OPEN ACCESS

International
Journal
Of Modern Engineering Research (IJMER)

Magneto hydrodynamic Rayleigh Problem with Hall Effect and Rotation in the Presence of Heat Transfer

P T Hemamalini[1] and M Deivanayaki[2]

[1]Professor, Department of Science and Humanities, Faculty of Engineering, Karpagam University, Coimbatore,
[2]Research Scholar, Department of Mathematics, Karpagam University, Coimbatore.

ABSTRACT: *The main objective of the paper is to investigate the magneto hydrodynamic flow version of the Rayleigh problem including Hall effect and Rotation in the presence of Heat transfer. Analytical solution has been found depending on the physical parameters including the Hall parameter* N, *Hartmann number* M, *Grashof number* Gr, *Prandtl number* $\Pr$ *and the Rotation parameter* K^2. *The influence of these parameters on velocity profiles and temperature profiles are demonstrated graphically and the results are discussed. Further, it is observed that increase in the Hall parameter and Rotation parameter leads to decrease in the velocity profiles. It is also found that the increase of Grashof number results in the decrease of primary flow and increase of secondary flows.*

Keywords:- *MHD flow, Hall Effect, viscous fluid, Heat transfer, Rotation*

I. INTRODUCTION

In recent years, the analysis of hydromagnetic flow has applications in diverse fields of Science and Technology such as soil sciences, astrophysics, nuclear power reactors etc. The study of MHD flow problems has achieved remarkable interest due to its application in MHD generators, MHD pumps and MHD flow meters etc. Geophysics encounters MHD phenomena in interaction on conducting fluids and magnetic fluids. The rotating flow of an electrically conducting fluid in presence of magnetic field has got its importance in Geophysical problems. The study of rotating flow problems is also important in the solar physics dealing with the sunspot development, the solar cycle and the structure of rotating magnetic stars.

Stoke's analysed the flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate in its own plane. Rossow[1] initiated Rayleigh's problem for non-conducting plate while Chang&Yen[2] have taken the plate to be perfectly conducting in a various transverse magnetic field. The effect of coriolis forces on Rayleigh Problem was considered by Sathi [3]. Rayleigh problem in MHD with suction was considered by Girish Chandra Pandey[4].

The Hall effect is due merely to the sideways magnetic force on the drifting free charges. The electric field has to have a component transverse to the direction of the current density to balance this force. In many works on plasma physics, the Hall effect cannot be ignored as it has a significant effect on the flow pattern of an ionized gas. Hall effect results in a development of an additional potential difference between opposite surfaces of a conductor for which a current is induced perpendicular to both the electric and magnetic field. This current is termed as Hall current. The effect of Hall current on MHD Rayleigh's problem in ionized gas was discussed by Mohanty[5].Solution of Rayleigh problem for conducting fluid was considered by Abd-el-Malek etal[6]. Magneto hydrodynamic Rayleigh problem with Hall effect was studied by Haytham Sulieman[7]. Deivanayaki etal[8] studied the MHD Rayleigh problem with Hall effect and Rotation. In this study the effect of the Hall current and rotation on the magneto hydrodynamic flow version of the classical Rayleigh problem in the presence of Heat transfer was considered.

II. FORMULATION OF THE PROBLEM

The scenario under investigation comprises an incompressible electrically conducting, viscous fluid past an infinite vertical plate occupying the plane $y = 0$. The x-axis is taken in the direction of the motion of the plate and z – axis lying on the plate normal to both x and y – axis. Initially it is assumed that the plate and the fluid rotate in unison with a uniform angular velocity Ω about the y - axis normal to the plane are at the same temperature T everywhere in the fluid. At time $t > 0$, the plate starts moving impulsively with the uniform velocity in its own plane along the x-axis. Also the temperature of the plate is raised/lowered to T_∞. A uniform magnetic field H_0, parallel to y - axis is imposed. The schematic diagram is given by

