), (2.6) and (2.5) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U_s\frac{dU_s}{dx}+\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^3\psi}{\partial y^3}-\frac{\sigma H_0^2}{\rho}\left(\frac{\partial\psi}{\partial y}-U_s\right) \tag{2.8}$$

$$\rho c_p\left(\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\mu\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{2.9}$$

The boundary conditions are

$$\frac{\partial\psi}{\partial y}=N_1\upsilon\frac{\partial^2\psi}{\partial y^2},\frac{\partial\psi}{\partial x}=0,T=T_w+D_1\frac{\partial T}{\partial y}\quad\text{at}\quad y=0$$

$$\frac{\partial\psi}{\partial y}\to U_s,T\to T_\infty\qquad\text{as}\quad y\to\infty \tag{2.10}$$

Now, the dimensionless variable for the stream function is implemented as

$$\psi=\sqrt{cv}xf\left(\eta\right) \tag{2.11}$$

where the similarity variable η is given by $\eta=y\sqrt{c/\upsilon}$.

Using relation (2.11) and similarity variable, (2.8) to (2.10) finally takes the following self-similar form:

$$\left(1+\frac{1}{\beta}\right) f''' + ff'' - f'^2 - M\left(f' - C\right) + C^2 = 0 \qquad (2.12)$$

$$\left(3R+4\right)\theta'' + 3R\,\mathrm{Pr}\, f\theta' + \mathrm{Pr}\, Ecf''^2 = 0 \qquad (2.13)$$

The transformed boundary conditions can be written as

$$f = 0,\ f' = Sf'',\ \theta = 1 + b\theta' \quad \text{at} \quad \eta = 0$$
$$f' \to C,\ \theta \to 0 \quad \text{as} \quad \eta \to \infty \qquad (2.14)$$

where primes denote differentiation with respect to η, $M = \sigma H_0^2 / \rho c_p$ is the magnetic parameter, and $C = a/c$ is the velocity ratio parameter, $\mathrm{Pr} = \mu c_p / k$ is the Prandtl number, and $R = k^* k_1 / 4\sigma T_\infty^3$ is the thermal radiation parameter.

The physical quantities of interest are the wall skin friction coefficient C_{fx} and the local Nusselt number Nu_x, which are defined as

$$C_{fx} = \frac{\tau_w}{\rho U_w^2(x)},\ Nu_x = \frac{xq_w}{\alpha\left(T_w - T_\infty\right)} \qquad (2.15)$$

where τ_w is the shear stress or skin friction along the stretching sheet and q_w is the heat flux from the sheet and those are defined as

$$\tau_w = \left(\mu_B + \frac{p_y}{\sqrt{2\pi}}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$q_w = \alpha\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (2.16)$$

Thus, we get the wall skin friction coefficient C_{fx} and the local Nusselt number Nu_x as follows:

$$C_{fx}\sqrt{\mathrm{Re}_x} = \left(1+\frac{1}{\beta}\right) f''\left(0\right)$$
$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}} = -\theta'\left(0\right) \qquad (2.17)$$

where $\mathrm{Re}_x = \dfrac{U_w x}{\upsilon}$ is the local Reynolds number.

III. Solution Of The Problem

The set of equations (2.12) to (2.14) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called **bvp4c**. This program solves boundary value problems for ordinary differential equations of the form $y' = f\left(x, y, p\right), a \le x \le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g\left(y(a), y(b), p\right)$. Here ***p*** is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with **bvp4c**. The first step is to write the *ODEs* as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka[47].

IV. Results And Discussion

The abovementioned numerical scheme is carried out for various values of physical parameters, namely, the velocity ratio parameter (C), the magnetic parameter (M), the Casson parameter (β), momentum slip parameter (S), thermal slip parameter (b), the Prandtl number (Pr), Eckert number (Ec), and the thermal radiation parameter (R) to obtain the effects of those parameters on dimensionless velocity and temperature distributions. The obtained computational results are presented graphically in Figures 1-15 and the variations in velocity and temperature are discussed.

Firstly, a comparison of the obtained results with previously published data is performed. The values of wall skin friction coefficient $f''(0)$ for Newtonian fluid case $(\beta = \infty)$ in the absence of momentum slip parameter and external magnetic field for different values of velocity ratio parameter (C) are compared with those obtained by Mahapatra and Gupta [48], Nazar et al. [49] and Bhattacharya [23] in Table 1 in order to verify the validity of the numerical scheme used and those are found in excellent agreement.

