

# Converting Daily Solar Radiation Tabulated Data with Different Sky Conditions into Analytical Expression using Fourier Series Technique

Mohammed Hadi Ali

Assistance Professor University of Mustansiriyah

**ABSTRACT:-** This paper presents a method to convert the Meteorological tabulated data into an analytical expression in which it can be used in engineering application.

The paper uses a Fourier series technique to convert the solar radiation variation during the day for clear days and for a beclouded intermittent days to predict an analytical expression for solar radiation at any time of that day by finding the weights for both ( $a_n$  &  $b_n$ ) parameters for Fourier series upto ( $n=10$ ).

It is found an encouraged result that will help the researchers to use Fourier series to convert any tabulated data into an analytical expression which gives a useful indication for using such method in solar applications.

The results which are drawn in the plots for ( $n$  upto 10) showing the accuracy in coincidence of this series increases for the given function as the ( $n$  terms) increases from ( $n=0$  to  $n=10$ ), for more precision the weights for both ( $a_n$  &  $b_n$ ) parameters ( $n$  more than 10) should be found.

## I. INTRODUCTION

The sun is probably the most important source of renewable energy available today. Solar energy is the energy force that sustains life on Earth for all plants, animals, and people. It provides a compelling solution for all societies to meet their needs for clean, abundant sources of energy in the future [1].

Solar radiation as it passes through the earth's atmosphere undergoes to many factors such as absorption, scattering and reflection by air molecules, water vapor and clouds. The direct solar beam that is arriving directly to the earth's surface is considered as direct solar radiation. The total amount of solar radiation received and falling on a horizontal surface (which is the summation of the direct solar beam plus diffuse solar radiation on a horizontal surface) is called global solar radiation.

Solar radiation, incident to the earth's surface, is a result of complex interactions of energy between the atmosphere and surface [2].

Accurate measurement devices for solar energy are not available. Devices like Pyranometers are capable of measuring solar intensity but, they fail to draw the exact picture of the incident energy. There are hundreds of meteorological stations distributed around the world directly or indirectly measuring solar radiation. These stations work to derive spatial databases from these measurements with different interpolation techniques are used to get the required accurate information or estimation. Other techniques of generating spatial databases are solar radiation models integrated within geographical information systems (GIS) [3].

In addition, solar energy systems require collecting these records of solar radiation and other meteorological data like wind speed, temperature, humidity ratio and cloud which covers the sky. The constructed hourly data base where tabulated and arranged in Handbook which are available in computer compatible form [4].

These tabulated data base is needed from time to time to applied in engineering works, so it is necessary to construct an analytical expression to be defined instead of a table of value. Various methods have been derived to do this work; the simplest one which is the most obvious in evaluating of the integrals in the Euler formulas is done by using the (Trapezoidal Rule). But since for the occasional application that is confronting the most researchers and those who are dealing and handling with numerical functions regularly, the Fourier expansion is quite satisfactory for such application and it will be discussed in this paper.

Because so many problems in harmonic analysis involve functions, such as meteorological or economic quantities, whose period is either a day or a year, it is customary to assume that data are available at interval of ( $1/12$ ,  $1/24$ , or sometime  $1/48$  of a period [5].

## II. THE RESEARCH GOAL

The goal of this research is to carry out a study to change the tabulated data base for solar radiation intensity into a developed analytical expression or formula that can be used in any engineering applications required in solar energy systems or in any meteorological data base such as temperature, wind speed and any parameters that take place regularly and in harmonic way.

## III. THEORETICAL ANALYSIS

Fourier series is that method used to approximate functions with sums of sine and cosine functions. It is capable and well suited to analyze periodic and discontinuous functions, such as communication signals and alternative current, for solving heat transfer applications, and for many other problems in engineering [6].

$$f(t) = \frac{a_0}{2} + a_1 \cos \frac{\pi t}{p} + a_2 \cos \frac{2\pi t}{p} + \dots + a_n \cos \frac{n\pi t}{p} + b_1 \sin \frac{\pi t}{p} + b_2 \sin \frac{2\pi t}{p} + \dots + b_n \sin \frac{n\pi t}{p} \dots (1)$$

We shall consider a function [  $f(t)$  ] of a period ( $2p$ ) for which values are available at intervals of ( $1/24$ ):

$$\Delta t = \frac{2p}{24} = \frac{p}{12}$$

Referring to [ figure ( 1 ) ] any function [  $f(t)$  ] can be expressed as the sum of an even functions and an odd functions, simply by writing:

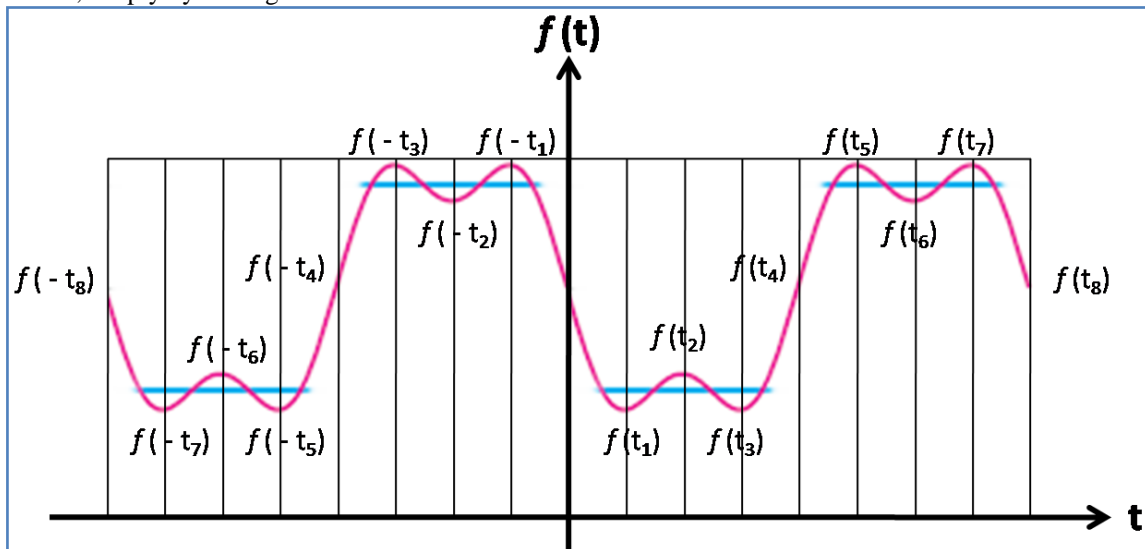


Figure ( 1 ): the schematic drawing showing [  $f(t)$  and  $f(-t)$  ].

$$f(t) = \frac{f(t) + f(-t)}{2} + \frac{f(t) - f(-t)}{2} = g(t) + h(t) \dots (2)$$

Where:

$$g(t) = \frac{f(t) + f(-t)}{2} \dots (3) \quad \text{and} \quad h(t) = \frac{f(t) - f(-t)}{2} \dots (4)$$

Since [  $g(t)$  ] is clearly even and [  $h(t)$  ] is clearly odd, hence the cosine terms in the expansion of [  $f(t)$  ] are just the terms in the half range cosine expression of [  $g(t)$  ], and the sine terms in the expansion of [  $f(t)$  ] in the half range sine expansion of [  $h(t)$  ].

For the even function [  $g(t)$  ], we have as usual:

$$a_n = \frac{2}{p} \int_0^p g(t) \cdot \cos \frac{n\pi t}{p} \cdot dt \dots (5)$$

By applying the expression of trapezoidal rule, with [  $\Delta t = p/12$  ], it is easily can obtain that:

$$a_n = \frac{2}{p} \left[ \frac{p}{12} \left( \frac{g_0}{2} \cos \frac{n\pi}{p} \cdot 0 + g_1 \cdot \cos \frac{n\pi}{p} \cdot \frac{p}{12} + \dots + g_{11} \cdot \cos \frac{n\pi}{p} \cdot \frac{11p}{12} + \frac{g_{12}}{2} \cdot \cos \frac{n\pi}{p} \cdot \frac{12p}{12} \right) \right]$$

$$a_n = \frac{1}{6} \left[ \left( \frac{g_0}{2} + g_1 \cdot \cos \frac{n\pi}{12} + \dots + g_{11} \cdot \cos \frac{11n\pi}{12} + \frac{g_{12}}{2} \cdot \cos n\pi \right) \right]$$

$$a_n = \left[ \left( \frac{g_0}{12} + \frac{g_1}{6} \cdot \cos \frac{n\pi}{6} + \dots + \frac{g_{11}}{6} \cdot \cos \frac{11n\pi}{12} + \frac{g_{12}}{12} \cdot \cos n\pi \right) \right] \dots (6)$$

But we have:

$$g_0 = \frac{f(0) + f(-0)}{2} = \frac{2f(0)}{2} = f(0)$$

$$g_1 = \frac{f(1) + f(-1)}{2}$$

$$g_2 = \frac{f(2) + f(-2)}{2}$$

$$g_{11} = \frac{f(11) + f(-11)}{2}$$

$$g_{12} = \frac{f(12) + f(-12)}{2}$$

Thus:

$$a_n = \left( \frac{f(0)}{12} + \frac{[f(1) + f(-1)]}{12} \cdot \cos \frac{n\pi}{12} + \dots + \frac{[f(11) + f(-11)]}{12} \cdot \cos \frac{11n\pi}{12} + \frac{[f(12) + f(-12)]}{24} \cdot \cos n\pi \right)$$

$$a_n = 0.08333 f(0) + 0.08333 [f(1) + f(-1)] \cdot \cos \frac{n\pi}{12} + \dots + 0.08333 [f(11) + f(-11)] \cdot \cos \frac{11n\pi}{12} + 0.04167 [f(12) + f(-12)] \cdot \cos n\pi \dots (7)$$