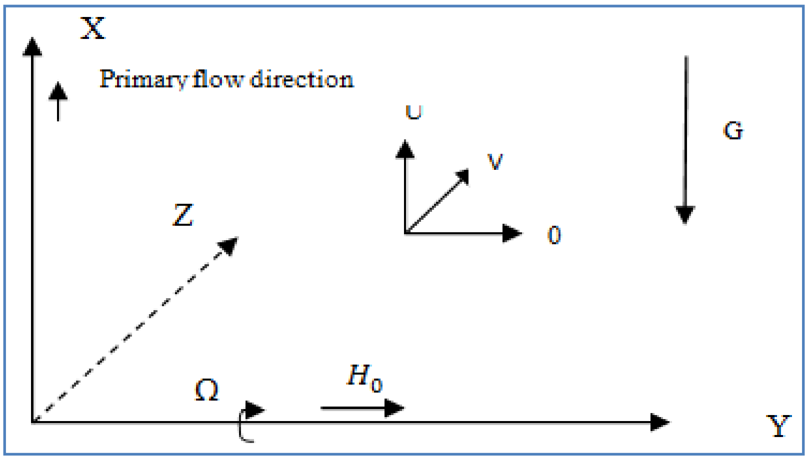

Fig 1: Schematic representation

Under the above assumptions, in the absence of an input electric field the governing boundary layer equations are

Equations of continuity $\nabla.\bar{q}=0$ (2.1)

Equation of motion

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q} + 2\Omega \times \bar{q} = -\frac{1}{\rho}\nabla P + \gamma\nabla^2\bar{q} + \frac{1}{\rho}\bar{J} \times \bar{H} + \rho\bar{g} \quad (2.2)$$

The energy equation

$$\kappa\, \nabla^2 T = \rho\, C_p \frac{DT}{Dt} \quad (2.3)$$

The generalized Ohm's law, neglecting ion-slip effect but taking Hall current is,

$$\frac{J}{\sigma} = (E + \bar{q} \times H) - \frac{J \times H}{n.e} \quad (2.4)$$

where $\sigma = \frac{e^2 m}{m}$ (is the electrical conductivity). Here J is the current density, t is the time, ρ is density, γ is kinematic viscosity, *e* is electric charge, *m* is mass of an electron, *n* is the electron number density, τ is the mean collision time and μ is magnetic permeability.

Let $T - T_\infty = \theta(y,t)$

The initial and boundary conditions are

$t \leq 0$: $u = 0, v = 0, \theta = 0$ for $y \geq 0$

$t > 0$: $u = U_0, v = 0, \theta(0,t) = ae^{i\omega t}$ for $y = 0$

$u = 0, v = 0, \theta(\infty,t) = 0$ as $y \to \infty$

$H_x \to 0, H_y = H_0, H_z \to 0$ as $y \to \infty$ (2.5)

At infinity the magnetic induction is uniform with components (0,H,0), and hence the current density vanishes and since the free stream is at rest, it follows from generalized Ohm's law that $E = 0$ as $y \to \infty$. Assuming small magnetic Reynolds number for the flow, the induced magnetic field is neglected in comparison to the applied constant field H_0.

Introducing the non-dimensional quantities:

$$y^* = \frac{U_0.y}{\gamma},\quad u^* = \frac{u}{U_0},\quad v^* = \frac{v}{U_0},\quad t^* = \frac{U_0^2 t}{\gamma}, \theta^* = \frac{\theta}{a}, Gr = \frac{g\beta\gamma a}{U_0^3}, N = \omega\tau, M^2 = \frac{\sigma H_0^2 \gamma}{\rho U_0^2},$$

$$Pr = \frac{\gamma\rho C_p}{\kappa}, K^2 = \frac{\gamma\Omega_y}{\rho U_0^2}, \psi = \frac{\gamma\omega}{U_0^2} \quad (2.6)$$

All the physical variables are defined in the Nomenclature.