The velocity and temperature profiles for various values of velocity ratio parameter C are plotted in Figs. 1 and 2, respectively. Depending on the velocity ratio parameter, two different kinds of boundary layers are obtained as described by Mahapatra and Gupta [50] for Newtonian fluid. In the first kind, the velocity of fluid inside the boundary layer decreases from the surface towards the edge of the layer (for C < 1) and in the second kind the fluid velocity increases from the surface towards the edge (for C > 1). Those characters can be seen from velocity profiles in Fig.1. Also, it is important to note that if C = 1 (a = c), that is, the stretching velocity and the straining velocity are equal, then there is no boundary layer of Casson fluid flow near the sheet, which is similar to that of Chiam's [51] observation for Newtonian fluid and also similar to that of Bhattacharya [23]. From Figure 2, it is seen that in all cases thermal boundary layer is formed and the temperature at a point decreases with C, which is similar to that of Bhattacharya [23] observation for Casson fluid. The effects of Casson parameter β on the velocity and temperature fields are depicted in Figs. 3 and 4. It is worthwhile to note that the velocity increases with the increase in values of β for C = 2 and it decreases with β for C = 0.1. Consequently, the velocity boundary layer thickness reduces for both values of C. Due to the increase of Casson parameter β, the yield stress p_y falls and consequently velocity boundary layer thickness decreases, these results are similar to that of Bhattacharya [23]. The influences of Casson parameter on the temperature profiles are different in two cases, C = 2 and C = 0.1. Temperature at a point decreases with increasing β for C = 2 and increases with increasing β for C = 0.1, which is similar to that of Bhattacharya [23] observation for Casson fluid.

In Figs. 5 and 6, the velocity and temperature profiles are presented for several values of magnetic parameter M. Similar to that of Casson parameter, due to the increase of magnetic parameter the dimensionless velocity at fixed η increases for C = 2 and for C = 0.1 the velocity decreases. Consequently, for both types of boundary layers, the thickness decreases. The Lorentz force induced by the dual actions of electric and magnetic fields reduces the velocity boundary layer thickness by opposing the transport phenomenon. Also, for C = 2, the temperature decreases with M and increases with M for C = 0.1, which is similar to that of Bhattacharya [23] observation for Casson fluid.

The dimensionless temperature profiles (η) for several values of Prandtl Number Pr and thermal radiation parameter R are exhibited in Figs. 7 and 8, respectively, for two values of C. In both cases (C = 0.1 and 2), the temperature decreases with increasing values of Prandtl number and radiation parameter and the thermal boundary layer thickness becomes smaller in all cases. Actually, the rate of heat transfer is enhanced with Prandtl Number and radiation parameter and this causes the reduction of thermal boundary layer thickness, which is similar to that of Bhattacharya [23] observation for Casson fluid.

The effects of momentum slip parameter S on the velocity is depicted in Figs. 9. It is worthwhile to note that the velocity increases with the increase in values of S for C = 2 and it decreases with S for C = 0.1. The dimensionless temperature profiles for several values of thermal slip parameter b and Eckert number Ec are depicted in Figs. 10 and 11, respectively, for two values of C. In both cases (C = 0.1 and 2), the temperature decreases with increasing values of thermal slip parameter and the thermal boundary layer thickness becomes smaller in all cases (Fig.10), where as temperature increases with rising the values of Eckert number and the thermal boundary layer thickness becomes larger in all cases (fig.11).

The physical quantities, the wall skin friction coefficient C_{fx}, and the local Nusselt number Nu_x, which have immense engineering applications, are proportional to the values of $\left(1+\frac{1}{\beta}\right)f''(0)$ and $-\theta'(0)$, respectively. The values of $\left(1+\frac{1}{\beta}\right)f''(0)$ against the momentum slip parameter S are plotted in Figs.12 for different values of C. From the figure, it is observed that the magnitude of wall skin friction coefficient decreases with increasing values of velocity ratio parameter C when C < 0.1, whereas for C > 0.1 the magnitude of skin-friction increases with C. Due to higher values of Casson parameter β, the magnitude of $\left(1+\frac{1}{\beta}\right)f''(0)$ decreases (Figure 13) for the values of C>1 whereas $\left(1+\frac{1}{\beta}\right)f''(0)$ increases for the value of C<1. The influence of Magnetic parameter M, the magnitude of $\left(1+\frac{1}{\beta}\right)f''(0)$ increases (Figure 13) for the values of C>1 whereas $\left(1+\frac{1}{\beta}\right)f''(0)$ decreases for the value of C<1. The local Nusselt number (Figure 14) increases with C; that is, the heat transfer rate is enhanced with C, whereas reduced with the influence of Eckert number *Ec*. On the other hand, the value of $-\theta'(0)$, that is, the heat transfer (Figure 15), increases with β for C = 2 (>1) and decreases with β when C = 0.1 (<1) whereas $-\theta'(0)$ decreases with the influence of thermal slip parameter *b* for the values of C (C<1 as well as C>1).

V. Conclusions

The MHD stagnation-point flow of Casson fluid and heat transfer over a stretching sheet with radiation are investigated taking into consideration the viscous, momentum and thermal slip effects. Using similarity transformations, the governing equations are transformed to self-similar ordinary differential equations which are then solved using Bvp4c MATLAB solver. From the study, the following remarks can be summarized.

