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# A Modified PSO Based Solution Approach for Economic Ordered Quantity Problem with Deteriorating Inventory, Time Dependent Demand Considering Quadratic and Segmented Cost Function

[1] Ashutosh Khare, ,[2] Dr. B.B. Singh, [3] Dr. R.K.Shrivastava,

*[1] PhD department of Mathematics, SMS Govt. Science College, Gwalior*
*[2] H.O.D. Department of Computer Science, Govt. K.R.G. college, Gwalior*
*[3] H.O.D. Department of Mathematics, SMS Govt. Science College, Gwalior*

***ABSTRACT:*** This paper presents a modified PSO algorithm that utilizes the PSO with double chaotic maps to solve the Economic Ordered Quantity (EOQ) problem with different cost functions. In proposed approach, we employ the PSO method that involves the alternating use of chaotic maps in estimating the velocity of the particle. Presently the PSO method has been widely used in complex designs problems with multiple variables where the optimization of a cost function is required. The use of pure analytical method for the Economic Ordered Quantity (EOQ) problem is widely studies however the best solution may require problem dependent derivations, complex mathematical calculations and at the worst it fails to achieve global best solution, hence to overcome the problem this paper presents a different approach by using PSO with irregular velocity updates which is performed by some kind of random function which force the particles to search greater space for best global solution. However the random function itself derived from a well-defined mathematical expression which limits its redundancy hence in the paper we are utilizing the two different chaotic maps which are used alternatively this mathematically increased the randomness of the function. The simulation of the algorithm verifies the effectiveness and superiority of the algorithm over standard algorithms.

***Keywords:-*** Economic Ordered Quantity (EOQ) problem, PSO, Chaotic Maps, Logistic Map, Lozi Map.

## I. INTRODUCTION

The deterioration of goods in present inventory system is an unavoidable problem, which is applicable from common goods like, foods, vegetables and fruit to the specialized goods like medicines. Hence it is important to consider the deterioration loss during the EOQ problem. Although the a number of deteriorating inventory models have been studied and proposed in recent years but the consideration of cost change due to ordering size has not been studied in best of our knowledge. Hence this paper presents a modeling and solution approach for such systems. Furthermore these proposal tackles the EOQ as the optimization problem and solved using numerical and analytical solution approach like lambdaiteration method, quadratic programming and the gradient method. In order to use any of these methods for EOQ should be capable of handling the constrains(cost change due to ordering size) which are inherently highly non-linear this leads to multiple local minimum points of the cost function. In order to overcome these limitations this paper presented the PSO technique enhanced by alternative use of two different chaotic maps. The rest of the paper is arranged as second section presents a brief review of the related literatures in third section discusses the problem formulation while fourth section explains the PSO and the variants used in the paper finally in chapter five and six respectively presents the simulated results and conclusion.

## II. LITERATURE REVIEW

In recent years, a lots of research have been performed on EOQ models and evolutionary systems some of them found useful during writing this paper are presented here. Lianxia Zhao [7] studied an inventory model with trapezoidal type demand rate and partially backlogging for Weibull-distributed deterioration items and derived an optimal inventory replenishment policy. Kuo-Lung Hou et al. [10] presents an inventory model for deteriorating items considering the stock-dependent selling rate under inflation and time value of money over a finite planning horizon. Liang YuhOuyanget al. [1] presented an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. Their model also handles the shortages and variable rate partial back ordering which dependents on the waiting time for the next replenishment .Kai-Wayne Chuang et al. [2] studied pricing strategies in marketing, with objective to find the optimal inventory and pricing strategies for maximizing the net present value of total profit over the infinite horizon. The studied two variants of models: one without considering shortage, and the other with shortage .Jonas C.P. Yu [4] developed a deteriorating inventory system with only one supplier and one buyer. The system considers the collaboration and trade credit between supplier and buyer. The objective is to maximize the total profit of the whole system when shortage is completely backordered. The literature also discuss the negotiation mechanism between supplier and buyer in case of shortages and payment delay. The model allows shortages and partially backlogging at exponential rate. Ching-Fang Lee et al. Al [14], considered system dynamics to propose a new order system for integrated inventory model of supply chain for deteriorating items among a supplier, a producer and a buyer. The system covers the dynamics of complex relation due to time evolution. Michal Pluhacek et al [15] compared the performance of two popular evolutionary computational techniques (particle swarm optimization and differential evolution) is compared in the task of batch reactor geometry optimization. Both algorithms are enhanced with chaotic pseudo-random number generator (CPRNG) based on Lozi chaotic map. The application of Chaos Embedded Particle Swarm Optimization for PID Parameter Tuning is presented in [16]. Magnus Erik et al [17] gives a list of good choices of parameters for various optimization scenarios which should help the practitioner achieve better results with little effort .