Equations (2.1), (2.2) and (2.3) transform to the following non-dimensional forms, respectively.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta - \frac{M^2}{(1+N^2)}(u + Nv) - 2vK^2 \quad (2.7)$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial y^2} + \frac{M^2}{(1+N^2)}(Nu - v) + 2uK^2 \quad (2.8)$$

$$\frac{\partial^2 \theta}{\partial y^2} = Pr\frac{\partial \theta}{\partial t} \quad (2.9)$$

The corresponding boundary conditions in non-dimensional form are

For $t \leq 0$: $u(y,0) = v(y,0) = 0, \theta(y,0) = 0\ for\ all\ y$

For $t > 0$: $u(0,t) = 1, v(0,t) = 0, \theta(0,t) = e^{i\psi t} for\ y = 0$.

$u(y,t) = 0, v(y,t) = 0, \theta(\infty,t) = 0$ as $y \to \infty$ (2.10)

III. METHOD OF SOLUTIONS

The equations (2.7) and (2.8) can be combined using the complex variable $q = u + iv$

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \left[\left(\frac{M^2}{1+N^2}\right)(1 - iN) - 2iK^2\right] q + Gr\theta \qquad (3.1)$$

The boundary condition (2.10) are transformed to

$$q(y,0) = 0, \quad q(0,t) = 1, \quad q(\infty,t) = 0$$
$$\theta(y,0) = 0, \quad \theta(0,t) = e^{i\psi t}, \quad \theta(\infty,t) = 0 \qquad (3.2)$$

Substitute $\theta(y,t) = e^{i\psi t} f(y)$ in equation (2.9), we have

$$f''(y) - i\psi \Pr f(y) = 0 \qquad (3.3)$$

The equation (3.3) can be solved under the boundary condition

$$f(0) = 1, f(\infty) = 0 \qquad (3.4)$$

Hence the solution is $f(y) = e^{-y\sqrt{i\psi Pr}}$ (3.5)

Separating the real and imaginary parts and taking the real part only we get

$$\theta(y,t) = e^{-yx_1} Cos(\psi t - x_1 y) \qquad (3.6)$$

where $x_1 = \sqrt{\frac{\psi Pr}{2}}$.

Also substitute $q(y,t) = e^{i\psi t} g(y)$ in (3.1), we have

$$g''(y) - (i\psi + \beta) g(y) = -Gre^{-y\sqrt{i\psi Pr}} \qquad (3.7)$$

The equation (3.7) can be solved under the boundary conditions,

$$g(0) = e^{-i\psi t}, g(\infty) = 0 \qquad (3.8)$$

Therefore the solution is

$$g(y) = \left(e^{-i\psi t} + \frac{Gr}{i\psi Pr - (\psi i + \beta)}\right) e^{-y\sqrt{i\psi + \beta}} - \frac{Gr}{i\psi Pr - (i\psi + \beta)} e^{-y\sqrt{i\psi Pr}} \qquad (3.9)$$

Separating equation (3.9) into real and imaginary parts, we have

$$u = e^{-yx_3} Cos(\psi t) Cos(\psi t - yx_4) + e^{-yx_3} Sin(\psi t) Sin(\psi t - yx_4) + \frac{(\psi Pr - \psi - b) Gr\, e^{-yx_3} Sin(\psi t - yx_4)}{(\psi Pr - \psi - b)^2 + a^2} - \frac{a\, Gr\, e^{-yx_3} Cos(\psi t - yx_4)}{(\psi Pr - \psi - b)^2 + a^2} - \frac{(\psi Pr - \psi - b) Gr\, e^{-yx_1} Sin(\psi t - yx_1)}{(\psi Pr - \psi - b)^2 + a^2} + \frac{a\, Gr\, e^{-yx_1} Cos(\psi t - yx_1)}{(\psi Pr - \psi - b)^2 + a^2} \qquad (3.10)$$