(a) The velocity boundary layer thickness reduces with velocity ratio parameter, magnetic parameter and momentum slip parameter.

(b) The velocity boundary layer thickness for Casson fluid is larger than that of Newtonian fluid.

(c) For Casson fluid, that is, for decrease of Casson parameter, the thermal boundary layer thickness decreases for C = 0.1 (<1) and, in contrast, for C = 2 (>1) the thickness increases.

(d) Due to thermal radiation, the temperature inside the boundary layer decreases and the temperature increases with the influence of Eckert number.

(e) The magnitude of wall skin-friction coefficient decreases with Casson parameter β.

(f) The magnitude of the wall skin-friction coefficient increases with momentum slip parameter S for C<1 whereas the wall skin-friction coefficient decreases with S for C>1.

(g) The rate of heat transfer increases with C whereas decreases with the influence of Eckert number.

Table 1 Comparison for $-f''(0)$ for *M=S=0*

C	$-f''(0)$			
	Present Study	Bhattacharyya [23]	Nazar et al. [49]	Mahapatra and Gupta [48]
0.1	-0.969386	-0.969386	-0.9694	-0.9694
0.2	-0.918107	-0.918107	-0.9181	-0.9181
0.5	-0.667264	-0.667263	-0.6673	-0.6673
2.0	2.017503	2.017503	2.0176	2.0175
3.0	4.729282	4.729284	4.7296	4.7293