The cosine factors in the last expression can be evaluated once and for all and combined with the other numerical factors, including the [ 1/2 ] in the definition of [ g (t) ] to yield a set of weights by which the successive values of the sum [ f (t) + f (-t) ] are to be multiplied before they are added. In this way, weights of [ a<sub>0</sub> (n = 0) & a<sub>1</sub> (n = 1) ] for example take the form of:

$$a_0 = \left[ \left( 0.08333 f(0) + 0.08333 [f(1) + f(-1)] \cdot \cos \frac{0 * \pi}{12} + \dots + 0.08333 [f(11) + f(-11)] \cdot \cos \frac{11 * 0 * \pi}{12} + 0.04167 [f(1) + f(-1)] \cdot \cos 0 * \pi \right) \right]$$

$$a_0 = 0.08333 f(0) + 0.08333 [f(1) + f(-1)] + \dots + 0.08333 [f(11) + f(-11)] + 0.04167 [f(1) + f(-1)]$$

$$a_1 = 0.08333 f(0) + 0.08049 [f(1) + f(-1)] + \dots - 0.08049 [f(11) + f(-11)] - 0.04167 [f(1) + f(-1)]$$

In similar manner, the other of (a's) weights can be calculated. Table (1) shows the weights involved in computation of the first ten of [ a's ].

**Table ( 1 ): The weights for the first ten of ( a<sub>n</sub> ). [5]**

Terms	Weight for determination of ( a <sub>n</sub> )										
	n = 0	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
<i>f<sub>t</sub> + f<sub>-t</sub></i>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.08333</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>	<b>0.0833</b>
<i>g<sub>0</sub> = f<sub>0</sub></i>	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.08333</b>	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3	<b>0.0833</b> 3
<i>f<sub>1</sub> + f<sub>-1</sub></i>	<b>0.0833</b> 3	<b>0.0804</b> 9	<b>0.0721</b> 7	<b>0.05893</b>	<b>0.0416</b> 7	<b>0.0215</b> 7	<b>0.0000</b> 0	- 0.0215 7	- 0.0416 7	- 0.0589 3	- 0.0721 7
<i>f<sub>2</sub> + f<sub>-2</sub></i>	<b>0.0833</b> 3	<b>0.0721</b> 7	<b>0.0416</b> 7	<b>0.00000</b>	- 0.0416 7	- 0.0721 7	- 0.0833 3	- 0.0721 7	- 0.0416 7	<b>0.0000</b> 0	<b>0.0416</b> 7
<i>f<sub>3</sub> + f<sub>-3</sub></i>	<b>0.0833</b> 3	<b>0.0589</b> 3	<b>0.0000</b> 0	- 0.05893	- 0.0833 3	- 0.0589 3	<b>0.0000</b> 0	<b>0.0589</b> 3	<b>0.0833</b> 3	<b>0.0589</b> 3	<b>0.0000</b> 0
<i>f<sub>4</sub> + f<sub>-4</sub></i>	<b>0.0833</b> 3	<b>0.0416</b> 7	- 0.0416 7	- 0.08333	- 0.0416 7	<b>0.0416</b> 7	<b>0.0833</b> 3	<b>0.0416</b> 7	- 0.0416 7	- 0.0833 3	- 0.0416 7
<i>f<sub>5</sub> + f<sub>-5</sub></i>	<b>0.0833</b> 3	<b>0.0215</b> 7	- 0.0721 7	- 0.05893	<b>0.0416</b> 7	<b>0.0804</b> 9	<b>0.0000</b> 0	- 0.0804 9	- 0.0416 7	<b>0.0589</b> 3	<b>0.0721</b> 7
<i>f<sub>6</sub> + f<sub>-6</sub></i>	<b>0.0833</b>	<b>0.0000</b>	-	<b>0.00000</b>	<b>0.0833</b>	<b>0.0000</b>	-	<b>0.0000</b>	<b>0.0833</b>	<b>0.0000</b>	-

	3	0	0.0833		3	0	0.0833	0	3	0	0.0833
			3				3				3
$f_7 + f_{-7}$	0.0833	-	-	0.05893	0.0416	-	0.0000	0.0804	-	-	0.0721
	3	0.0215	0.0721		7	0.0804	0	9	0.0416	0.0589	7
		7	7			9		7	7	3	
$f_8 + f_{-8}$	0.0833	-	-	0.08333	-	-	0.0833	-	-	0.0833	-
	3	0.0416	0.0416		0.0416	0.0416	3	0.0416	0.0416	3	0.0416
		7	7		7	7		7	7	7	7
$f_9 + f_{-9}$	0.0833	-	0.0000	0.05893	-	0.0589	0.0000	-	0.0833	-	0.0000
	3	0.0589	0		0.0833	3	0	0.0589	3	0.0589	0
		3			3			3	3	3	
$f_{10} + f_{-10}$	0.0833	-	0.0416	0.00000	-	0.0721	-	0.0721	-	0.0000	0.0416
	3	0.0721	7		0.0416	7	0.0833	7	0.0416	0	7
		7			7		3	7	7	0	
$f_{11} + f_{-11}$	0.0833	-	0.0721	-	0.0416	-	0.0000	0.0215	-	0.0589	-
	3	0.0804	7	0.05893	7	0.0215	0	7	0.0416	3	0.0721
		9			7	7		7	7	3	7
$f_{12} + f_{-12}$	0.0416	-	0.0416	-	0.0416	-	0.0416	-	0.0416	-	0.0416
	7	0.0416	7	0.04167	7	0.0416	7	0.0416	7	0.0416	7
		7			7	7		7	7	7	7