$$v = e^{-yx_3} Cos(\psi t) Sin(\psi t - yx_4) - e^{-yx_3} Sin(\psi t) Cos(\psi t - yx_4) - \frac{(\psi Pr - \psi - b) Gr\, e^{-yx_3} Cos(\psi t - yx_4)}{(\psi Pr - \psi - b)^2 + a^2} - \frac{a\, Gr\, e^{-yx_3} Sin(\psi t - yx_4)}{(\psi Pr - \psi - b)^2 + a^2} + \frac{(\psi Pr - \psi - b) Gr\, e^{-yx_1} Cos(\psi t - yx_1)}{(\psi Pr - \psi - b)^2 + a^2} + \frac{a\, Gr\, e^{-yx_1} Sin(\psi t - yx_1)}{(\psi Pr - \psi - b)^2 + a^2} \qquad (3.11)$$

The constants involved in the above discussion have been obtained but not presented here for the sake of brevity.

In order to attain a physical insight into the problem, we have carried out numerical calculations for the velocity fields and temperature field at the plate due to time, Hartmann number, Hall parameter, Rotation parameter, Prandtl

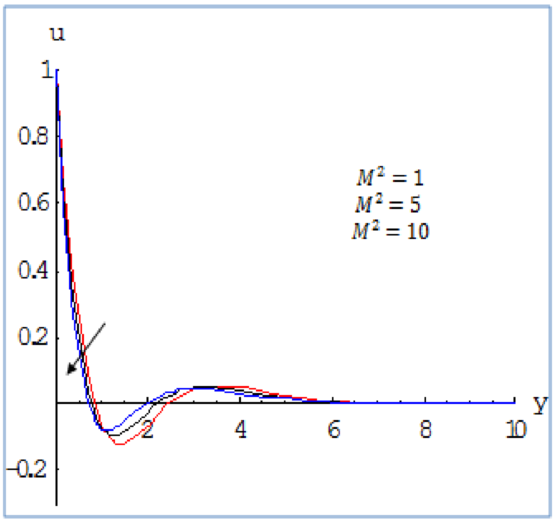

Fig2: Variation of u when M^2 increases with