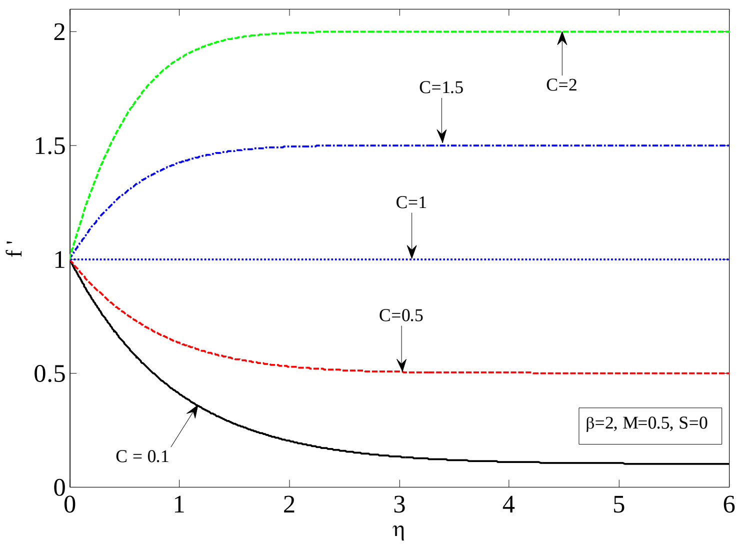

Fig. 1 Velocity for different *C*

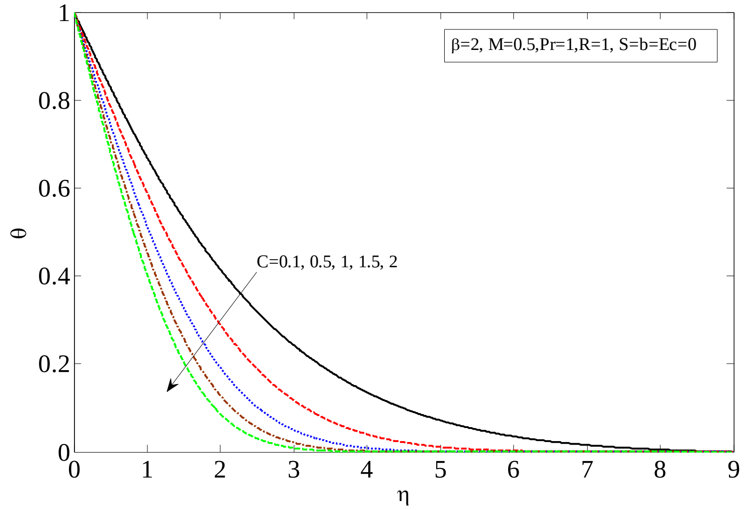

Fig. 2 Temperature for different *C*

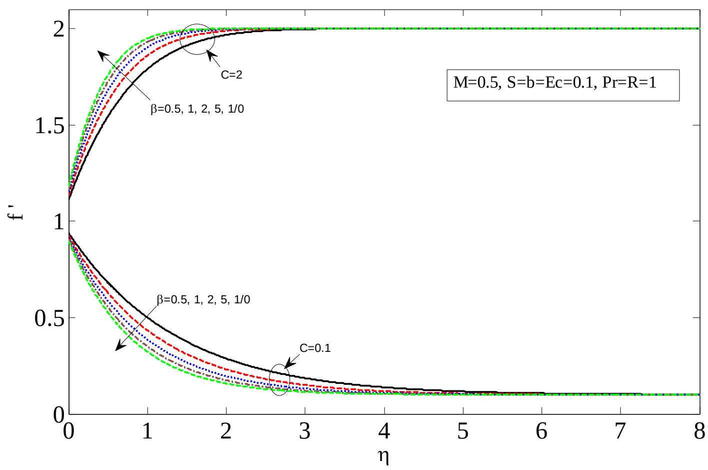

Fig.3 Velocity for different *C* and *β*

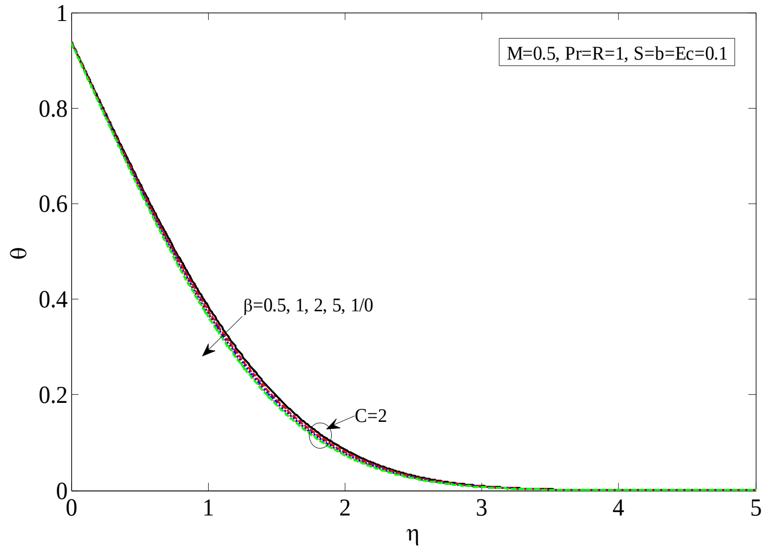

Fig.4(a) Temperature for different *β* for *C=2*

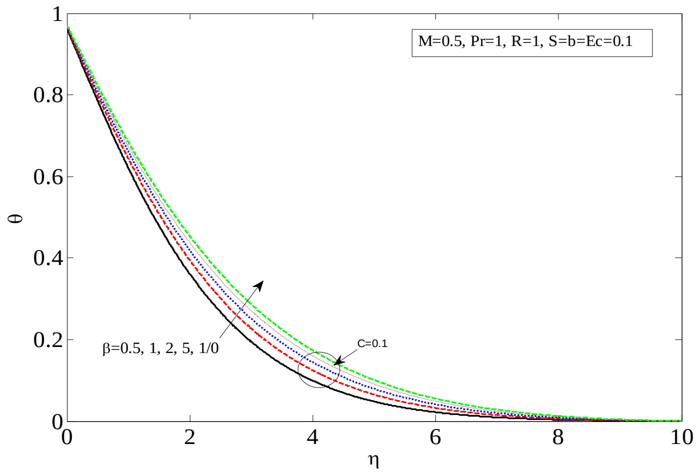