Similarly, for [ h ( t ) ], it can easily obtain that:

$$b_n = \frac{2}{p} \int_0^p h(t) \cdot \sin \frac{n \pi t}{p} \cdot dt$$

$$b_n = \frac{2}{p} \left[ \frac{p}{12} \left( \frac{h_0}{2} \sin \frac{n \pi}{p} * 0 + h_1 \cdot \sin \frac{n \pi}{p} \cdot \frac{p}{12} + \dots + h_{11} \cdot \sin \frac{n \pi}{p} \cdot \frac{11p}{12} + \frac{h_{12}}{2} \cdot \sin \frac{n \pi}{p} \cdot \frac{12p}{12} \right) \right]$$

$$b_n = \frac{1}{6} \left[ \left( h_1 \cdot \sin \frac{n \pi}{12} + h_2 \cdot \sin \frac{n \pi}{6} \dots + h_{11} \cdot \sin \frac{11n \pi}{12} \right) \right]$$

$$b_n = \left( \frac{h_1}{6} \cdot \sin \frac{n \pi}{12} + \frac{h_2}{6} \cdot \sin \frac{n \pi}{6} \dots + \frac{h_{11}}{6} \cdot \sin \frac{11n \pi}{12} \right) \dots (8)$$

$$h_1 = \frac{f(1) - f(-1)}{2}$$

$$h_{11} = \frac{f(11) - f(-11)}{2} \quad \text{and} \quad h_{12} = \frac{f(12) - f(-12)}{2} = 0$$

$$b_n = \frac{[f(1) - f(-1)]}{12} \cdot \sin \frac{n \pi}{12} + \frac{[f(2) - f(-2)]}{12} \cdot \sin \frac{n \pi}{6} \dots \dots + \frac{[f(11) - f(-11)]}{12} \cdot \sin \frac{11n \pi}{12}$$

$$b_n = 0.08333 [f(1) - f(-1)] \cdot \sin \frac{n \pi}{12} + 0.08333 [f(2) - f(-2)] \cdot \sin \frac{n \pi}{6} \dots \dots$$

$$+ 0.08333 [f(11) - f(-11)] \cdot \sin \frac{11n \pi}{12} \dots (9)$$

In similar way for [ ( n = 0 ) ,  $b_0$  will equal zero ] since [  $\sin ( 0 ) = 0$  ], and the weight of (  $b_1$  ) for example takes the form of:

$$b_1 = 0.08333 [f(1) - f(-1)] \cdot \sin \frac{1 * \pi}{12} + 0.08333 [f(2) - f(-2)] \cdot \sin \frac{2 * \pi}{6} + \dots \dots$$

$$+ 0.08333 [f(11) - f(-11)] \cdot \sin \frac{11 * 1 * \pi}{12}$$

$$b_1 = 0.02157 [f(1) - f(-1)] + 0.04167 [f(2) - f(-2)] + \dots \dots + 0.02157 [f(11) - f(-11)]$$

The weights involved in computation of the first ten of [  $b$ 's ] required for the evaluation of this expression are given in table (2).