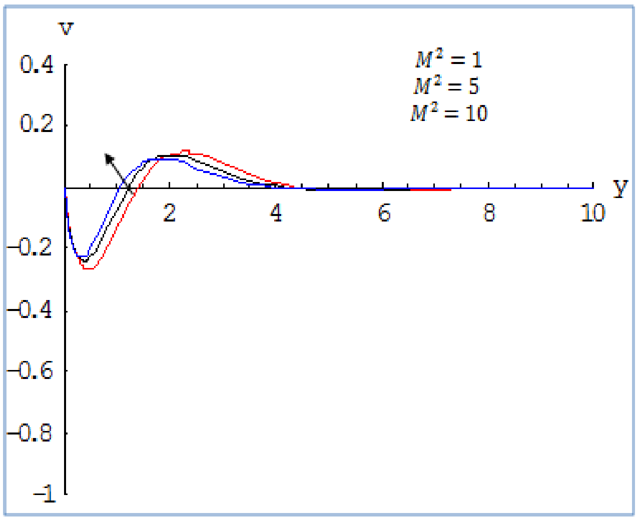

Fig3: Variation of v when M^2 increases with y

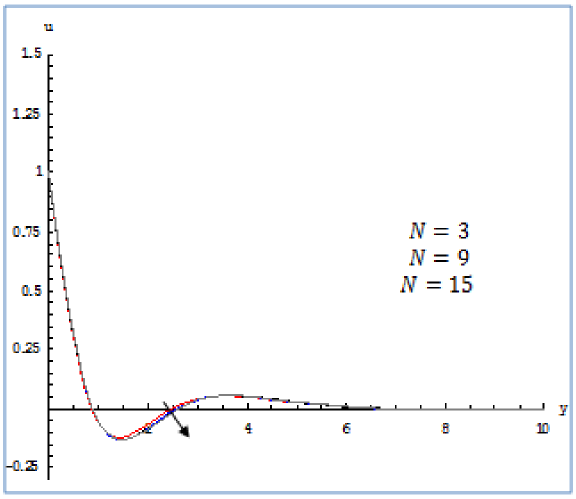

Fig4: Variation of u when N increases with y

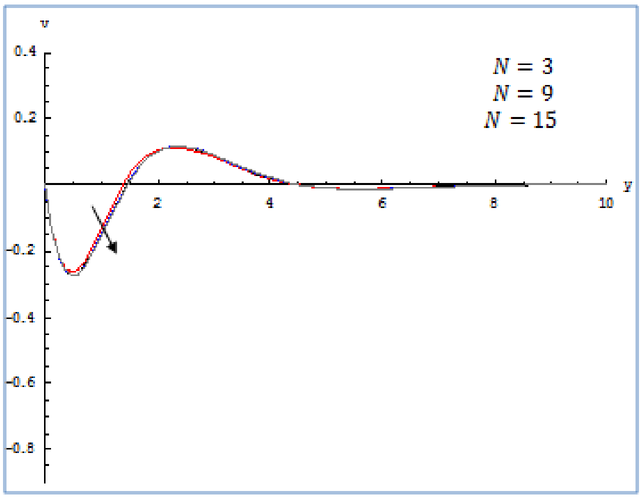

Fig5: Variation of v when N increases with y

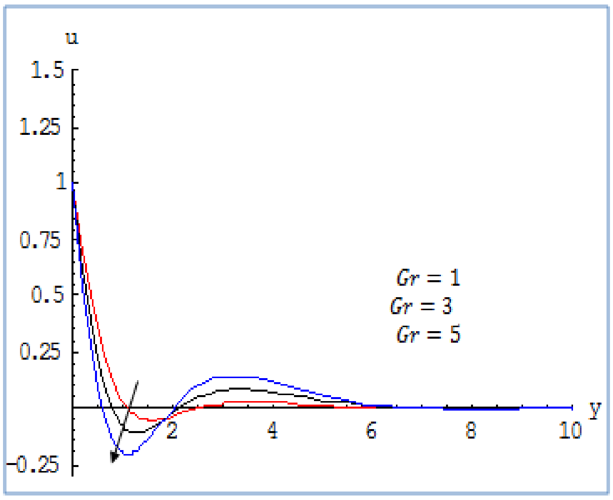

Fig6: Variation of u when Gr increases with y

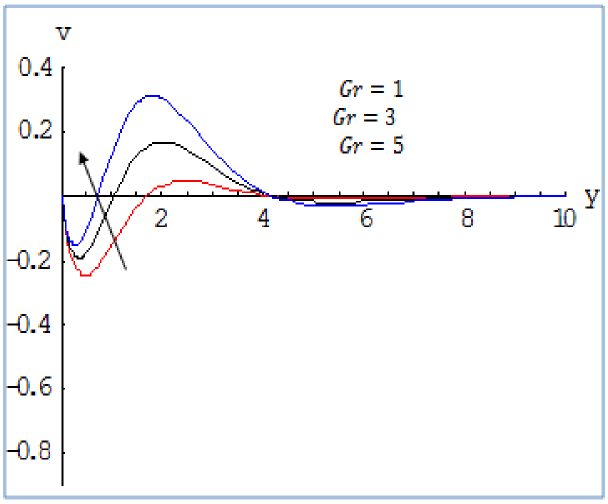

Fig7: Variation of v when Gr increases with y

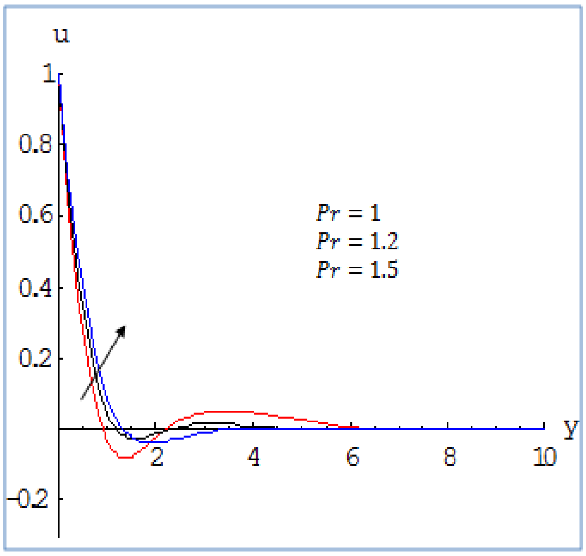

Fig8: Variation of u when Pr increases with y

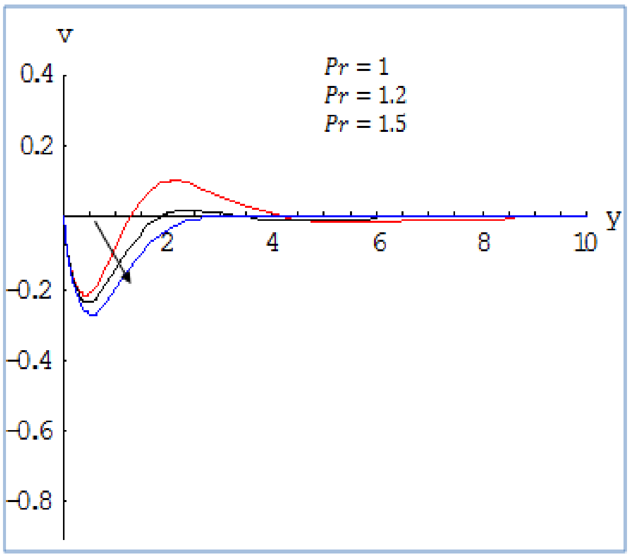

Fig9: Variation of v when Pr increases with y

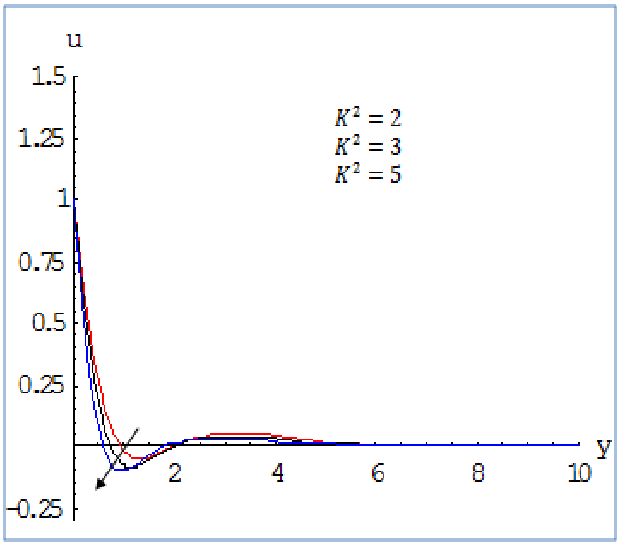

Fig10: Variation of u when rotation increases with y

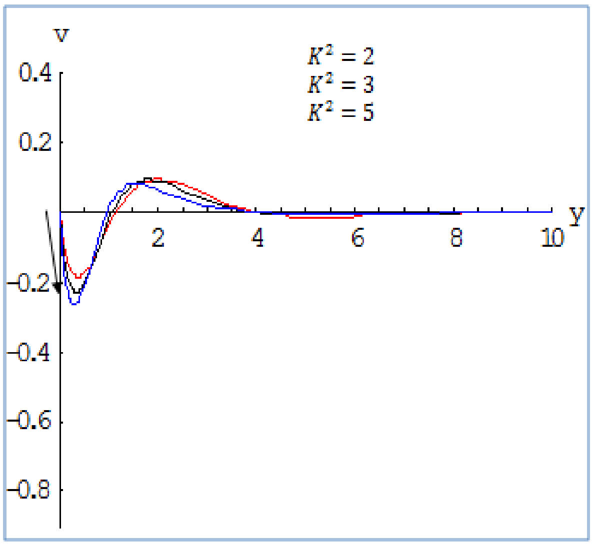

Fig11: Variation of v when rotation increases with y

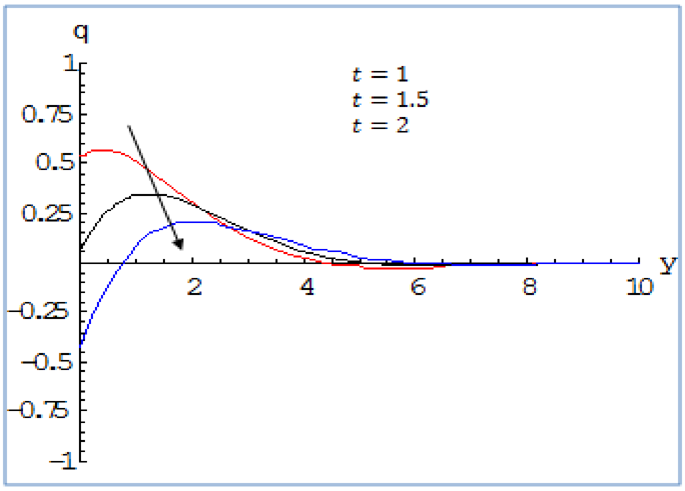

Fig 12: Variation of θ when t increases with y