Fig.4(b) Temperature for different *β* for *C=0.1*

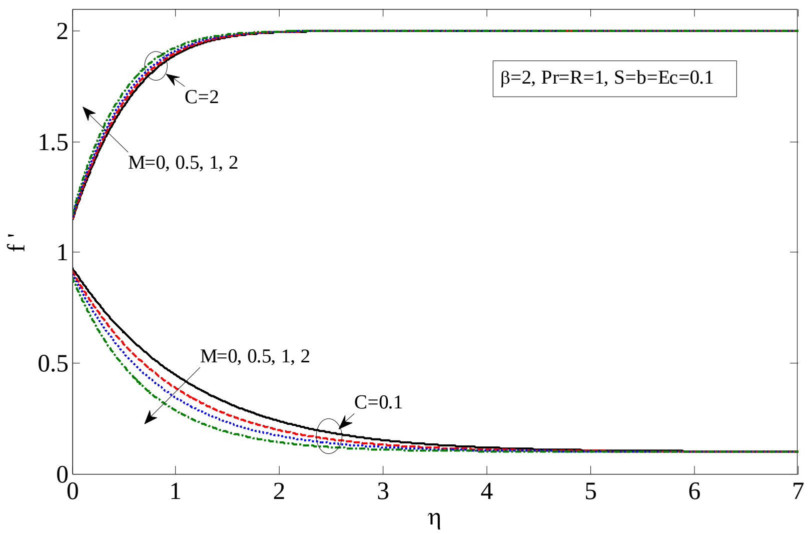

Fig.5 Velocity for different *C* and *M*

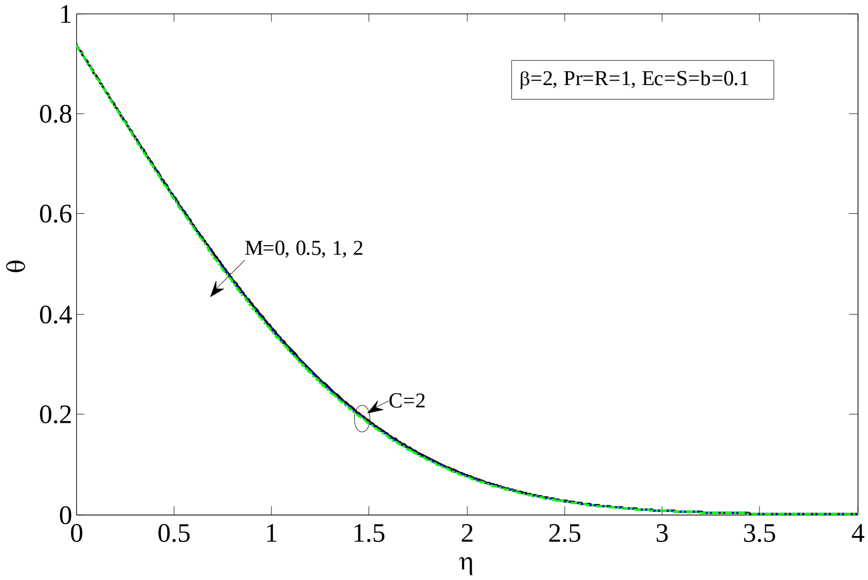

Fig.6 (a) Temperature for different *M* for *C=2*

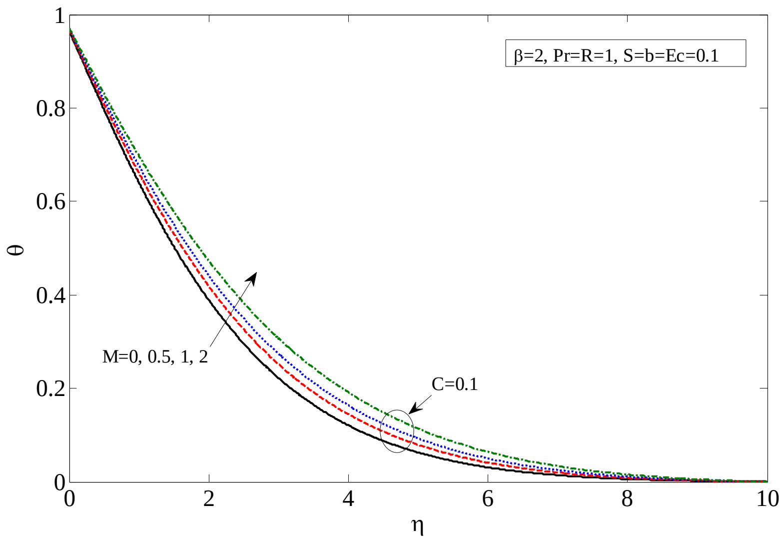

Fig.6 (b) Temperature for different *M* for *C=0.1*

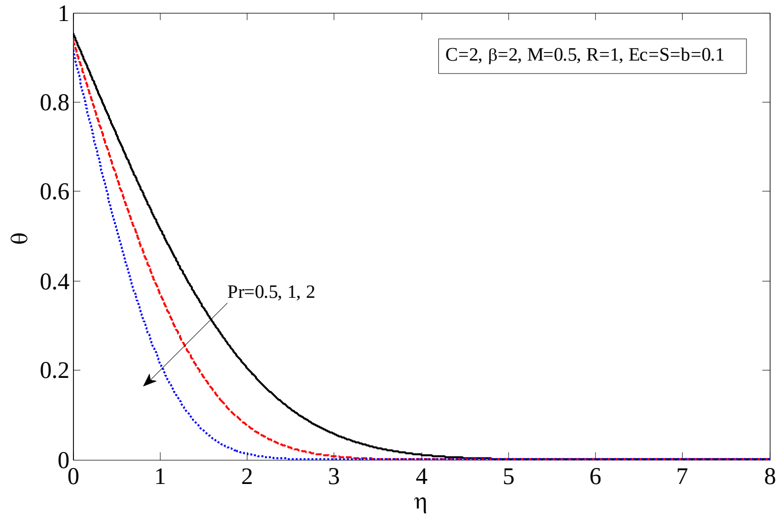