**Table ( 2 ): The weights for the first ten of (  $b_n$  ). [5]**

Terms	Weight for determination of ( $b_n$ )									
	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9	n = 10
$f_t + f_{-t}$										
$f_1 - f_{-1}$	0.0215 7	0.0416 7	0.0589 3	0.0721 7	0.0804 9	0.0833 3	0.0804 9	0.0721 7	0.0589 3	0.0416 7
$f_2 - f_{-2}$	0.0416 7	0.0721 7	0.0833 3	0.0721 7	0.0416 7	0.0000 0	- 0.0416 7	- 0.0721 7	- 0.0833 3	- 0.0721 7
$f_3 - f_{-3}$	0.0589 3	0.0833 3	0.0589 3	0.0000 0	- 0.0589 3	- 0.0833 3	- 0.0589 3	0.0000 0	0.0589 3	0.0833 3
$f_4 - f_{-4}$	0.0721 7	0.0721 7	0.0000 0	- 0.0721 7	- 0.0721 7	0.0000 0	0.0721 7	0.0721 7	0.0000 0	- 0.0721 7
$f_5 - f_{-5}$	0.0804 9	0.0416 7	- 0.0589 3	- 0.0721 7	0.0215 7	0.0833 3	0.0216 7	- 0.0721 7	- 0.0589 3	0.0416 7
$f_6 - f_{-6}$	0.0833 3	0.0000 0	- 0.0833 3	0.0000 0	0.0833 3	0.0000 0	- 0.0833 3	0.0000 0	0.0833 3	0.0000 0
$f_7 - f_{-7}$	0.0804 9	- 0.0416 7	- 0.0589 3	0.0721 7	0.0215 7	- 0.0833 3	0.0216 7	0.0721 7	- 0.0589 3	- 0.0416 7
$f_8 - f_{-8}$	0.0721 7	- 0.0721 7	0.0000 0	0.0721 7	- 0.0721 7	0.0000 0	0.0721 7	- 0.0721 7	0.0000 0	0.0721 7
$f_9 - f_{-9}$	0.0589 3	- 0.0833 3	0.0589 3	0.0000 0	- 0.0589 3	0.0833 3	- 0.0589 3	0.0000 0	0.0589 3	- 0.0833 3
$f_{10} - f_{-10}$	0.0416 7	- 0.0721 7	0.0833 3	- 0.0721 7	0.0416 7	0.0000 0	- 0.0416 7	0.0721 7	- 0.0833 3	0.0721 7
$f_{11} - f_{-11}$	0.0215 7	- 0.0416 7	0.0589 3	- 0.0721 7	0.0894 9	- 0.0833 3	0.0804 9	- 0.0721 7	0.0589 3	- 0.0416 7

**IV. THEORETICAL APPLICATION**

In this paragraph, and in order to give an example to how apply the Fourier series to serve this method a sample of theoretical calculation will put into practice to find the way to convert the tabulated data for solar radiation intensity or any other meteorological data into an analytical expression, to do that we should go back for any tabulated data for solar radiation intensity.

We will choose any arbitrary data for clear sky day and the other for partially cloudy day. From the Meteorology Database for Baghdad City (2000), the solar radiation intensity of clear sky day for (21<sup>st</sup> of December) can be tabulated in the following table (3):

**Table ( 3 ): The Solar Radiation for 21<sup>st</sup> of December [7].**

Time (hour)																						
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
0	0	0	0	0	0	0	3	1	2	3	3	3	3	2	1	3	0	0	0	0	0	0
							4	5	5	0	7	9	5	7	5	4						
								3	5	6	4	1	7	2	3							
Solar Radiation Intensity ( W/m <sup>2</sup> )																						

The tabulated solar intensity was drawn as shown in [figure (2)], in which [  $f(t)$  &  $f(-t)$  ] are explained, so the functions [  $f(t)$  &  $f(-t)$  ] can be calculated easily and tabulated in table (4):

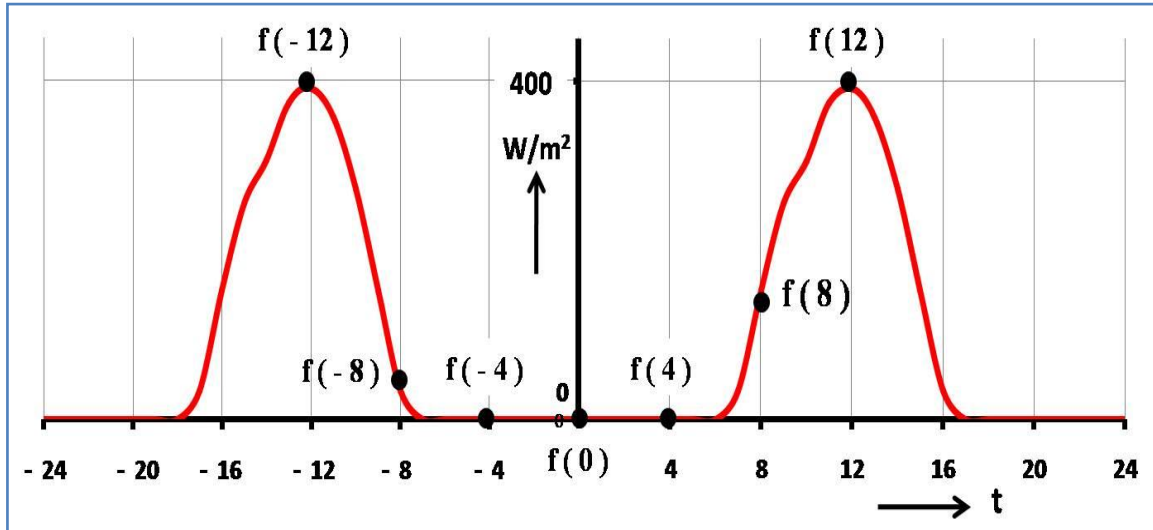


Figure ( 2 ): the solar radiation intensity for (21<sup>st</sup> of Dec.) showing [  $f(t)$  and  $f(-t)$  ].

Table ( 4 ): The values of [  $f(t)$  &  $f(-t)$  ].