IV. CONCLUSION

Fig. 2, 4 and 6 depict the variation of the velocity component u against y under the influence of Hartmann number, Hall parameter and Grashof number of heat transfer respectively. The velocity component decreases in all the cases. From fig. 3 and 7 it is clear that the velocity component v increases against y under the influence of Hartmann number and Grashof number. In fig. 5, the effect of Hall parameter on the velocity component v against y are shown and v decreases.

We see from fig. 8 and 9 that u increase and v decrease when the effect of Prandtl number is considered. Fig.10 and 11 shows that the velocity components u and v decreases with the increase of Rotation parameter against y. The temperature profile decreases with the increase of time against y as shown in Fig.12.

REFERENCE

[1]. V. Rossow, On Rayleigh Problem in Magnetohysrodynamics. *Phys. Fluids.* 3, 1960, 395.
[2]. Chang and J. Yen. Rayleigh Problem in Magnetohydrodynamics. *Phys. Fluids*. 2, 1959, 239.
[3]. H.L. Sathi, Effect of Coriolis Forces on Rayleigh's Problem, *F.N.I.*,1967, 57-62.
[4]. Girish Chandra Pandey, Rayleigh Problem in Magnetohydrodynamics with Suction, Def. Sci. J., Vol 20, 1970, 71-75.
[5]. H. Mohanty Hydromagnetic Rayleigh problem with Hall effect. *Czech. J. Phys.* 27, 1977, 1111-1116.
[6]. B. Abd-el-Malek , Nagwa A. Badran and Hossam S. Hassan, Solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method, *International Journal of Engineering Science*, 40, 2002, 1599-1609.

[7]. Haytham Sulieman and Naji A. Qatanani (2012) Magnetohydrodynamic Rayleigh Problem with Hall effect. *IJMER*., 1, 2012, 390-402.

[8]. M. Deivanayaki and P.T. Hemamalini, Magnetohydrodynamic Rayleigh Problem with Hall effect and rotation, *IJERT*, vol 12, 2013, 3042-3046.