Fig.7 (a) Temperature for different *Pr* for *C=2*

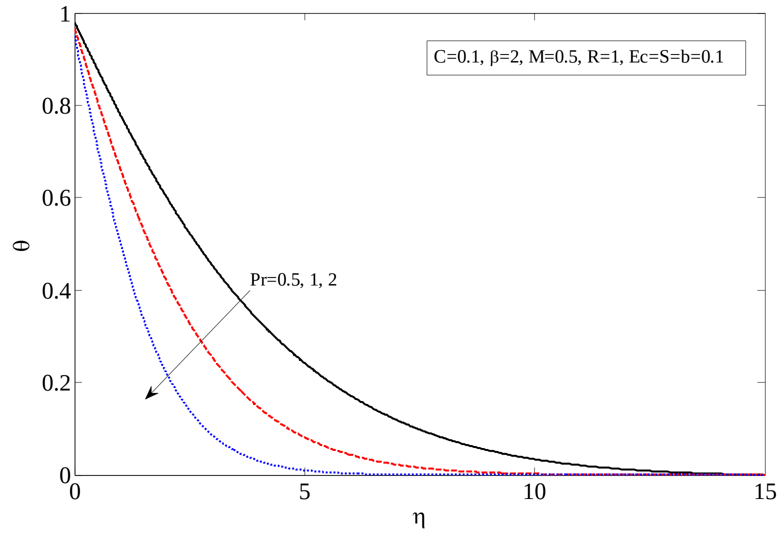

Fig.7 (b) Temperature for different *Pr* for *C=0.1*

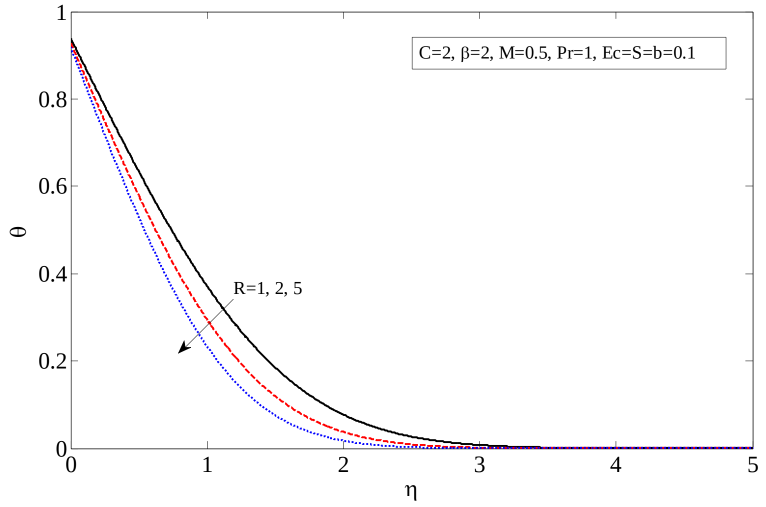

Fig.8 (a) Temperature for different *R* for *C=2*

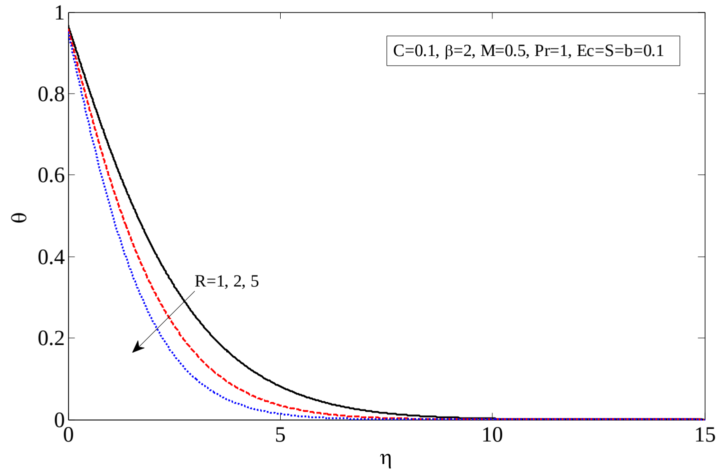

Fig.8 (b) Temperature for different *R* for *C=0.1*

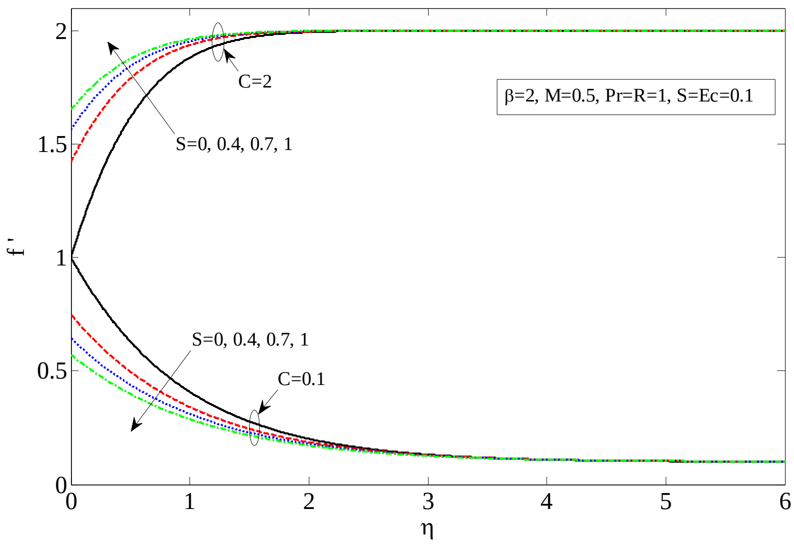