## III. PROBLEM FORMULATION

The objective of an EOQ problem is to minimize the total inventory holding costs and ordering costs which should be found within the equality and inequality constraints (operational constrains) limitations. The simplified cost function of each inventory item can be represented as described in (2)

$$C_T = \sum_{i=1}^{n} c_i(S_i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

$$c_i(S_i) = \alpha_i * S_i \ldots\ldots\ldots (3.2)$$

*where*

$$C_T = Total\ Inventory\ Cost$$

$$c_i = Cost\ Function\ of\ Invertory\ i$$

$$\alpha_i = per\ unit\ cost\ of\ Invertory\ i$$

$$S_i = Ordered\ size\ of\ Invertory\ i$$

3.1.1Order Size Dependent Cost Model 1: The inventory with order dependent cost shows a greater variation in the total inventory holding cost and ordering cost function. Since the cost depends upon order size, a cost function contains higher order nonlinearity .The equation (3) presents a cost function quadratic behavior:

$$C_i(S_i) = \alpha_i + \beta_i S_i + \gamma_i S_i^2 \ldots . (3.3)$$

*where*

$$\alpha_i, \beta_i\ and\ \gamma_i are\ the\ cost\ coefficient\ of\ inventory\ i,$$

And can be explained as

$\alpha\ is\ the\ minimum\ cost\ and\ should\ be\ a\ positive\ constant$

$\beta\ is\ the\ price\ per\ unit\ and\ should\ be\ a\ positive\ constant$

$\gamma\ is\ the\ discount\ and\ should\ be\ defined\ as\ follows$

Let a single inventory system

$$C(S) = \alpha + \beta S + \gamma S^2 \ldots\ldots\ldots\ldots\ldots (3.4)$$

Since the cost must always be higher for higher quantities even after adding discount hence it should be a monotonic increasing function in the range $S \in [S_{min} S_{max}]$. Because the minimum ordered quantity must be equal to or larger than 1 we can say that the

$$\frac{dC(S)}{dS} > 0, S \in [S_{min} S_{max}] \ldots\ldots (3.5)$$

$$\beta + 2\gamma S > 0, \qquad S \in [S_{min} S_{max}]$$

$$\gamma > -\frac{\beta}{2S}, \qquad S \in [S_{min} S_{max}] \ldots\ldots\ldots\ldots (3.6)$$

The equation (6) states the condition required for monotonic function however in the present case on more consideration is required because the cost per unit should be decreased when ordered at higher amount but the total cost should never fall below the total cost of lowered ordered quantity as shown in (7).

$$C(S_i) > C(S_j), for\ i > j, and\ \{S_i, S_j\} \in [S_{min} S_{max}] \ldots (3.7)$$

Or

$$\frac{d^2C(S)}{dS^2} < 0, S \in [S_{min} S_{max}] \ldots\ldots (3.8)$$

$$2\gamma < 0, \qquad S \in [S_{min} S_{max}]$$

$$\gamma < 0, \qquad S \in [S_{min} S_{max}] \ldots\ldots\ldots\ldots (3.9)$$

Combining the equation (6) and (9) we can get the possible values for the $\gamma$ *as follows*

$$-\frac{\beta}{2 * S} < \gamma < 0 \ldots\ldots\ldots (3.10)$$

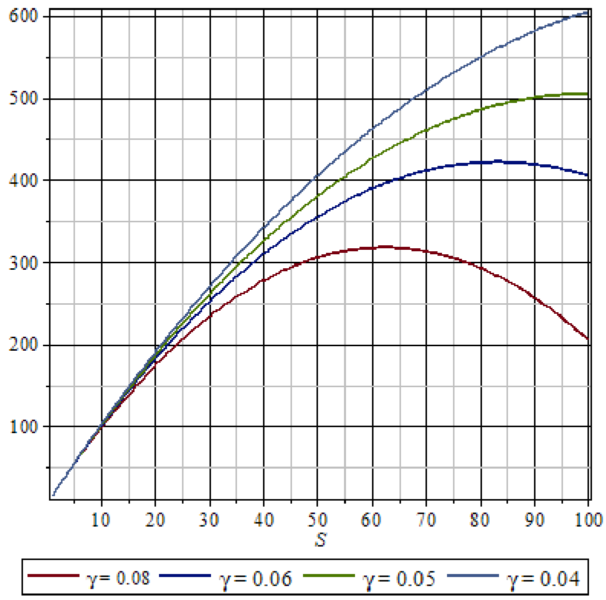



**Figure 1: the effect of $\gamma$ on cost $C(S)$, where $\alpha = 5, \beta = 10$, and $S_{max} = 100$.**

3.1.2Order Size Dependent Cost Model 2: this model represents the model where items per unit cost depends upon order size segment which can be represented with several piece wise quadratic functions reflecting the effects of different inventory ordering costs according to order size segment it belongs to .In general, a piecewise quadratic function is used to represent the input-output curve of an inventory with lumped order size [4] and described as
Let the inventory is divided into $N$ equal segments then the size of each segment.

$$S_{seg} = \frac{S_{max} - S_{min}}{N}$$

Now the quantities under the $j^{th}$ segment

$$S_j = \{S_{min} + (j-1) * S_{seg}, S_{min} + (j-1) * S_{seg} + 1,$$
$$S_{min} + (j-1) * S_{seg} + 2, \ldots..S_{min} + (j) * S_{seg} - 1\} \ldots..(3.11)$$