$f(-12)$	$f(-11)$	$f(-10)$	$f(-9)$	$f(-8)$	$f(-7)$	$f(-6)$	$f(-5)$	$f(-4)$	$f(-3)$	$f(-2)$	$f(-1)$	
391	357	272	153	34	0	0	0	0	0	0	0	
$f(0)$	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(7)$	$f(8)$	$f(9)$	$f(10)$	$f(11)$	$f(12)$
0	0	0	0	0	0	0	34	153	255	306	374	391

The values of (  $a_n$  and  $b_n$  ) now can be found by utilizing table (4) in conjugation with equations ( 7 & 9 ), the (  $a_n$  and  $b_n$  ):

$$a_n = 0.08333 * 0 + 0.08333 [0 + 0]. \cos \frac{n \pi}{12} + \dots + 0.08333 [374 + 357]. \cos \frac{11n \pi}{12} + 0.04167 [391 + 391]. \cos n \pi$$

$$b_n = 0.08333 [f(1) - f(-1)]. \sin \frac{n \pi}{12} + 0.08333 [f(2) - f(-2)]. \sin \frac{n \pi}{6} \dots + 0.08333 [f(11) - f(-11)]. \sin \frac{11n \pi}{12}$$

For (  $n = 0$  ), [  $a_0$  ] was found to be equal to ( 194.08 ) and for (  $n = 1$  ), [  $a_1$  ] was found to be equal to ( - 165.71 ).

For (  $n = 0$  ), [  $b_0$  ] was found to be equal to ( 0 ) since [  $\sin(0) = 0$  ], and for (  $n = 1$  ), [  $b_1$  ] was found to be equal to ( 19.12 ).

For other values of (  $a_n$  &  $b_n$  ) involved for the calculation for the first ten of them (upto  $n = 10$ ), they were tabulated and shown in table ( 5 ):

Table ( 5 ): The values of first ten of [  $a_n$  &  $b_n$  ].

Values of ( $a_n$ )										
$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
194.08	-165.7	99.18	-34.03	-1.41	6.88	0.00	-4.20	2.83	0.03	-1.42
Values of ( $b_n$ )										
$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
0	19.12	-21.67	7.84	7.36	-10.93	4.25	3.27	-4.91	2.18	0.42

Substituting the values the first six of [  $a_n$  and  $b_n$  ] into equation (1), it yields to an analytical expression for the solar radiation [  $f(t)$  ] at any required time:

$$f(t) = 97.04 - a_1 165.7 \cos \frac{\pi t}{12} + 99.18 \cos \frac{2 \pi t}{12} + \dots - 0 * \cos \frac{6 \pi t}{12} + 19.12 \sin \frac{\pi t}{12} - 21.67 \sin \frac{2 \pi t}{12} + \dots + 4.25 \sin \frac{6 \pi t}{12}$$

To obtain for example the value for solar radiation [  $f(11)$  ] at time (  $t = 11$  hour), it can be done simply by substituting for (  $t = 11$  ) in this equation to get its value, which is:

$$f(11) = 97.04 - a_1 165.7 \cos \frac{\pi * 11}{12} + 99.18 \cos \frac{2\pi * 11}{12} + \dots + 6.88 \cos \frac{5\pi * 11}{12} + 19.12 \sin \frac{\pi * 11}{12} - 21.67 \sin \frac{2\pi * 11}{12} + \dots + 4.25 \sin \frac{6\pi * 11}{12}$$

$$f(11) = 346.5 \frac{W}{m^2}$$

Comparing the actual tabulated value for the solar radiation at (  $t = 11$  hour ) which is (  $374 \text{ W/m}^2$  ) with the value obtained from the analytical expression  $346.5 \text{ W/m}^2$ , the values are too close for each other.

### V. RESULTS AND DISCUSSION

In this paragraph, the analysis for this research shows the possibility to predict the solar radiation intensity in analytical expression rather than in tabulated data. The analysis can be carried out by choosing the solar radiation for a dedicated day which want to convert into analytical expression from a Meteorological database and performing the procedure of Fourier series technique to obtain the analytical expression for that day.

For a clear shining day of (  $21^{\text{st}}$  of December ), the Meteorological database was shown in table (no. 1). Performing the Fourier series procedures that was explained in theoretical application paragraph upto (  $n = 6$  ), it shows a good precision coincidence between the tabulated database and the analytical expression.

Figure (3) shows the solar radiation intensity for (  $21^{\text{st}}$  of December ) in its analytical expression and Meteorological database for (  $n = 4$  ), the figure shows the coincidence between them for time period of (  $t = 6$  to  $t = 15$  ) is too close, but it is not for the rest of time periods.

Figure (4) shows the solar radiation intensity for the same day but for other values of (  $n$  ) for (  $n = 2$  and  $n = 6$  ), it is clear from the figure that as (  $n$  ) increases the coincidence precision increases to obtain a good fitness between the values for both of them.