Fig.9 Velocity for different *S* and *C*

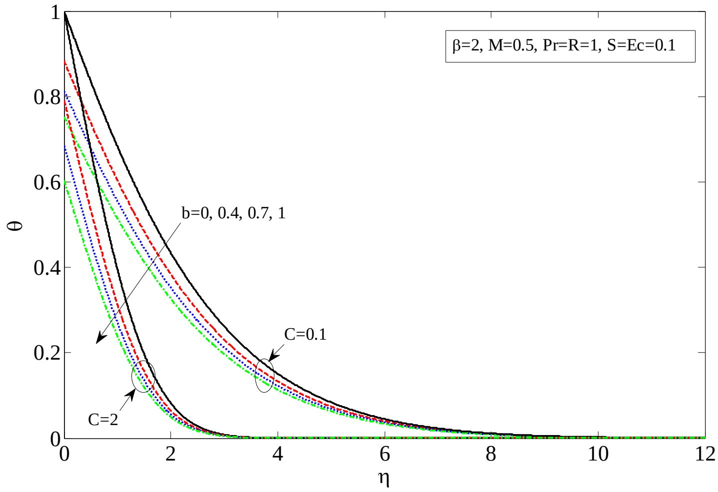

Fig.10 Temperature for different *b* and *C*

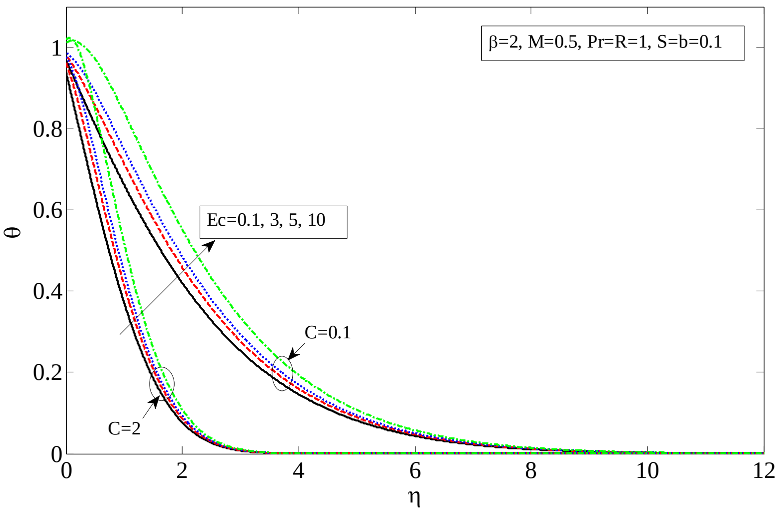

Fig.11 Temperature for different *Ec* and *C*

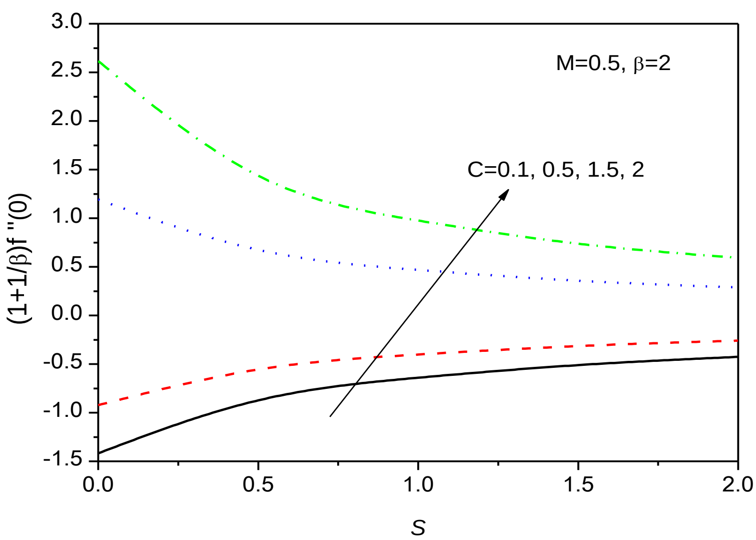

Fig.12 Effect of *S* and *C* on $(1+1/\beta)f''(0)$

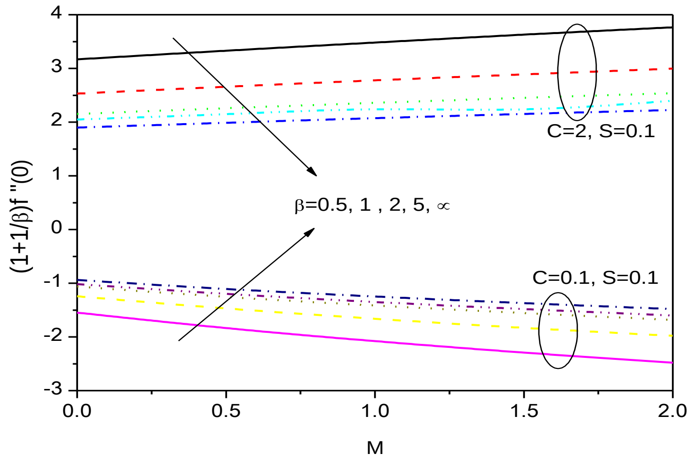