$\beta_j$ Be the cost per unit when ordered under$j^{th}$ segment size, hence the ordering cost

$$C_j = \beta_j * S_j \ldots\ldots\ldots\ldots.(3.12)$$

The$\beta_j$ should be estimated as

$$\beta_i < \beta_j, for\ i > j$$

And

$$C_i > C_j, for\ i > j$$

Now the maximum number of quantity that can ordered under the $j^{th}$ segment

$$S_{j,max} = S_{min} + j * S_{seg} - 1$$

While the minimum quantity that can ordered under the $(j+1)^{th}$ segment

$$S_{(j+1),min} = S_{min} + ((j+1) - 1) * S_{seg}$$

Now the cost for $S_{j,max}$ $and$ $S_{(j+1),min}$ must follow

$$S_{(j+1),min} * \beta_{(j+1)} > S_{j,max} * \beta_j, \beta_{j+1} < \beta_j$$

$$(S_{min} + \big((j+1) - 1\big) * S_{seg}) * \beta_{(j+1)} > (S_{min} + j * S_{seg} - 1) * \beta_j$$

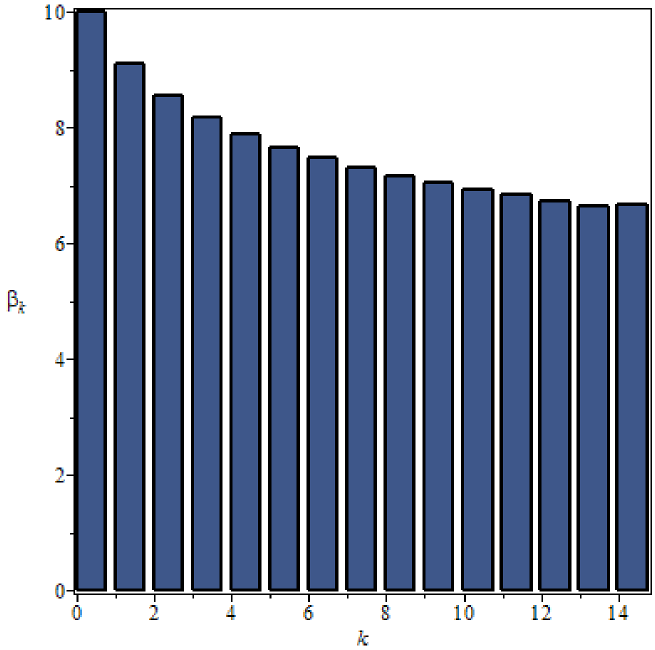
$$S_{min} * \beta_{j+1} + j * S_{seg} * \beta_{j+1} > S_{min} * \beta_j + j * S_{seg} * \beta_j - \beta_j$$

$$S_{min}\left(\beta_{j+1} - \beta_j\right) + j * S_{seg}\left(\beta_{j+1} - \beta_j\right) > -\beta_j$$

$$\left(\beta_{j+1} - \beta_j\right)\left(S_{min} + j * S_{seg}\right) > -\beta_j$$

$$\frac{\beta_{j+1}}{\beta_j} < 1 - \frac{1}{\left(S_{min} + j * S_{seg}\right)}$$

$$\beta_{j+1} < \beta_j\left(1 - \frac{1}{\left(S_{min} + j * S_{seg}\right)}\right) \ldots\ldots\ldots.(3.13)$$



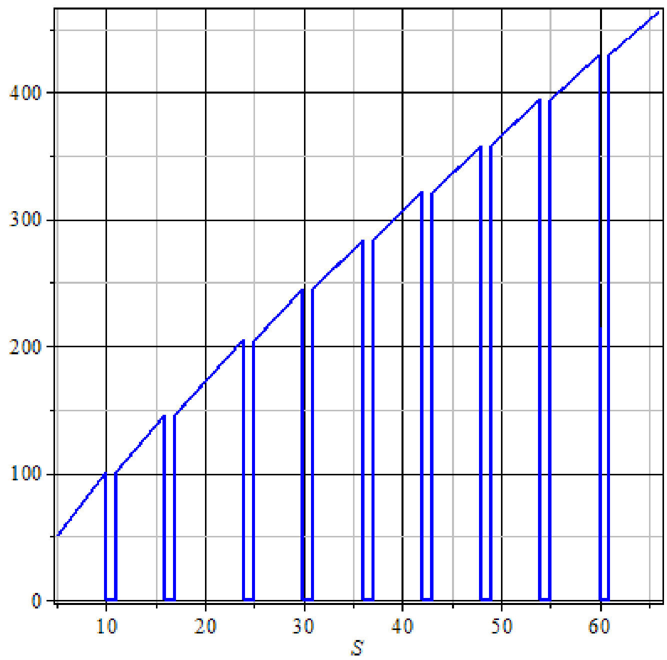




**Figure 3: shows the cost per unit for different segments ($\beta_k$)(above) and cost of ordered quantity under different segments for $\beta_1 = 10, S_{min} = 5, S_{max} = 95, S_{seg} = 6, N = 15$.**

In general, total inventory ordering costs is calculated by equation (4).Determining the selection of total inventory order quantity for each unit is dictated by the demand and backlogging rate, and can be solved by economic total inventory holding and ordering cost. This paper assumes that such selection is given a-priori. Therefore, to obtain an accurate and practical EOQ solution, the total inventory holding and ordering cost function should be considered with both multi-item total inventory holding and ordering costs with order size segment effects simultaneously [7]. Thus, the total inventory holding and ordering cost function (3) should be combined with (4), and can be represented as follows:

$$c_i(S_i) = \begin{cases} c_{i1}(S_i), & segment\ 1, \quad S_{i,min} \le S_i \le S_{i,1} \\ c_{i2}(S_i), & segment\ 2, \quad S_{i,1} \le S_i \le S_{i,2} \\ & \dots\dots. \\ & \dots\dots. \\ c_{ik}(S_i), & segment\ k, \quad S_{ik-1} \le S_i \le S_{i,max} \end{cases} \quad \dots\dots\dots\dots\dots (3.12)$$