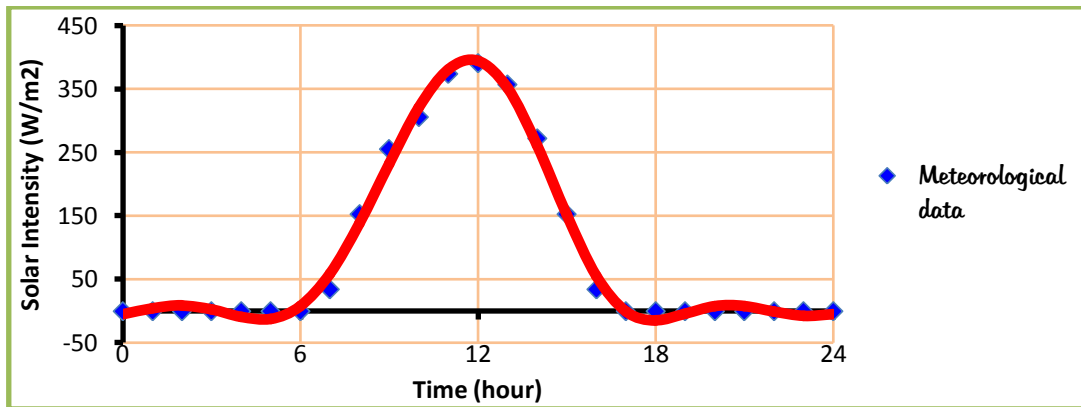


Figure ( 3 ): the Meteorological data for solar radiation intensity for (  $21^{\text{st}}$  of Dec.) and the analytical expression for (  $n = 4$  ).

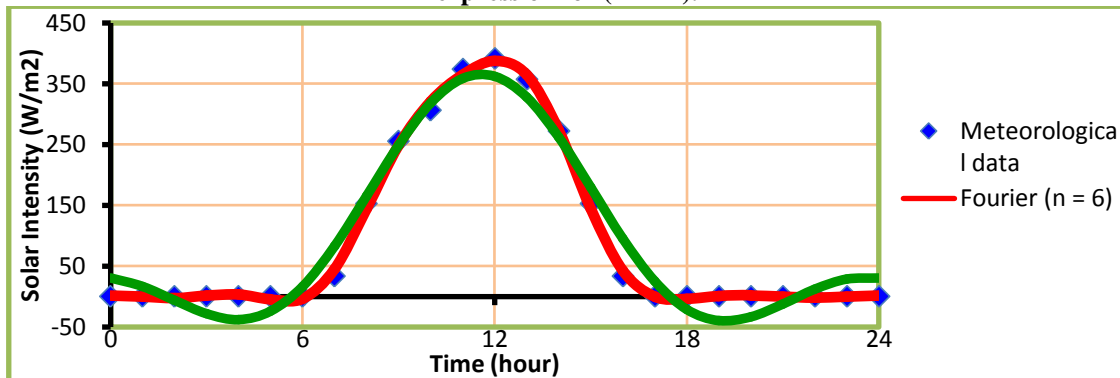


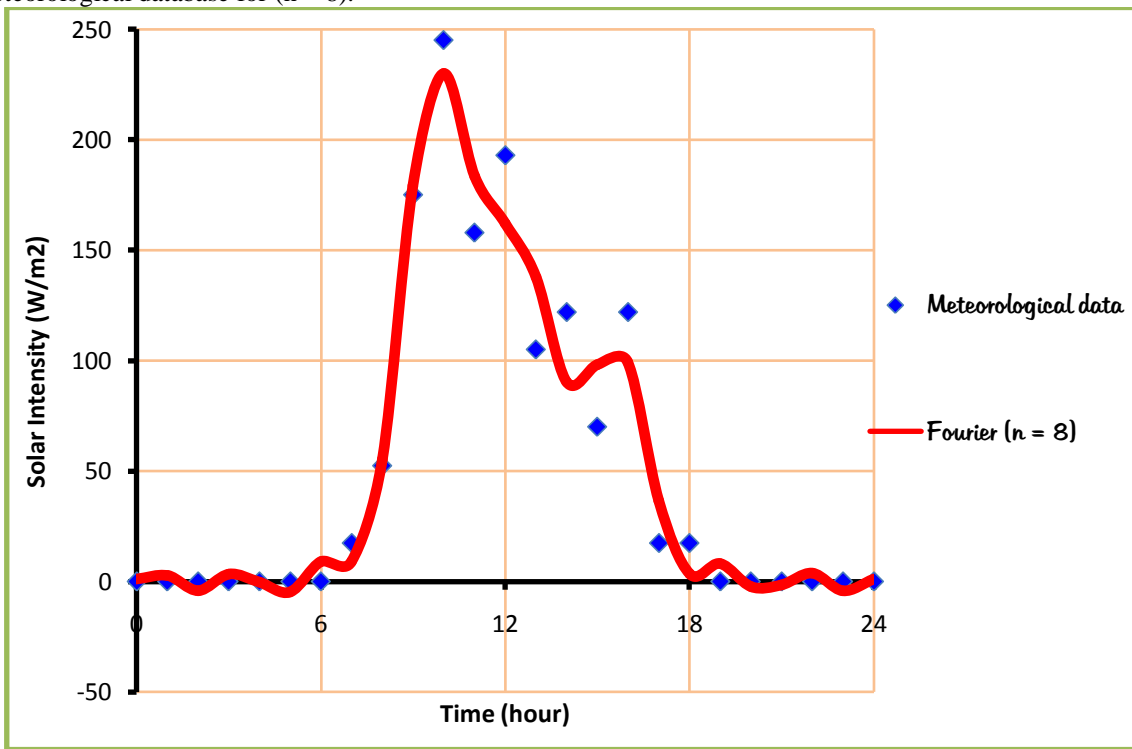
Figure ( 4 ): the Meteorological data for solar radiation intensity for (  $21^{\text{st}}$  of Dec.) and the analytical expression for (  $n = 2$  &  $n = 6$  ).