Fig.13 Effect of *β*, *M* and *C* on $(1+1/\beta)\,f''(0)$

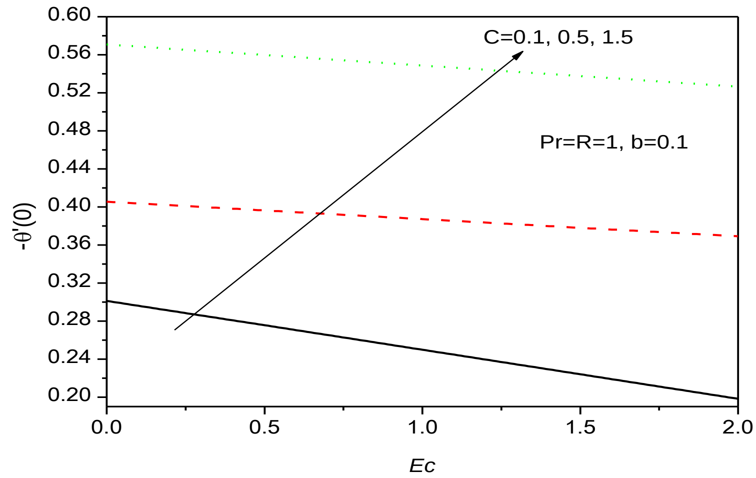

Fig.14 Effect of *Ec* and *C* on the Nusselt number

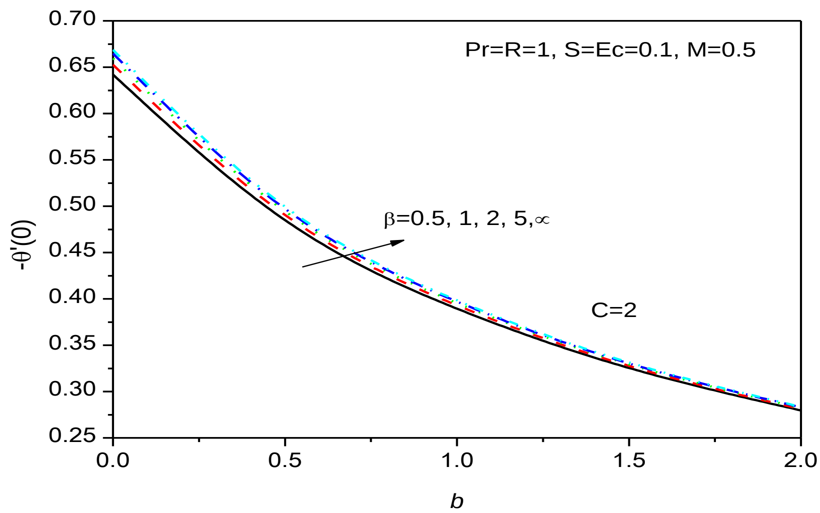

Fig.15(a) Effect of *β* and C on the Nusselt number

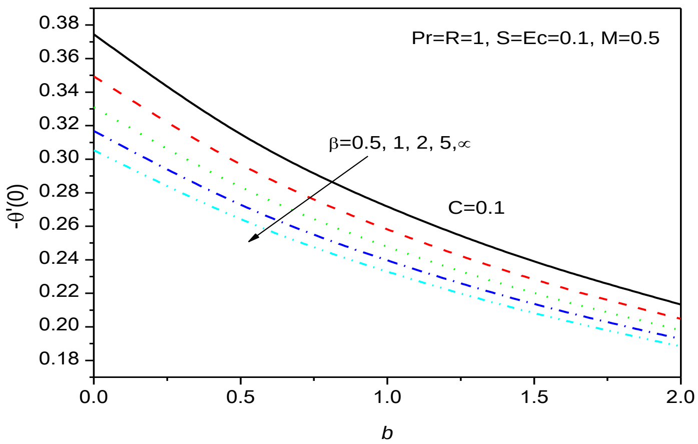

Fig.15(b) Effect of β and C on the Nusselt number

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