$Where$

$$c_{ik}(S_i) = \alpha_{ik} + b_{ik}S_i + c_{ik}S_i^2 \ \dots.. (3.13)$$

$and\ S_{i,min}\ is\ the\ minimum\ order\ size\ of\ nventory\ i.$
$S_{i,max}\ is\ the\ maximum\ order\ size\ of\ nventory\ i$

3.2. Equality and Inequality Constraints

3.2.1 Demand and Stock Balance Equation: For Demand and Stock balance ,an equality constraint should be satisfied . The total stock should be equal or greater than the total demand plus the total Deterioration loss

$$\sum_{i=1}^{n} S_{i,demand} + S_{i,loss} \ \dots\dots\dots\dots.. (3.14)$$

$where\ S_{i,demand}\ S_{i,loss}\ represents\ the\ total\ demand\ and$
$Deterioration\ loss\ of\ i^{th} inventory\ is\ a\ function\ of\ the$
$units\ ordered\ that\ can\ be\ represented\ using\ Demand\ (D_i)$
$and\ Deterioration\ (L_i)\ coef$ficients [2] $as\ follows$:

$$\sum_{i=1}^{n} S_i D_i + \sum_{i=1}^{n} S_i L_i \ \dots\dots\dots. (3.15)$$

## IV. MATHEMATICAL MODELING

The mathematical model in this paper is rendered from reference [1]which derived the following conclusive equations:
Notation Used:

$c_1$: Holding cost, (\$/per unit)/per unit time.
$c_2$: Cost of the inventory item, \$/per unit.
$c_3$: Ordering cost of inventory, \$/per order.
$c_4$:Shortage cost, (\$/per unit)/per unit time.
$c_5$:Opportunity cost due to lost sales, \$/per unit.
$t_1$ :Time at which shortages start.
$T$ :Length of each ordering cycle.
$W$ :The maximum inventory level for each ordering cycle.
$S$ :The maximum amount of demand backlogged for each ordering cycle.
$Q$ :The order quantity for each ordering cycle.
$Inv(t)$: The inventory level at time t.

The inventory level at any given time $(t)$ , $t_1 \le t \le T$:

$$Inv(t) = \frac{D}{\delta}\{\ln[1 + \delta(T - t)] - \ln[1 + \delta(T - t_1)]\}, \dots\dots\dots. (4.1)$$

Maximum amount of demand backlogged per cycle as follows:

$$S = -Inv(T) = \frac{D}{\delta}\ln[1 + \delta(T - t_1)] \ \dots\dots\dots\dots\dots (4.2)$$

The ordered quantity per cycle is given by

$$Q = W + S = \frac{A}{\theta-\lambda}\left[e^{(\theta-\lambda)t_1} - 1\right] + \frac{D}{\lambda}\ln[1+\delta(T-t_1)] \dots\dots\dots\dots..(4.3)$$

The inventory holding cost per cycle is

$$HC = \frac{c_1 A}{\theta(\theta-\lambda)} e^{-\lambda t_1}\left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda}\left(e^{\lambda t_1} - 1\right)\right] \dots\dots\dots..(4.4)$$

The deterioration cost per cycle is

$$DC = c_2 A\left\{\frac{1}{\theta-\lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e_1^{-\lambda t}\right)\right\} \dots\dots..(4.5)$$

The shortage cost per cycle is

$$SC = c_4 D\left\{\frac{T-t_1}{\delta} - \frac{1}{\delta^2}\ln[1+\delta(T-t_1)]\right\} \dots.(4.6)$$

The opportunity cost due to lost sales per cycle is

$$BC = c_5 D\left\{(T-t_1) - \frac{1}{\delta}\ln[1+\delta(T-t_1)]\right\}, \dots\dots(4.7)$$

Considering the scenario 3.1.1 the cost is given by the equation considering single inventory type

$$C = \alpha + \beta * S + \gamma * S^2 \dots\dots\dots..(4.9)$$

Now replacing $c_2$ in equation (4.5) by (4.9)

$$DC = \alpha + \beta * A\left\{\frac{1}{\theta-\lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e^{-\lambda t_1}\right)\right\} + \gamma * A\left\{\frac{1}{\theta-\lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e^{-\lambda t_1}\right)\right\}^2 \dots\dots\dots\dots.(4.10)$$

Considering the scenario 3.1.2 the cost is given by the equation considering single inventory type

$$C_j = \beta_j * S_j \dots\dots\dots\dots(4.11)$$

Now replacing $c_2$ in equation (4.10) by (4.11)

$$DC = \beta_j A\left\{\frac{1}{\theta-\lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e^{-\lambda t_1}\right)\right\}, for\ A\left\{\frac{1}{\theta-\lambda}\left(e^{(\theta-\lambda)t_1} - 1\right) - \frac{1}{\lambda}\left(1 - e^{-\lambda t_1}\right)\right\} \in S_j \dots\dots\dots\dots.(4.12)$$

Therefore, the average total cost per unit time per cycle can be expressed as:

$$TVC \equiv TVC(t_1, T)$$

= (holding cost +deterioration cost + ordering cost +shortage cost+ opportunity cost due to lost sales)/ length of ordering cycle