For other day which is not clear or shiny day, and it could be cloudy and sun shining beclouds intermittently, it can also using Fourier series to get and analytical expression for that day. For instance, the Meteorological data for the solar radiation intensity for (27<sup>th</sup> of Feb.) which is tabulated in table (6) can be used to find an analytical expression for this day.

**Table ( 6 ): The Solar Radiation for 21<sup>st</sup> of December [7].**

Time (hour)																						
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
0	0	0	0	0	0	0	1	5	1	2	1	1	1	1	7	1	1	1	0	0	0	0
Solar Radiation Intensity ( W/m <sup>2</sup> )																						
0	0	0	0	0	0	0	8	3	7	4	5	9	0	2	0	2	8	8	0	0	0	0
									5	5	8	3	5	2		2						

It is clear that it is beclouded day, and the sun is shining intermittently, so by using the Fourier series an analytical expression can be found and used in any engineering application. Figure (5) shows the solar radiation intensity for (27<sup>th</sup> of February) in its analytical expression and Meteorological database for (n = 8).



**Figure ( 5): the Meteorological data for solar radiation intensity for (27<sup>th</sup> of Feb.) and the analytical expression for ( n = 8 ).**

This figure shows the coincidence between them for time period of ( t = 6 to t = 18 ) isn't too close, and the coincidence doesn't gives a precise fitting between them.

Figure (6) shows the solar radiation intensity for the same day but for other values of (n) for (n = 6 and n = 10), it is clear from the figure that the plot for (n = 10) is more precision than the plot for (n = 6). As a conclusion it is obvious that as ( n ) increases the coincidence precision increases to obtain a good fitness between the values for both the tabulated values and the analytical expression.



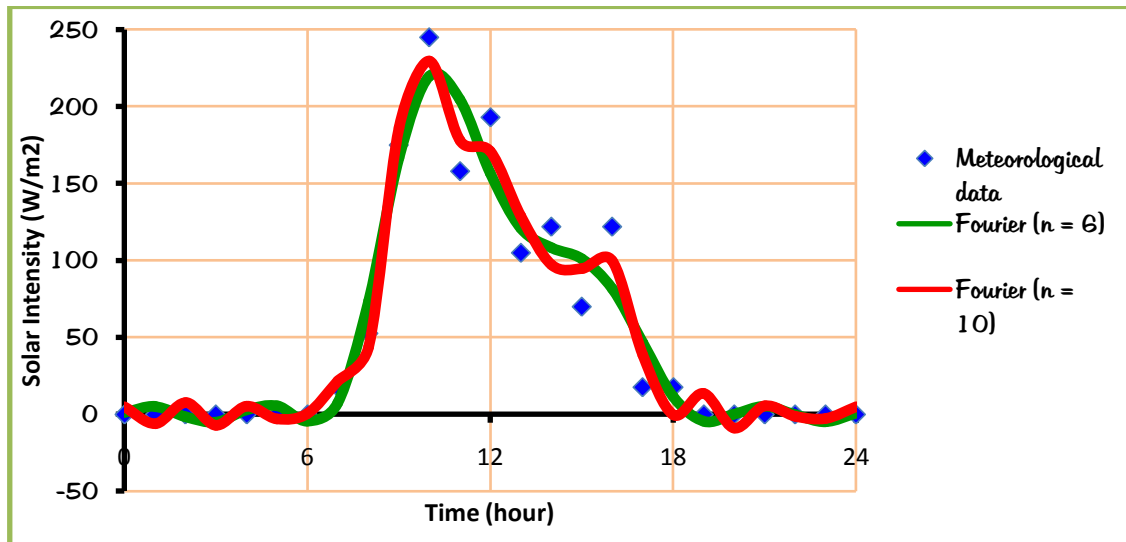


Figure ( 6 ): the Meteorological data for solar radiation intensity for (27<sup>th</sup> of Feb.) and the analytical expression for ( n = 6 & n = 10 ).

The plots showing the accuracy with which the (n terms) of this series increase for the given function and they are almost indistinguishable from the graph of  $f(t)$ .

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