For scenario 3.1.1

$$TVC = \frac{1}{T}\left(\frac{c_1 A e^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(e^{\theta t_1} - 1 - \frac{\theta\left(e^{\lambda t_1}-1\right)}{\lambda}\right) + c_3 + c_4(D)\left(\frac{T-t_1}{\delta} - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right)\right.$$
$$+ c_5(D)\left(T - t_1 - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta}\right) + \alpha + \beta A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)$$
$$\left. + \gamma A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)^2\right) \quad \ldots\ldots(4.13)$$

For scenario 3.1.2

$$TVC = \frac{1}{T}\left(\frac{c_1 A e^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(e^{\theta t_1} - 1 - \frac{\theta\left(e^{\lambda t_1}-1\right)}{\lambda}\right) + c_3 + c_4(D)\left(\frac{T-t_1}{\delta} - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right)\right.$$
$$\left. + c_5(D)\left(T - t_1 - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta}\right) + \beta A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)\right) \ldots(4.14)$$

The objective of the model is to determine the optimal values of $t_1$ and $T$ in order to minimize the average total cost per unit time, TVC. The optimal solutions $t_1^*$ and $T^*$ need to satisfy the following equations:

For scenario 3.1.1

$$\frac{\partial\, TVC}{\partial t_1} = \frac{1}{T}\left(-\frac{c_1 A \lambda e^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(e^{\theta t_1} - 1 - \frac{\theta\left(e^{\lambda t_1}-1\right)}{\lambda}\right) + \frac{c_1 A e^{-\lambda t_1}\left(\theta e^{\theta t_1} - \theta e^{\lambda t_1}\right)}{\theta(\theta-\lambda)}\right.$$
$$+ c_4(D)\left(-\delta^{-1} + \frac{1}{\delta\left(1+\delta(T-t_1)\right)}\right) + c_5(D)\left(-1 + \left(1+\delta(T-t_1)\right)^{-1}\right)$$
$$\left. + \beta A\left(e^{(\theta-\lambda)t_1} - e^{-\lambda t_1}\right) + 2\gamma A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)\left(e^{(\theta-\lambda)t_1} - e^{-\lambda t_1}\right)\right)$$
$$= 0 \ldots\ldots\ldots(4.15)$$

and

$$\frac{\partial\, TVC}{\partial T} = \frac{1}{T}\left(c_4(D)\left(\delta^{-1} - \frac{1}{\delta\left(1+\delta(T-t_1)\right)}\right) + c_5(D)\left(1 - \left(1+\delta(T-t_1)\right)^{-1}\right)\right)$$
$$- \frac{1}{T^2}\left(\frac{c_1 A e^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(e^{\theta t_1} - 1 - \frac{\theta\left(e^{\lambda t_1}-1\right)}{\lambda}\right) + c_3 + c_4(D)\left(\frac{T-t_1}{\delta} - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right)\right.$$
$$+ c_5(D)\left(T - t_1 - \frac{ln\left(1+\delta(T-t_1)\right)}{\delta}\right) + \alpha + \beta A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)$$
$$\left. + \gamma A\left(\frac{e^{(\theta-\lambda)t_1}-1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda}\right)^2\right) = 0 \ldots\ldots\ldots\ldots(4.16)$$

For scenario 3.1.2

$$\frac{\partial\, TVC}{\partial t_1}=\frac{1}{T}\left(-\frac{c_1 A\lambda \mathrm{e}^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(\mathrm{e}^{\theta t_1}-1-\frac{\theta\left(\mathrm{e}^{\lambda t_1}-1\right)}{\lambda}\right)+\frac{c_1 A\mathrm{e}^{-\lambda t_1}\left(\theta \mathrm{e}^{\theta t_1}-\theta \mathrm{e}^{\lambda t_1}\right)}{\theta(\theta-\lambda)}\right.$$
$$+c_4(D)\left(-\delta^{-1}+\frac{1}{\delta\left(1+\delta(T-t_1)\right)}\right)+c_5(D)\left(-1+\left(1+\delta(T-t_1)\right)^{-1}\right)$$
$$\left.+\beta A\left(\mathrm{e}^{(\theta-\lambda)t_1}-\mathrm{e}^{-\lambda t_1}\right)\right)=0 \ldots\ldots\ldots\ldots\ldots(4.17)$$

$$\frac{\partial\, TVC}{\partial T}=\frac{1}{T}\left(c_4(D)\left(\delta^{-1}-\frac{1}{\delta\left(1+\delta(T-t_1)\right)}\right)+c_5(D)\left(1-\left(1+\delta(T-t_1)\right)^{-1}\right)\right)$$
$$-\frac{1}{T^2}\left(\frac{c_1 A\mathrm{e}^{-\lambda t_1}}{\theta(\theta-\lambda)}\left(\mathrm{e}^{\theta t_1}-1-\frac{\theta\left(\mathrm{e}^{\lambda t_1}-1\right)}{\lambda}\right)+c_3+c_4(D)\left(\frac{T-t_1}{\delta}-\frac{\ln\left(1+\delta(T-t_1)\right)}{\delta^2}\right)\right.$$
$$\left.+c_5(D)\left(T-t_1-\frac{\ln\left(1+\delta(T-t_1)\right)}{\delta}\right)+\beta A\left(\frac{\mathrm{e}^{(\theta-\lambda)t_1}-1}{\theta-\lambda}-\frac{1-\mathrm{e}^{-\lambda t_1}}{\lambda}\right)\right)$$
$$=0\ldots\ldots\ldots\ldots(4.18)$$

## V. PARTICLE SWARM OPTIMIZATION (PSO)

The PSO algorithms inspired by the natural swarm behavior of birds and fish. In PSO each particle in the population represents a possible solution of the optimization problem, which is defined by its cost function. In each iteration, a new location (combination of cost function parameters) of the particle is calculated based on its previous location and velocity vector (velocity vector contains particle velocity for each dimension of the problem).
Initially, the PSO algorithm chooses candidate solutions randomly within the search space. The PSO algorithm simply uses the objective function to evaluate its candidate solutions, and operates upon the resultant fitness values.

Each particle maintains its position, composed of the candidate solution and its evaluated fitness, and its velocity. Additionally, it remembers the best fitness value it has achieved thus far during the operation of the algorithm, referred to as the individual best fitness, and the candidate solution that achieved this fitness ,referred to as the individual best position or individual best candidate solution. Finally, the PSO algorithm maintains the best fitness value achieved among all particles in the swarm, called the global best fitness ,and the candidate solution that achieved this fitness, called the global best position or global best candidate solution.

The PSO algorithm consists of just three steps, which are repeated until some stopping condition is met:

1. Evaluate the fitness of each particle

2. Update individual and global best fitness's and positions

3. Update velocity and position of each particle

4. Repeat the whole process till the

The mathematical equations involved in above operations are as follows:

The velocity of each particle in the swarm is updated using the following equation:

$$v(i+1)=w*v(i)+c_1*\left(pBest-x(i)\right)+c_2*\left(gBest-x(i)\right)\ldots\ldots\ldots(5.1)$$

Modified PSO with chaos driven pseudorandom number perturbation

$$v(i+1) = w * v(i) + c_1 * Rand * \left(pBest - x(i)\right) + c_2 * Rand * \left(gBest - x(i)\right) \dots \dots \dots (5.2)$$

A chaos driven pseudorandom number perturbation ($Rand$) is used in the main PSO formula (Eq. (13)) that determines new ‘‘velocity’ ’and thus the position of each particle in the next iterations (or migration cycle). The perturbation facilities the better search in the available search space hence provides much better results.

$Where$:

$v(i+1) - New\ velocity\ of\ a\ particle.$

$v(i) - Current\ velocity\ of\ a\ particle.$

$c_1, c_2 - Priority\ factors.$

$pBest - Best\ solution\ found\ by\ a\ particle.$

$gBest - Best\ solution\ found\ in\ a\ population.$

$Rand - Random\ number, interval\ (0,1). Chaos\ number\ generator\ is\ applied\ only\ here.$

$$x(i) - Current\ position\ of\ a\ particle.$$

The new position of a particle is then given by (5.3), where $x(i + 1)$ is the new position:

$$x(i+1) = x(i) + v(i+1) \dots \dots \dots \dots .. (5.3)$$

Inertia weight modification PSO strategy has two control parameters $w_{start}$ and $w_{end}$. A new $w$ for each iteration is given by (5.4),where$i$standforcurrentiteration number and $n$ for the total number of iterations.

$$w = w_{start} - \frac{\left((w_{start} - w_{end}) * i\right)}{n} \dots \dots \dots \dots \dots . (5.4)$$

## VI. CHAOTIC MAPS

A chaotic map is an evolution function) that shows some sort of disordered behavior. These maps may be discrete-time or continuous-time. The terms is came from chaos theory where the behavior of dynamical systems which greatly depends upon the initial conditions. In such cases even a small change in initial conditions may cause the exceptionally diverging results which makes the long-term prediction for such system impossible .Presently there are many such maps have been discovered however in our work we have selected the Lozi and Logistic maps because of their simplicity. The logistic map is a polynomial mapping (equivalently, recurrence relation) of degree 2, shows the complex, chaotic behavior from very simple non-linear dynamical equations. Mathematically, the logistic map is written as:

$$X_{n+1} = \mu X_n (1 - X_n) \dots \dots \dots \dots \dots (6.1)$$

The Lozi map is a simple discrete two-dimensional chaotic map presented by following equations:

$$X_{n+1} = 1 - x|X_n| + bY_n \dots \dots \dots \dots . (6.2a)$$

$$Y_{n+1} = X_n \dots \dots \dots \dots \dots \dots . (6.2b)$$

## VII. IMPLEMENTATION OF IMPROVED PSO ALGORITHMFOR ECONOMIC ORDERED QUANTITY (EOQ) PROBLEMS

Since the decision variables in EOQ problems are $t_1$ and $T$ with $S = \{S_1, S_2, \dots., S_n\}$ where $S_i$ ordering quantity of $i^{th}$ inventory, the structure of a particle is composed of a set of elements corresponding to the $[t_1, T, S]$. Therefore, particle’s position at iteration $k$ can be represented as the vector$X_i^k = \left(P_{i1}^k, P_{i2}^k \dots \dots .., P_{im}^k\right)$where $m = n + 2$ and $n$ is the number of inventories. The velocity of particle $i$corresponds to the generation updates forall inventories. The process of the proposed PSO algorithm can be summarized as in the following steps.

1. Initialize the position and velocity of a population at random while satisfying the constraints.
2. Update the velocity of particles.
3. Modify the position of particles to satisfy the constraints, if necessary.
4. Generate the trial vector through operations presented in section 4.
5. Update and Go to Step 2 until the stopping criteria is satisfied.

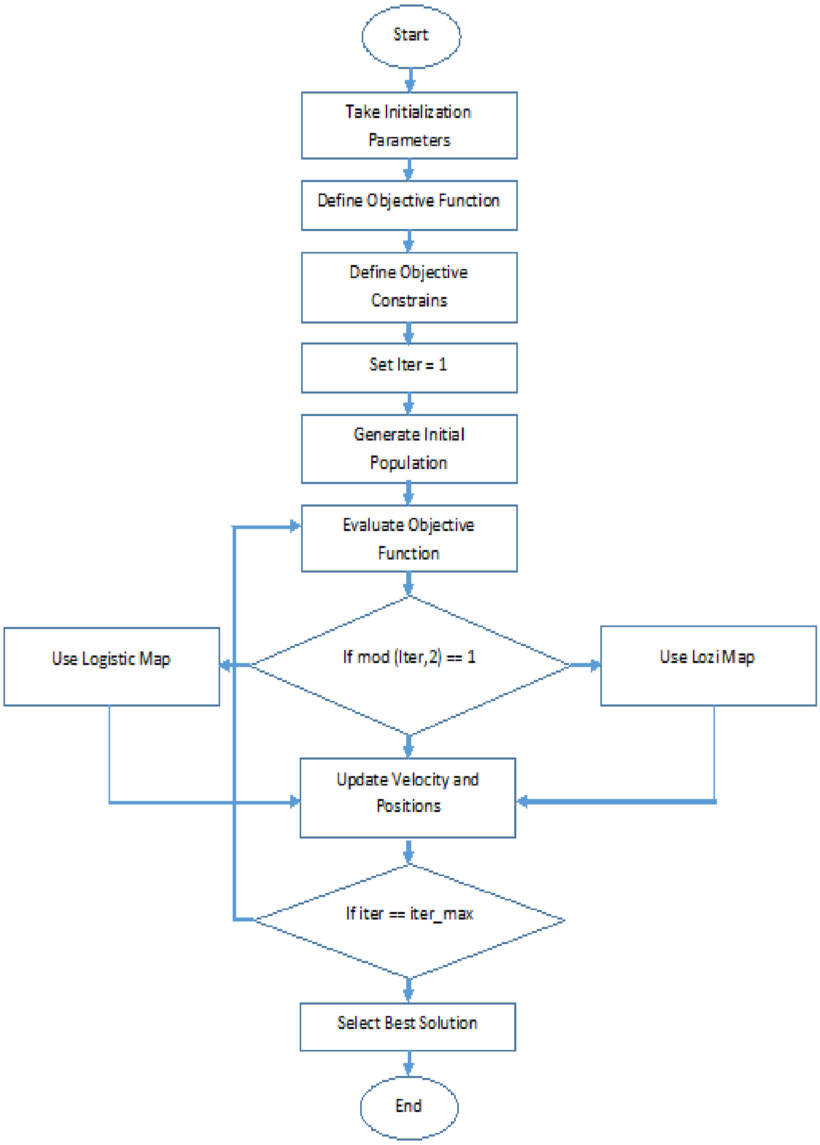



**Figure 8: Flow Chart of the Proposed Algorithm.**

## VII. SIMULATION RESULTS

The proposed IPSO approach is applied to three different inventory systems explained in section 3 and evaluated by all three PSO models as follows:

• The conventional PSO
• The PSO with chaotic sequences
• The PSO with alternative chaotic operation

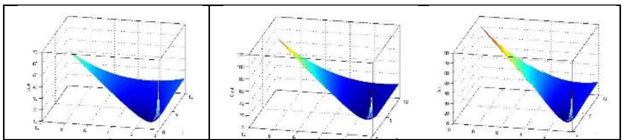
The simulation of all algorithms is performed using MATLAB. The population size$N_P$ and maximum iteration number $iter_{max}$ are set as 100 and100, respectively.$w_{max}$ and$w_{min}$ are set to 0.9 and 0.1 respectively because these values are widely accepted and verified in solving various optimization problems. The list of all values used for the system are shown in the table below

**Table 1: parameter values used for different PSO algorithms**

| Name of Variable | Value Assigned |
|---|---|
| $c_1$ | 2 |
| $c_2$ | 1 |
| $w_{max}$ | 0.9 |
| $w_{min}$ | 0.1 |
| $\mu$ (logistic map) | 4.0 |
| $k$ (logistic map) | 0.63 |
| $a$ (lozi map) | 1.7 |
| $b$ (lozi map) | 0.5 |
| Total Particles | 100 |
| Maximum Iterations | 100 |

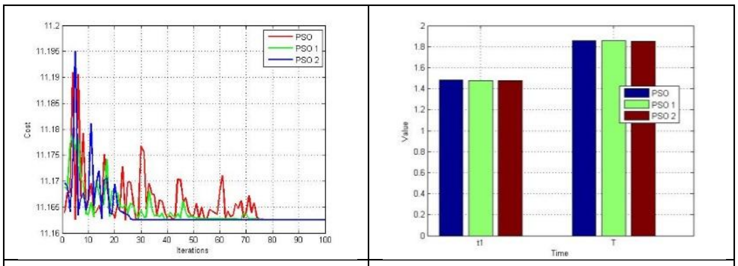
**Table 2: values of system variables:**

| | Variable Value | |
|---|---|---|
| Variable Name | Scenario 1 | Scenario 2 |
| $A$ | 12 | 12 |
| $\theta$ | 0.08 | 0.08 |
| $\delta$ | 2 | 2 |
| $\lambda$ | 0.03 | 0.03 |
| $c_1$ | 0.5 | 0.5 |
| $c_2$ | 1.5 | 1.5 |
| $c_3$ | 10 | 10 |
| $c_4$ | 2.5 | 2.5 |
| $c_5$ | 2 | 2 |
| $D$ | 8 | 8 |
| $\alpha$ | 5 | 5 |
| $\beta$ | 10 | 10 |
| $\gamma$ | 0.04 | N/A |
| $S_{min}$ | 1 | 1 |
| $S_{max}$ | 100 | 95 |
| $N$ | N/A | 15 |
| $S_{seg}$ | N/A | 6 |
| $\beta_1$ | N/A | 10 |



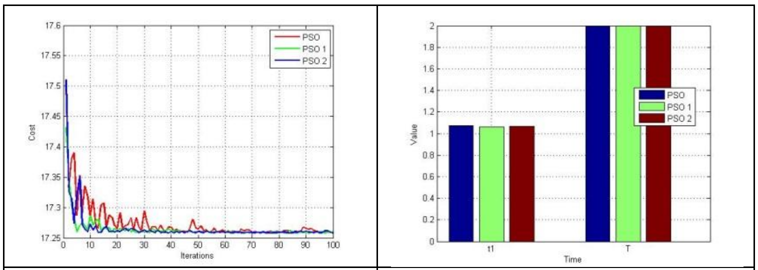
**(a)** **(b)** **(c)**

Figure 9 (a): surface plot for the normal inventory system. With respect to $t_1$ (the time at which shortage starts) and $T$(ordering cycle time) the figure shows a smooth and continuous curve and hence can be solved by analytical technique also .Figure 9(b): surface plot for the order dependent inventory cost type model. With respect to $t_1$ (the time at which shortage starts) and $T$(ordering cycle time) the figure shows much steep variations and continuous curve and hence can be solved by analytical technique with difficulties .Figure (c): surface plot for the order segment dependent inventory cost type model. With respect to $t_1$ (the time at which shortage starts) and $T$(ordering cycle time) the figure shows much abrupt variations and many discontinuities in the curve and hence can be very difficult to solve by analytical techniques.



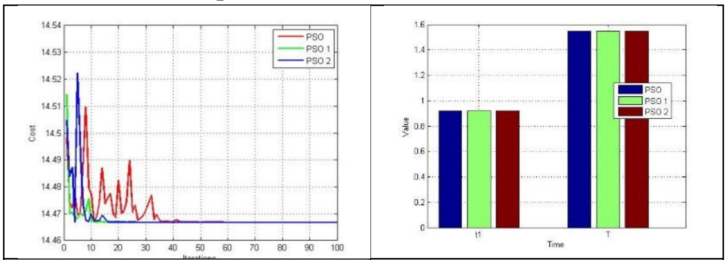

| (a) | (b) |
|---|---|

Figure 10 (a): the value of objective function (fitness value or TVC) at every iteration of PSO for model 1. Figure 10(b): the best values of variables $t_1$ and $T$ for all three PSO for model 1.




| (a) | (b) |
|---|---|

Figure 11(a): the value of objective function (fitness value or TVC) at every iteration of PSO for model 2. Figure 11(b): the best values of variables $t_1$ and $T$ for all three PSO for model 2.




| (a) | (b) |
|---|---|

Figure 12: the value of objective function (fitness value or TVC) at every iteration of PSO for model 3. Figure 12(b): the best values of variables $t_1$ and $T$ for all three PSO for model 3.

**Table 3: Best Fitness Values by all three PSO for model 1, 2 and 3.**

| **Type of PSO** | **Best Fitness (TVC)** | | |
|---|---|---|---|
| | Model 1 | Model 2 | Model 3 |
| **PSO** | 11.6625 | 17.7682 | 14.9668 |
| **PSO1** | 11.4125 | 17.5094 | 14.7168 |
| **PSO2** | 11.2736 | 17.3695 | 14.5779 |

## VIII. CONCLUSION AND FUTURE SCOPE

In this paper presents the mathematical model for three different inventories systems where the inventory cost depends on order size by some continuous function of discontinuous function the paper also presents the derivations for evaluation of the function parameters for practical applications and finally it proposes an efficient approach for solving EOQ problem under the mentioned constrains applied simultaneously. Which may not be solved by analytical approach hence the meta-heuristic approach has been accepted in the form of standard PSO furthermore the performance of standard PSO is also enhanced by alternative use of two different chaotic maps for velocity updating finally it is applied to the EOQ problem for the inventory models discussed above and tested for different systems and objectives. The simulation results shows the proposed approach finds the solution very quickly with much lesser mathematical complexity. The simulation also verifies the superiority of proposed PSO over the standard PSO algorithm and supports the idea that switching between different chaotic pseudorandom number generators for updating the velocity of particles in the PSO algorithm improves its performance and the optimization process. The results for different experiments are collected with different settings and results compared with other methods which shows that the proposed algorithm improves the results by considerable margin.

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